Digital Signal Processing
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Lecture 12:
Elementary Signals (4), Systems and their Properties (1)
System definition
System Properties: Linearity

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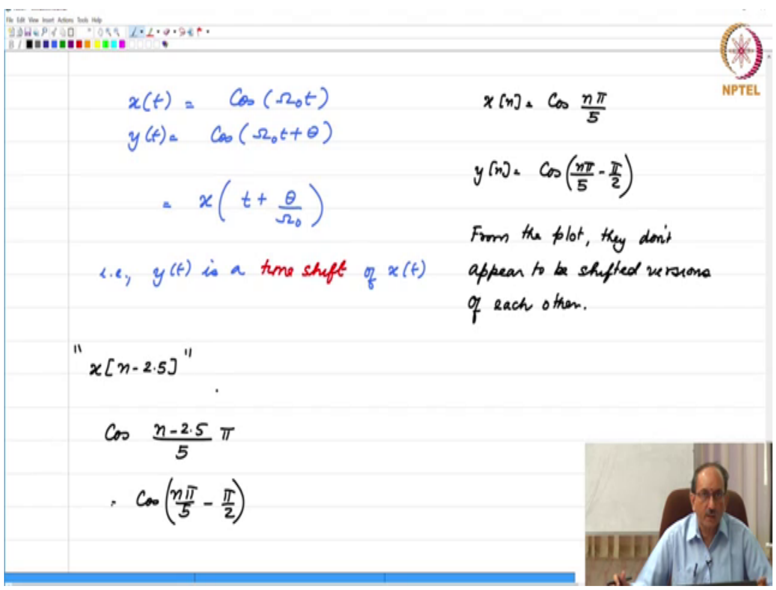

Now, let us move on to systems. Again, this will follow its very similar path compared to continuous-time systems. So, we are going to define what a system is, what the system properties are, and then later we will focus on one particular subclass namely the class of linear time-invariant systems exactly what happened along the lines of in signals and systems, but there you focused only on continuous-time systems.

Now we will focus on discrete-time systems and in LTI, we will again revisit convolution quickly. It is very similar to continuous-time convolution. Again, we will point out some of the properties or some of the observations that you may or may not have taken note of or made a note of before.

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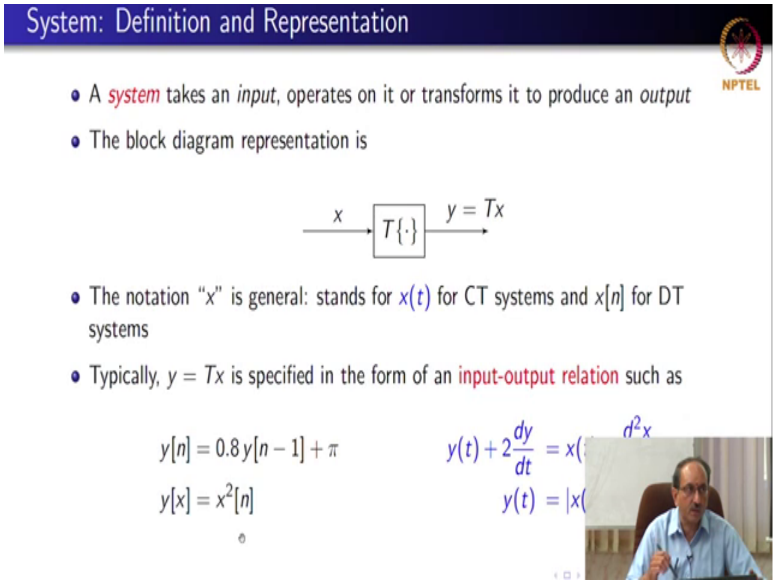

So, let us look at the systems. So, for this topic and LTI systems, I have a set of slides which I will go through, but since this parallels what you have learnt before to see the similarities and if there are any differences you note them down should not be very difficult. So, system takes an input, operates on it or transforms it and produces an output and in typically this is represented as a block diagram as x, the system is represented by an operator $T\{.\}$ and $y = Tx$ is the output.

