Linear Systems Theory
Prof. Ramkrishna Pasumarthy
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 01
Lecture - 02
System Models

Hello everybody, welcome to this 2nd lecture on Linear Systems Theory. The 1st weeks of lectures we will just learn about writing down or developing models for a given system. So, last time we saw a couple of models from circuits till simple mechanical systems two systems which were called the predator prey models, which essentially were a set of a non-linear equations in the discrete time. So, we continue on those things and do a couple more examples related to non-linear systems ok.

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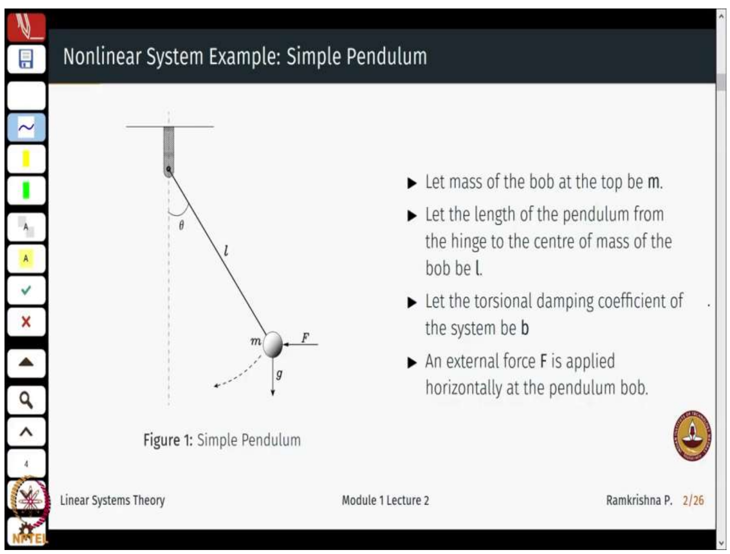

So, one of the simplest examples which we encounter even while we do high school physics is the example of simple pendulum. So, we start with a setting over here where I have a simple pendulum it is like a maybe a point mass of mass m, length is at a distance of l from a pivot point here and so, you have gravitational forces, you have damping coefficient over here and so on ok.

So, assume that this is a external perturbation to the system. So, the dynamics of the system are well simply given by equations of this form. So, my variables here are the angular displacement and the angular velocity. It was x the linear displacement $\dot{x}$ the linear velocity in the case of a mass spring the damper system. So, here I am just talking of a rotational motion, so it would be θ, the angular displacement and $\dot{\theta}$ the angular velocity ok.

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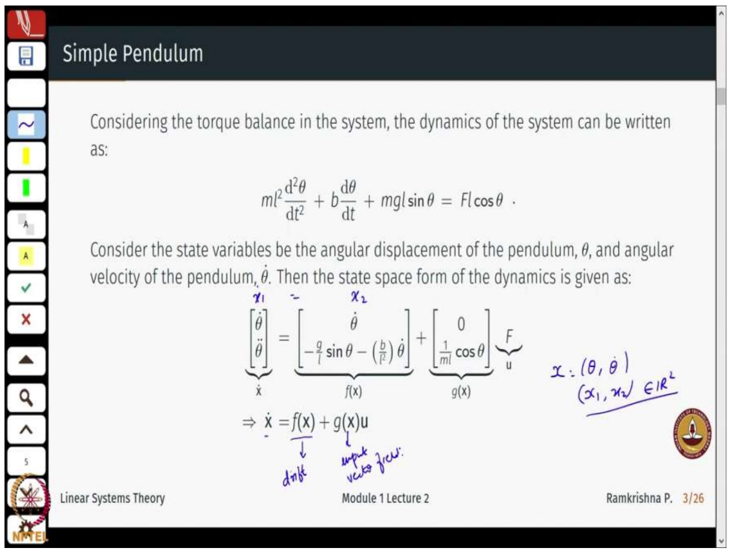

I will not derive the equations for this which we like I presume that you would know this these equations and they are like a pretty straightforward to derive ok. So, this writing down in the state space form; so, my states here would be in two dimensions. So, x would be θ and $\dot{\theta}$ right so, this x_1 1 and x_2, if am equal and this; these are like belongs to R^2 ok. We slowly familiarize ourselves with notations of this kind. So, what I have here is $\dot{\theta}$ and $\dot{\theta}$ the same if I write this will be $\dot{x_1} = x_2$, if I map θ to x_1, $\dot{\theta}$ to x_2 and so on ok.

So, now in general, so I can write this system of equations as well the evolution of states $\dot{x}$ is some non-linear function here, this is called the drift term usually non-linear dynamical systems language, this is called the drift and this is the input vector field. So, what do we know about this system? Right.

