Linear Systems Theory
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Module - 09
Lecture - 02
Canonical Forms and State Feedback Control

Hi everyone, so welcome to this lecture 2 of week 9 on the on the course of Linear System Theory. So, just the previous lecture we had seen explicitly how to deal with systems which could be both uncontrollable and an observable at the same time, and how could we actually get split each of those parts into the completely controllable plus observable mode, till the modes which are neither controllable nor observable and so on.

We also had a nice exposition to what it meant by minimal realization of the system that the system is or a certain realization is minimal if and only if it is both controllable and observable. And we also saw few examples related to that. So, what we will focus on in this lecture is to look at problems more from a design perspective right. So, we spend a lot of time building up tools starting from the algebra till analysis starting from solutions to the state space equations, deriving conditions for stability, checking for controllability, observability and so on right.

It is so much of results we had for the analysis phase. And of course, we also towards the end had some results on stabilized ability and what does it mean by existence of a stabilizing feedback and so on. So, we will today start with some basic exposition to solving basic design problems. How are these related to the design methods which we did by root locus or even via the Bode plot essentially to design lead like compensators which were in a way is some kind of an approximation of PID controllers right.

So, there was a nice relationship between a PID controller and a lead like compensator and so on. If we remember correctly so, much of the analysis there was based on the dominant poles, so everything we said if I were to choose a controller for a certain overshoot, a settling time, possibly also for a certain steady state response with respect to the error and so on. So, we focus a lot of it based on the dominant poles. And then we say the other poles are fairly to the left, so their dominance is minimal and there are also nice results on how to ensure dominance and so on which were of course a little approximation of the exact problem that we were supposed to handle.

Now, we did not really explicitly look at can I not only look at the dominant poles, but can I also look at placing all the other poles at the exact locations. We actually did not encounter problems like that. We were happy just by looking at the dominant pole analysis, and where the response of the dominant poles was fairly close to the actual system right.

Now, can we have a little finer control of the system or can I place all of the poles together I will tell you what pole placement means in terms of eigenvalues, why are some state feedback what if I want to handle problems were. I may just want to do place few of the poles or not say I have 10 poles I may just want to place two or three right. So, this is also referred to as partial pole placement.

So, I can have a little more finer control of my closed loop poles or I can do a fine finer control of my design procedure right. So, I can have a finer control of the system and can be design appropriate techniques for to achieve complete pole placement ok. So, we will slowly build upon those results and see what are the methods of for doing this ok. So, I will quickly go through the state space canonical forms. And, much of the results we will derive based on these canonical forms ok.

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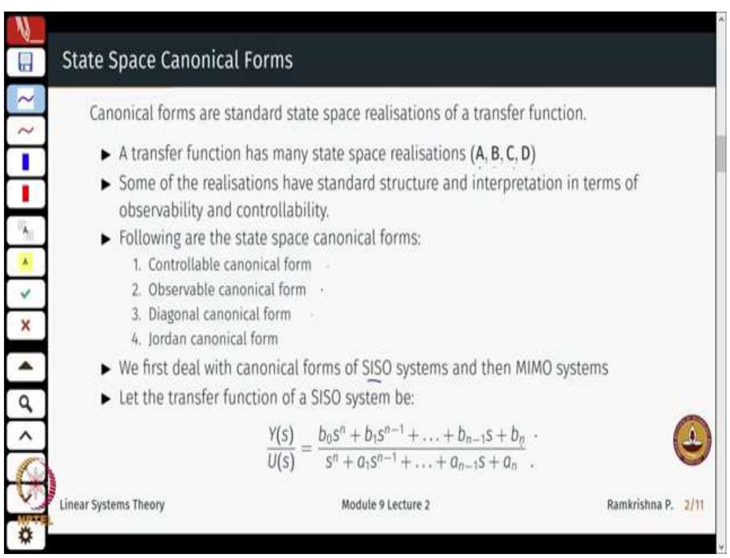

So, the general canonical forms would be again I am interested in systems of state space realizations with A, B, C and D matrices, and some of these realizations will have a standard structure and interpretation in terms of controllability and observability, and also the controller design and so on with the observer design which will come up in module number 10. So, we have standard forms of the controllable from, the observable form, the diagonal form and the Jordan form ok.