And the notation x is general. It can either stand for $x(t)$ or $x[n]$ based on whether you are looking at CT or DT systems and typically $y = Tx$ is specified in the form of an input-output relationship and here is one example of an input-output relationship. Now, here is another in continuous-time that you may have seen.

So, this is a linear constant coefficient differential equation whereas here this is a linear constant coefficient difference equation. And then here are two more examples in which the input-output relationship is not in the form of an LCCDE. Just to show that the input-output relationship is general LCCDE is a very specific subclass of the general class.

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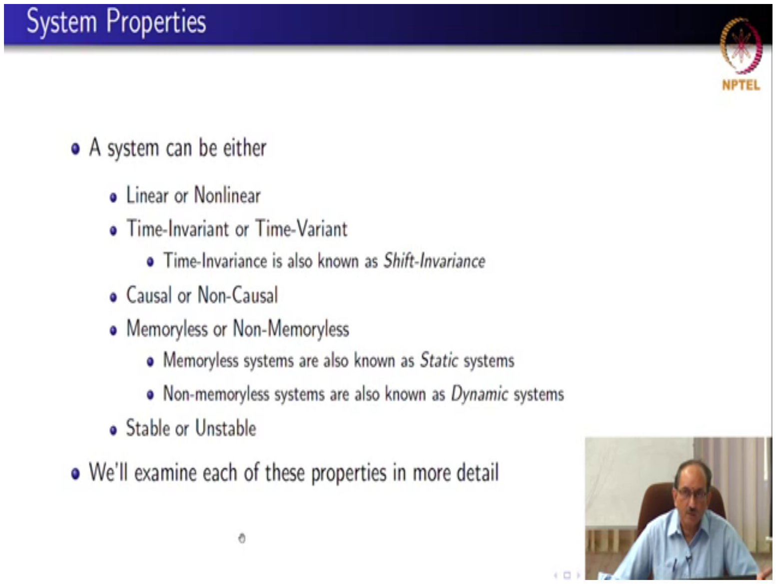

And properties that you would have seen is the context of continuous-time can either be linear or non-linear, time-invariant or time-variant. By the way, time-invariance is also known as shift-invariance. So, LTI or LSI, Linear Shift Invariant systems is another common terminology and system can be causal or non-causal, memory less or non-memoryless. Memoryless systems are also known as static systems and systems with memory, what is the other name that you may have known from your earlier course? The fact that the memoryless systems are called static should give you a clue.

Student: Dynamic.

Dynamic, ok. So, non-memoryless systems are also called dynamic. System can be stable or unstable. Stability of course, is a very important property for both man and machine. So, we will examine each of these properties in more detail. So, we will look each of these, define them and give examples.

So, typically as you must have done exercises in continuous-time, you would be given an input-output relationship and you have to test whether the system satisfies these properties. And to show that the system satisfies a property you have to give a general proof, to show that it does not just one counter example is enough.

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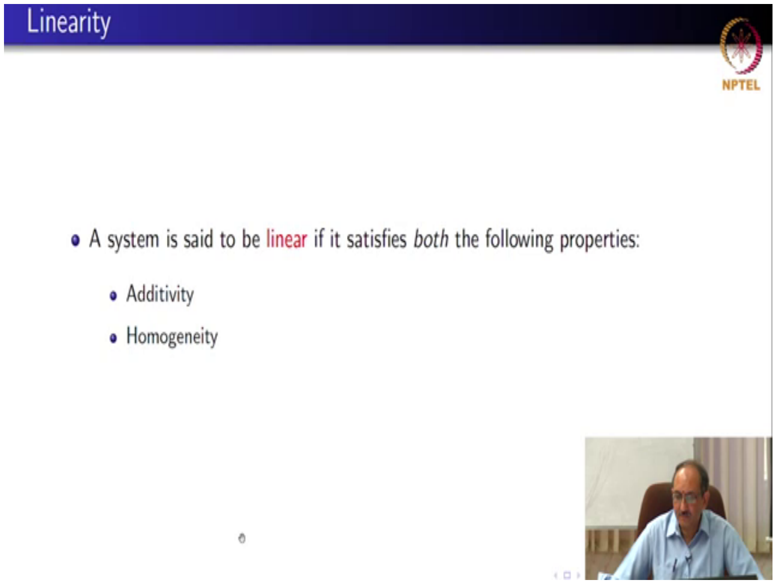

So, linearity involves two sub properties which are, not superposition.

