Linear Systems Theory
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Module - 11
Lecture - 01
Optimal Control

Hi everyone. So, welcome to this week 11s lectures on the course on Linear Systems Theory. I am Ramkrishna from IIT, Madras. If you have survived this course till week 11, I think you are doing pretty good. So, just a very quick recap of what happened over the past 2 or 3 weeks was at least from a design perspective we had formulated design problems.

I certain say pole placement problems or we also at some point of time ask ourselves the question what if the states are not completely measurable, then we had an equivalent observer design. We also looked at simultaneous controller and observer design. And so, how design of a controller does not really affect what is going on in the observer design and vice versa.

So, this can be treated as independent problems and how also towards the end we saw that the observer dynamics should be faster than the controller dynamics for obvious performance reasons, ok. So, what we were interested in? We were interested in one stability, second performance in terms of placing the closed loop poles at desired locations and of course, at times when we required an observer design. We actually also followed the procedure of designing the observer.

And starting from the definition of controllability right so, loosely speaking week. So, the control definition was can I start from any point in the state space and come back to the origin with help of some unconstrained control. I was not really interested in what is the control effort required or how much time does it really take I was just happy saying; well,. you should reach from point a to point b in some finite time right and also an unconstrained control input. So, we were only interested whether or not the system is say for example controllable.

We never talked about what is the cost of the control in what so, can I say, ok with a control restricted to some quality $\bar{u}$, can I reach from point a to point b in say some 5 seconds for example, right. We never really talked about quantifying those quantities right of the inputs, the time and so on. So, what we will today do is to look slightly beyond just pole placement or just beyond looking at stabilization pro kind of properties. Or beyond looking at just a control problem as just u = -k x such that the poles of the closed loop system are at the appropriate locations.

So, we will do something slightly different today and of what is called as an optimal control. So, what is a good control law? A good control law is something, is an expensive control law good control law right, it is like, if I were to draw an analogy to check; what is a good car for me to buy, ok? Then, you just maybe just out of curiosity just ask Google or you ask Alexa for example, she is also quite competent these days. So, what is a good car? and that in sometimes is also not a well posed problem; because you do not really specify your requirements of why do you need a car? What is the cost that you can afford and so on.

It is also looking in a different way if I say, I get a salary per month, I want to maximize my savings, I also want to maximize my comforts, right. So, what is a good solution? So, if I say well I want to maximize my savings and I should not spend at all or if I say I want to maximize my comfort or luxury rebels I should not save at all.

So, this is like in some sense a contradictory problem right of maximizing my savings versus maximizing my comfort levels. So, what does that mean in the control sense? That can we actually solve problems like this, when we have a conflict between good performance plus some constraints which occurred to us.

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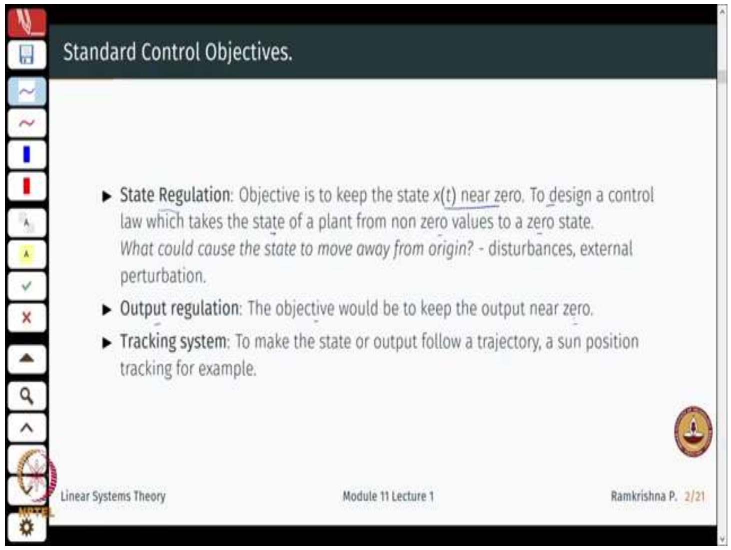

So what are the control problems that we typically encounter or say so, one is what we call as the state regulation problem, which is to keep the state x near zero right so to speak. So, what is the aim of the controller? Aim of the controller is to design a control law, which takes the state of a plant from some nonzero values to the zero state equilibrium for example.

Now, well why does this actually occur, why do I even need to design the controller? So, if I am at the equilibrium might always be at the equilibrium. So, what could cause the state to move away from the origin? There could be disturbances, there could be external perturbations. Say, if I am looking at a power grid a sudden change in load patterns might actually trigger my system in such a way that it will move away from the set point. Or if I am looking at IRCTC website sudden a booking pattern might take my system away from my set point so, to speak.

So, we can also similarly look at not only state regulation, but output regulation where the objective would be to keep the output in near zero or some. So, the zero is that I do not want to really drive the system to a zero output. But, some kind of a reference value which I set and you know usually, you can also talk of it in terms of the error will being you know in towards zero, ok.

Now, the third thing could be looking at a tracking problem, right. Where I can make the state follow or even the output in some case follow a desired trajectory; say I am I want to design a solar panel then it seems. So, if I would maybe face it at the east in the morning towards the end of the day I may not maximize the output of my solar generator right. So, I have to really track the position of the sun and this has been kind of a very interesting and well said it problem in control literature or even from power engineering perspective, right.

So, these are typical control problems that we encounter. So, so far we are like can I design a controller u -k x such that poles are given, performance is given, and so on, ok.

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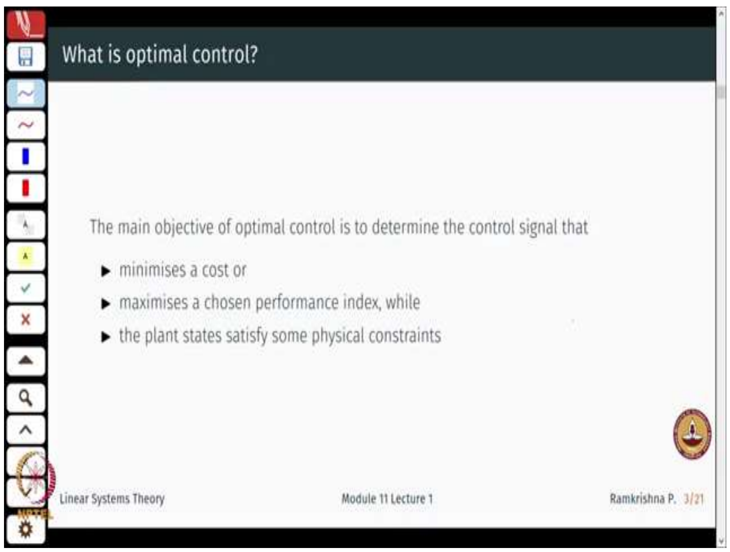

So, when I look at a little further, yes I can design a controller, but now I need to answer a little more questions right. So, one question would be; well, the main objective here if I look at an optimal control is to design a control signal that minimizes a certain cost, right at the same time does it maximize a certain performance index.