So, we are do most of our analysis based on single input single output system that is where we could actually write down expressions neatly and understand for ourselves. And let us put up some notes related to MIMO on the course platform, and we could discuss that whenever there are some difficulties in that ok. So, I start with a transfer function of the order n and given these coefficients b_o till b_{n_1} , a_1 till a_n ok.

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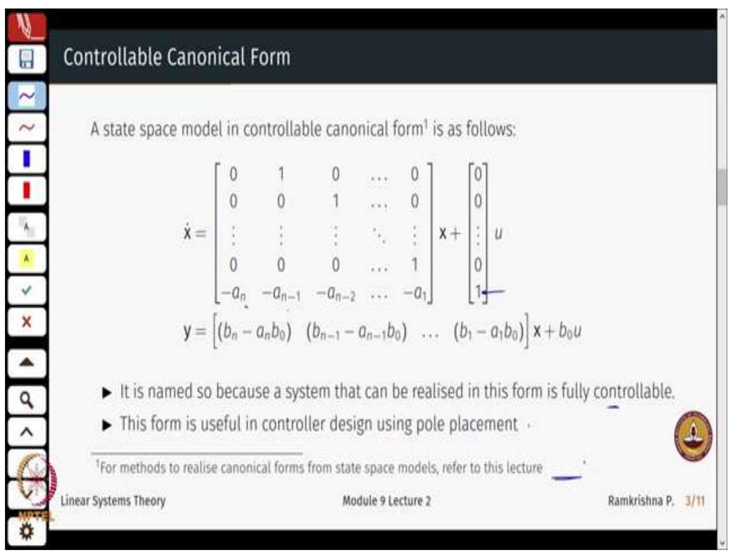

So, I have something like this. So, the controllable canonical form would just be like this at its $\dot{x_1}$ is x_2, $\dot{x_2} = x_3$ and so on and $\dot{x_n}$ will have all the coefficients of the denominator terms over here, and the input will just disappear in the last entry of the B matrix ok. It is named this way because if I can write a system in this form it is always fully controllable.

And I know that given the system in one form I can always write it into to some other forms via some canonical transformation. So, I will also introduce what kind of transformations we need to write a system into its controllable canonical form. And I will see also its direct implication on pole placement techniques, so the design techniques. So much of this how to derive that you can refer to my earlier lectures, the reference is missing I will put up in the slides when I when I upload them on the portal.

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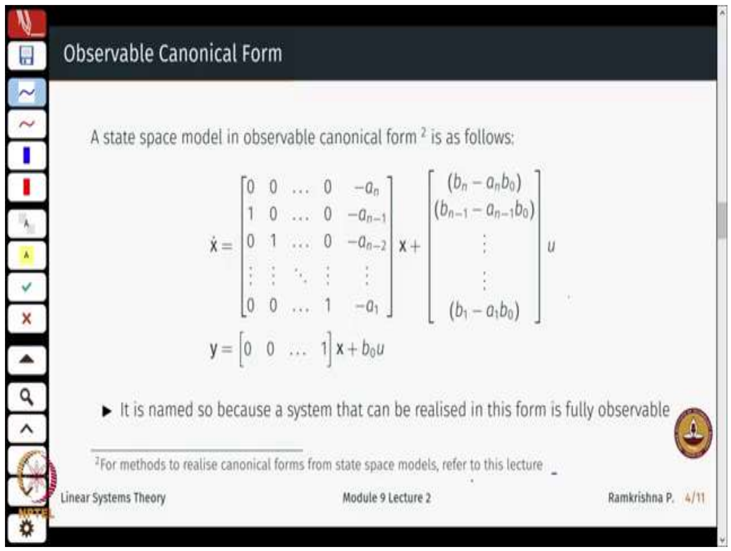

Similarly, we will have something also called as an observable canonical form. And wherever I can add the system this way it is fully observable. I will not go into the details of this, but again you could refer to our earlier lectures where things were rederived as starting from scratch. So, I will avoid repetition of those discussions again ok.