Student: Additivity and homogeneity.

Yes, additivity and homogeneity, all right. So, it has to be additive as well as homogeneous; additivity is very simple.

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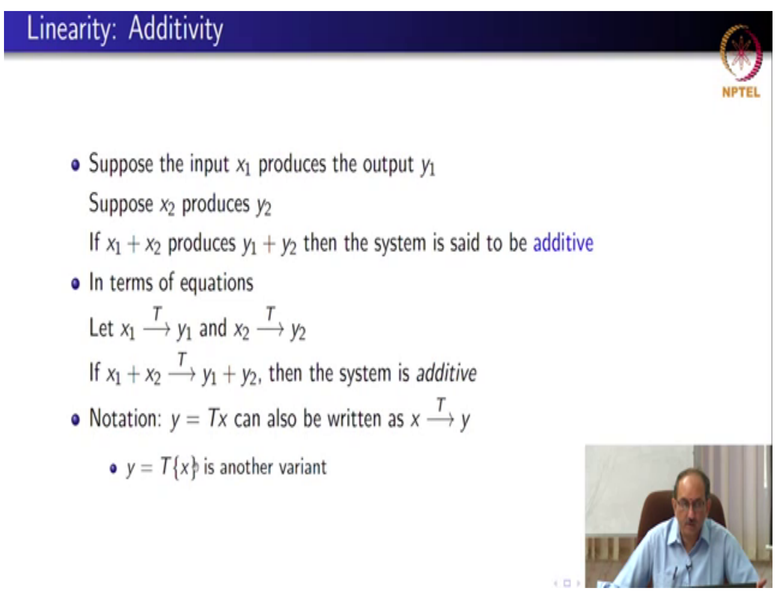

If x_1 produces y_1, x_2 produces y_2; you require $x_1 + x_2$ to produce $y_1 + y_2$ which means the system is additive. In terms of equations, this is the notation that is used; $x_1 + x_2$ produces $y_1 + y_2$, the system is additive. And the, this also brings in the notation instead of using $y = Tx$, you can also use the notation $x \xrightarrow{T} y$ and this is another variant of; another variant that you will see in this context.

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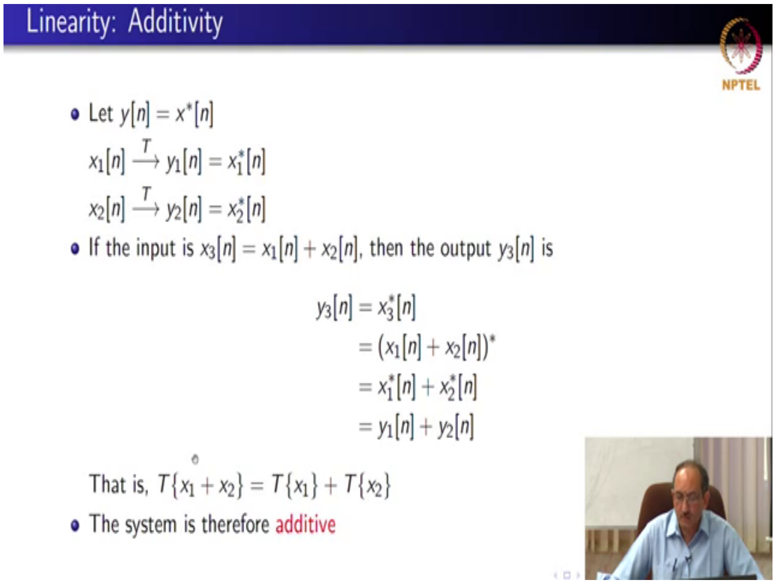

So, this is additivity. So, here is a simple example $y[n] = x^*[n]$. So, if $x_1 \xrightarrow{T} y_1$, y_1 in this case is x_1^*, y_2 is x_2^*. If you now give it $x_3[n]$ which is $x_1[n] + x_2[n]$, $y_3 = x_3^*$, x_3 is nothing but $x_1[n] + x_2[n]$. Therefore, $(x_1[n] + x_2[n])^* = x_1^*[n] + x_2^*[n]$, this is $y_1[n] + y_2[n]$. Therefore, the system is indeed additive, ok. So, here is an example of a system that is additive.

