Linear Systems Theory
Prof. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module – 09
Lecture – 01
Kalman Decomposition and Minimal Realisation

Hi, everyone. So, welcome to this week 9th lecture on the course on Linear Systems Theory. So, much of the first 8 weeks were focused on of course, gathering tools sfrom math and the remaining part of it was devoted to lots of analysis of systems before even we start a design procedure or go about designing a system for a certain requirements could be on transients, could be on steady state and so on. And that kind of gives you also importance of the analysis part which includes starting from stability till verifying conditions for stability, controllability, observability and so on.

So, now, we will slowly try to build up a case for why these tools were useful or why this rigorous analysis first was useful even before we start solving design problems alright. And then the next 4 weeks will exclusively deal on several design aspects of linear systems, starting from a controller design to an observer design, optimal control or a very basics of optimal control. And towards the end we will look at some of the computational aspects of the analysis tools that we had learnt earlier in the course, right.

So, just before we go into design process we will just do the last part of the analysis which kind of turns out to be quite useful and also elegant in its representation, ok. So, we start with what is called the Kalman decomposition. So, the question that we will ask today is, in week 7 we had talked about a controllable decomposition and defined for ourselves the notion of stabilizability, and the system is good to stabilize if and only if the uncontrollable part is it stable and so on we had a weaker version of observability called the called the condition on detectability and we also had a process of how we go about doing the observable decomposition of the system into its observable and an observable part.

And we kind of viewed these two as two independent features of systems which in practice may not be the case all the time. We may have a system which is both uncontrollable and an observable at the same time, right.

So, we will today look at methods or how to arrive for this decompose models when I have both the system losing controllability and the system also losing observability, what is the appropriate transformation that will take me from take me to take me from the original system to the system which is decompose explicitly in terms of its uncontrollable, unobservable modes and so on.

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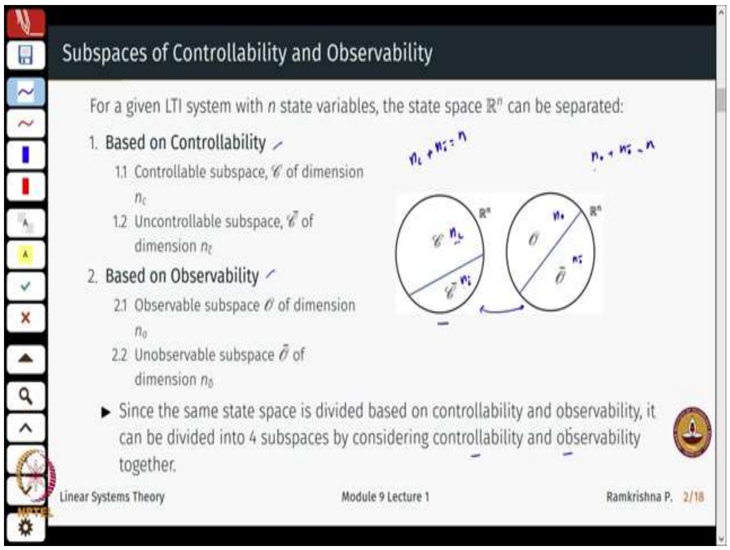

So, there could be 4 possibilities that that could exist. So, if I just look at the standard controllability notion, I could decompose my system into the controllable part and the uncontrollable part. I will call this $n_{\bar{c}}$ in such a way that n_c +$n_{\bar{c}}$ was equal to n. So, I am just looking at the dimension of the controllable and the uncontrollable part, similarly with the observable and the unobservable part, ok.

Again, in such a way that the observable, the total dimension of the observable space plus the unobservable subspace is the dimension of the state space, right. So, that is what we did separate analysis both for controllability and observability, ok. So, now, a system can have both the properties together, right. So, we can now decompose or divide the system based on it is both of its controllability and observability properties by control by considering controllability and observability together, ok.

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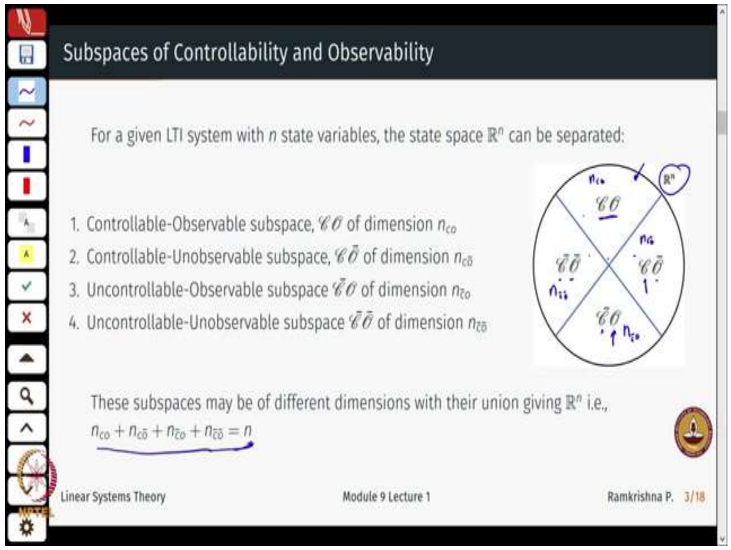

How that looks like? So, it should be kind of pretty intuitive to arrive at this picture, right. So, we could have a system which has both modes which are controllable and an observable, like a part of the system could be both controllable and observable. A part of the system just could be controllable, but not observable, similarly a part of the system could be observable, but not controllable. And there could be a part of the system which is neither controllable nor observable. And we say let us decompose the system into its 4 of this kind of subspaces where the part which is both controllable and observable like all eight of dimension n co.

Here which is neither which is controllable, but not observable I call it $n_{\overline{co}}$ and the part which is observable, but not controllable I call it n c bar o, and this is a part which is neither controllable nor observable, ok. So, this would be its respective dimensions. Now, these 4 dimensions together will satisfy this relation that the all of these sub space will constitute R^n and a nice pictorial view of this decomposition, ok.

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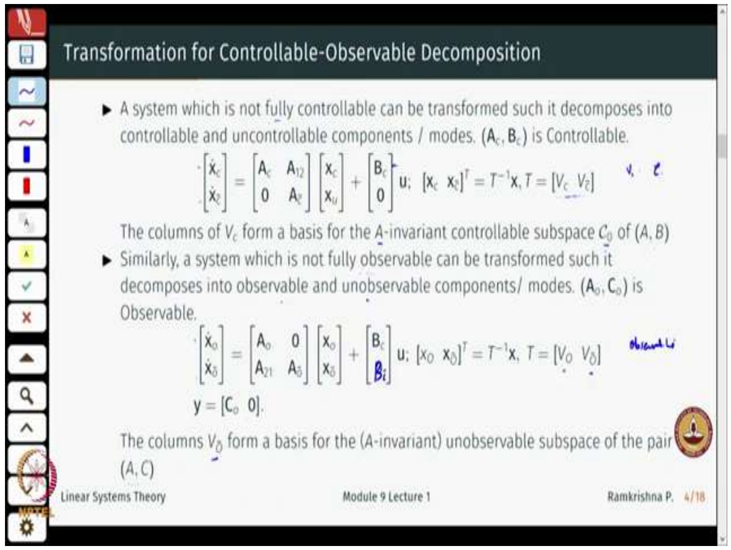

So, before we come to the overall a decomposition let us individually revisit the controllable and the observable decomposition techniques. So, first what we what we know is that a system which is not fully controllable can be transformed into its controllable mode, I call it x_c and the uncontrollable modes I call them $x_{\bar{c}}$, right, in such a way that the $\dot{x_c}$, the B_c influences only the first k components and then the remaining n - k are uncontrollable, right. And then this will come via some kind of a for transformation which looks something like this where V_c forms a basis for the A invariant controllable subspace C_0 of A B, right, so this V_c was constructed through the independent the k independent columns of the controllability matrix C, ok.

