Linear Systems Theory
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Module - 01
Lecture - 04
General Representation

In the concluding lecture of week 1 I just introduced you to the general formulation of system. So, as said we will deal with finite dimensional systems or systems which are represented by ordinary differential equations, we will not deal with general formulation of infinite dimensional systems or hybrid systems. So, we will restrict ourselves to finite dimensional systems.

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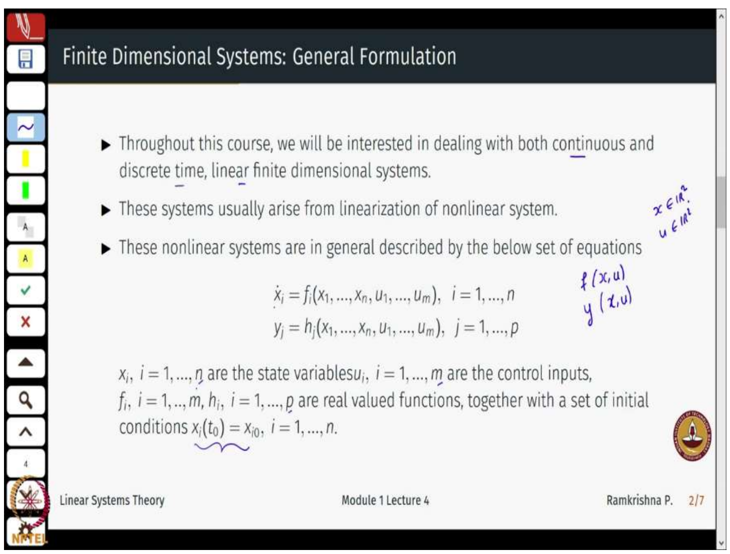

So, we will be talking both with interested both in continuous and discrete time linear finite dimensional systems and these systems usually arise from linearization of non-linear system. So, we will sometime later in the course talk about methods of linearization and also limitations of linearizations, we will talk about some different methods also of linearization right. So, in general so, all systems are described by set of non-linear equations right. So, you will have a if I just go to write it in more general terms, you will have n state variables related via a non-linear relation ‘f(.)’ between the states and the inputs. I would also be interested in measuring the outputs for obvious reasons, which are also some non-linear functions of x and u.

So, usually my notations will be I will have a system of n states, m inputs and p outputs together with some initial conditions which may be required for me to and some say compute the solutions or we will see how that goes a little later in the course.

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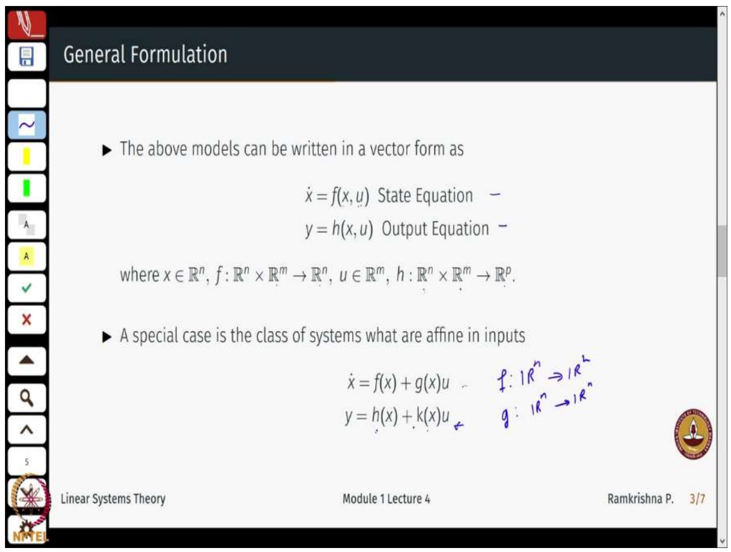

So, I can just write this down in a compact form as $\dot{x} = f(x, u)$; where f() takes values from $R^{n\,x\,m}$ to give me again some values in R^n . So, this will be a general notation that we will follow right. So R^n would be an n dimensional vector space, we will learn those properties of vector space in the next lecture, R^nwill be an n dimensional vector space where my set of inputs come from $R^n$is where my set of states come from. In the example for example, in the case of the inverted pendulum my states were 2 dimensional input was just R^1 right just a little perturbation or a torque that was applied ok.