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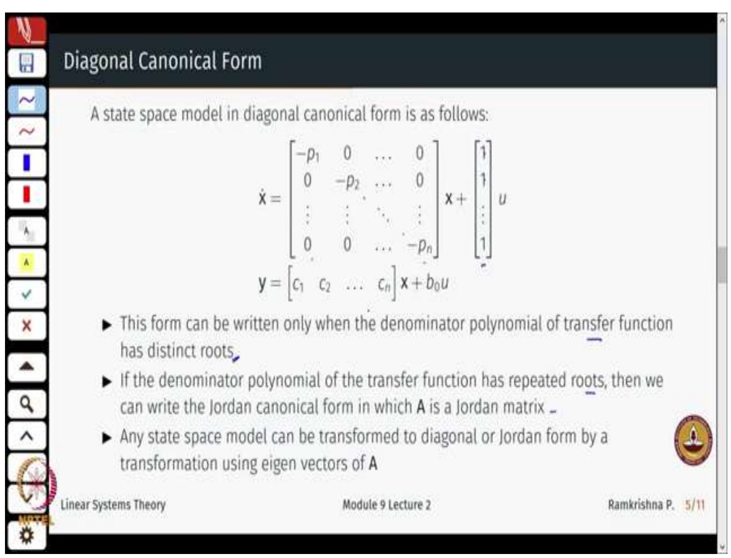

So, an interesting aspect is the diagonal form right, where I can just split the A matrix into the diagonal entries which are the eigenvalues of the A matrix. And, well and interesting to note is that if I write the system in the diagonal form, it is easier to check controllability because as you I have all non-zero eigenvalues on the diagonal, then the system is controllable if and only if each entry of the U matrix has a has a entry one right or a non-zero entry so to speak right.

So, now again when can I do this, well this can be written only when the denominator polynomial of the transfer function has distinct roots. Well, if it has repeated roots, then we can look at the at the Jordan form we know also the answer to what to do with repeated roots ok. So, again we know how to actually give in a system how to transfer it into or transform it into a diagonal form or a Jordan form. We did we did spend a lot of time analyzing the diagonal, diagonal form and also even the Jordan form. So, we will skip again that that discussion ok.

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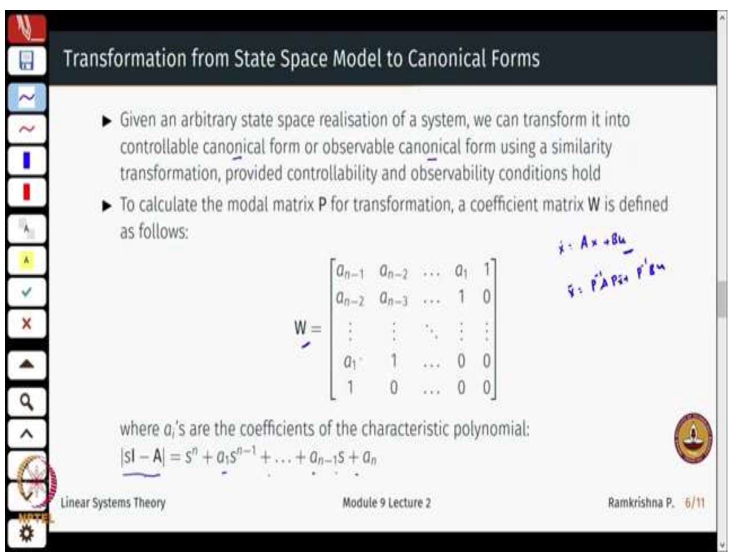

Now, given any system of the form $\dot{x}$ = A x + B u, how do I write it in the all controllable canonical form ok. So, given any arbitrary state space realization, we can transform it into either a controllable or observable canonical form using similarity transformation. So, similarity transformation where you know in ever coordinates you will have $\dot{\bar{x}} = P^{-1}$A P$\bar{x}$+P^{-1}Bu here and so on.