And the plant could have some physical constraints. Let us say for example, if I drive a car, I know that I possibly cannot go beyond say 150 kilo meter per hour or 200 kilo meters per hour, it just comes with some kind of a physical constraint. So, even though the car can move as a controllable relate analysis would suggest that the car can go from point a to point b in some finite time, ok.

Now, if I say can I go from say Chennai to Delhi by my car in say 2 hours; well, that is that is a difficult question to answer, right as. The answer may always not be possible.

because my car is physically constrained by its maximum speed. Then in most cases I also want to maximize a performance index. Performance index could be in terms of say, if I go back to the web server problem which I which we had in the week once lectures.

Can I serve maximum requests per second right and not only that can I also serve requests as fast as possible, There is a contradiction also in that particular problem right. And then of course, I want to minimize the overall cost, right. So, ideally I would want a maximum performance at zero cost and that is like the ideal situation. Well, but unfortunately that does not happen.

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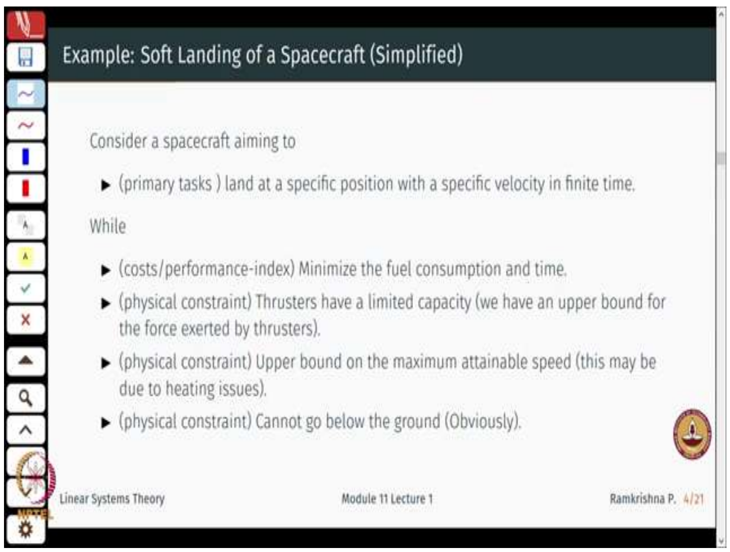

So, is there something that will help us a bit in this, ok? What could be other examples? So, something which is being talked about everywhere is about soft landings so, to speak ok. So, I will not tell you why we could not get to a soft landing ok, we will leave that for somebody else to say. But if I just looking at say just to draw some parallel between what I learn here to what actually see on TV each day, right.

So, one is if we if I look at a soft landing of a spacecraft I am going to see simplified right, you can do better than those scientists. So, what is the primary task? The primary task is to land at a specific position with specific velocity in finite time, ok. While minimizing well the fuel consumption and minimizing the time, this could be one of the cost or the performance index.

At the same time there could be physical constraints, that the thrusters might have a limited capacity and so on. And there could also be upper bound on the maximum attainable speed right, for whatever reasons right. For example, if I look at a current carrying capacity of a transformer or a voltage rating it knows not necessarily due to the conductor there, but it is mostly the properties of the insulator that kind of motivate that rating right.

So, these are essentially physical constraints which come with the system ok. Now, of course, there are another physical constraint you cannot go below the ground, right. This is kind of obvious, but this is a real time physical constraint, ok.

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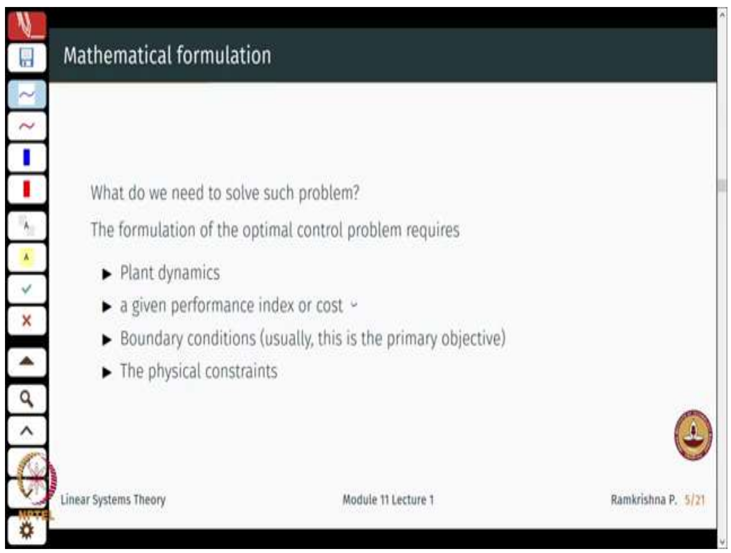

Now, what do we need to solve such problems? It is kind of fancy to talk about these problems of what they call also as soft landing and so on; what do we need to solve this problems? Well, we from our control engineers perspective we need a few things I need some few information.

First is I should know the plant dynamics as closely as possible. I need to know what is my performance specification, ok. For example, my performance specification as a teacher should be on my teaching abilities, but not my football skills or like of that, right. So, given a performance specifications boundary conditions right what are my limitations of what can be worked out what cannot be worked out, right. And then of course, physical constraints of the system, right.

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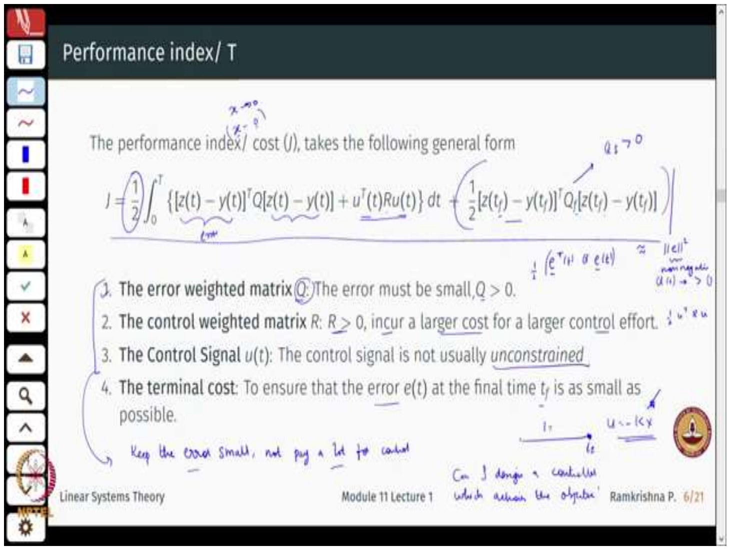

So, in general if I were to look at formulating this problem mathematically. So, all the time what we were interested is given a physical system can be write a model of that, a model was a mathematical abstraction of the system in terms of linear ordinary differential equations in this mostly in this in this course, right. So, from that model can I derive a controller and then put it back to my original physical system. That was my very straightforward you know design procedure set in a simplified way ok.

