Linear Systems Theory
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Module - 11
Lecture - 02
Optimal Control

Hello everybody. So, welcome to this lecture number 2 of week 11 on the course on Linear Systems Theory. As we saw last in the previous lecture, we redefined our control problem in terms of certain optimality conditions to be fulfilled. Then we had a couple of methods to derive conditions for what we called as LQR controller, which essentially ended up in solutions or in the form of some kind of equation called a Riccati equation.

So, today we will look at one more method to derive that particular equation. And also we will look at what if what to do with the Riccati equation when the said there exist solutions if at all and what are the how do we derive those conditions. For existence of solutions of Riccati equations ok.

So, to begin with so, this method will be based on what is called as a feedback invariants. And will closely relate to what we had done also in the previous lecture and why well both methods are kind of similar ok.

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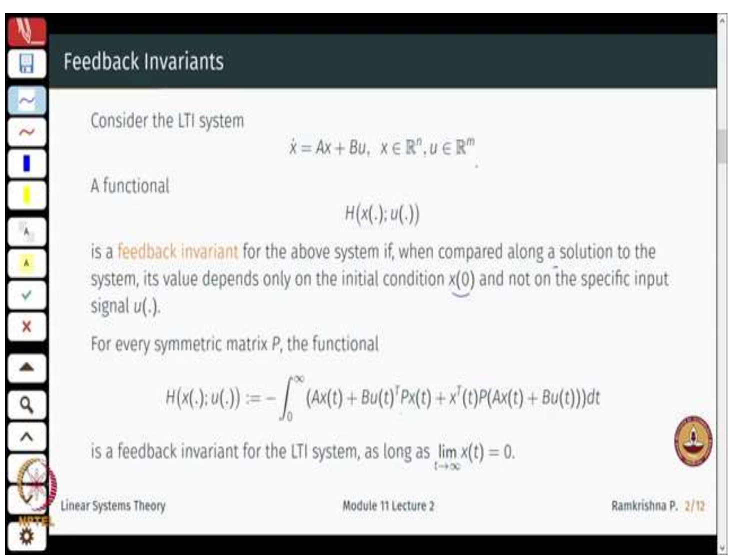

So, the first thing here is to look at the following. So, if I have a system again I will just look at a standard LTI system a functional H; defined on x and u is a feedback invariant for the LTI system. If when compared or when computed along the solution to the system, its value depends only on the initial condition and not at and not on the specific input signal u.

For example, for every symmetric matrix P; this particular functional is a feedback invariant for the LTI system as long as the system is stable or asymptotically stable or in other words limits limit x tends to infinity sorry limit t tends to infinity x of t goes to 0 so.

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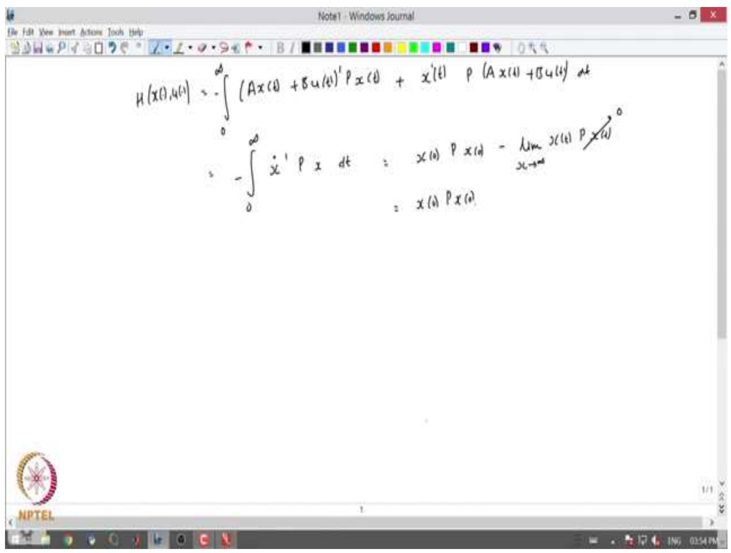

We will do a little proof of this right. So, what we have to show is we are given a function of the form H(x,u) = $\int_0^{\infty}(Ax(t) + Bu(t))^T Px(t) + x(t)^T P(Ax(t) + Bu(t))dt$ ok.

So, this thing inside the integral is nothing but $\int_0^{\infty} \dot{x}^T Px \, dt$ right. So, and this integral looks take some very nice formula. So, x(0) P x(0) now - $\lim_{x->\infty} x(t)Px(t)$ and the system is stable this will go to 0 as t goes to infinity and what I am left with is just this one P x of 0 ok.

So, this functional therefore, is a feedback invariant right. So, what was the definition is that, its value depends only on x at 0 and not on this specific input signal u. So, if you remember in the lecture number 1 we had used something very similar here right. So, this is this exactly was what we derived.

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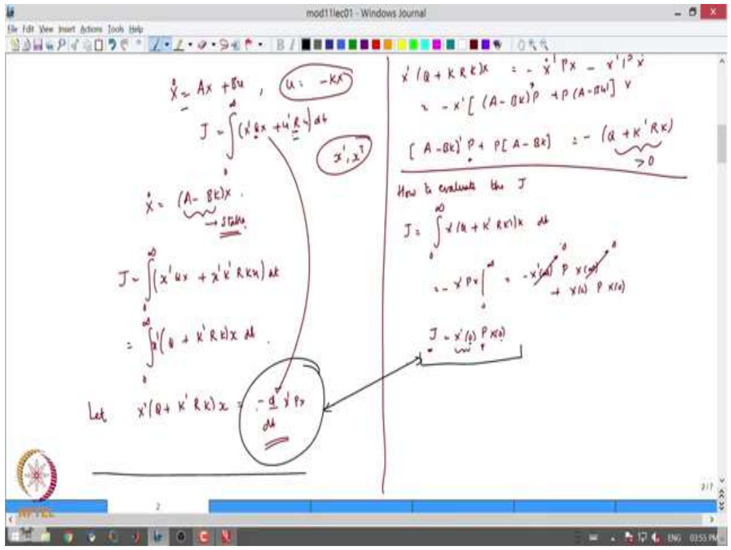

So, now if we give it a little formal notion in terms of feedback invariant set. So, nothing really new happening here, but we are just trying to give it some more general or give it a little more systematic meaning right ok.