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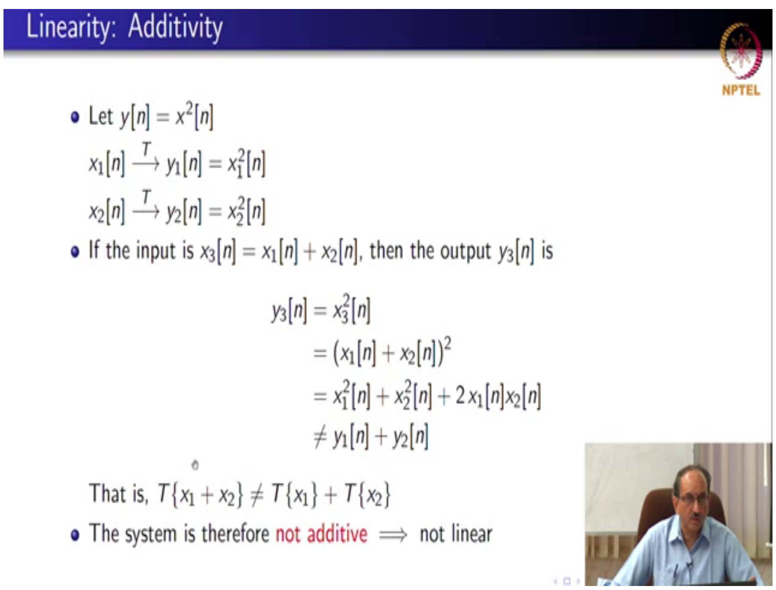

Suppose you have $y[n] = x^2[n]$, then $x_1[n] \xrightarrow{T} y_1[n] = x_1^2[n]$, $x_2[n] \xrightarrow{T} y_2[n] = x_2^2[n]$ and the combination is $x_1[n] + x_2[n] \xrightarrow{T} (x_1[n] + x_2[n])^2$ which is not the same as, does not produce $y_1[n] + y_2[n]$. So, here is a system that is not additive. Therefore, this is not linear and the other property associated with linearity is homogeneity.

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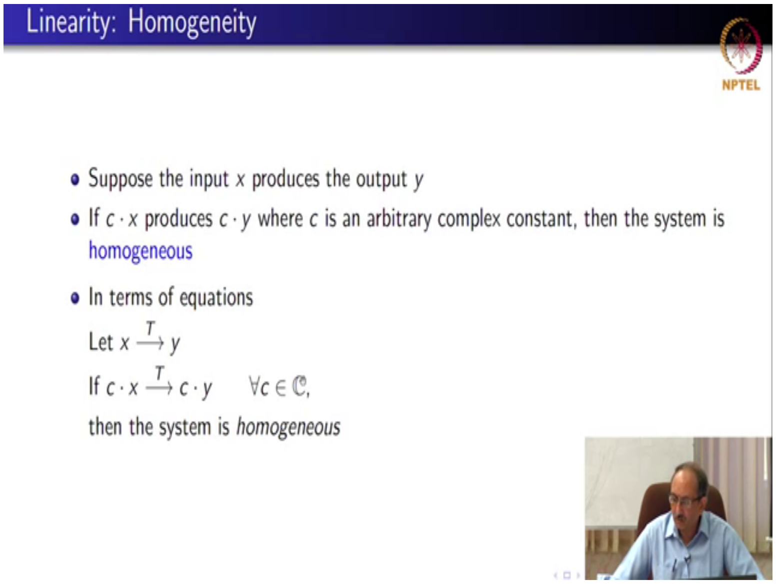

So, if $x \xrightarrow{T} y$, then $c.x$ must produce $c.y$ where c is an arbitrary complex constant in which case the system is homogeneous. So, these are the definitions, $x \xrightarrow{T} y$, then $c.x \xrightarrow{T} c.y$ for all c belonging to the set of complex constants.