So, let us say one of the more general forms of cost function, I think different books will start with different motivation. Because ok, we are not really doing an optimal control course here, but this is just a little introduction to optimal control. You are just like not even like scratching the surface, you are just maybe watching this, the surface a bit from business, ok. So, what are; what am I my interested in, right. So, what does this entire integral here mean right. So, what is let us analyze each of these terms a little in detail, ok.

So, first I have ok, some difference here is z - y, z - y something to do with the inputs and something which has some t f here, right. So, we will just analyze what are each of these terms ok. So, let me say that the first term in general could correspond to some kind of a of a tracking error, right. I mean if I just want to my state to go to 0, then it will just be that I am just looking at x - 0, so this is my reference this is my actual and so on. So, this is something to do with the error ok, error some term Q here I will tell you what this means this is the input again some matrix R here ok. So, we will come to this term a little later, ok.

So, first suppose is the first one, the first is the error weighted matrix Q first is that well usually who you would want or we would want the error to be small. So, this usually has the form e^T, ok. So, lots of people use also this 1 by 2 just because when you differentiate the 2 will go away, but ok. Now without loss of generality you can also remove the 1 by 2.

So, I am looking at a signal which is of this form $e^T(t)$Qe(t), ok. Now typically so, this could also have some kind of interpretation in terms of the energy of the signal, ok. Now; well, this should always be non negative ok. So, this is a little approximation when say Q is say identity for example, this is. So, this quantity is always non negative and therefore, the nature of Q and sometimes it could also depend on t; is such that it is positive definite, ok.

So, what do you have to pay attention to we may have to pay attention to large errors and they are not they are not usually permissible. Now, second thing I have is the control weighted matrix. So, what; in the first case what we know is that the error weighted matrix Q should be positive for definite, right. So, we just get some kind of a quantification of. the error, right and then that the error should be small and so on ok.

So, the control weighted matrix it also has a form like this. So, you have half u^TR u. So, what this means is that well if R is typically greater than 0 that one has to incur a higher cost for a larger control effort. That is kind of natural that if I want to put in more control effort I would incur a natural larger cost. Say if I want to have a curve which goes as at say 250 kilometers an hour; I would possibly not get that for a price of a Maruti Alto for example, right ok. So, and then the control signal right, the control signal as I said earlier is usually unconstrained.

Even though while in our basic analysis we kind of assume that there is a let us see well if there is an unconstrained control can actually go from point a to point b, ok. Now what is the conclusion from this three points? The conclusion is that we would like to keep the error small; keep the error small and not pay a lot for control, right.

So, my error should be small, but it should also not incur a large cost on me, ok. And then last we have what is called as the asset terminal cost. So, that is taken care of by this term ok. So, the terminal cost is to ensure that the error at the final time is as small as possible, say I want to go from point a to point b in some t_f.

So, what; where am I at how far am I from the desired performance objectives at the final time is what this qualifies, ok? And as usual again this matrix Q_f should be positive definite, ok. So, in general what I would like to then say that I want to keep errors small not pay large cost. And therefore; and at the same time also have the error at the final time is as small as possible. Now can I design a controller, which achieves these objectives? Ok.

So, again my controller looks like a controller we will just look at you look like this, very similar to what we had previously now what is a good key, ok. Now, is there a standard solution to this? So, what will be important here is how much importance I give to each of those quantities? For example, I may want to reach from point a to point b in a shorter time, which means I am willing to pay longer, right. So, I am willing to pay a larger amount of money, ok.

And on the other hand if I have constraints on the money of going from point a to point b I am to go by a slow moving transport or a public transport for example, right. So, I fixed those weights know things why are these terms and then decide how am I going to achieve my control objective given those certain constraints. A solution may always not exist for example, going from Chennai to Delhi in 2 hours with my little car, it made it will not access, right. So, these are possibilities that we could we could encounter, ok.

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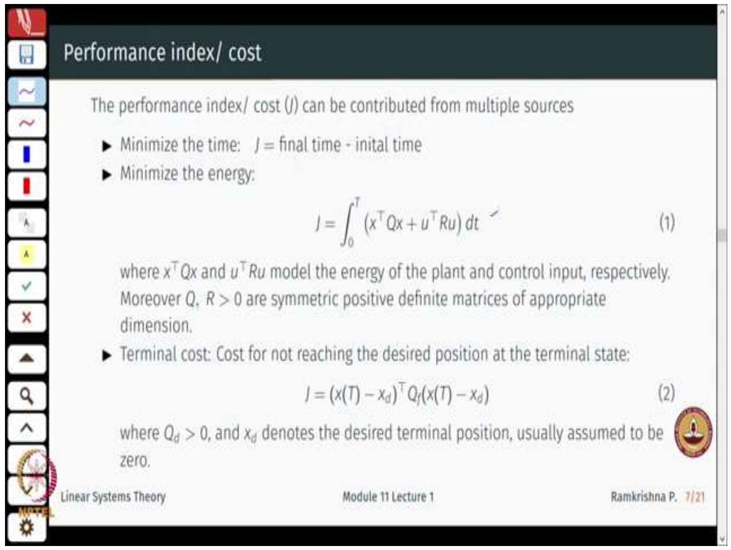

Now, most books will talk of a little simplify it cause function in this form, right. So, we will start just with something which looks like this right, where J is x^TQ x + u^T R u remains the same as before. So, first is minimize a time that is also and also at the same time minimize the energy, right.

So, as usual x^TQ x model the energy of the plant and u^TRu; what is the control input and so on right, a little general thing that we at most books would. So, we are here talking more like say a problem where I am looking at a some kind of a state regulation problem where I just want the states to go to 0, ok.

And then as similarly I will have some kind of a terminal cost, the cost is the cost of not reaching the desired position at the terminal state and as usual I want this to be as close to 0 or x(T) to be equal to x desired at least as closely as possible, ok.

So, we start with bit of a assumption that ok. Sometimes the terminal cost is assumed to be 0. So, what we start now first is, is a finite horizon problem. Where I know that I want to reach from a certain initial time and a certain final time is fixed right, ok.

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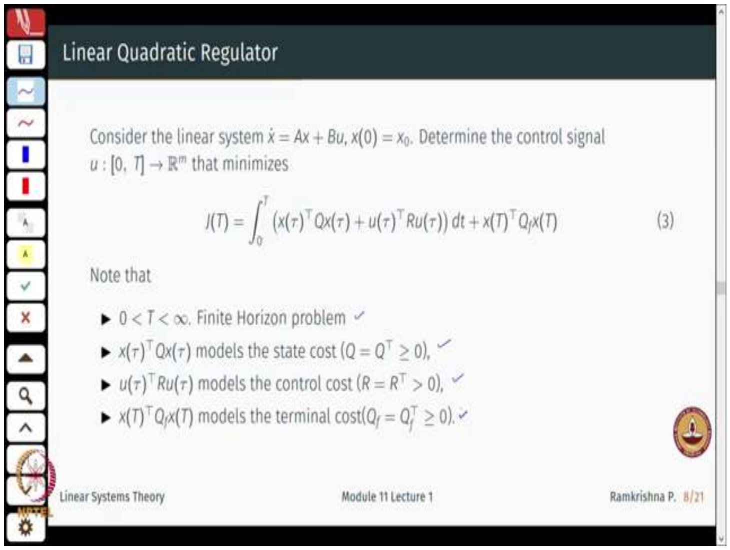

So, what is the problem here? The problem is that given a linear system $\dot{x}$=Ax + Bu with some initial state. Determine the control signal u that minimizes this particular cost function, ok. Now, minimize this cost function.