So, does there so the question to ask ourselves is does there exists a similarity transformation which will take my state space system from a general form to a controllable canonical form? And if the answer is yes, what is that form that takes us to that thing. So, I will just read out the steps again will not spend much time on the proofs of this, but we will spend time on doing on realizing the form the controllable canonical form by itself ok.

So, let me compute A matrix which has entries in the following form right. So, we are all these a_1 till a_n n, they come from here ok. Now, once I have so these are the these s are these just the coefficients of the characteristic polynomial ok.

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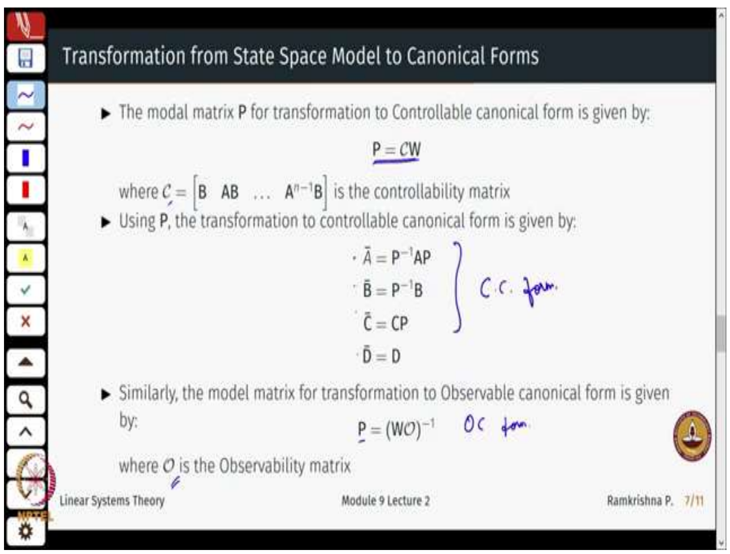

Now, once I have this matrix W, I can always compute the controllable matrix the controllability matrix B till A^{n-1}B and I define now a transformation P which looks as this C multiplied with this W over here ok. Now, so once I do this I just plug in into my original system, and I compute what is P^{-1}AP, I find that this $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ or always in the controllable canonical form ok. Similarly, I can use the same matrix W with the observability matrix to get the equivalent transformation to write my system in the observable canonical form ok.

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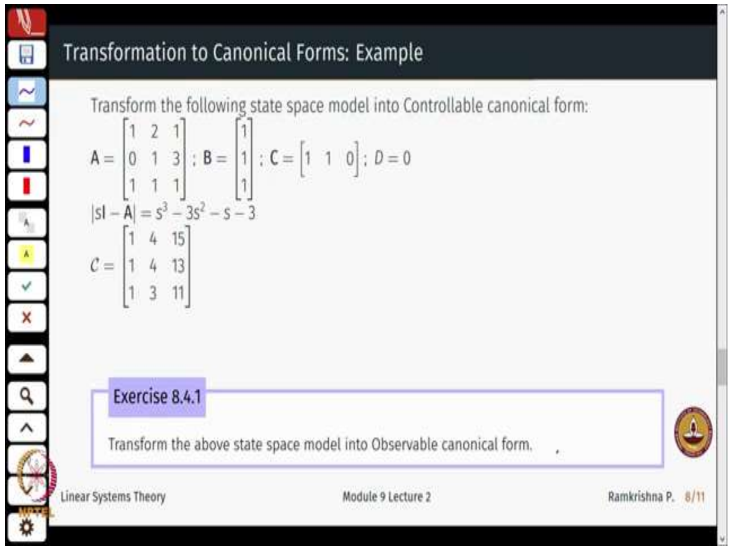

As a little exercise, you can just try solving, solving this I will not spend time doing this it is a kind of a trivial exercise. So, transform this into the controllable form and the observable form, I just do some computations here to find what are these coefficients of sI - A or the determinant of sI - A and also the controllability matrix.