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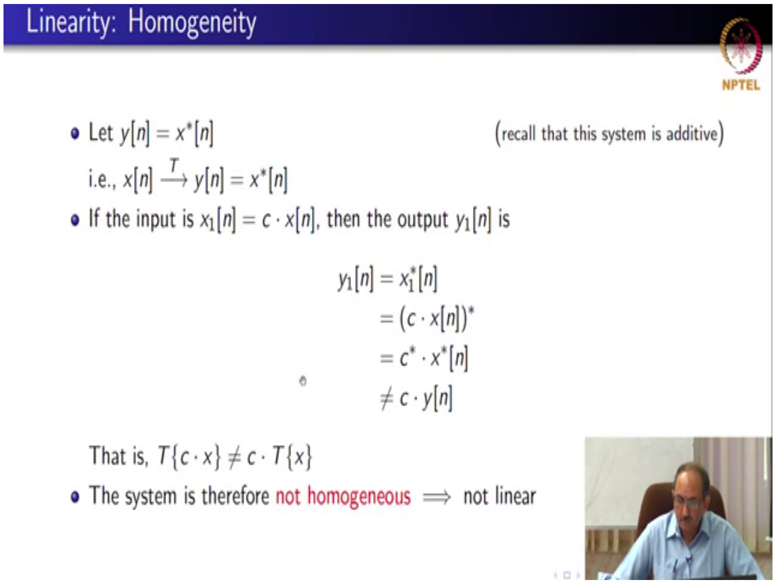

So, now let us take the previous example; $y[n] = x^*[n]$ and this system was shown to be additive, right? So, if the input is now $c.x[n]$, the output is input times, you have to take the input and complex conjugate it. Therefore, the output is $x_1^* = (c.x[n])^*$ which is $c^*.x^*[n]$. Clearly, the system is not homogeneous, all right? So, here we have a system that is additive, but not homogeneous.

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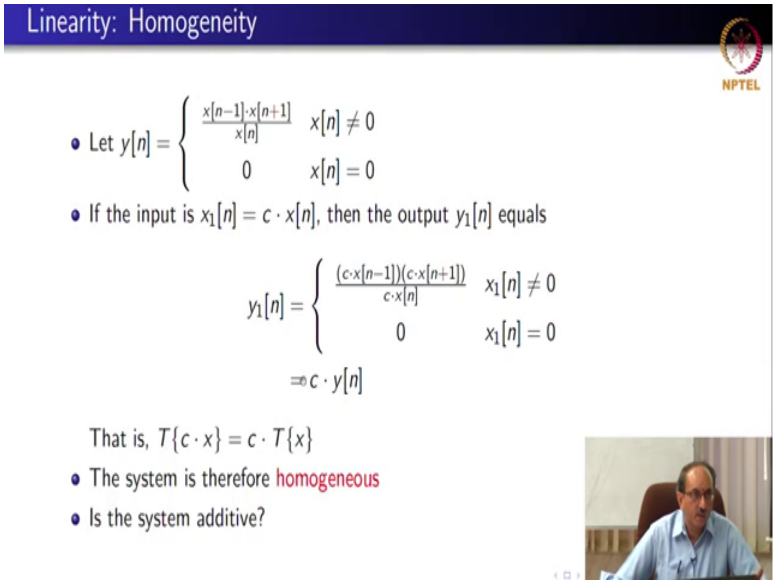

Now, let us consider this particular system, $y[n] = \begin{cases} \frac{x[n-1].x[n+1]}{x[n]}, & x[n] \neq 0 \\ 0, & x[n] = 0. \end{cases}$ Now, if you put, if you take an input $x_1[n]$ which is $c.x[n]$, you see that $y_1[n]$ when the input is $x_1[n]$, $x_1[n]$ in turn is $c.x[n]$, $y_1[n]$ turns out to be $c.y[n]$, correct? So, pretty simple, the c's cancel and you get $c.y[n]$.