So, what is the observation here? First is look I am looking at 0 to T finite horizon problem, right. And then I have something which models the state cost something which models the control cost and something which more money is bit models the terminal cost, right.

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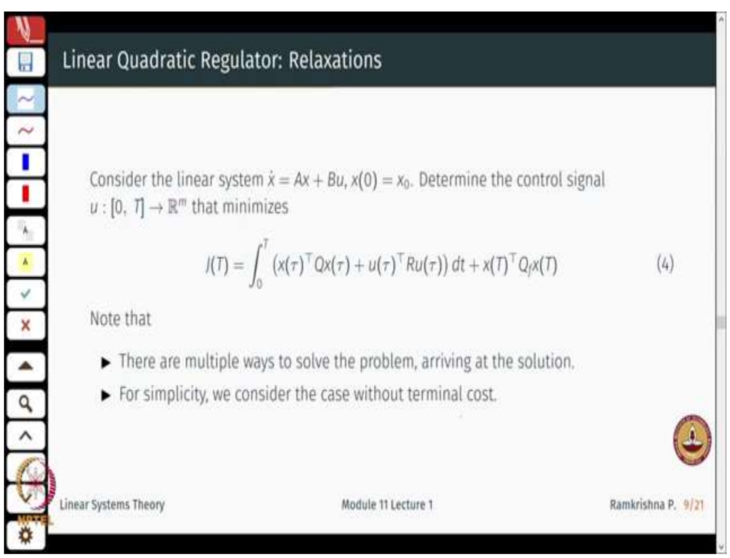

And, what we will see and that, ok; there are multiple ways to solve this problem. So, I will present you two or three ways which you will regularly encounter in some textbooks. There might be in more ways to do that there are lots of detail analysis, but we will try to get for our self for a good understanding of what is happening here, right. Two-three different methods and again you can follow whichever method you want and all methods give me the same solution right, ok. So, first is let us start for a case which may possibly not have a terminal cost.

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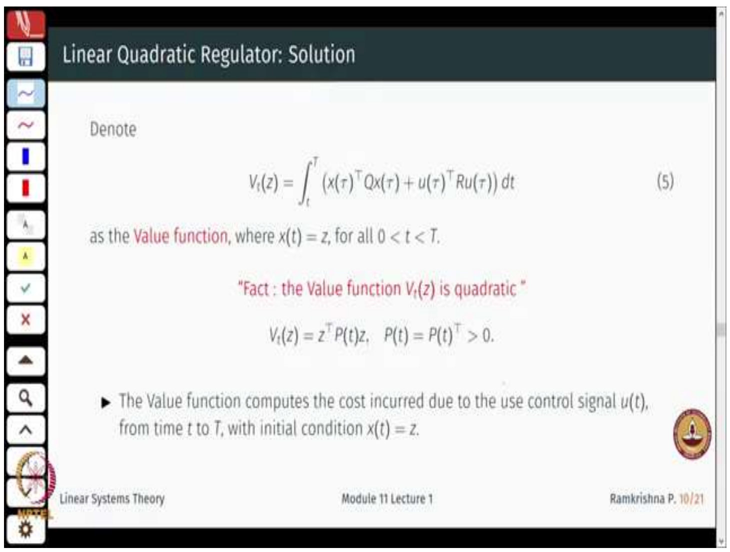

So, which means I have the cost function which is also sometimes called as the value function written in this following way should be, ok. So, first is let me call this my cost function or the value function, right. And let us say x(t) is z for all t between 0 and T, ok. And well, you can see that from here something which is obvious is that the value function or the cost function is has some kind of a quadratic form.

Its z^TP(t) z and this is also symmetric like this it is a straightforward computation from here to here. So the value function here computes the cost incurred due to the use of the control signal from t to T, with some initial condition given x(t) = z, right.

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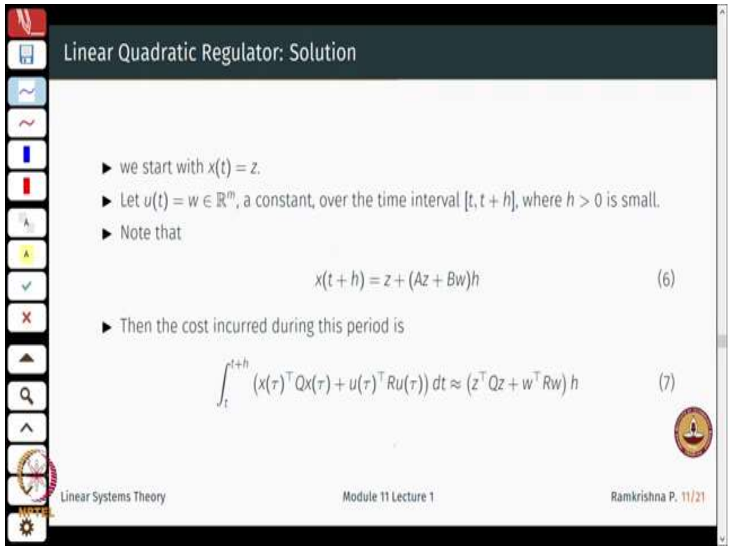

so we start with x(t) = z, let u t let me say that it is let u(t) be a constant function over the time interval t to t + h with some very small time h right. And within this interval I can write x(t + h) in the following little Taylor series expansion write x(t + h) is x(t + h) $\dot{x}$, ok. So, this is kind of obvious and what is the state did not attend the state is said and the input is w ok.

What is the cost incurred during that time well the cost incurred during this period is the following set x from $\int_t^{t+h}(x(\tau)^T Qx(\tau) + u(\tau)^T Qu(\tau))d\tau$ which can be loosely approximated to this, ok. Again this is just a little time interval from t to t+h. This should again be like easy to verify.

