Digital Signal Processing

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Lecture 01:
Introduction to Signals

Keywords: discrete-time signal, continuous-time signal, energy signal, power signal

Let us get started. First, we will look at discrete-time signals and then we will drop parallels to the continuous-time counterpart. Things that are similar, we will reinforce them so that the ideas get set in more. And there will also be important differences. So, we will highlight the differences so that you see what is true in continuous-time probably is not true in discrete-time. So, we will compare and contrast and see the similarities and differences.

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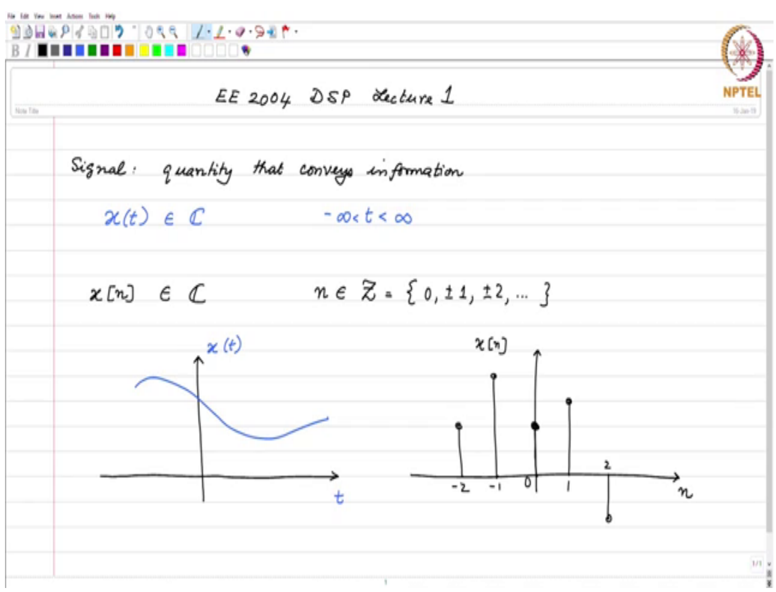

Signal as you all know is a quantity that conveys information. So, this is the generic definition. In our context, we typically use the notation $x(t)$ to denote a signal. This is a signal whose independent variable is called t and this in general can take on complex values i.e., $x(t) \in \mathbb{C}$. So, mathematically $x(t)$ is a scalar function of one variable and the independent variable is called time though it need not be time but we will use the word time as a generic term. In general,

$$x(t) \in \mathbb{C}, \qquad \infty \leq t \leq \infty,$$

t belongs to a set of real numbers.

The corresponding counterpart in discrete-time is $x[n]$ and this is again in general can take on complex values.

$$x[n] \in \mathbb{C}, \qquad n \in \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\},$$

n belongs to the set of integers. So, this is the discrete-time sequence of signal. So, we will interchange the use of the word sequence and signal. Unlike in the continuous-time case where the independent variable can take on all values between $-\infty$ to ∞, in the discrete-time case you are limited to n belonging to the set of integers.

Typically, what you have for continuous-time is something like this. In the discrete-time case, the independent variable is n and, n belongs to the set of $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$. So, I have clearly plotted real valued signals for ease of plotting and this is how a typical discrete-time sequence looks like and this is what is called a stem plot in Matlab notation. Just to reinforce some of the differences that can arise, now let us look at something.

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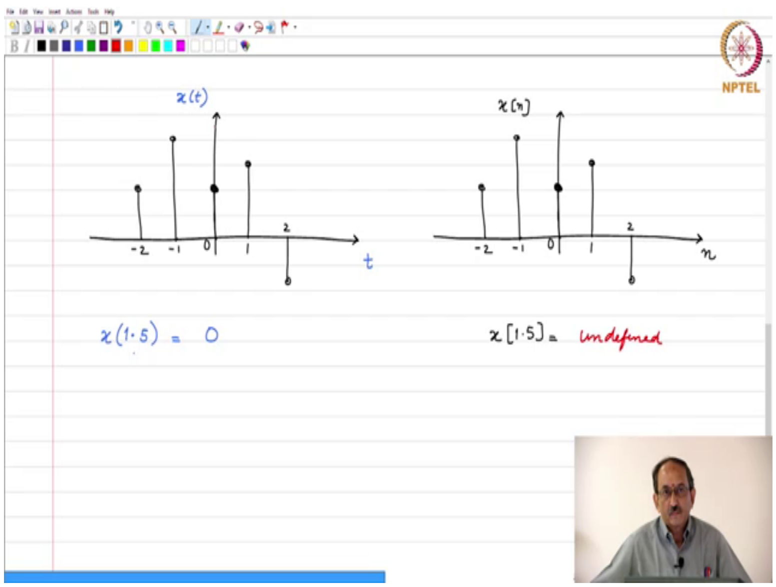

Now, let us look at this particular signal where I am going to make some changes and then I am going to point out the differences. So, this is now continuous-time signal. This looks very similar to the discrete-time sequence. In fact, I cut and pasted that figure, but I merely changed the axis.