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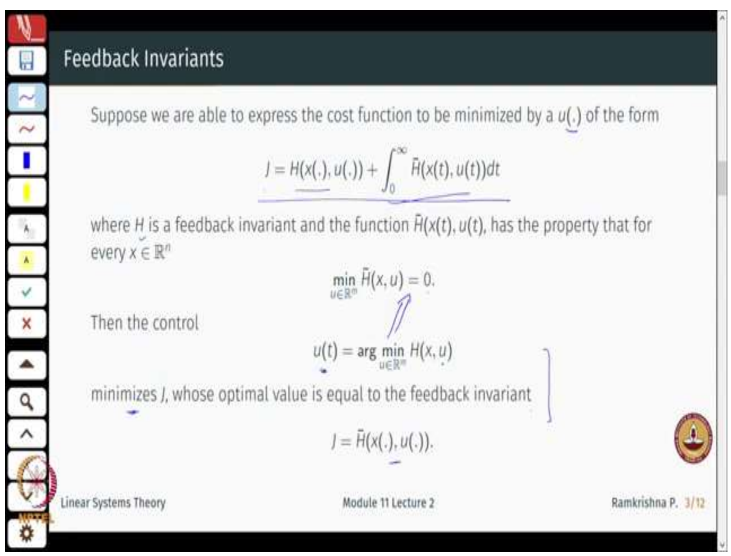

So, now, how do we derive the optimal control law from here? So, suppose we are able to express the cost function to be minimized by a certain u in this format. So, J is the cost function to be minimized. H here is a feedback invariant and this term in the integral is such that it has. So, that the minimum is 0 right the control u of t this minimizes J whose optimal value is equal to the feedback invariant ok.

So, we will just read it out again. So, I suppose I am able to express my cost function right to be minimized in this form right. So, the previously defined cost functions could easily be written in this form and we will see shortly why that is true right where the first term is a feedback invariant. And the second term this integral, is such that for every x belongs into n the minimum is 0.

In that case the control u(t) which is the arg min which is the u that minimizes this to 0. And this u also minimizes J whose optimal value is equal to the feedback invariant. And we will revisit this statement once we once we derive what this means here ok.

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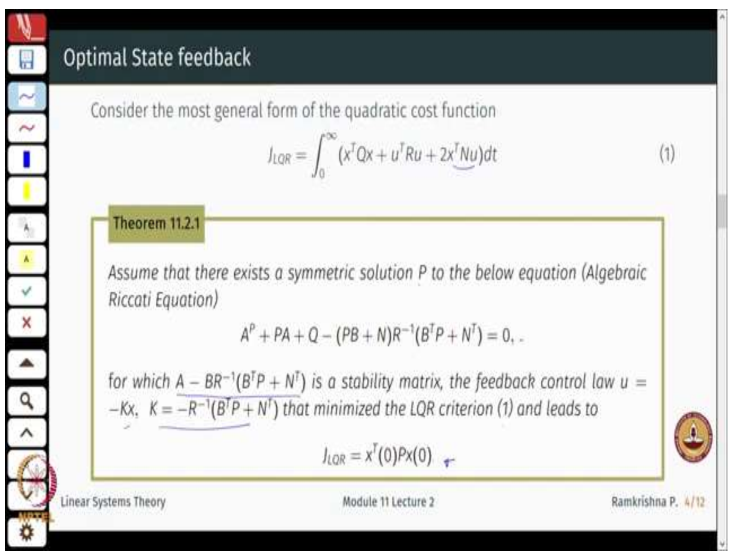

So, we start with a little more general form of quadratic function. We just add now in addition to what we had previously just some cross terms between with between x and u ok. So, the theorem that we will prove again we will derive the same Riccati equation, but in a slightly different way.

So, we assume that there exists a symmetric solution to the below Riccati equation, for which this particular thing is a stability matrix. The feedback, control law u = -K x which with this K. So, this u minimizes the LQR criterion one and leads to J LQR of this form ok, very very similar to what was here right.

This was a J LQR which we are derived. And we will just derive it from right from a slightly different point of view ok. So, let us do quickly the proof of this. And it will turn out that its not really difficult to understand or even interpret ok.

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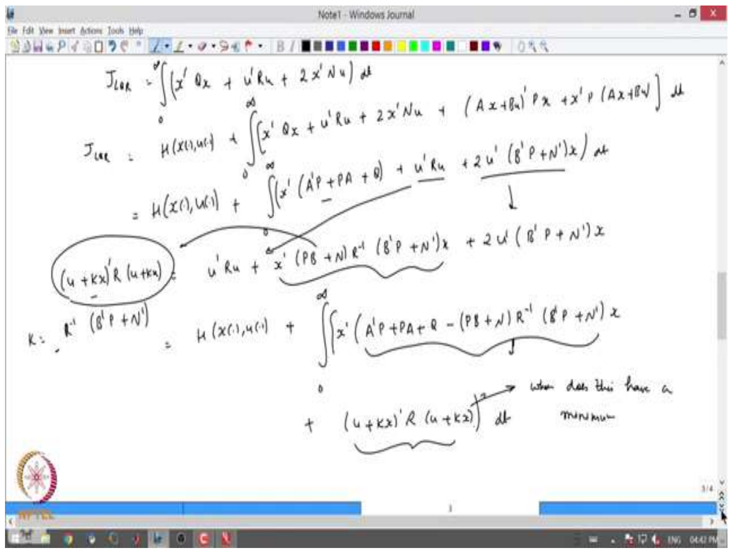

So, the objective again is to minimize the cost function. So, I have J LQR is 0 to infinity. So, this is already given at x^TQx; u^TR u + 2x^TNu d t ok. Now can I so, the first step would be to just check, if I can write it in this way if I can write the J as something like this right. Where H is a feedback invariant and something inside the integral which eventually goes to 0 and the u that makes it 0 will be my optimal u. So, we will start from that ok.

So, before that let us again revisit this yes again revisit this result. So, from this result what we can see is that I can just write I take this to the left hand side. And I can I write this as H plus the term in the integral is equal to 0. So, I will just export this relation and use it for the proof. So, this J LQR now can be written as in the following way.

So, I have H(x,u) + $\int_0^\infty [x^T Qx;\ u^T R\, u\ +\ 2x^T Nu + (Ax + Bu)^T Px + x^T P(Ax + Bu)]$ dt. So, this is the additional term just come from here ok. So, I just rearranged terms. So, I have H (x,u) plus $\int_0^\infty [x^T\ (A^T P + PA + Q) + u^T Ru + 2u^T (B^T P{+}N^T) x] dt$ ok. So, let me first see what this quantity looks like $(u + kx)^T$R(u+kx). With K as $R^{-1} B^T$P + N^T.