Similarly, in the unobservable case we know that a system which is not fully observable can be transformed such that it decomposes the observable and unobservable components in such a way, right. So, these are my first say k observable modes, these are my unobservable modes and see, this need not be 0, but there can be something here like B_c bar and y will be decomposed into its observable and the un observable form, an observable and the unobservable components.

Again, the transformation will be a V_0 and $V_{\bar{o}}$, where the columns of V naught bar form a basis for the A invariant, unobservable subspace of the form of the transformation A C and this V_0 comes from again the observability matrix, ok. So, now, we will see how to combine these two and get a decomposition for a system which can be both uncontrollable in some modes and unobservable in some modes, ok.

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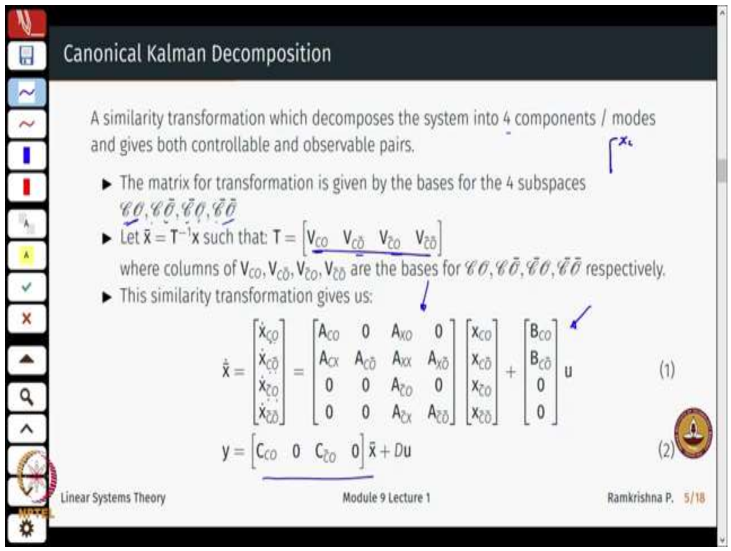

So, this Canonical Kalman Decomposition would give us a similarity transformation which decomposes the system into it is its 4 components or modes and gives both controllable and observable pairs. So, we know what is the transformation that gives us a controllable decomposition that is if I say x_c, ok. Let me write this all over here again.

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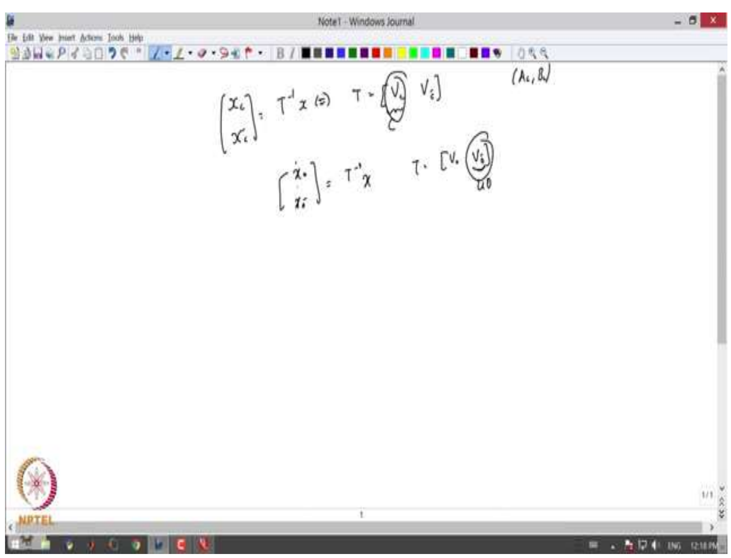

So, I have $\begin{bmatrix} x_c \\ x_{\bar{c}} \end{bmatrix} = T^{-1}$ x which was such that this T was a result of $[V_c\ V_{\bar{c}}]$. So, I could just split it into, so this come comes from the controllable subspace with the pair A_c, B_c being controllable, and similarly what we saw was with the observability get I have x_0, $x_{\bar{o}}$ is in such a way that, so this decomposition comes why are again some T^{-1}x sorry, T^{-1}x where T was $[V_0\ V_{\bar{0}}]$, ok. So, this columns found the subspace or this columns from a basis for the unobservable subspace and these columns from the basis for the controllable subspace, as we saw just in the previous slide, ok.

So, now, can I write this or can I find a transformation which combines both of these things, right, so in such a way that I have this four subspaces controllable, observable, controllable but not observable, observable but not controllable and either controllable nor observable. So, can I find a basis for each of these 4 subspaces? So, V co denoting the basis for the controllable and observable subspace and so on, right. Can I find a transformation which then transforms my system to something like this? Ok.

So, I have the decomposed system into its parts which are both controllable observable, neither controllable not observable or either of them, ok. So, this is how this is what we will derive today or how to find a basis which will transform a system to a form which looks like this, ok.

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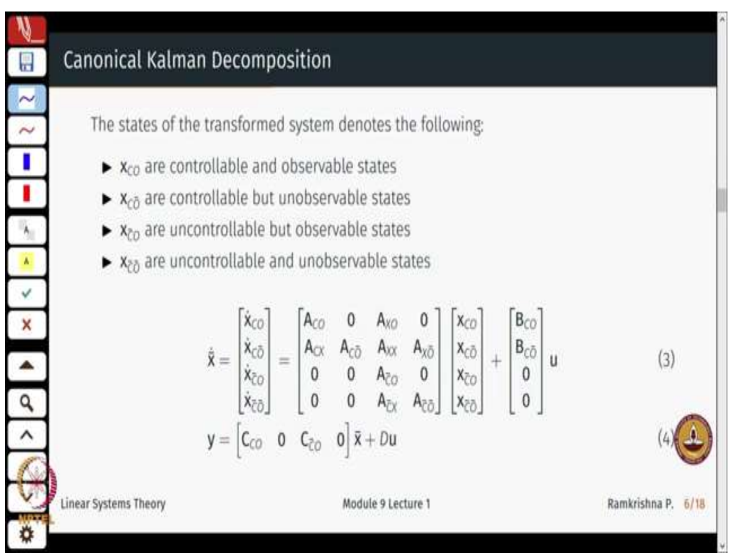

So, again x co are the controllable and observable states and so on, ok.