Similarly, with the output right so, outputs again the it is like a map from $R^n\;x\;R^m$the state space the input space to the output space. You could have p outputs or you know we have not so, far explicitly defined outputs, but for example, in the case of the pendulum I could measure for example, the velocity as the output; the angular velocity or even the position maybe for example, as the output ok.

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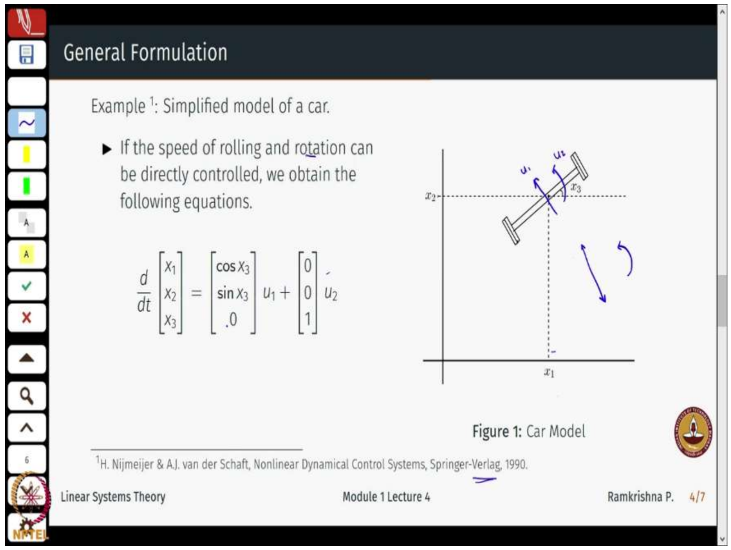

So, let us do a little example of this one right. So, let us say that I take a simplified model of a car, which is also called a unicycle again from this famous book on non-linear control systems. So, in this control I have two control signals, one is like the rotation I will call this u_2 right and then one is just the translational motion it is a just u_1 here again just go. So, what I am allowed is movements which are like this and also movements which are like this and that will define also it will cover all the state space of my car and it can go anywhere in the R^2plane.

So, the speed of the rolling right this is controlled by u_1 and the rotation is controlled by u_2 , then I can write down the equations in the following way right. So, the change in x_1 direction, $\dot{x_1}$ dot the change in $x_2$is $\dot{x_2}$ and then $\dot{x_3}$ is assessable relation which we can derive at $\cos(x_3)$, $\sin(x_3)$, 0 here and then $\dot{x_3}$ is simply the directly proportional to u_2 right.

So, this is an example which looks like this right of a general form of a non-linear system $\dot{x}$ = f(x,u). Special class of systems which are affine inputs as we saw in the case of the linear pendulum right. So, again I do not; I will not derive that again. So, here instead of f being from $R^n x R^m$ to R^nf is a simply a map from $R^n \ to \ R^n$, similarly g() is also from R^n to R^n right.

Similarly, with the maps h() and k() we can see this as an exercise you can just write down what did you, what the domain and the range of these maps R ok.

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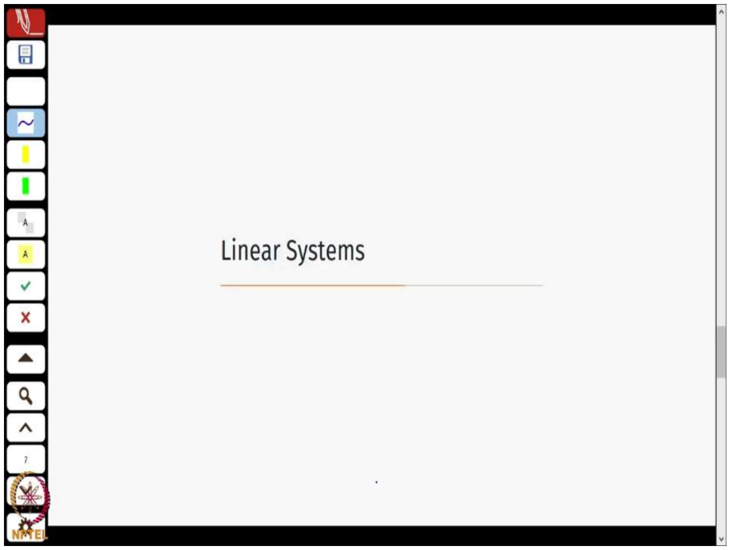

So, once we perform the process of linearization.