This comes from the theorem statement here this one here right ok. So, this will expand to the following. So, I have u^TR u + x^T(PB + N) $R^{-1}(B^T$P + N^T)x +2u^T(B^TP +N^Tx)ok.

So, I will write these two terms here which correspond to the term here and here in terms of this quadratic function here and then this term corresponding to this xx^T and x ok. So, and then substitute for this value of K. So, the J LQR we will now take the following forms I have H(x,u) plus integral 0 to infinity x^T all this things will come as it is A^TP + PA + Q minus this is well its suppose this side right.

So, minus (PB + N) $R^{-1}(B^T$P $+N^T)$x. Plus this term here $(u + kx)^T$R(u + kx) ok. Now look at the term inside here right this term ok. So, all this will be under dt. So, all this will be under the integral dt; look at this term carefully and then also this term right. So, first let us check this right.

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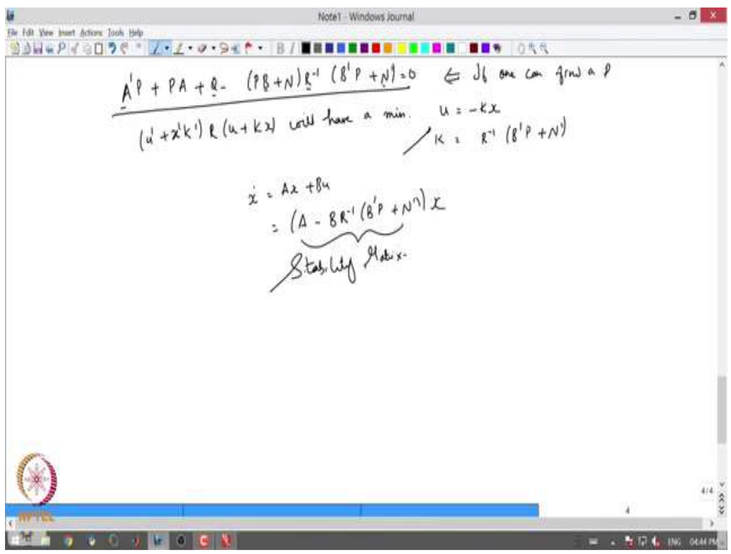

So, if I put this to 0, which means A^TP + PA + Q – (PB + N) $R^{-1}(B^T$P + N) = 0 right. So, if I put this to 0 how can I put this to 0 well a is given to me I know what this Q is given to me B is given to me R also N everything is given to me except P right.

So, if I can find an P if one can find a P which sets this term inside the bracket here to 0. So, my you what is my eventual aim? My eventual aim is to set this term inside the integral to 0 ok. So, the first question is well can I find a P which sets this to 0 ok. If the answer is yes, then what else is remaining; the other term that is remaining here is ok.

So, this entire thing will be in dt the other term that is remaining in the integral is this guy $(u + kx)^T$ R(u + K x). And I ask the question where does this have a minimum ok? So, it is a straightforward answer a this will have a minimum when this write down little more semantically. So, this term u^TRu + k x will have a minimum.

When precisely u = -K x with K given as $R^{-1}(B^T$P + $N^T)$ in such a way that the closed loop system now which is $\dot{x}$ = A x + B u with u = -K x is (A - B $R^{-1}(B^T$P + $N^T)$ x. So, that this is a stability matrix ok.

So, now, let us go back, the u which is equal to -K x with k being of this form; this we show that this minimizes this J LQR ok. And for which this closed loop system is now a stability matrix that is pretty neat I will say to show right. So, this was another way of looking at the Riccati equation. Now what we need to show is well do solutions exists or a bunch of questions. We need to answer once I have derived this theorems like, there is a P which this is, then u equal to -kx minimizes a certain function and so on right ok.

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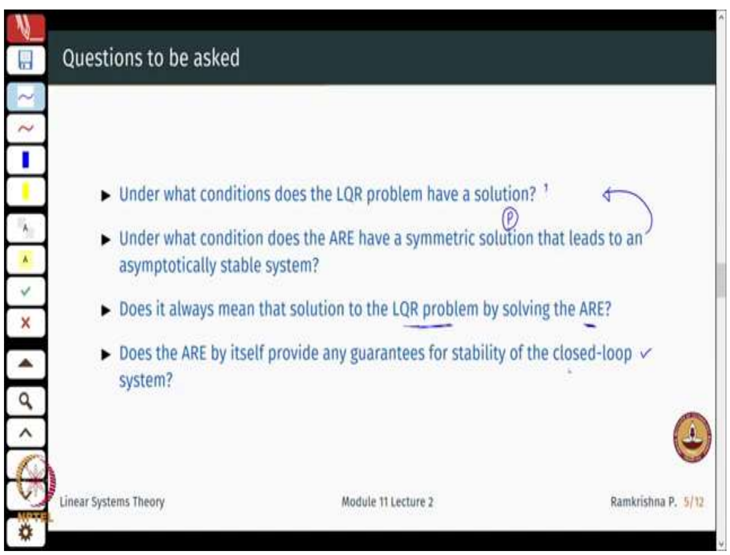

So, what are the questions to be asked right the first is under what conditions does the LQR problem have a solution ok? Now, similarly under what conditions does there exist a P or you go back to the slide? Under what conditions does there exists should be A^TP thus there exists a P which. So, under what conditions is ARE or the algebraic Riccati equation?

I have a symmetric solution which is precisely do to do with the existence of P that leads to an asymptotically stable system right, the closed loop system must be stable ok. Now does it also always mean that if I can find a solution to the LQR problem it also so sorry.

So, does it also always mean that the solution to the that one can find the solution to the LQR problem by only solving for the ARE by Riccati equation.

So, if I can find a P does it also mean that I have solved the LQR problem ? And lastly a question that we will also ask is does the Riccati equation by itself provide any guarantees for the closed loop system ok. So, these are the questions that we will try to answer ok.