So, clearly this system is homogeneous and one look at the definition, you can see that the system is in terms of additivity, is the system additive or not? What can you say about the system's behavior when you add two signals? It is not, because you see $x[n-1].x[n+1]$, your product there you will have cross terms.

So, this system which is homogeneous, very easy to see that this is not additive. So, the point of this example is to show that additivity and homogeneity or independent properties, you can have a system that is additive, but not homogeneous. You can have a system that is homogeneous, but not additive. Only when both additivity and homogeneity are satisfied is the system linear.

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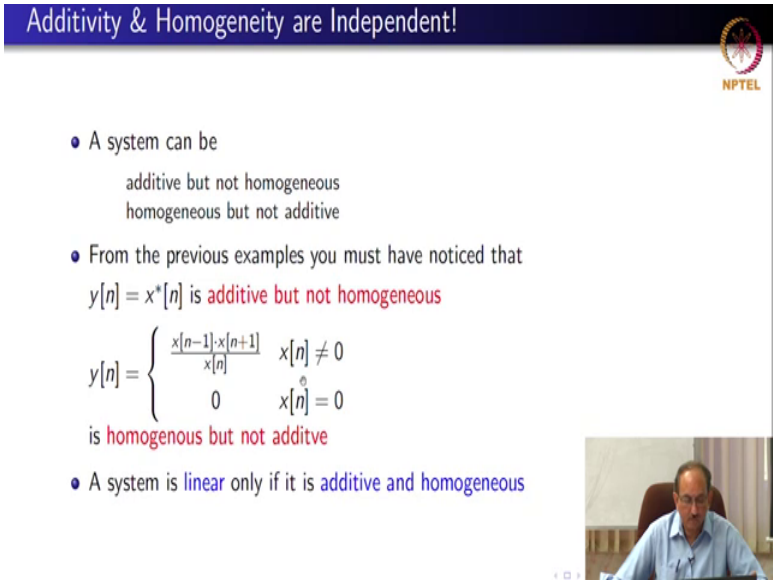

So, this slide just puts in words, puts in writing what I had said in words.

So, system is linear if and only if it is both additive and homogeneous.

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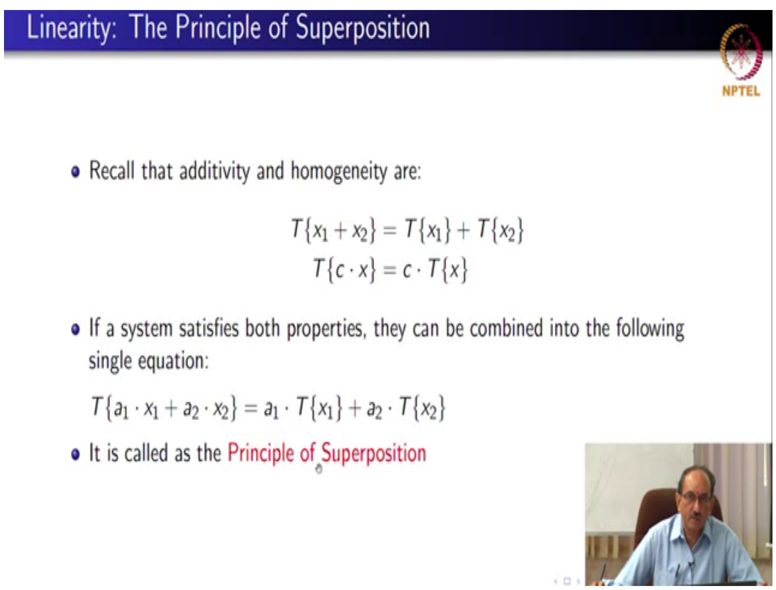

And somebody said superposition, that superposition principle captures in one statement both additivity and homogeneity. So, you say that the system is linear, if $T\{a_1.x_1 + a_2.x_2\}$ gives you $a_1.y_1 + a_2.y_2$ therefore, the system is linear. When you make that statement, you are capturing two sub-properties togethert. You are capturing both of these in one statement and this is called the Principle of Superposition.

So, when you say the system is linear, it obeys the principle of superposition. You are implicitly stating that the system satisfies both additivity as well as homogeneity.