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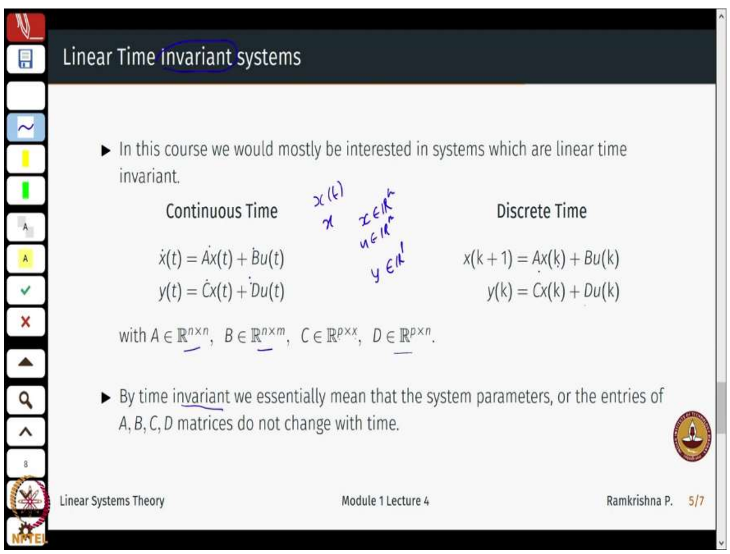

I would be interested essentially in two kinds of systems right, one is continuous time systems. I use the word here invariant I will elaborate on this shortly, continuous time systems $\dot{x} = Ax(t) + Bu(t)$, y= $Cx(t) + Du(t)$.

So, in most cases when it is obvious I will just write x(t) as just x. So, similarly even if it is obvious I will just kind of omit this argument just to make the notations a bit compact. So, well you can see that A is a n x n matrix where the system is x is in R^nsay u is in R^m.

So, B will be an n x n matrix C will be if I have p inputs, then C will be a matrix which is a p x n dimension and similarly with D let us say and this will be same even if I am looking at a discrete time system.

So, what is the time invariance in these cases? So, by time invariant we essentially mean that the system parameters A B C and D are they do not really change with time. So, they are they are just constant for example, say a mass or a resistance or an inductance say that they are just constant over time. So, I do not really have to worry about any change in the system parameters.

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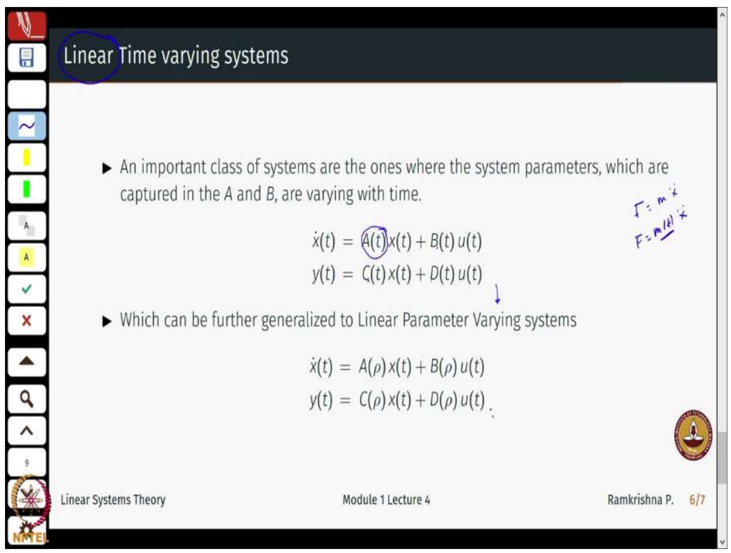

Time varying systems where I know well maybe that say maybe my fitness level will change with respect to my age. So, I can say well that that is can be modeled as a time varying system where the parameters essentially are the ones, which are varying with time A B C and D are now also some functions of time. Still these are C linear systems, there is nothing that is adding to the complexity, yet I am still looking at linear systems, but where the parameters are changing with time.