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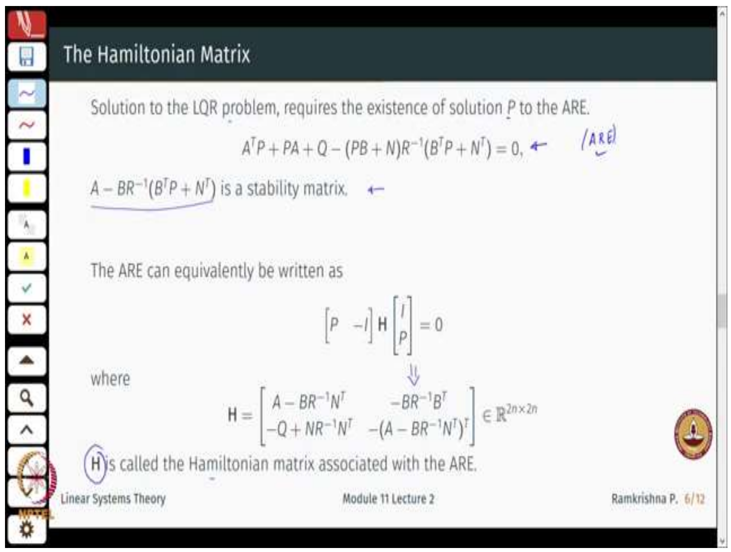

So, first we the first step to here to analyze this problem regarding the solution of the Riccati equation. You should determines or to define something called the hamiltonian matrix ok. So, what does it mean when I say I am looking at the solution to their to the LQR problem? Well, the solution to the LQR problem it requires existence of the solution P to this equation right. Everything else apart from P in this equation is given.

And such that this closed loop system is or the closed loop a matrix is indeed a stability matrix ok. So, I just rearrange terms a little bit and say that this ARE the algebraic Riccati equation can you currently be written in this form. So, P identity was a negative H and I P and this H will takes this form. So, I will skip the details, but you can just derive this for yourself ok. So, first definition here is that this matrix H is called the Hamiltonian matrix associated with this algebraic Riccati equation.

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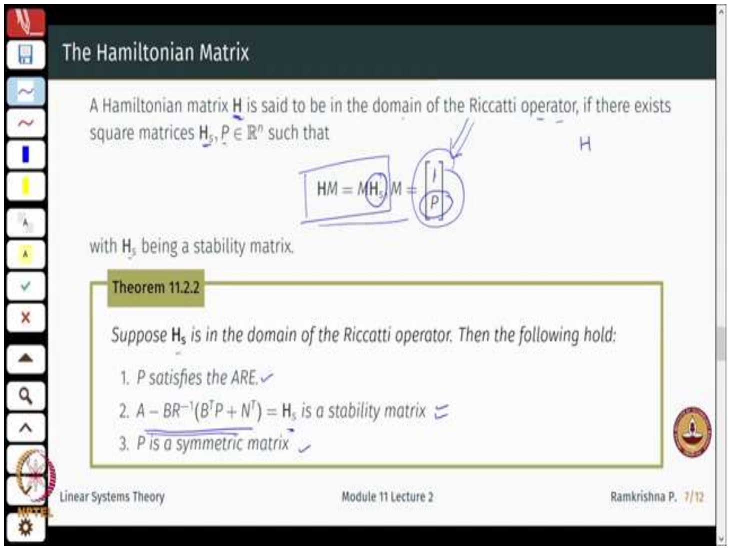

Now, what is good thing about this. So, the first thing that we will further define is that a Hamiltonian matrix H is said to be in the domain of the Riccati operator. If there exists a square matrix H and the square matrix P such that this relation holds HM where M is this identity and P equal to M H_s with this H s being a stability matrix.

So, the first thing what does it mean by this Hamiltonian matrix being in the domain of the Riccati operator ok. If something like this holds where H is the stability matrix then well P satisfies the algebraic Riccati equation right. The P here and they will slowly see how to derive this P also. If I can write a structure of this form H M = M H_s this expression actually means that the matrix H is in the domain of the Riccati operator ok.

With an M which takes this form P is a square matrix then this P which exists here satisfies the ARE. We will find later what this P precisely is second this H_s here is precisely equal to the closed loop matrix that we derived here this one A - BR P transpose and so on. This is a stability matrix and towards the end that P is also a stability matrix. So, let us do spend some time deriving that.

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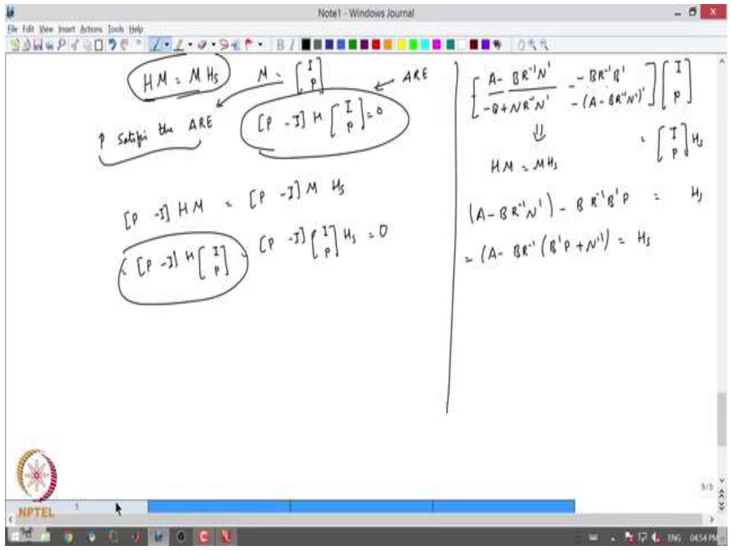

The first thing is so, H M = MH_s with M of the following form $\begin{bmatrix} I \\ P \end{bmatrix}$. So, first is we must prove that P satisfies the ARE P satisfying the ARE we wrote down it in terms of the Hamiltonian matrix in this form [P -I] H$\begin{bmatrix} I \\ P \end{bmatrix}$ = 0. This is this is my ARE right in terms of the Hamiltonian perfect.

So, just to prove what was the first statement to prove that? This P which is which comes from here that this actually satisfies the ARE ok. So, take this expression again here H M and then left multiply this with [P -I] ok. On the right hand side I will still do the same have [P -I] MH_s.

So, what does this mean? This is [P -I] on the left hand side H my M is of the form I and P. On the right hand side I have [P -I]M is of the form I and P and here I will have H_s and this is equal to 0 right. So, this here and this are the same right. And therefore, this P satisfies the Riccati equation ok. Now say what is what was statement number 2? Statement number 2 was to show that this H_s is precisely the closed loop closed loop matrix ok.

So, how does H look like let me just write down how H looks like again. This is $\begin{bmatrix} A - BR^{-1}N^T & -BR^{-1}B^T \\ -Q + NR^{-1}N^T & -(A - BR^{-1}N^T)^T \end{bmatrix}$ ok. So, these are this is how the H looks like ok. Now substitute this H into this equation ok.