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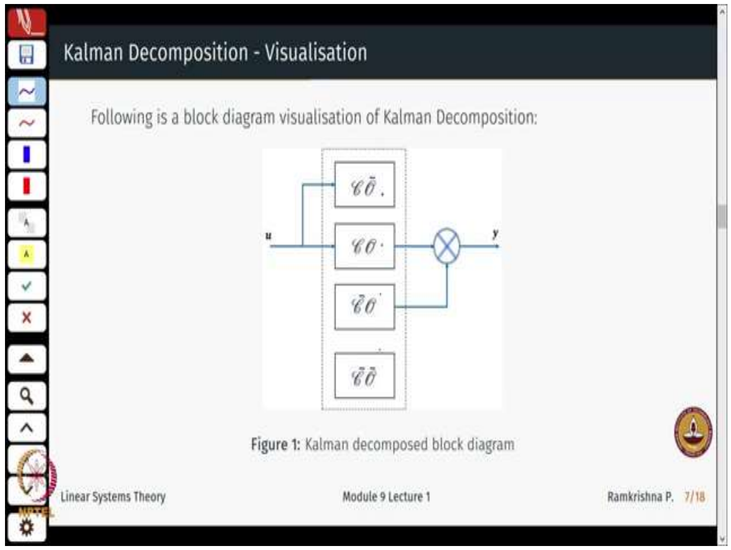

Now, I can just draw a block diagram of this which looks like this, right. So, I have all the modes here sorting from controllable not observable till neither controllable, nor observable, ok.

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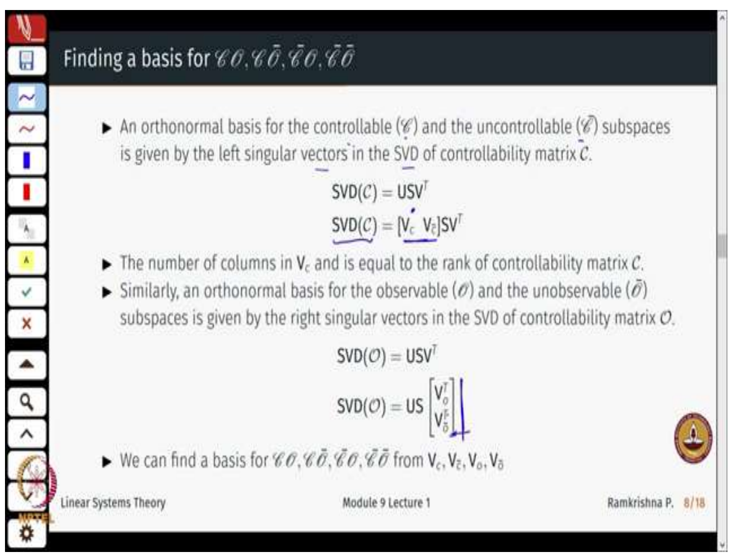

So, the other way of looking at how to arrive at this V_c and $V_{\bar{o}}$ is also via the singular value decomposition. So, I take the controllable matrix and so I can I kind of try to find ortho normal basis for the controllable and the uncontrollable subspace, and this is given by the left singular vectors of in the SVD of the of the controllability matrix. So, the singular value decomposition of the C would precisely have these terms here like V_c and $V_{\bar{c}}$ which we derived earlier. And similarly for the observability matrix here I will have ${V_o}^T$ and ${V_{\bar{o}}}^T$ this was the basis for the invariant controllable subspace, this was the basis for the a invariant unobservable subspace, ok. Now, ok; so, this is another way of looking at how to how to derive V_c and $V_{\bar{o}}$ all, right, ok. It is the same process, you could just verify it quickly, ok.

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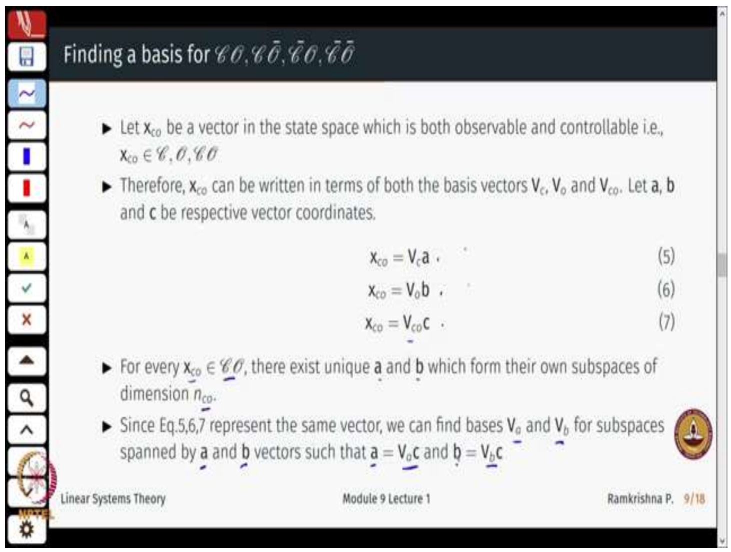

Now, how do we start for this? Right. So, let us also write this down as we go by, ok.

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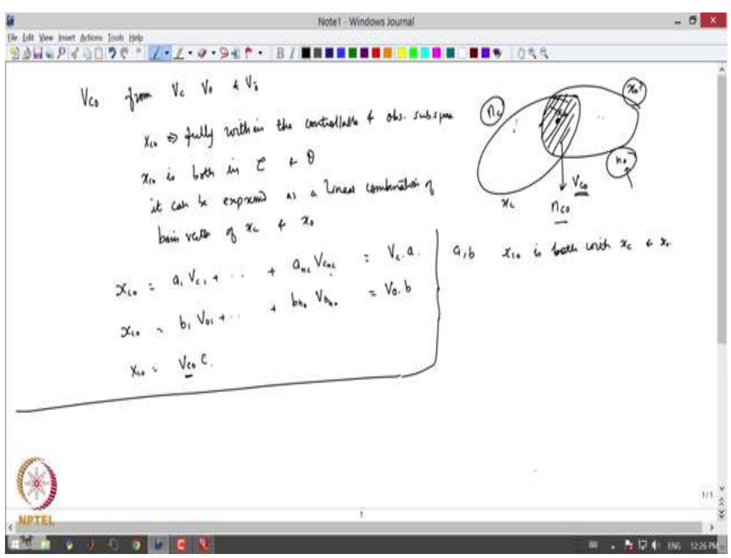

So, let me just draw a little picture here, right. So, which will say, I have this as my controllable subspace of say some dimension n_c and this is an observable subspace of some dimension n_o, and there is some intersection here which will say give me the subspace which is both controllable and observable, ok. Now, the aim is to find these 4 basis here. This V_{co}, $V_{c\bar{o}}$, $V_{\bar{c}o}$ and $V_{\overline{co}}$, ok. So, now let us say I have, let us start with say that I want to find V c o from, I know these two things I know V_c I also know V_r from this the slides here, right or even the previous ones, and possibly also maybe $V_{\bar{o}}$, ok.