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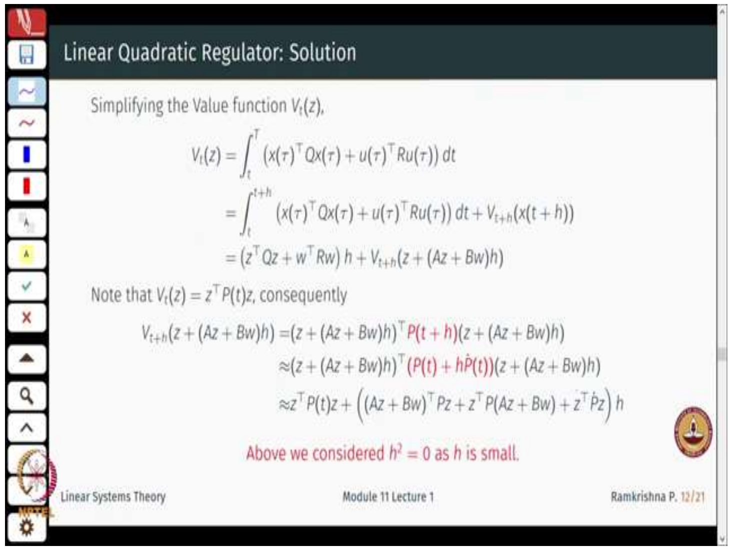

Now ok; so, let us without reading the steps let us actually try to derive all these things and I will come back to the explanations again, ok.

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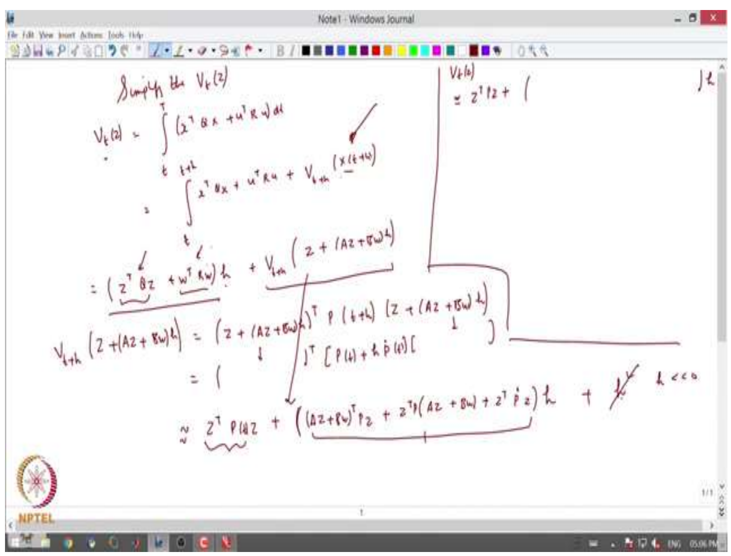

Now first so, simplify the value function V_t(z). So, what was V_t(z)? $\int_t^T (x(\tau)^T Qx(\tau) + u(\tau)^T Qu(\tau))d\tau$, ok. The second right has two parts right one is an integral starting from t to t + h, the time the same things inside $x(\tau)^T Qx(\tau)$, $u(\tau)^T Ru(\tau)$ plus the term integrated from now t + h to T with the same thing in the bracket.

So, what will this mean? This will actually be the value function V computed at t + h with x(t+h). So, this is just I am just using the definition of the value function, ok. Now, from this approximation here I can rewrite this as $(z^T Qz + w^T Qw)$h + V_{t+h}.

Now, again go back to this expression right, x(t+h) is just a simple expansion who tell me that this looks something like this. So, $(z + (Az + Bw)h)^T$ ok. Now let us see what this thing looks like V_{t+h}(z +$(Az + Bw)h$) is ok. Now what is V like, V is this V_t is a quadratic in P and z.

So, this will be $(z + (Az + Bw)h)^T$ P(t+h), right. So, that is where we start from right times $z + (Az + Bw)h)^T$. I am not doing anything special here I am just writing V_t right this, for this particular V with this particular h where this particular x where the x is approximated as this function right, ok.

So, I will ok, so what is the next step. So, this will remain as it is P, I can again further approximate as P(t+h) times $\dot{P}(t)$, ok. You know this guy coming and sitting here again, ok. Now this can be a further approximated as z^T P z plus I will have a bunch of terms here A sorry, $(Az + Bw)^T$Pz + z^TP(t)(Az + B w) + I $z^T \dot{P}$ z , right.

So, I am only taking the terms which have a common factor of h. And then you will have something of h square which I will ignore because h is like really small right. So, I can ignore these terms so ok. So, where are we now? So, we started with computing sampling the value function I said I will first integrate from t to t h. And then I have the remaining term over here which is approximated like this right, ok.

So, where do I go from here? So, this $V_t(z)$ can now be written in the following way. So, the approximation of it, so, I am just taking this term plus what I derived over here which is equivalent to this. So, I will have a bunch of things here z^TP z plus all the terms with a common factor of h, ok.

So, there is so, what we will add here is that this term and this term also have a common factor of h. So, they will just go here right. So, this term will be as it is a transpose P z. So, whatever is in this big bracket here we will have two additional terms this one and this one ok. So, then this is how it will look like, ok.

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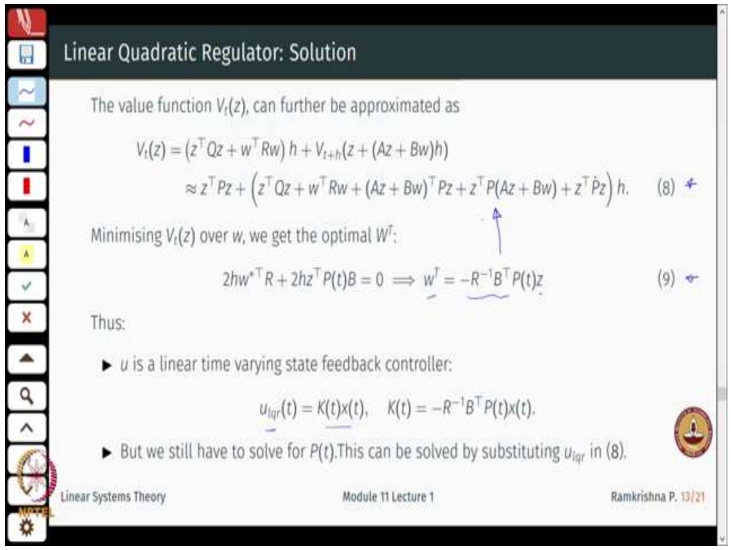

Now, what we want to be so, we want to minimize this cost function over what we want to find the minimum u, ok. So, what is the u here; so, I am looking at minimizing it over w. So, I get the optimal w small w just a function in one variable which I want to minimize I just take the derivative equal to 0 and this is what I get at.

The optimal control w^T = -$R^{-1}B^T$P(t) z. Now, what do I have now? So, once I have w, I can now get a state feedback control which is of this form ok, u is. So, in general right if I look at. So, for a small w it looked this way. So, what will the general u look like this is u this is -K(t) x, right ok.

Now, what do I know from here? I know what is R because I fix the matrices here or I fix the weights of how much say constraints I have on the input and how much constrains I have possibly on the state right. So, I know this B is given to me I know x, right. So, what is unknown here is this term P ok. Now to find this optimal input which depends on P, via this relation I still need to solve for this P(t), ok. Now how does this look like well this can be solved by substituting for this. So, the u which was Kx(t) or w which was this Kz I can put it back here ok. So, what does that that give me right, ok.