For example, if I can write as $F = m\ddot{x}$ is a linear system $F = m(t)\ddot{x}$ is also a linear system just that it is time varying maybe the mass changes with time for example, if I look at an automobile the mass would change it with time because of the fuel which is being consumed.

Now, this time varying can be generalized to something called a linear parameter varying systems. We will not really explicitly deal with these systems, we will deal a bit with time varying systems, but it is interesting to know what are these parameter varying systems. So, if you buy any equipment right say you will say well its operating region is between say 10° Celsius to 60° Celsius, what happens outside the range? Well this it is not guaranteed to perform because some parameters might change.

For example I know that resistance can vary with temperature or some other you know parameters of the system can also depend on pressure or humidity and things like that. So, whenever the specifications of the equipment are to work within a certain atmospheric conditions could be temperature or pressure, it essentially means that outside those range the parameters might change and therefore, my performance of the system will also change.

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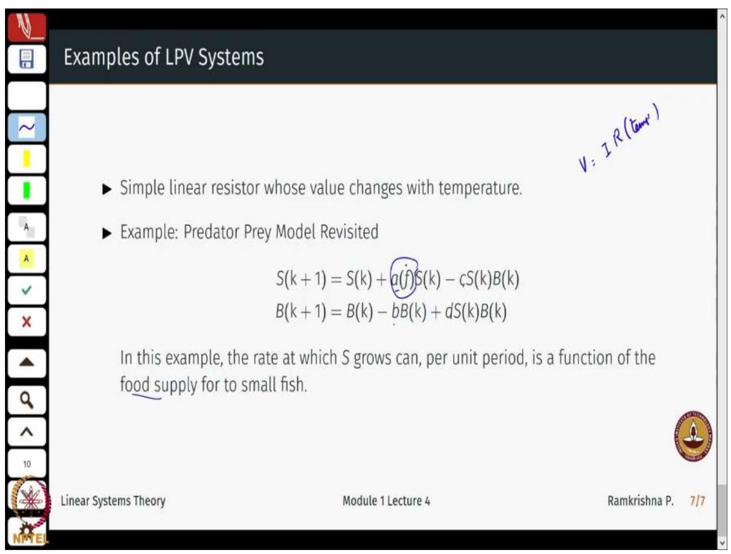

A little simple example would be; so, in most operating regions V would be a linear relation with R and R would be fairly constant. But if I say R is depending on temperature, you know then that is a parameter varying resistant it is actually depending on the temperature.

Let us revisit the predator prey model right of what parameter varying could actually mean in some of those cases. So, where we had that a b, c and d governed how the system actually behaves. So, a was the rate at which my small fish was increasing so or multiplying. Some obvious observation would tell us that 1 way of modeling this could be or one way of influencing the rate of growth of the small fish could directly depend on the amount of you know food that is available right.

So, the rate of which S grows is a function of a food supply. So, this I can just add an extra term or extra maybe flexibility to model this as a parameter varying system. And you could visualize many of this; I can maybe sometime during the course quote you some literature on parameter varying systems even though that will not really be our aim to comprehensively look at those.

So, this ends a module 1; so, from module 2 we will start learning some basic tools from linear algebra starting from vector spaces till some matrix algebra, which will form a bulk of tools that we will use in this courses through the rest of the course.

So, as I suggested it would be nice for you to looked at some comprehensive lectures on linear algebra, even though we will teach you what all we need in through the course we will give you a set of problems to work out apart from the regular assignments. And I really hope that you spend lot of time doing the module 2 and 3 in detail so, as to have a smooth transition through the rest of the course. So, see you in module 2.

Thank you.