Now, for some vector, right, so some vector which comes from the, so this vector x_{co} is from that is this is fully within the controllable and observable subspaces, ok. Now, this x_{co}, so let us say, there is this, this has a point and call this x_{co}, ok. Now, this x_{co} is both in the controllable subspace and also in the observable subspace and therefore, it can be expressed as, so x_{co} is both in the controllable subspace and the observable subspace, ok.

I am just I am using the notation a bit, but it is good for understanding. And because it is in both observable subspace it can be expressed as a linear combination of let us say this I call this as x_c and I call this as x_o of the basis vectors of the controllable subspace x_c and the observable subspace x_o which means that this x_{co} can be written as; so, we will we just try to derive for these expressions.

So, this x_{co} is say some $a_1 V_{c1}$ till a_{nc} V_{cnc}, this can be written as V_c a, ok. So, this is again you see that this x_{co}. So, x_c is of dimension n_c and therefore, I will have n_c basis vectors. This x c naught can also be written in terms of this space with its corresponding basis as say some $b_1 V_o$, right; V_{o1} + $b_{no} V_{ono}$ like a this is V_c, sorry V_o b, right. This is the dimension of the observable subspace, ok.

And this is also in this subspace, right with I can also write this as some basis of V_{co} which is the subspace which is both controllable and observable plus some time some vector c, ok. So, this vectors a and b are such that x_{co} is both within the controllable and observable subspace, ok. Now, what happens? Ok. How do we write? Now, given these two vectors how can I find our basis for V_{co} like for this intersection space here? Ok.

So, where we are now, right. So, for every x_{co} in the controllable and observable subspace there will excess a unique a and b which form their own subspaces of dimension n_{co}. So, this dimension is the dimension of this subspace which is both controllable and observable and these are all the same vectors x_{co} which is V_ca, V_ob and V_{c0}c these are all the same vectors.

Now, can we find again the basis for now, V_{co} that is what we will do right now, ok.

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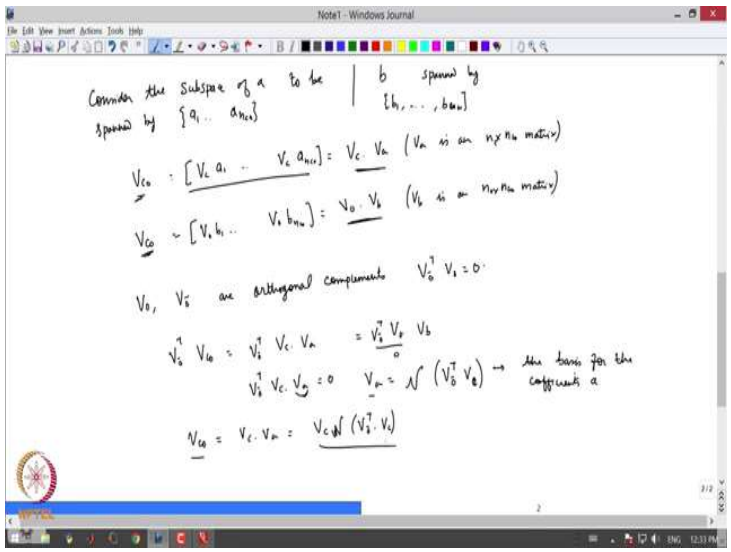

So, let me say that, ok. So, let us say the go to a new page say consider the subspace of a to be spanned by some vectors say a_1 till a_{nco}, and similarly for b to be spanned by b_1 till b_{nco}.

Now, the basis for V_{co} given the sets what do I have now; I have $V_c a_1$ $V_c a_{nco}$, this is $V_c V_a$. Similarly, V_{co} can also be written as $V_o b_1$, $V_o b_{nco}$, this is $V_o V_b$, ok. Now, what is this V_a here? V_a is an n x n_{co} matrix, and similarly n_c x n_{co}. Similarly, V_b is and n_o x n_{co} matrix, ok.

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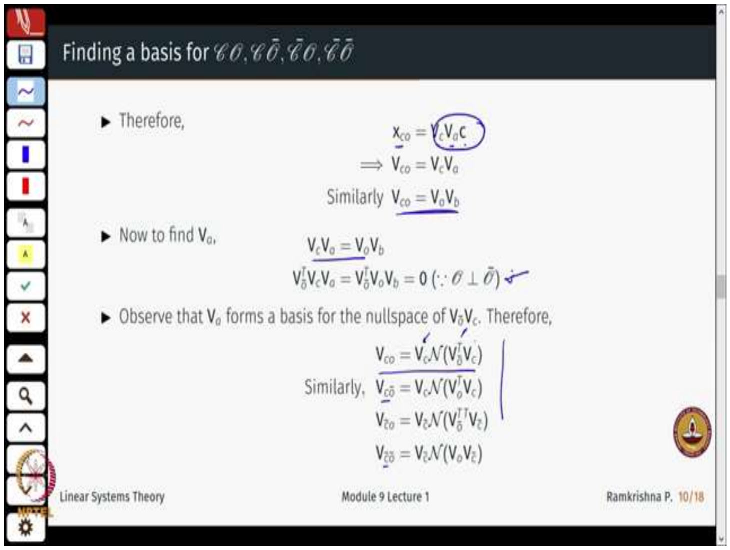

Now, what do I have now so far? So, this; so, the idea was, since 5, 6 and 7 to represent the same vector we can find a basis V_a and V_b for subspace is spanned by a and b vectors, such that a is V_ac and b is some V_bc, ok. Now, that is how this basis is derived. So, this means that now I can write these vectors, let us go here that x_{c0} as V_c, V_ac or this also comes from this thing here, right. So, V_{co} is $V_c V_a$. So, I just do this here.

So, therefore, x_{co} would be V_c, V_ac which essentially means that now compare this expression and sorry, and this expression which would kind of imply that there is something like this holds true alright which is also what we derived over here V_{c0} is V_ca and V_{c0} is also $V_o V_b$, right, ok.

Now, the next steps, right. So, what do we do? So, I have a this relation, and I have this relation. So, let us quickly do something else. So, if I know basis for V_0 or and we know $V_{\bar{o}}$ we know that these are orthogonal complements which means $V_{\bar{o}}{}^T V_o = 0$, ok. Now, what do I know? I know that V_{c0} is $V_c V_a$, this is also equal to V_oV b, ok.

Now, what do I do is just pre multiply this by this one, transpose, ok. Now, this is 0 therefore, $V_{\bar{o}}{}^T V_c V_a = 0$, ok. Now, which means that this V_a is in the null space of $V_o{}^T V_c$, from here, ok. Now, what is this V_a? This V_a is the basis for the coefficient say from here, ok. And therefore, now if I go one step further what is V_{co}? V_{co} was $V_c V_a$ or this is equal to V_c. Now, where does V_a come from? V_a is just from the null space of $V_o{}^T V_c$, right. So, this is the first one the relation that we derive, and this, the steps are written also here. So, this is what we derived, V_{c0} is V_cN($V_o{}^T V_c$). Now, I know each of this set I know V_c, I know this one and I also know this one from the from the decomposition which comes from somewhere over here, ok.