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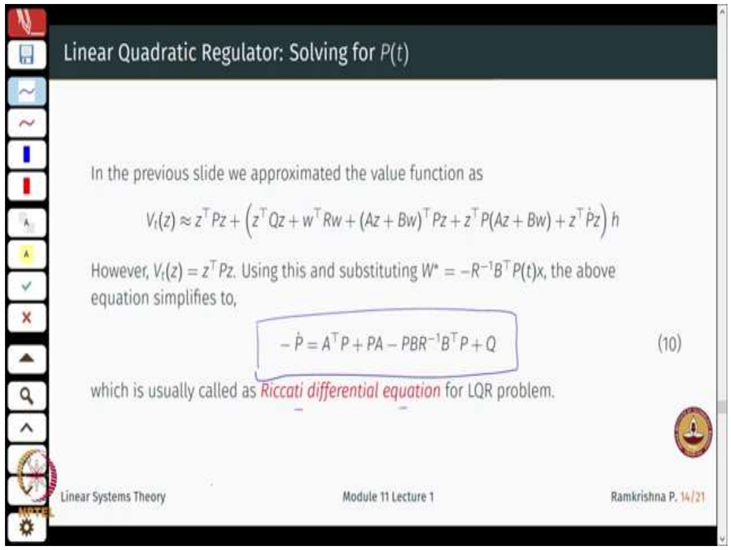

I will just skip the computations, but what will that give me that the equation now simplifies to something like this, ok. Now this is usually called as the Riccati differential equation for the LQR problem, ok. So, where did we start off with?

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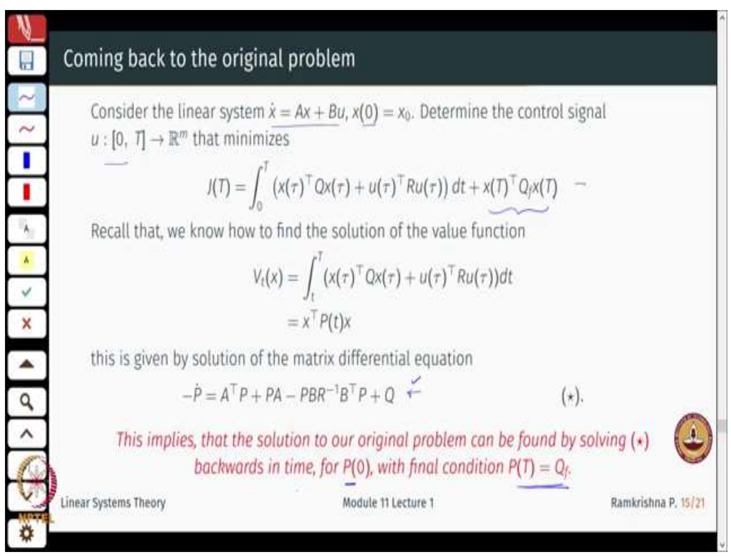

We started off by a system dynamics $\dot{x}$= A x + B u with some initial condition. We wanted to determine a control input that minimizes a certain cost, ok. Now we know that well the solution in the absence of this guy we know that the solution V_t is given by this matrix differential equation, ok.

Now this implies that the solution to the original problem can be found by solving for this equation backwards in time for P(0) with final condition P(T) = Q_f right. So, that is what is so now, I can look at what my terminal cost looks like, right. So, I can I know this P(T) is Q_f and I can actually solve for it now backwards in time. So, once I know this it becomes a little easy for me.

So, I can just substitute for that P over here, ok. Now this P is again this u of t which is time varying will be K(t) x(t) and so on right. So, now, what I know is, that to be get this value of K which is depending on time. I need to solve for this unknown P via this Riccati or this via this matrix differential equation, right.

So, that is a little basic building block to what we are up to here right. How do I solve the optimization problem or the optimal control problem? Well, that turns out to be a coming from a solution of the matrix differential equation. Is something analogous to the matrix the Lyapunov matrix equation that we had A^TP + P A = -Q; well, does a solution always exist or not this depends on whether the system over there was stable or not.

It also depend here over the system is controllable or can I actually know the solution is not does not always exist. And we possibly we will see problems also where the solution may not exist ok. The next thing which I mean most books will talk about is the infinite horizon problem. In a way that ok, what will happen when I am looking at the infinite horizon problem is that well the final time is infinity, right.

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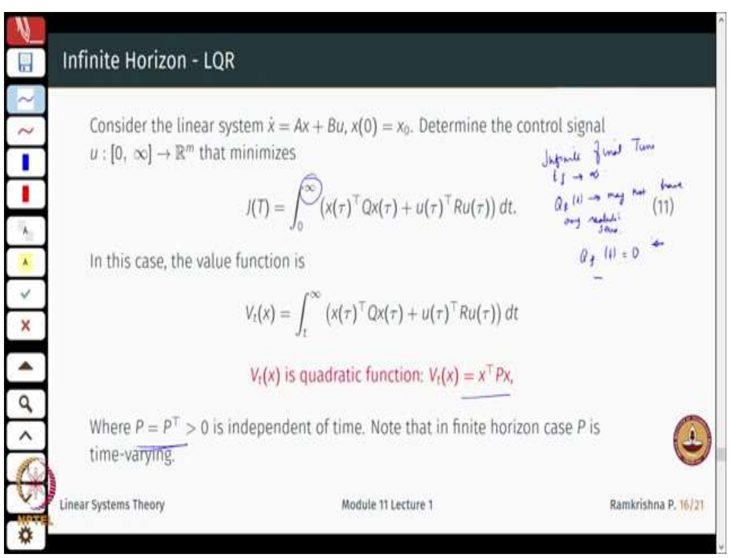

So, the, this is the case of infinite final time and the t_f is infinity, ok. And then the final cost term may not ok, this may not be or may not have any interpretation; may not have any realistic sense ok. Since we are interested you know usually solutions over finite time and just. Therefore, Q_f in this case must be equal to 0.

So, we are solving now for the same problem with no final cost and the cost J going from 0 to infinity, ok. So, in this case the value function or the cost function is simply quadratic it will now be independent of time because well there is nothing really associated here, ok. Now, the problem now turns out to be kind of pretty simple now. So, I am just looking now at a solution so, instead of a matrix differential equation.