And the point that I want to convey about this is that, this continuous-time signal is defined for all t. Therefore, if I now ask you what the value of the function x at 1.5; the answer would be?

Student: 0.

0, all right. So, even though this looks like a discrete-time sequence pictorially, since this is defined for all t, this is actually a continuous-time signal and $x(1.5)$ is indeed 0. All right?

Now in contrast; so, it is exactly the same plot that I drew. Now if you ask the question what $x[1.5]$ is, what would be your answer? So, this is undefined and it is not 0.

The analogy is you can think of a discrete-time sequence as belonging to an array with the array index corresponding to n, and the array value that stored in the array location has the value of the sequence itself. So, if you look at this analogy, it is meaningless to ask what is the array location 1.5. Array locations are by definition with respect to reference integer values. Therefore, just as it is meaningless to talk about array location of 1.5, the moment be array location of 1.5 becomes meaningless, then the content at that location also is a meaningless question. Therefore, in terms of discrete-time sequences, the independent variable takes on only the set of integers and hence, things like $x[1.5]$ is undefined or meaningless.

So, this is an important difference between continuous-time and discrete-time. And in this course we will be only focusing on one-dimensional signals i.e., the independent variable will be one-dimension and the value of the function will be also one dimensional. Can you give an example of, say in continuous-time of a signal that is vector valued; the common signal that happens in real world that is vector valued?

Student: (Refer Time: 08:31)

No, I am talking about the value of the function being vectors. Here $x(t)$ belongs to the set of complex numbers which is a scalar. Similarly, $x[n]$ belongs to a set of complex numbers which is a scalar. So, the dependent variable is a scalar and now asking the question what about an example of a real world signal where the dependent variable is a vector.

Student: (Refer Time: 08:55)

Yeah, electromagnetic field which has various components, anything else? And ECG signal in which several leads are stuck at different parts of your body and then recordings are made. The independent variable is time and yet at every instant of time, you have a whole bunch of related measurements which you can group them as a vector. So, that is an example of a signal that is vector valued.

And, the independent variable is multi-dimensional is, an image. An image, the independent variable are the x and y coordinates. The dependent variable is still a scalar here, a pixel value or a number whereas, the independent variable is two-dimensional. So, that is as far as a planar image goes. If you have an image in 3D, then the independent variables are x, y and z.

So, there are all these several classifications of signals. The other classification that you can think of is deterministic versus random and so on. So, in this course, we will be considering only the independent variable being scalar or one-dimensional. The dependent variable also will be scalar or one-dimension and the independent variable will be called as time although it need not be time.

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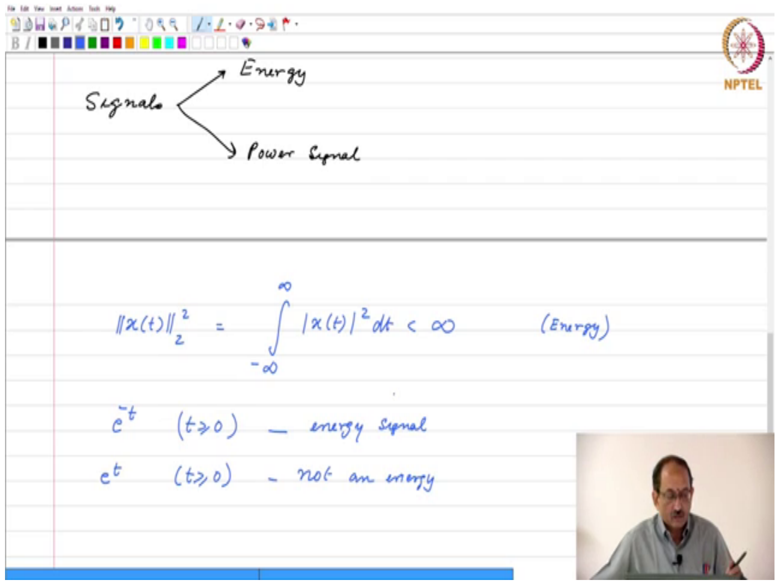

And another typical classifications of signals is this can be either what is called energy signal or it can be called as a power signal. Again these all review, you must have seen this in your signals and systems course and these are terms that are used by electrical engineers; whereas, the mathematician would use this. A signal is called an energy signal if

$$||x||^2 = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty \quad \text{(Energy Signal)}.$$

The notation should tell you what the terminology is that the math people use. What is this notation tell you? This is l_2 norm of the signal. And $e^{-t},\ t \geq 0$ is indeed an energy signal because its integral would be finite whereas, if we had $e^{t},\ t \geq 0$ is not an energy signal.