Similar, steps I can derive now $V_{c\bar{o}}$, $V_{\bar{c}o}$ and $V_{\overline{co}}$ very similar steps, ok. And then I just use this nice property here, right. And therefore, now with this steps I now know what is this T that will take me from a given system to its this decomposes form into explicitly into its modes which are both controllable and observable controllable, but not observable, but not controllable and either controllable nor observable, ok.

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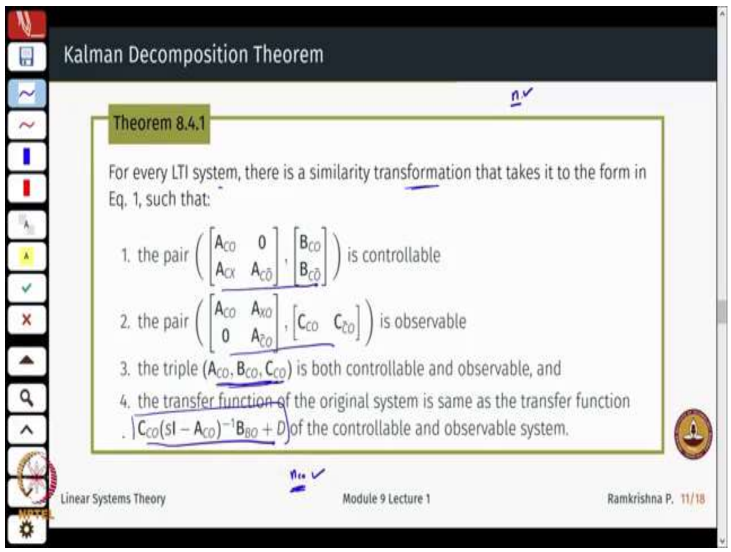

So, now, so what we have derived now so far is the following. Like for every LTI system there is a similarity transformation which we just derived that takes it to the following form in such a way that this pair is controllable and this pair is observable; you see from the expression, you see like this is the controllable.

So, you have controls x_c entering here and here and no controls going here. So, these pairs is; these two pairs are controllable then. So, this is a controllable pair, this is an observable pair, the triple A, B, C, with A_{co}, B_{co}, C_{co} is both controllable and observable and the transfer function of the original system is the same as the transfer function of this one, right of both the controllable and observable system.

So, when we did the controllable decomposition we proved that the controllable or the transfer function is the transfer function of only the controllable part. Similarly, if I do it for the observable decomposition I get the relation that the transfer function of the system is only a transfer function of its observable part. Now, if I combine these two I get this result that the transfer function of the system is the transfer function of both the controllable and the observable system that is what we read from this triple here, ok. So, that is that is kind of kind of kind of nice.

So, this leads us to now to define a concept of what is a minimal realization because if I look at this system here well the system is of dimension n, or if would expect that there will be n poles in the system whereas, if I derive the transfer function it will just have poles which just have this number, right from both the controllable and the a no the controllable part and the observable part. Well, this is also a state space representation of the system. Whereas, I can go back from this transformation and still derive the state space representation of the system and both are same, right. Even though here you define n states here you will define only n_{co} states, ok.

Now, we will see what is how to actually get to a system which has only you know which only shows me the controllable and the observable form and just discards the remaining parts, ok.

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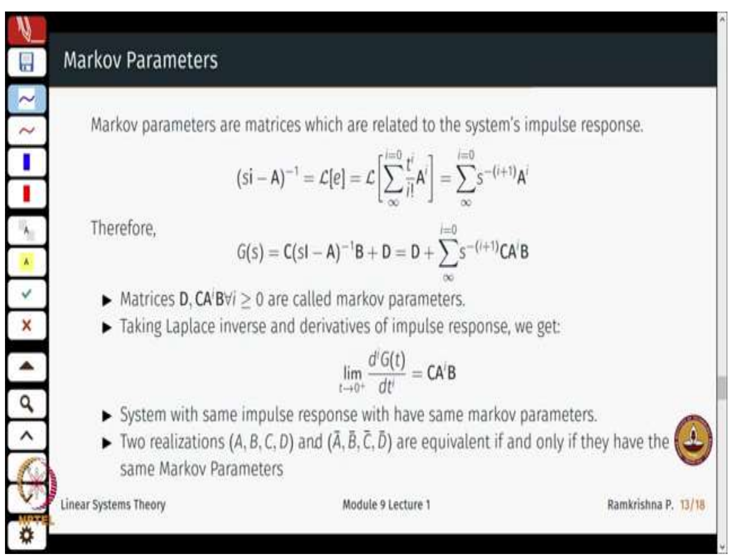

So, now what is the realization, and I guess we all know this derivation. So, given A, B, C, D this is called a realization of a transfer function, if I can do this. So, this is the relation between the state space and the transfer function model, ok.

Now, given the state space model I can construct the transfer function. The converse problem is then called the realization problem. Can I go from, this is a simple computation or can I go from here till here, ok? Now, the size n of the vector x is called the order of realization, right. So, here I started with a system which was like this, right, so this in this realization. So, this $\bar{x}$ was also of dimension n, whereas well if I look at the transfer function it might just show me n_{co} number of poles or that will that is a minimal order of the system that is what we will go we will derive now, ok.

So, now, well this is not surprising to know there are many possible state space realization for a give a transfer function, and they can be offer different orders. I will towards the end do some examples on this and see how it goes, ok.

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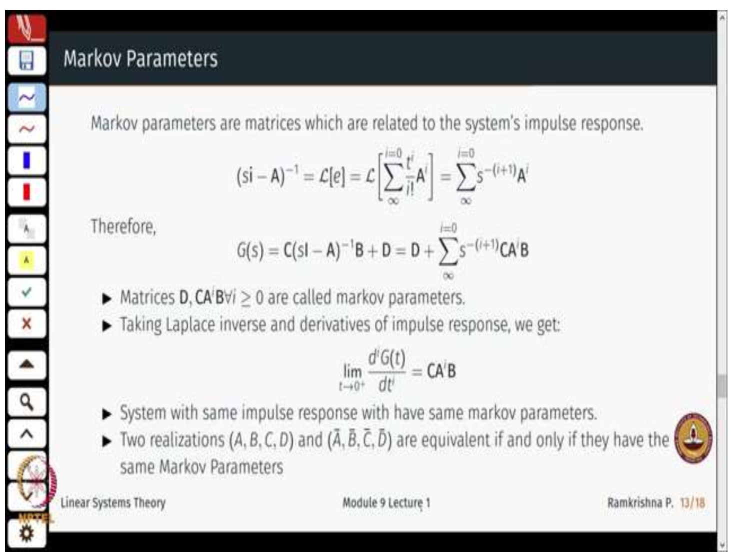

So, before this we needs to define some small concepts called the Markov parameters, ok.