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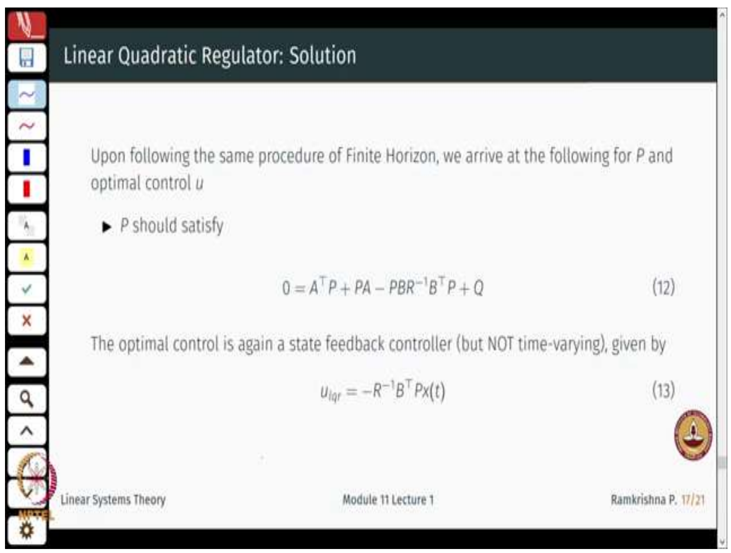

I just have some kind of a linear like looking equation or well maybe not, right. So, I am just having maybe a non-linear equation here there is no differential term here. So, the solution that will give me the optimal control law is -$R^{-1}B^T$P x, where P is now solution of this equation number 12 ok.

And what is important here to look at is that the state feedback controller that the gain K here is constant throughout. Whereas, in the previous case the gain K over here was varying with time because of the dependence of P on time right so, that is a little distinction between the infinite horizon problem and ok.

So, much of the textbooks will first start with the infinite horizon problem so, I just do a little reverse right. So, what I did so far was, what is also called as the dynamic programming approach. So, if you look at ogata he will have a different method which I will shortly tell you is in the following way.

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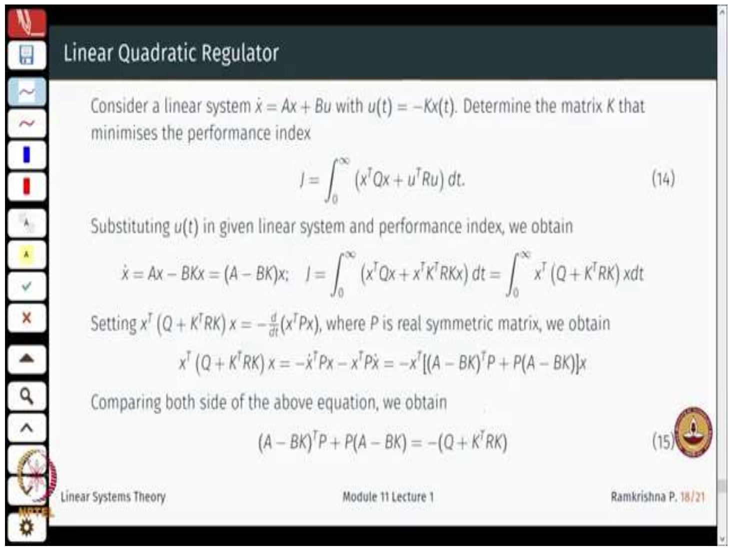

So, consider a linear system $\dot{x}$ = Ax + B u with some input that we need to design. And in such a way that the matrix K minimizes this cost function or the performance index, ok. Now, what do I know let us write down the steps here.

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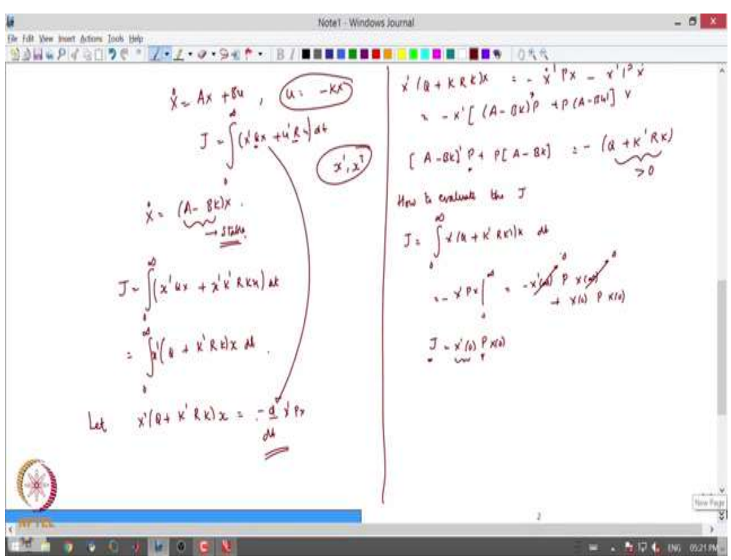

So, what is given to me is again $\dot{x}$ = Ax + B u. And I want to minimize or find a control law u = -k x, which minimizes the cost function is defined from $\int_0^\infty (x^T Qx + u^T Ru)dt$. So, this is like the linear quadratic regulator and that is where the name comes from possibly, ok.

So, this is the control law right and then what I have, what do? I have as a closed loop system looks something like this $\dot{x}$ = (A - B K)x right we have seen this already a lot. Now, it will be safe for us to assume that this is stable as there is no point designing this controller. So, let us assume that A - B K is stability matrix, ok.

And therefore, the cost function now J looks like this $x^T Qx$; so, x this also means transpose this also means transpose. So, this is $x^T Qx$ plus u is K x. So, I will have $x^T K^T$R Kx d t ok. So, this is of the form$\int_0^\infty x^T (Q + K^T RK)xdt$ ok.

Now, let us play some tricks here right. So, let us set this number $x^T (Q + K^T RK\ )$x is - $\frac{d}{dt}(x^T Px)$ ok. Now ok, so what do we know right? So, this x^T(Q + K^TRK) x is this is differentiate this and get the following. So, this is -$\dot{x}^T$P x - x^TP$\dot{x}$ here.

And this will look like this -$x^T (A - BK)^T P$ + P(A - B K)x here right ok. So, this is something what we had also for the Lyapunov equation right. Now what do I know that A - B K is stable right. And therefore, given a matrix Q + K^TR K now this is because Q and R grater than 0 this is also greater than 0.

For this Q well, I should be able to find a P such that $(A - BK)^T$P + P(A - B K). So, some equation some Lyapunov equation like this holds, ok. Now, what is to be found out here while minimizing the cost function is does there exist a P which satisfies the solution given a certain optimal K, ok.

Now, how to evaluate what is the P or what is the appropriate input? So, J here is now 0 to infinity now from here right I just do a substitution. So, J was x^T(Q + K^TR K)x d t was x^TP x 0 to infinity this is x at infinity it will here it will be minus sign.

Here because of the minus here -$x^T(\infty)$P x(∞)+ x(0) P x(0). Now, since A-B K is stable this will go to 0 and what I am left is J is sorry x$x(0)^T$P x(0) ok. So, this means that I can find the performance index J; in terms of the initial condition x(0) which is given to me and P ok. Now the next step to the solution is the following.