So, a little guess that you could do now is that I can do this if and possibly a necessary condition is that the system must be completely controllable right. And if it is not, then maybe I might this P may not be full rank and so on. This only build up on these on these results and really derive towards the end necessary and sufficient conditions for which I can write a system into controllable canonical form, and therefore design a controller for the system.

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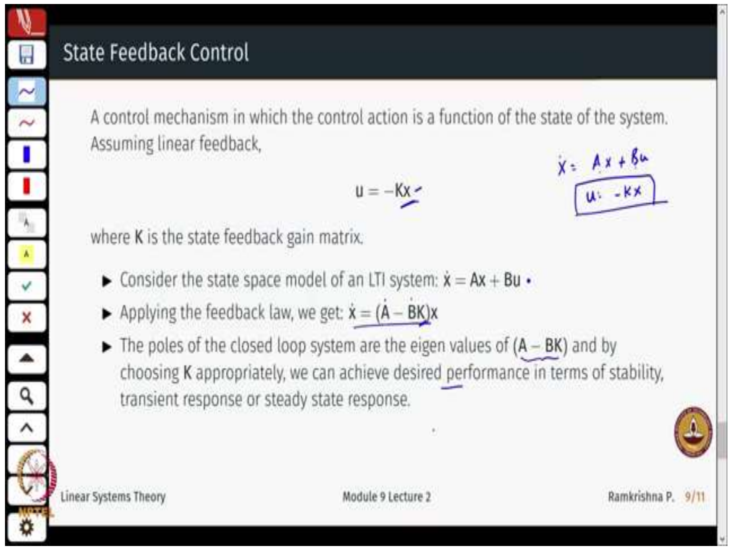

So, we in our previous lectures, spend a lot of time on systems of this form $\dot{x}$ = A x + B u what does it mean by designing a stabilizing controller, and what are the conditions on A, B and so on such that the closed loop system is or that the system is actually stabilizable ok. So, here we again we take our motivation from this expression which is called the state feedback controller.

Again the assumption is that I can actually the assumption here is that I can actually know what the states are. If I do not know the states that discussion will be in module number 10. But, for now I can assume let me assume safely that I know what the states are ok. So, I start with a system of this form together with a control my feed my closed loop system with this feedback control law looks something like this ok.

Now, what does this mean that the poles of the closed loop system are the eigenvalues of A - BK. And by choosing K right, so the eigenvalues of the closed loop system depends on of course, A is given to me and in the open loop, B is also the input matrix I cannot do much with this, but I can always change the eigenvalues based on the values of K, so that is the problem which we will solve.

So, can I choose K appropriately in such a way that I can achieve the desired performance that could just be starting from an open loop unstable system to stable closed loop system. The specifications could also be in terms of the transient response of the system it also be in terms of the steady state values of the system and so on.

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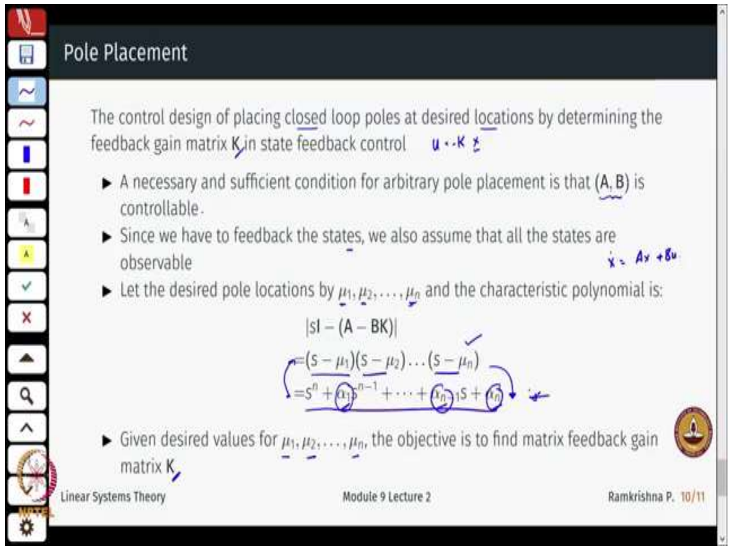