And we call this energy because we are used to thinking of these as voltages and currents and, if you these were to represent a voltage or a current and if these were passed in a 1 ohm resistor, then the power, energy. So, that is the reason why we are using these terms in this context.

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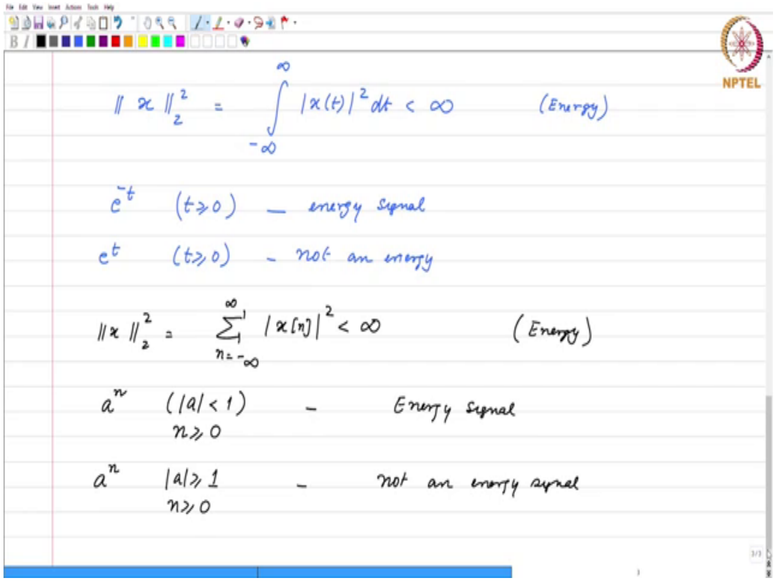

For continuous-time case, if it is an energy signal, the signal is square integrable. In terms of the discrete-time case, if the signal is an energy signal, it is square summable.

$$||x||_2^2 = \sum_{-\infty}^{\infty} |x[n]|^2 < \infty \quad \text{(Energy signal)}.$$

And typical examples: a^n, $|a| < 1, n \geq 0$ is an energy signal. On the other hand; if I had a^n, $|a| \geq 1, n \geq 0$ and if you look at the range $n \geq 0$, this is not an energy signal.

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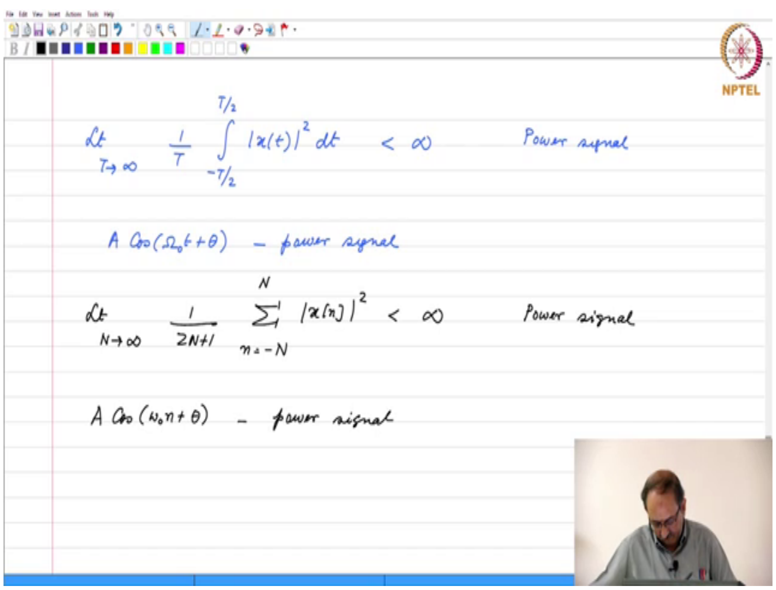

And in terms of power signals,

$$\lim_{T\to\infty} \quad \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} |x(t)|^2 dt < \infty \quad \text{(Power Signal)}.$$

And $A\cos(\Omega_0 t + \theta)$ is an example of a power signal. The discrete-time counterpart for this is

$$\lim_{N\to\infty} \quad \frac{1}{2N+1}\sum_{n=-N}^{N} |x[n]|^2 < \infty \quad \text{(Power Signal)}.$$

This is the discrete-time counterparts of a power signal and $A\cos(\omega_0 n + \theta)$ is an example of a discrete-time power signal. And typically signals that are periodic are power signals. Can you think of a periodic signal that is not a power signal?

Student: (Refer Time: 16:31)

Very good, if you had tan, then over one period, it is not integrable. It is a very good answer. So typically, power signals are periodic signals, but there are periodic signals that are not power signal. Now, let us again continue our comparisons between discrete-time and continuous-time signals.