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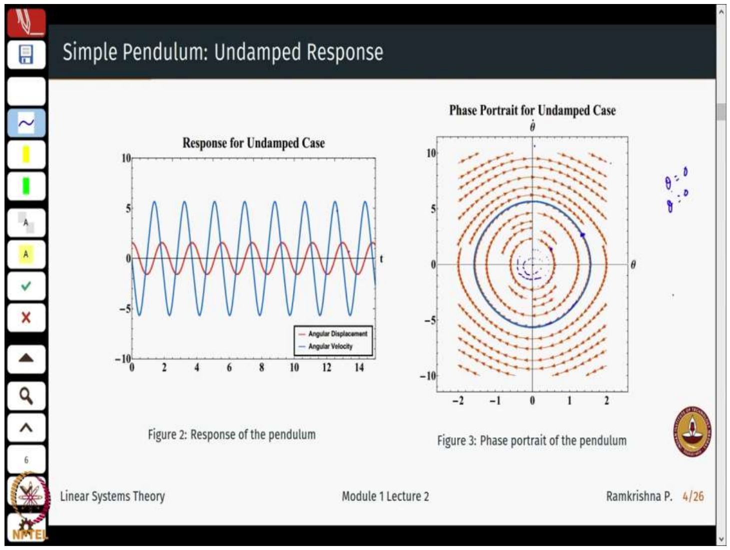

So, if I were to draw the phase plane. So, phase plane essentially is that I am just checking how these two x_1 and x_2 evolve with each other. So, how do I have a plot between theta and theta dot for any usual condition, right. So, this will give me the trajectory wave starting from initial condition here or here or here and so, on. So, I just; so, if you do not know phase space do not worry I just put some lectures online with the relevant code.

So, that you could run it run it by yourself, it is kind of straightforward there is nothing really that you would not know, it just some consequence of the calculus which we already know ok. So, let us start with physics here right. So, if I subject these to some excel perturbation and assuming there is no friction what I would see is that, will the pendulum will be oscillating forever right. So, that is captured if I look at the evolution of the θ and $\dot{\theta}$ then just be periodic right.

So, it just be oscillating forever so, in; so, with the time period defined by the parameters of the system ok. If I just look at plot of θ vs $\dot{\theta}$, you see they are just like forming close orbits I see if I say what happens if our initial condition is here, it will just go around going on or be defined determined by this line.

So, if it is here it just goes here, now around this blue line over here. What if the initial condition is such that $\theta = 0$, $\dot{\theta} = 0$? It will just be here forever. It will just be at this point, the side perturbation here then it will just have a close orbit here. If I perturbate slightly further then it will be something like this ok. There is a reason why I am doing this, I will just let you know in a bit. So, am just putting θ vs $\dot{\theta}$ in case of what we call as an undamped system, I just have my phase portrait just following some close orbits ok.

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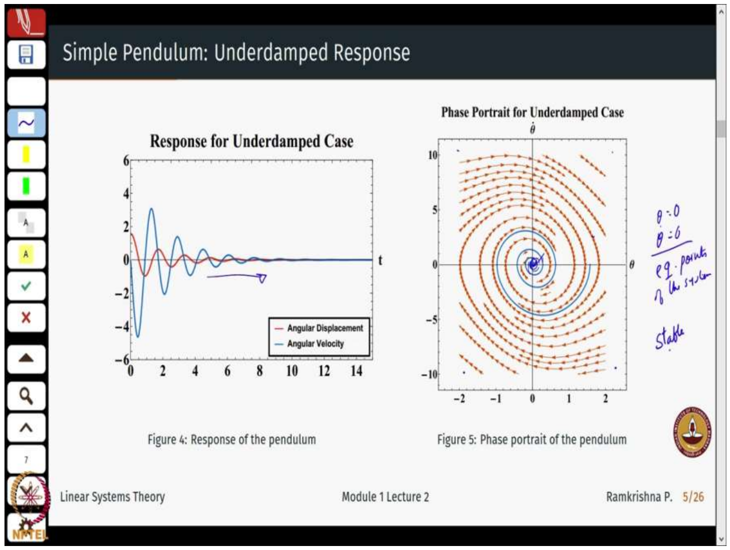

If the system is under damped I know that you know I start from an initial conditions and my oscillations will die out as time evolves and then I will just go to a steady state of θ = 0 and $\dot{\theta}$ = 0. So, assuming I start here, say an initial condition here, I will slowly spiral to the origin here. Again if I started the origin again with θ is 0 and $\dot{\theta}$ is say 0 I always remain then.

What happens if I perturbate slightly from here till this point? We will again all it will I will again spiral back; which actually means that if you use the pendulum slight. So, slight perturbation in the effect of damping it will just oscillate and the oscillations will die down with time in terms of some dissipation right of in terms of friction for example. So, for all initial conditions starting from anywhere in this phase plane, I will just you know go here to the equilibrium point.