So, this H let me just put it in this HM = MH_s. Now what is my M of the form M is of the form $\begin{bmatrix} I \\ P \end{bmatrix}$ ok. This is equal to I with P and H s no not I times P H_s I and P ok. Now I am just only interested now in this the first row side. So, this is A $-$ B$R^{-1}N^T$ - $BR^{-1}B^T$P is I H_s = H_s.

So, this I can re write again as A $-$ BR^{-1} ok. So, I have term say B^TP + N^T is H_s exactly what was in the theorem statement this one right; therefore, H s precisely take this takes this form ok.

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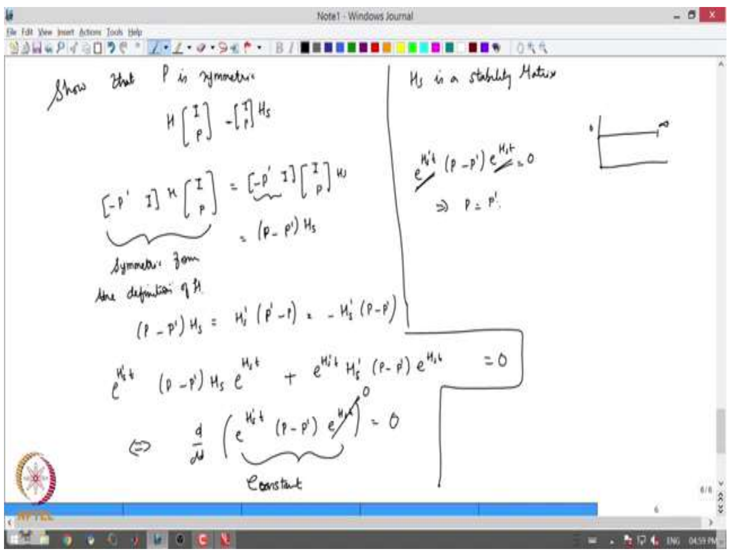

So, the last step is to show that P is symmetric ok. So, I again make you start from this expression HM = M H_s this M is a here $\begin{bmatrix} I \\ P \end{bmatrix}$ ok. Again I left I left multiply this by [-P^T I]H$\begin{bmatrix} I \\ P \end{bmatrix}$, = [-$P^T$ I] $\begin{bmatrix} I \\ P \end{bmatrix}$ H_s I just I just pre multiply both sides by this term. And here this will look like the following this is (P - P^T) H_s ok.

So, this can be shown to be symmetric from the definition of H ok. I will just leave that is for you to prove very safe forward process ok. So, the left hand side is symmetric the right hand side should also be symmetric which means, (P - P^T)H_s = ${H_s}^T$(P^T-P) this is also equal to -${H_s}^T$(P-P^T).

So, which means that (P - P^T)H_s + ${H_s}^T$(P-P^T) = 0 ok. Now, I just multiply this expression in the left by this term on the right hand side I will multiply this by this similarly here on the left hand side by this and the right hand side by this. And this will precisely turn out to be your following that $\frac{d}{dt}(e^{{H_s}^T t}$(P - P^T) $e^{H_s t}$) = 0.

Similar sums similar proof that we also did while we were proving stability right ok. So, will be this will still use the same concepts here ok. Now what does this tell me that d by dt of this term is 0 that this is term inside is a constant ok. Now what do we know? We know that H_s is a stability matrix if H_s is a stability matrix, this term will go to 0 as t goes to infinity ok.

And therefore, so this function is in this way right it is something like this constant ok. So, whatever is the value at infinity should also be the value at 0 right. So, therefore, the value of this function is should be 0 for all. So, you have that $e^{{H_s}^T t}$(P - P^T) $e^{H_s t}$ = 0 ok; because at infinity it is 0 it should also be 0 over here.

Now this is invertible e power H power H. So, the only option left is for P to be symmetric ok. So, we have proved the following right. So, if H is in the domain of the Riccati operator and which means an expression like the source and pre satisfies the the algebraic Riccati equation. This H_s is precisely equal to the closed loop stability matrix and P is also a symmetric matrix ok.

Now the next thing would be ok. How do we find this P are there ways to? So, what do I know what is the information that I have? I have H. So, given this H of this form can I find out a P? Now look at closely write all the terms in H are known to me a if comes from the system matrix B comes from the system matrixes Q R and N all come from the from my cost function. So, given this H can I now find a P? Making use of the expression that we have here ok.

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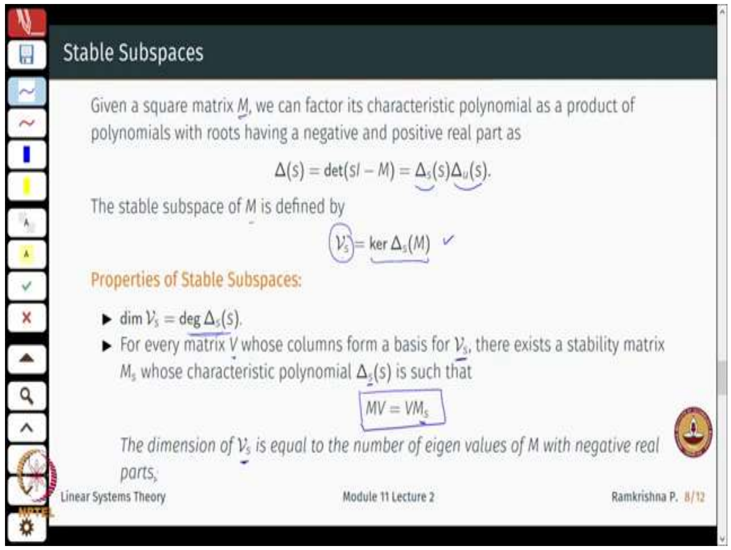

So, let us define something quickly. So, given a square matrix M; we can always factor its characteristic polynomial as a product of polynomials with roots having a negative real part and a positive real part. I will call them the stable and unstable or this s and u ok. Something that is easily observed easily observed is at the stable subspace of M is just the kernel of $\Delta_s(M)$ ok. So, this is also easy.

So, what is easy to check here is that, the dimension of this is also the degree of Δ_s of s all the all the stable subspace. Now in addition for every matrix v whose columns form a basis for V_s the stable subspace there x is a stability matrix M_s whose characteristic polynomial Δ_s is such that this relation holds. Some things similar to the to the controllable decomposition kind of thing that we did ok.