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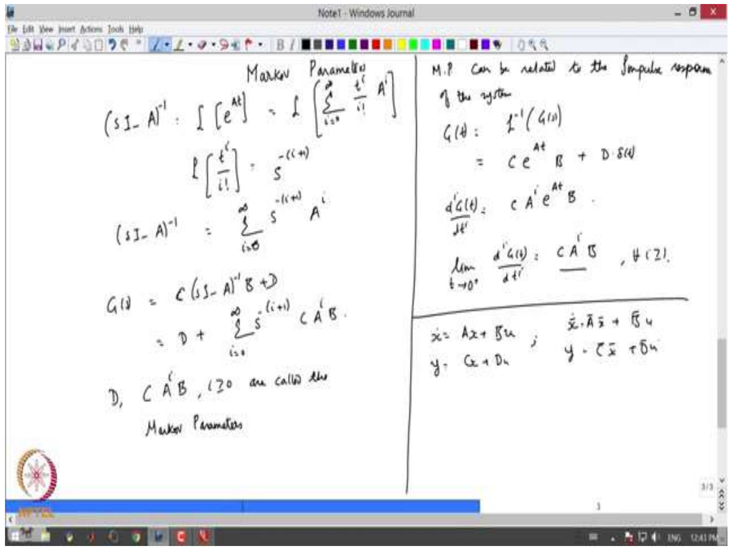

So, what are this Markov parameters? Ok. So, what I know from the relation between state space and transfer function and the tools from Laplace transforms is the following, right. This I think I know this one. Now, what is e^{At}? e^{At} usually shows up as some infinite series, ok. Now, Laplace of $\frac{t\,i}{i!}$ is $s^{-(i+1)}$, I think it should be able to derive this and therefore, $(sI - A)^{-1}$ is nothing, but the summation of now, $\sum_{i=0}^{\infty} s^{-(i+1)} A^i$, ok. And therefore, the transfer function G of s which was usually computed in the following form that here C$(sI - A)^{-1}$B + D plus, then I substitute for $(sI - A)^{-1}$ I get $\sum_{i=0}^{\infty} s^{-(i+1)} C A^i\, B$, ok. So, these two matrices D and C A^iB for i ≥ 0 are called the Markov parameters, ok.

And this Markov parameters are these parameters; Markov parameters can be related to the impulse response of the system impulse the responses also usually referred to as the natural response. So, therefore, G(t) would be the inverse of G(s) and I do all the steps and what I get is the following C e^{At} B plus D with the impulse function δ(t), ok.

Now, just take some derivative, so $\frac{d^i G}{dt^i}$ = is C$A^i e^{At}$B and, the D will no longer be here, ok. From which we obtain the following relationship between the impulse response and the Markov parameters. So, the limit of t, ok, sort of this $\frac{d^i G}{dt^i}$ is CA^iB. So, e^{At} is the identities for all i ≥ 1, ok. So, these are the Markov parameters and this is a little procedure of how they are derived, ok.

So, something which I will not prove, but just state as a result is the following say I have a system $\dot{x}$ = Ax + Bu, y = Cx + Du and another realization which looks like this $\dot{\bar{x}}$ b= $\bar{A}\bar{x}$ +$\bar{B}\bar{u}$, the u will be the same. Now, y = $\bar{C}\bar{x}$ + $\bar{D}\bar{u}$, ok. These two realizations denoted by A, B, C, D and $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ are equivalent if and only if they have the same Markov parameters, ok. We will, I will not do the proof of this, but we will use this relation to prove results relating to the minimal realization, ok.

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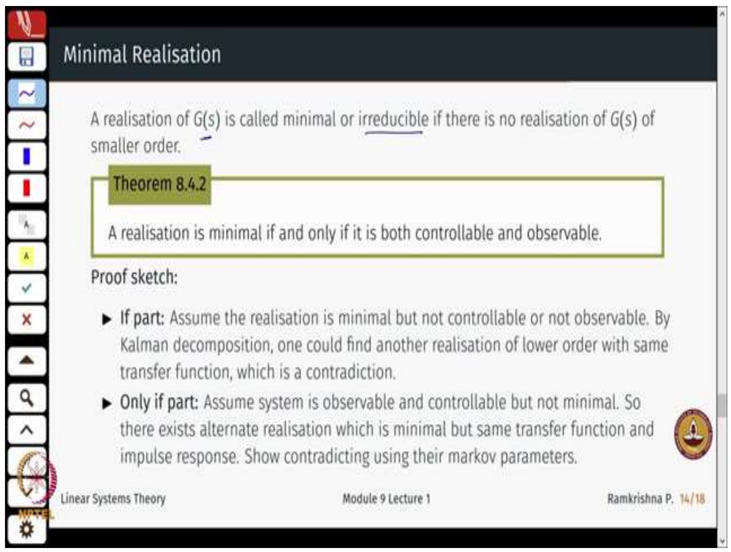

So, what does the minimal realization now mean? Realization G(s) of a realization of G(s) is called minimal or irreducible if there is no realization of G(s) of smaller order, right or if I go from the now transfer function to the state space it must just have n states, the minimum number of states. They should not be another n' which is less than n, ok

Now, when it is possible? Well, a realization or is minimal if and only if it is both controllable and observable, ok. So, we will do, spend some time just doing the proof of this.

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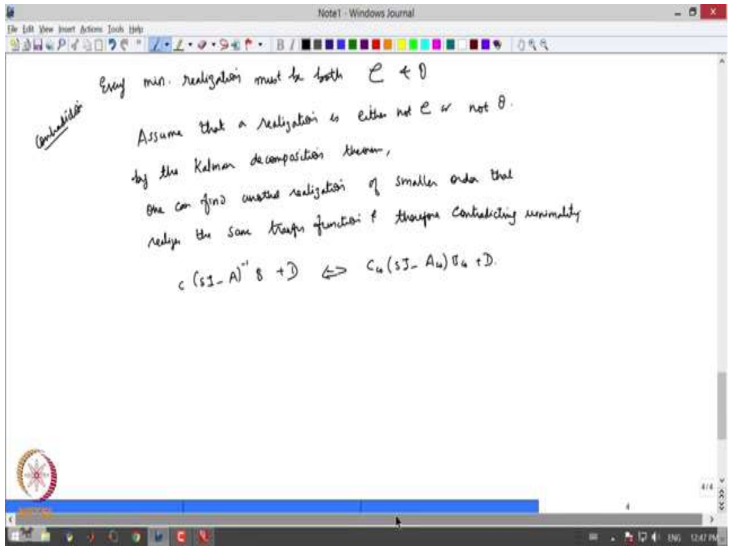

So, what does it say? Ok. First we will prove that every minimal realization must be both controllable and observable, ok. So, we again use the method of contradiction, ok. So, assume that realization is either not controllable or not observable, ok. And therefore, by the Kalman decomposition, ok, we can always find realization of smaller order that realize the same transfer function and therefore, contradicting minimality. Because what does the Kalman decomposition theorem also say is at $C(s I - A)^{-1}B + D$ is the same as C which corresponds only to the controllable and observable form, ok. So, that is kind of easy to check, ok.

