Digital Signal Processing
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Lecture 11:
Elementary Signals (4), Systems and their Properties (1)
Relationship between Time-shift and phase-shift in the DT sinusoids case

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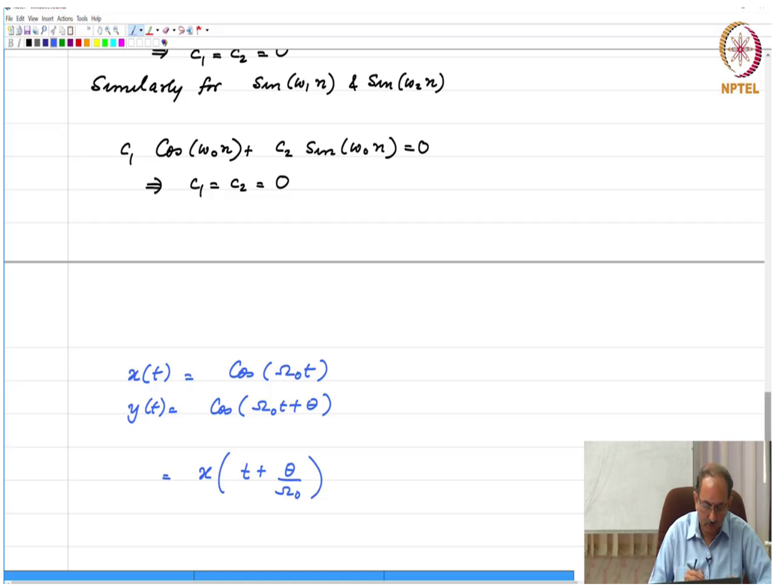

In terms of sinusoids, let us compare another aspect of continuous-time and discrete-time sinusoids and see where they are similar and different. Suppose, I have $x(t) = \cos(\Omega_0 t)$ and then I have $y(t) = \cos(\Omega_0 t + \theta)$. So, this is a sinusoid that is a phase shift of the previous case. If you want to express $y(t)$ in terms $x(t)$; what would be the relationship? This is nothing, but $x(.)$?

t plus?

Student: (Refer Time: 01:15) θ.

$x\left(t + \dfrac{\theta}{\Omega_0}\right)$ right; wherever t is there, if you replace t by $t + \dfrac{\theta}{\Omega_0}$, you will get this. So, what is the implication of this?

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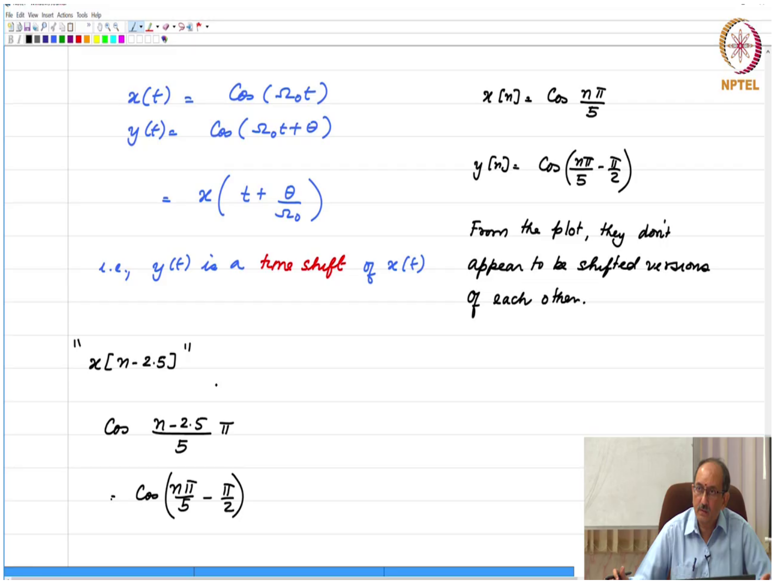

$y(t)$ is a time shift of $x(t)$, alright. Now, let us see how this compares on the discrete-time case. Now let us consider this particular example; so let us consider $x[n] = \cos\left(\frac{n\pi}{5}\right)$ and then $y[n] = \cos\left(\frac{n\pi}{5} - \frac{\pi}{2}\right)$. Now, what I am going to do is let me plot this in MATLAB.