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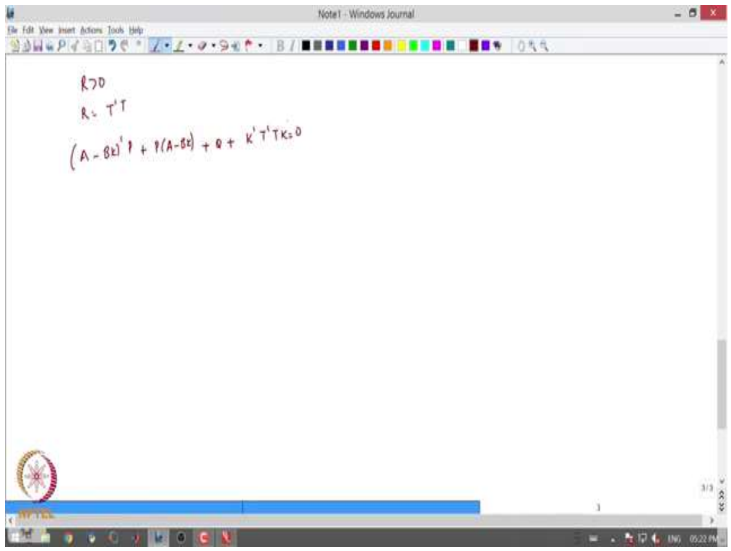

So, because R > 0 one can always write R as T^TT, ok. Now, again T is a non singular matrix and so on and therefore, I can write down my Lyapunov kind of equation which is like this right $(A - BK)^T$P + P(A - B K) + Q + $K^T T^T$T K = 0 where did this come from? So, there is a we recently opened here. So, for R I just substitute R = T^TT ok.

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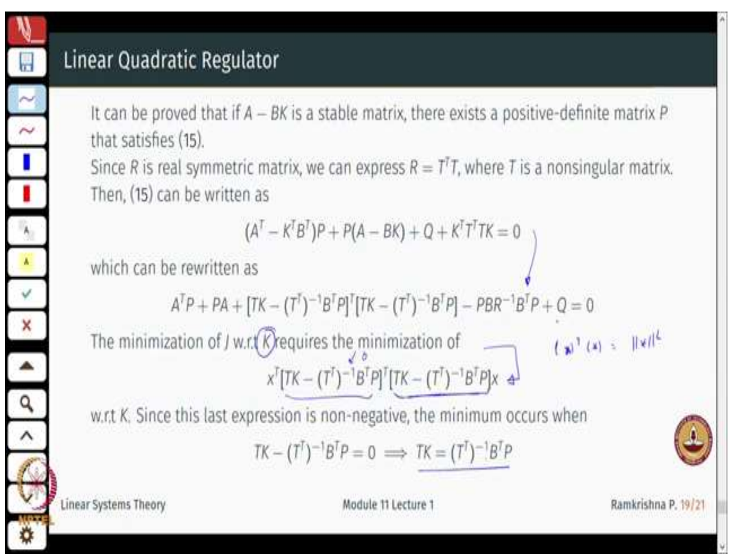

So, next what do I do is I just write down from here to here right I am just expanding terms. And minimization of this terms right again with respect to K right. So, this is what I am finding what is what is a good K this requires that this particular quantity be the minimum, ok.

Since the last expression or so, once we have so, the minimization of J with respect to K which means we may have to minimize this term and this occurs only when look at this end. So, I have x^T I have something here with the transpose something here and x. So, this is possible if and only if this term here goes to 0, yet because this expression in general will be non negative; because I have something like something with a transpose. And this will always be non negative. So, the minimum value here is only 0 and this 0 occurs when the term in the bracket becomes 0 which essentially means that T K is something like this, ok.

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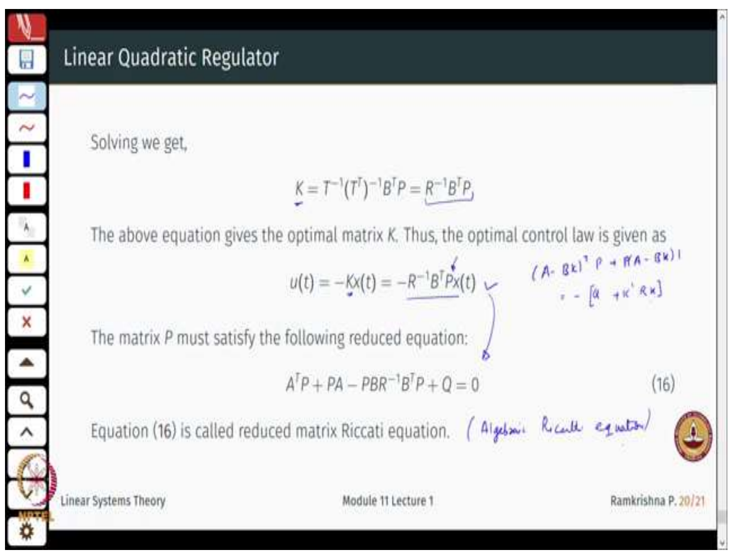

Now, I can back substitute and get a good looking value of K which is $R^{-1}B^T$P, ok. Let us keep the same and $R^{-1}B^T$P is what we also had here this was my K here at $R^{-1}B^T$P, right. So, therefore, now I have the optimal control law which is of this form.

And so, again the unknown here is this matrix P again how do I find this P well I have this expression for K in addition I also have an expression which looks something like this $(A - BK)^T$P + P(A - B K). This was equal to -Q + K^TR K.

Now substitute for this K in this equation right I am just using this equation again or this equation I substitute for a k=K and what we end up is that this reduces to solving this equation in P, right. So, this is called the reduced matrix Riccati equation or also the algebraic Riccati equation, ok. So, this is the second method for solving.

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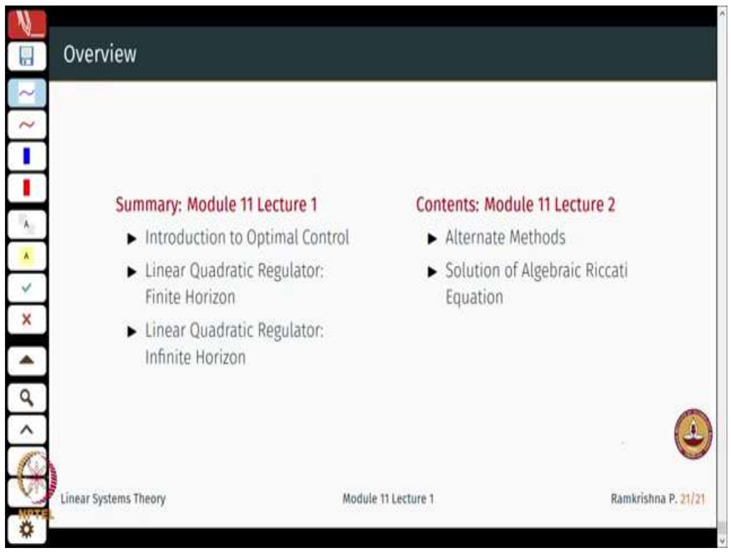

So, in the next class what we will also do in the or in the next lecture. To look at another method of solving the same problem same optimal control problem. And I will give you a brief introduction of this solvability of this Riccati equations under what conditions do I get the solution. Are there simplified methods to check whether or not there will exist a solution? How much is controllability important? How much is stabilizability important and so on, right? So, that will come up in the in the next lecture.

Thanks for listening.