So, now the second set that only if part assume, so we do the reverse thing that is, the reverse proof that assume that the system is both controllable and observable, but the realization is not minimal, ok.

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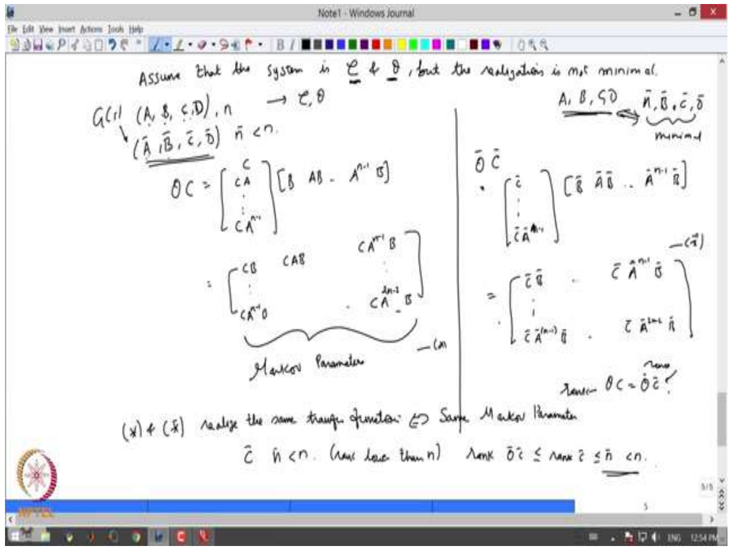

So, what does this say? Ok. So, I assume that when the system is both controllable and observable I can find a realization A, B, C, D that is not minimal which means there exist some other $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ that is actually minimal and then we will show that these two are actually the same, ok. There cannot be a realization which is minimal and then this one, it is actually the realization which we get from this assumption is actually the minimal one, ok.

So, again I have a, right. So, this assume that this is the minimal realization with some dimension n and say this is, assume that, this is not the minimal realization, right that is what we proof by contradiction, right. That the system is controllable and observable, but this realization is not minimal which means there exists another realization $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ which has a realization $\bar{n}$ which is less than n, ok.

Now, I just look at the controllability and observability matrix of this guy and say that O C the observability matrix times the controllability matrix $\begin{bmatrix} C \\ CA \\ CA^{n-1} \end{bmatrix}$[B AB…. A^{n-1}] is CB here, CAB here, CA^{n-1} B and then here I get CA^{n-1}B all the way till CA^{2n-2}B, ok. And essentially or what are in these matrices are the Markov parameters, ok.

Now, look at the other realization, right which is which we claim is that this there is some $\bar{n}$ bar which is less than n which gives me the minimal realization. So, this will be again $\bar{C}$

till $\bar{C}\bar{A}^{n-1}$ I have $\bar{B}$, $A\bar{B}$, $\bar{A}^{n-1}\bar{B}$, which will be the following. Again, I have the Markov parameters, now in terms of $\bar{C}$, $\bar{B}$ and so on. So, it will $\bar{C}\bar{A}^{2n-2}\bar{B}$, ok.

Now, let me call this say star and star with a bar, ok. Now, this star and the star with a bar realize the same transfer functions, ok. Now, if they realize the same transfer functions they must have the same Markov parameters as say that. So, I assume that there is a realization A, B, C, D of one transfer function G(s), and the same transfer function also real also has some other realization $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$, ok. So, these two realizations are come from the same transfer functions if and only if these are equivalent, right just for the result we stated earlier, ok.

Now, when they have same Markov parameters what we have is O C is $\bar{O}\bar{C}$, ok. Now, let us say, that say $\bar{C}$ has only $\bar{n}$ columns which is less than n and therefore, which is rank is lower than n and therefore, rank of $\bar{O}\bar{C}$ is less than or equal to rank of $\bar{C}$ this is less than or equal to rank of less than or equal to $\bar{n}$. This is less than or equal to n which contradicts the fact that if these two are similar the rank here should be the rank here, ok.

And therefore, we conclude that, if I whenever I assume that there is another minimal realization well that actually is not true that will be a contradiction and therefore, we can conclude from both ways that a realization is minimal if and only if it is both controllable and observable.

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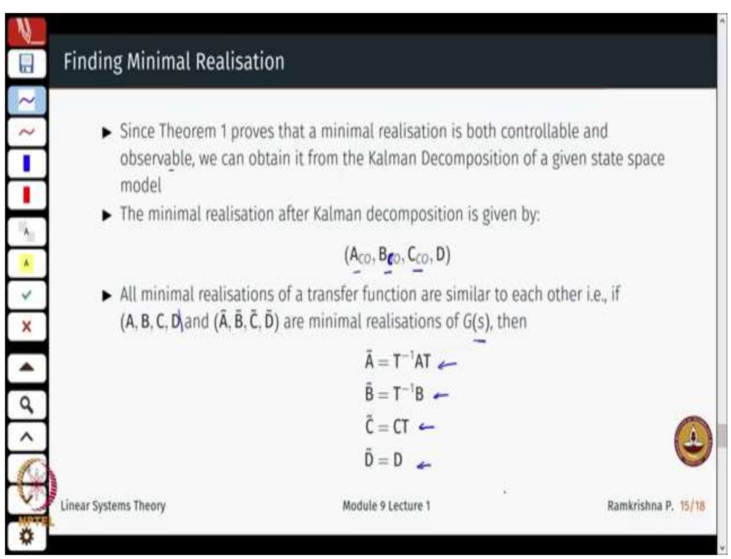

We now prove that a minimal realization is both controllable and observable we can therefore, obtain it from the Kalman decomposition. So, I have the Kalman decomposition matrices A_{co} both controllable observable both B also belonging to it should be C and so, the minimal realizations of all of a transfer functions are similar if, right. So, this A, B, C, D and $\tilde{A}$, $\tilde{B}$ etc are minimal realizations of G(s) then there will always exist a similarity transformation like this, ok. So, two minimal realizations of transfer functions are actually similar to each other. In the same way as similarity relations between systems which were transform maybe to the diagonal form or to the controllable form and so on, ok.