So, this technique is called the pole placement technique right so which essentially means we are placing the poles or the eigenvalues of the closed loop system at some desired locations or some predefined locations ok. So, the control design problem is of placing the closed loop system at desired locations by choosing an appropriate feedback gain matrix K right, and then just applying this control law u = -K x ok.

So, we will prove this a little later that a necessary and sufficient condition for arbitrary pole placement is that is fully controllable. So, before even proving this we first understand what does it mean by fluid, what does it mean of the system being fully controllable, and then what does it mean after that to actually assign the closed loop poles ok. Now, again as I said earlier we assume that we know all the states ok.

So, if I have a problem in such a way that let us say that I have a system $\dot{x}$ = A x + B u. And I say that a problem would state that place all the closed loop poles at these locations $\mu_1$1 till μ_n, μ_n in that case well I am just looking at well. So, this is how my closed loop system poles would look like and it will have a corresponding characteristic equation ok.

Now, this is given to me right place the poles at this desired locations. And therefore, with these poles, I can write the closed loop characteristic equation on this form, and I can expand it to a polynomials polynomial in s with this way this coefficients α_1 till α_n known to me right, they come from here till here ok. Now, given the desired values of μ_1 till μ_n through which I can also compute this αs.

So, the objective is to find a feedback gain matrix K such that the closed loop system satisfies in this characteristic equation right, so that will be the design problem here. Again the assumption here is that the system is fully controllable whatever system is not fully controllable, so we will come back to that a little later, but I think the you could actually guess what is coming up over there right. Can I actually place the poles of the controllable part at desired locations assuming that the uncontrollable part is a stability matrix, so that is what we did in lectures of week 7 right.

So, slowly we will come up with procedure of how to find this matrix this gain matrix K right. Given the location of the closed loop poles given my A matrix and given the B matrix which kind of determines how my system enters into or how my input enters into the system right.

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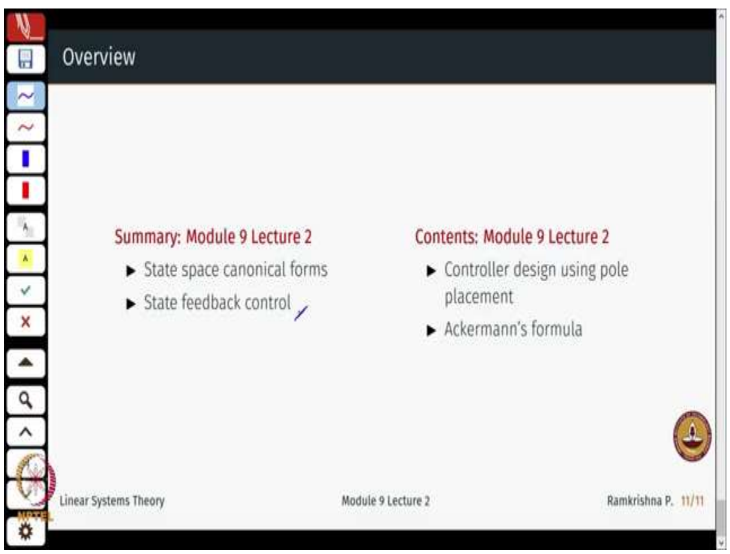

So, just to summarize we have looked at state space canonical forms and how to even formulate the feedback control problem. And in the next lecture, I will explicitly tell you methods of designing these controllers based on at least I have three methods, and also we will derive the famous Ackerman's formula.

Thanks for listening.