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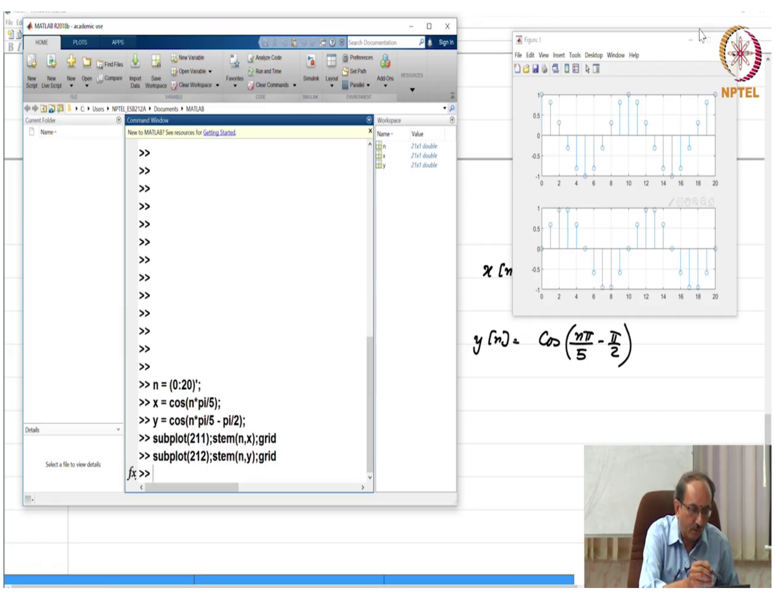

So, I am taking 20 samples here and then x is $\cos\left(\frac{n\pi}{5}\right)$; y is $\cos\left(\frac{n\pi}{5} - \frac{\pi}{2}\right)$. Now let me plot both these sequences; so this is the first plot and later zoom this so that you can see similarities and differences. So, this is the top plot is $x[n]$ which is $\cos\left(\frac{n\pi}{5}\right)$, the bottom plot is $\cos\left(\frac{n\pi}{5} - \frac{\pi}{2}\right)$.

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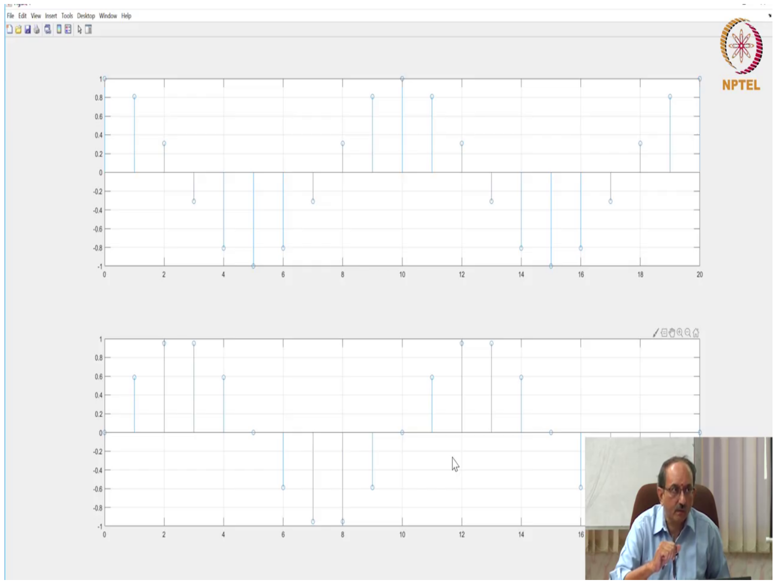

Now, let me zoom this up; in the bottom curve, you see the sequence takes on the value 0, at certain indices, in the top curve there is no sample that takes on the value 0. The other difference that you see is; here it hits the value $+1$ and here it hits the value -1; whereas, here the top; the peak does not quite hit $+1$ and the negative peak does not quite hit -1, correct?

So, these two do not appear as if they are a shifted version of each other, so let us go back and look at this. So, from the plot; they do not appear to be shifted versions of each other. Let us; in the continuous-time case very clearly we saw that phase shift equals time shift. Now, let us take $x[n]$ and then contemplate this $x[n-2.5]$. So, that is I have put that within quotes. So, if I take $\cos\left(\frac{n\pi}{5}\right)$ and then replace n by $n-2.5$; I get $\cos\left(\frac{n-2.5}{5}\pi\right)$ this; correct? And so, this is nothing, but $\cos\left(\frac{n\pi}{5} - \quad\right)$, minus?

Student: Minus.

$\frac{\pi}{2}$; $\cos\left(\frac{n\pi}{5} - \frac{\pi}{2}\right)$. Therefore, it looks as if the sequence $y[n]$ is a shifted version except that the shift is not an integer shift, it is a shift by 2.5 samples. Right now, we do not know how to interpret what are shift by 2.5 samples is, because we are used to things being defined on a set of integers. So, when you delay or advance a signal, you can shift to the left or to the right by integer number of samples. So, what does it mean at all to shift by an integer plus fraction; so we will make this notion precise later. We will come back to this notion later when we introduce the concept of fractional delay. But as far as equations are concerned, you can get one from the other by replacing n by $n - 2.5$.

So, this is as far as basic signals are concerned. So, most of it must have been review and hopefully you learned some things that you may not have encountered in your earlier course. Even though, you may have seen discrete-time sequences; some of the points that were made here, we hope where things that you are not thought of before.