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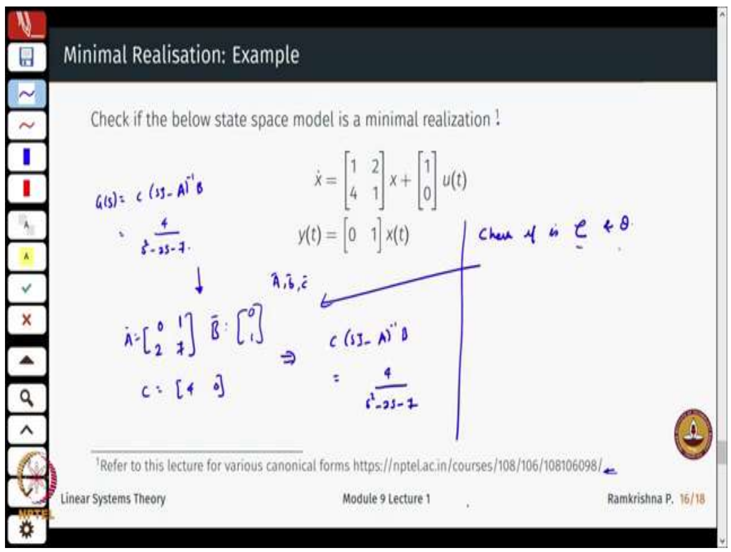

So, we will do a bit of an example and, ok; so, just to note the values a bit of the control, controller canonical form or the controllable canonical form here, that I just would like to refer you to this lecture nodes or the videos of our previous course on control engineering. So, I will not spend time doing the controllable canonical form all over again. But we will take a, we will use these examples to do a bit of illustration of the of the minimal realization here, ok. So, I have, so given this A, B, C, D sorry, this I will have G of s, right which comes from C$(sI - A)^{-1}$B. So, I can just compute this as $\frac{4}{s^2-2s-7}$ and this is an easy step for to compute, ok.

Now, I write this system equivalently in the controllable canonical form where let us say in the new form I have a pair $\bar{A}$, $\bar{B}$ and $\bar{C}$ which look like this. So, $\bar{A}$ will be, so $\bar{A}$ in the controllable canonical form would be $\begin{bmatrix} 0 & 1 \\ 2 & 7 \end{bmatrix}$ the new B would be $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and the C would be

[4 0], ok. So, again I do the transfer function from this I convert this to the transfer function where again $C(sI - A)^{-1}B$ and I get $\frac{4}{s^2-2s-7}$, right and therefore, I can write like verify this relation over here, right,. So, that is one simple example here, right, ok.

Now, what we can just to also add one step is check if this minimal realization here is both controllable and observable, right and you will actually find that this realization is both controllable and observable and therefore, the system which we get start from here is actually the minimal realization, ok. I will skip the skip the steps of finding computing the controllable, controllability and the observability matrix. It should be a very simple exercise to check, ok.

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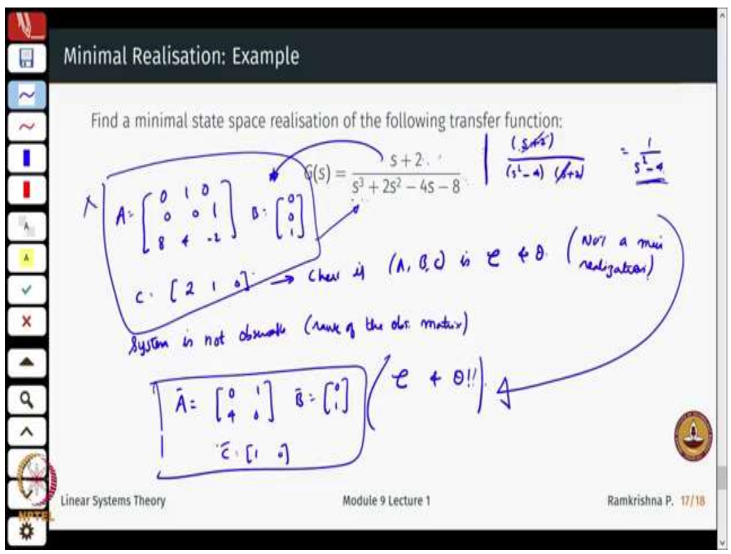

Now, we will go from the transfers function, right. Can I find a minimal realization for the following a transfer function? Ok. So, in the controllable canonical form if I write down the state space realization that will be something like this $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & 4 & -2 \end{bmatrix}$ a with B being $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and similarly, C being [2 1 0], ok.

Now, from if I go again back to the state from the state space transfer function I can realize this, but check if A, B, C is both controllable and observable, ok. And you can easily check that the system is not observable, you can just check from the rank of the observability matrix, ok. And therefore, this is not a minimal realization, ok.

Now, check this transfer function, right. So, we can also write this in the following way $\frac{s+2}{(s^2-4)(s+2)}$ is $\frac{1}{s^2-4}$, ok. And then I can now write down the controller a canonical form, the controllable canonical form and the observable canonical form where I have a new a let me call this $\bar{A}$ as say $\begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}$, $\bar{B}$ will be $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\bar{C}$ will be $[1\ 0]$ and this you can say that it is both controllable and observable. And therefore, this realization is actually a minimal realization and not this one, ok.

One thing we can you can immediately observe is whenever the transfer function you know if you have can spot the pole zero cancellation then, you if you go from here till here this may not necessarily be minimal realization because you kind of miss the pole zero cancellation here. So, whenever, you just factor out this terms here and take into account any cancellations then you will arrive at a minimal at a minimal realization of the system. And therefore, we also talked about the relation between the system losing controllability and or observability via pole zero cancellations and this is a little you know we can also look at it from that point of view of to get some intuitive relation of things between pole zero cancellations and lots of controllability and observability.

Similarly, you can also go from here till start with this realization and of course, it is easy to check that it is not controllable and observable, go back here, do the pole zero cancellation and then come back here, ok.

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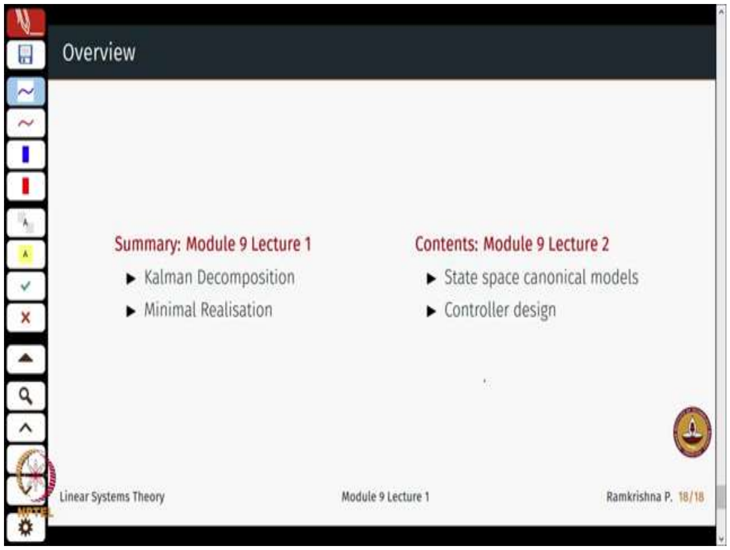

So, that is, that concludes todays lecture where we talked about decomposing the system into it explicit the forms are starting from all from the space which as this was controllable and observable till the space which is neither controllable and nor neither controllable and nor observable, and from that we derive what is the concept of a minimal realization of the system, ok.

So, next time we will start with our results on controller design, essentially this would mean can I find a state feedback to achieve certain system performances. So, we will slowly also relate to the performance specifications what we did for second order systems in an earlier control course on the peck over showed the damping and so on. So, we will begin a bit of motivation from there and then go about solving more complex, complex, design problems. Now, so that is coming up in the next lecture.

Thanks for listening.