Now what does this mean that the dimension of v s right. So, this is it comes from here or the stable subspace of M is equal to the number of eigenvalues of M with negative real parts ok. Its just in general right I do not know what is the matrix M I just know its a square matrix. And I can factor its characteristic polynomial as a product of polynomials which has roots on the left half plane which has roots on the right half plane and so on. And I can say that the dimension of V_s is equal to the number of eigenvalues of M with a negative real part ok.

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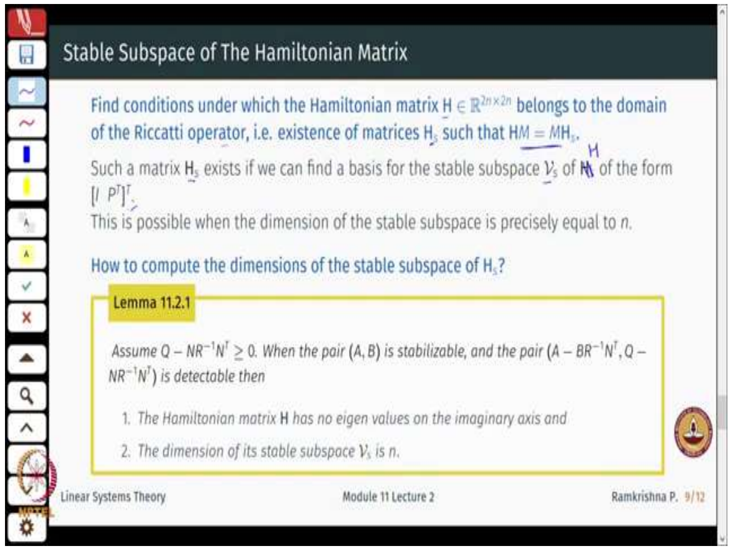

So, now, what we need to find here is. Find conditions under which the Hamiltonian matrix H belongs to the domain of the Riccati operator. That is existence of matrix H such that this holds right. So, we want to find when can I find a solution and how does the solution like these looks like? So, such a matrix H_s exists; if we can find a stable subspace of H should be not H_s stable subspace of H of the form something like this right.

So, it was here right this one. I need to find something like this right. So, what did what did I prove here is. If I can find something like this then this relation shows. Now I will ask a question when can I find some relation like this ok? So, now first is this is possible and we will see that shortly is this is possible, when the dimension of the subspace is dimension of the stable subspace is precisely equal to n ok.

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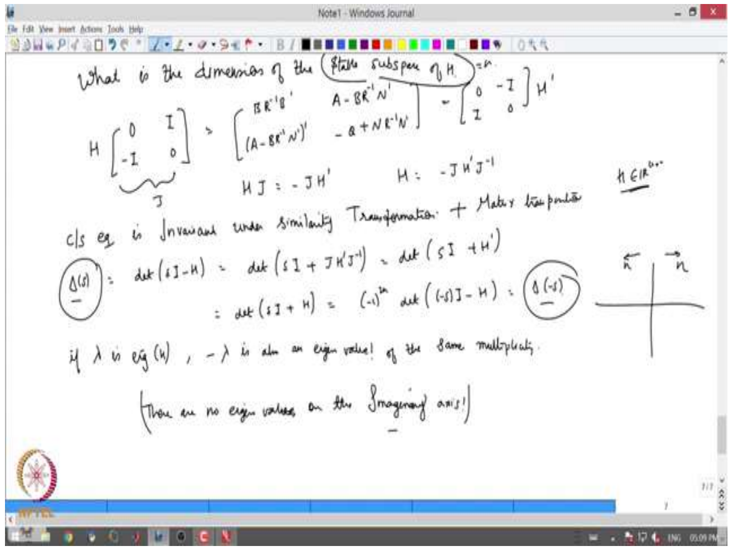

So, let us see how that looks like some small computations will show that. So, we need to find out what is the dimension of the stable subspace of H ok? So, let us do something. So, to take H multiply it with a skew symmetric matrix. So, this will give me$\begin{bmatrix} BR^{-1}B^T & \text{A} - \text{B}R^{-1}N^T \\ (A - BR^{-1}N^T)^T & -Q + NR^{-1}N^T \end{bmatrix}$.

The whole transpose - Q + $NR^{-1}N^T$ this is also equal to $\begin{bmatrix} 0 & -\text{I} \\ I & 0 \end{bmatrix} H^T$ ok. If I call this J what I have is HJ = JH the J is invertible. Therefore, H is it should be a minus a ok. So, H is - J HJ^{-1} ok. Now let us see what can be inferred from this. Now what do I know is that the characteristic equation is invariant, under similarity transformations. Now see so, the characteristic equation of H is the determinant of s I - H is equal to the determinant of s I + H s now can be written in this way. J H J inverse today with them is something is wrong here should be H^T here and I transpose here and here.

This is also equal to the determinant of s I +H^T s because the characteristic equation is invariant to similarity transformation plus also matrix transposition. That A and A^T will have the same characteristic equation ok. Now this is equal to determinant of s I + H ok, this is also equal to now $(-1)^{2n}$ determinant.

If I replace the s with the -H determinant of -s I -H. This is equal to $\Delta(-s)$ ok. So, $\Delta(s)$ was equal to $\Delta(-s)$ which shows what does this mean right so this and this. This means that if some λ is an eigenvalue of H; then - λ is also an eigenvalue right.

So, this is this is and moreover it is of the same multiplicity ok. And therefore, if n if H as so, H was what it was from R to n cross to n. So, if there are n eigenvalues here there will be n eigenvalues here also ok. Now this will be possible only if there are no eigen values on the imaginary axis ok. Now so, if this condition is satisfied that there are no eigenvalues on the imaginary axis.

Then H the Hamiltonian matrix will have n eigenvalues here and n eigenvalues here, which means that the dimension of the stable subspace of H is equal to n. Now, how do we ensure that there are no eigenvalues on the imaginary axis ok? So, the next result will prove that. So, we assume that if Q this is greater than 0; moreover if the pair A B is stabilizable and this pair is detectable. Then the Hamiltonian matrix has no eigenvalues on the imaginary axis. And therefore, this is obvious that the dimension of its stable subspace is n ok.

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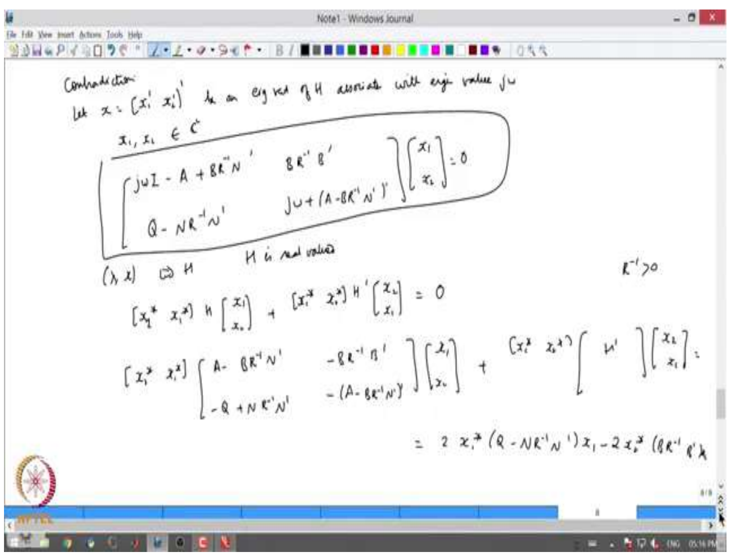

So, let us quickly do that ok. So, as usual we prove by contradiction ok. So, let x be $[x_1^T x_2^T]$be an eigenvector of H associated with eigen value jω ok. So, actually proved actually assumed this will belong to some complex numbers c. So, this x_1 and x_2 will both be in the set of cn ok. So, assume that let me assume that there is some eigen values which are which are sitting here ok.

And see what happens to the assumptions that we made or to the problems that we are dealing with ok. This means that if I go back to my Hamiltonian matrix this means that $\begin{bmatrix} j\omega I - A + BR^{-1}N^T & BR^{-1}B^T \\ Q - NR^{-1}N^T & j\omega I + (A - BR^{-1}N^T)^T \end{bmatrix}$ with x_1 x_2 is actually equal to 0 ok.

Now if again λ and x are eigen value eigen vector pairs of H right and H is real valued. So, I can rewrite this as $[x_1^* \; x_2^*]H \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$x+$[x_1^* \; x_2^*][H^T] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. This I can rewrite as $2x_1^* \, x_1 \, x_2$ plus $x_1^* \; x_2 \, x_1$ ok. The transposition and all it takes is this form and the stars denote the complex conjugate ok.

So, I will skip the steps, but this is easy to check that this will equate to 0. So, what does this mean that, if I take the left hand side of this expression that this entire thing will it will equate to 0. I have the following I have $[x_2^* \; x_1^*] \begin{bmatrix} A - BR^{-1}N^T & -BR^{-1}B^T \\ -Q + NR^{-1}N^T & -(A - BR^{-1}N^T)^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $+[x_1^* \; x_2^*][H^T] \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$.

= $2x_1^*(A - BR^{-1}N^T)x_1$- $2x_2^* \, BR^{-1}B^T x_2$ ok.

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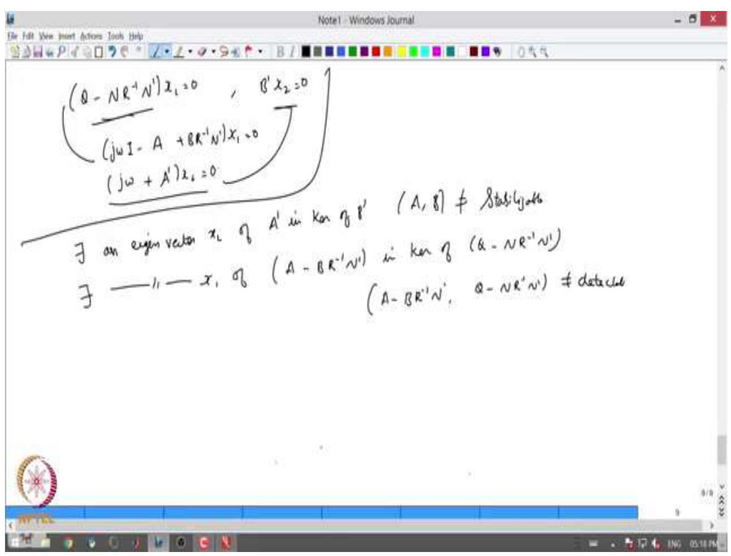

Now, this is equal to 0 right if this is equal to 0 what we conclude? If this should be equal to 0, (Q - NR^{-1}) x_1 = 0 and also $B'x_2$ should be equal to 0 ok. We know that R^{-1} is sorry that that R^{-1} is always greater than 0 ok. And therefore, the only possibilities are these two 1 and 2 ok.

Now this expression combined with this gives me a couple of additional conditions, which are that (jωI - A + BR^{-1}) x_1 = 0; not only that in addition I have (jω + A^T) x_2 = 0 ok. Now, let us see what each of these statements mean ok. First is there is an eigenvector there is an eigenvector and what is eigenvector this eigenvector x_2, of A' in kernel of B'.

This plus this, which means A , B is not stabilizable ok. Moreover then there is an eigenvector x_1 of from the these two expressions of Q - NR^{-1}N' sorry eigenvector of the off sorry now this one. Eigenvector of A - BR^{-1}N' in kernel of this thing Q - NR^{-1}N' from these two expressions these two first one was these 2. So, second one was these 2.

This means that the pair (A - B$R^{-1}N^T$), (Q - NR^{-1}N') is not detectable ok. Now, let us go back to the theorem statement, when A B is stabilizable and this pair is detectable then these things happen ok. Now when I assume that there are eigen values on the imaginary axis these two conditions are violated. And therefore, the Hamiltonian matrix H has no eigen values from the imaginary axis. And therefore, the dimension of its stable subspace is n one step we have we have proceeded further right ok.

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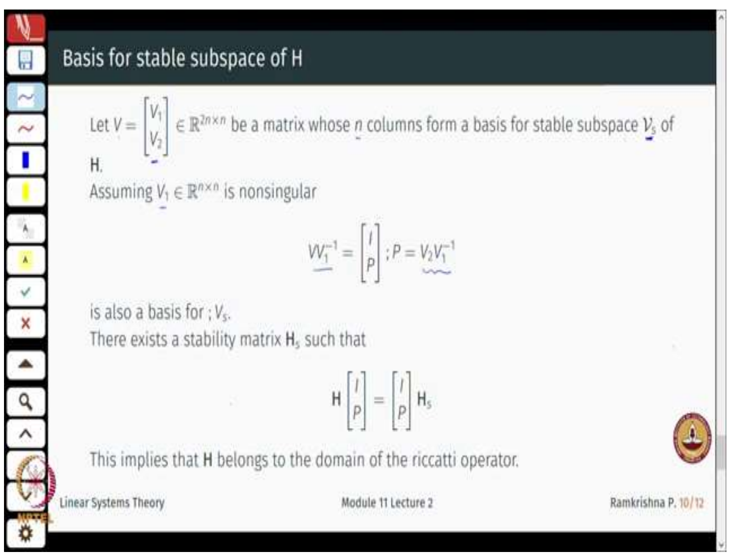

Now how do I find this the basis for the stable sub space of m of H? So, my eventual idea is to find how this P looks like ok. So, let us now I know that the dimension of V_s is n the stable subspace of H is n ok. Now let me just take let take some arbitrary basis right like the let V; which is V_1, V_2 be a matrix whose n columns form a basis for the stable subspace of H ok.

Now assuming that V_1 is non singular I can write V V_1^{-1} in the following way right I and P where P is $V_2 V_1^{-1}$. Now if v is a basis then VV^{-1} is also a basis for V s. And therefore, we conclude that from where do we so how what is the property that we will we will exploit now?.

We will exploit this property here from the property of stables subspaces for every matrix V; whose columns form a basis for v s here right sorry. For every whose columns form a basis for the stable sub space right, there exists a stability matrix this one whose characteristic polynomial $\Delta(s)$ is such that MV = VM s ok. I am just use writing the same.

There exists a stability matrix H_s such that H $\begin{bmatrix} I \\ P \end{bmatrix}$ this is this is my V right is $\begin{bmatrix} I \\ P \end{bmatrix} H_s$. Now this also implies now that H belongs to the domain of the Riccati operator that was from the definition right. The definition of the Riccati operator H is said to be in the domain of the Riccati operator if there exists square matrices such that this holds ok.

Now this so, what did we do? We just use the properties of stable subspaces one. Second is that the nature of the Hamiltonian matrix H is such that it has n eigenvalues with negative real parts n eigen values with positive real parts, which means that it has a stable subspace of dimension n right. Now we construct the basis for this for the stable subspace in the in this way.

And show that there exists a stability matrix such that this holds. And therefore, this H belongs to the domain of the Riccati operator. Now, once H belongs to the domain of Riccati operator, I know a bunch of things right that P satisfies the Riccati equation H_s is a stable matrix P is symmetric and so on.

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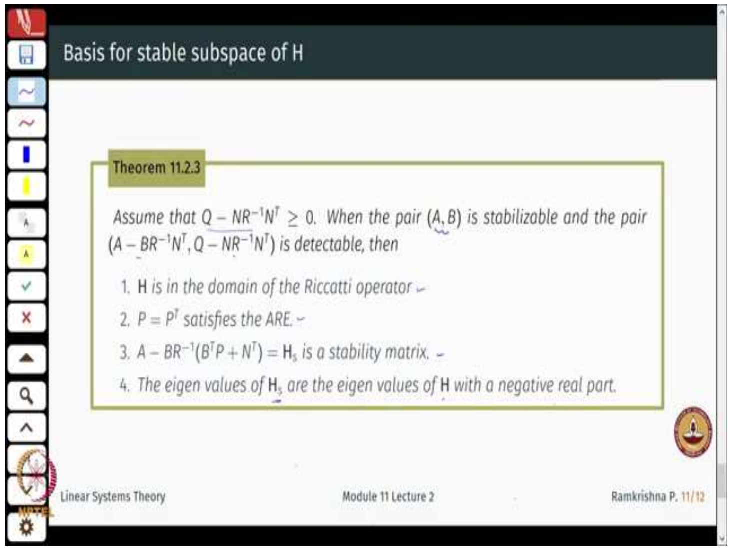

Now we can now sum up all the results into the following results from all the theorems and lemmas that. Assuming that this matrix is greater than 0; stability of A, B and detectability of the pair of this pair; then H is in the domain of the Riccati operator P is symmetric and it satisfies the algebraic Riccati equation this we already proved right. And then the eigenvectors of H eigen values of H s are the eigen values of H with a negative real part.

Now, I know under what conditions does P exists and I also know what P looks like right. So, P looks like they are exactly this find from here right. And all depends on identifying the stable subspace of H and saying that all its eigen values are equally distributed in the negative and the positive axis and nothing no eigen values exists on the imaginary axis ok. Now ok, we will do a little some examples maybe I will have a separate lecture on some of these on some problems related to this.

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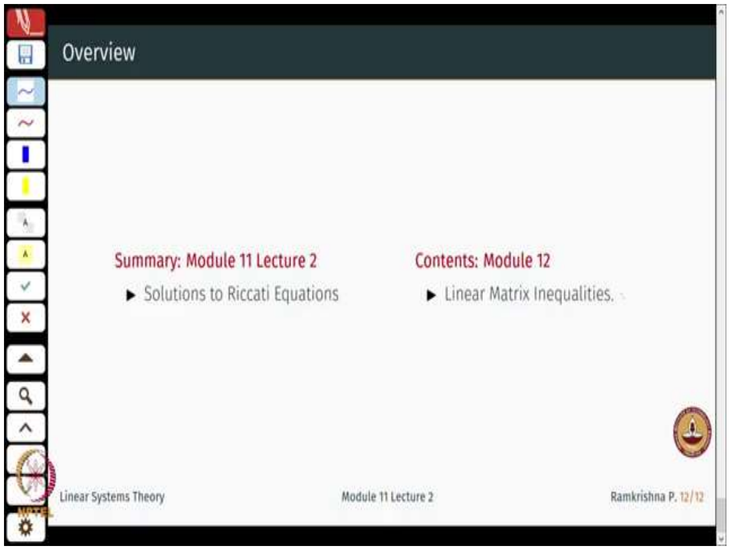

So, that kind of concludes the main topics that I wanted to cover in week 11. In next week we will really find off ok, what are the computational issues now associated with finding this P's; can we write those as linear matrix inequality starting from the solution to the Lyapunov equation till the Riccati equation and few other problems that.

In general control literature encounters. So, some computational tools in the form of linear matrix inequality is what we will do in week number 12. It is usually not covered in many of the standard textbook courses or any other courses or linear systems. But ok, since this is one of the first advanced level courses that we are doing in on NPTEL.

We would like to really even give you a little exposition to some other areas that you can; that you can reach out to from this. Just a little introduction to few other topics also as a result of this course. So, that is coming up next week.

Thanks for listening.