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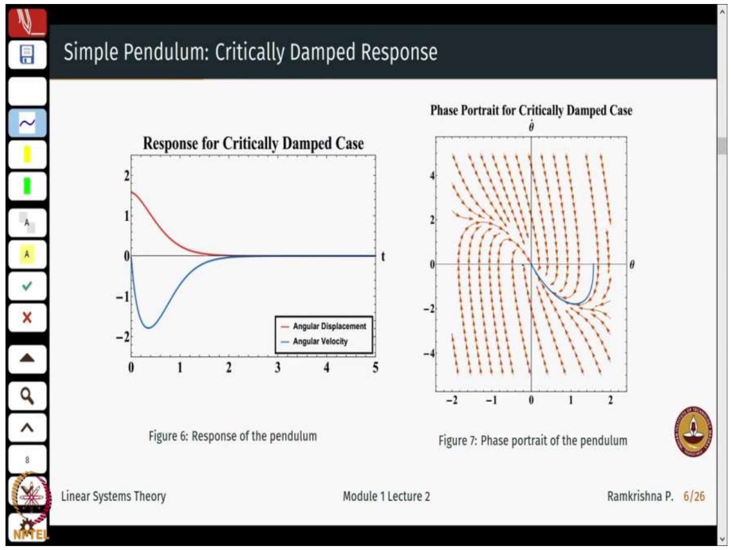

Similarly, if I have critically damped case, again I start; if I said starting at the original always be here and if I am start slightly closer to the original I just again be pulled back to the equilibrium. So, the idea here is, if I am at the equilibrium point, the slight perturbation will get me back to my original configuration. It is true in the underdamped case, what happens in the undamped case? It just and just like have and this revolving around the equilibrium point in some constant orbits or closed orbits.

Similar case happens even if I have an over damped case my response will be slower. So, if I start from here again am pulled back to the origin ok. So, this is what we know also from physics and the model which we develop here actually validates those claims right. So, check a bunch of videos on YouTube about some beautiful videos on how they mod or how you can actually visualize over damped, critically damped, under damped and the un-damped case ok. So, well, this is we know from our high school physics ok.

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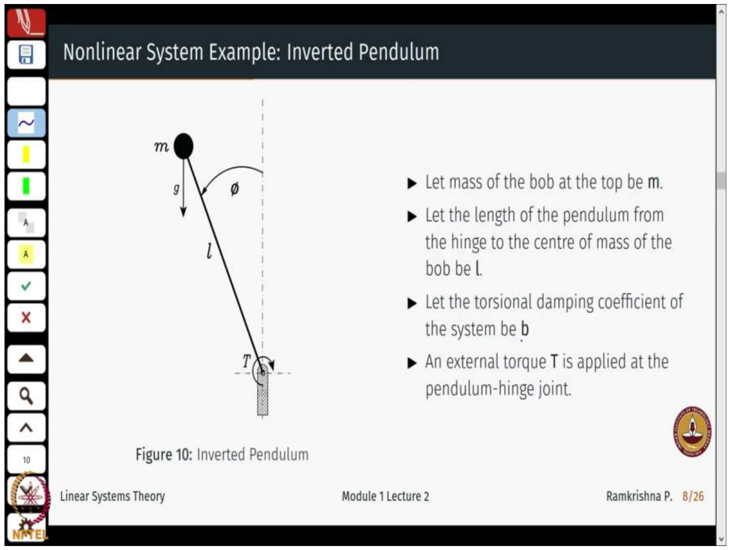

Things change when I just invert the pendulum that is so, like it is also called the inverted pendulum right. So, everything else remains the same I have the mass, the length L, the acceleration due to gravity and so, on ok.

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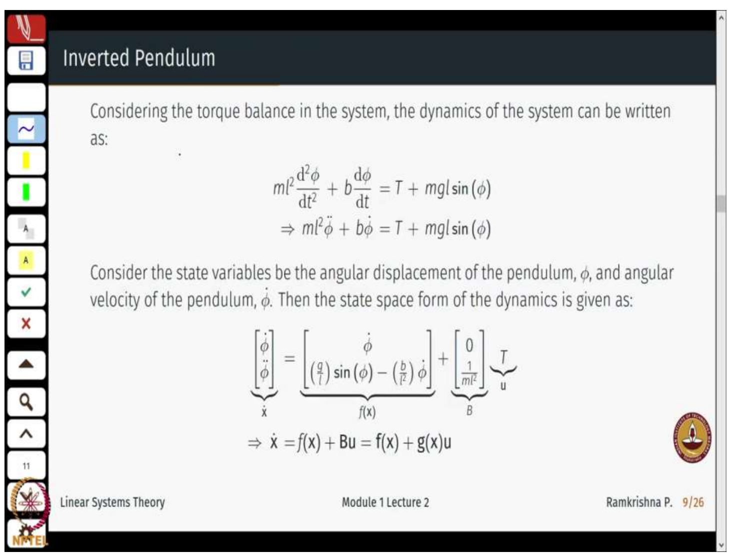

So, I write down the equations for the system same as before, my θ and $\dot{\theta}$ are as usual or φ and $\dot{\varphi}$ are as usual the states of the system, I again have a non-linear system the drift term and the input term, the input term is a constant here which more generally takes the form g(x) ok.

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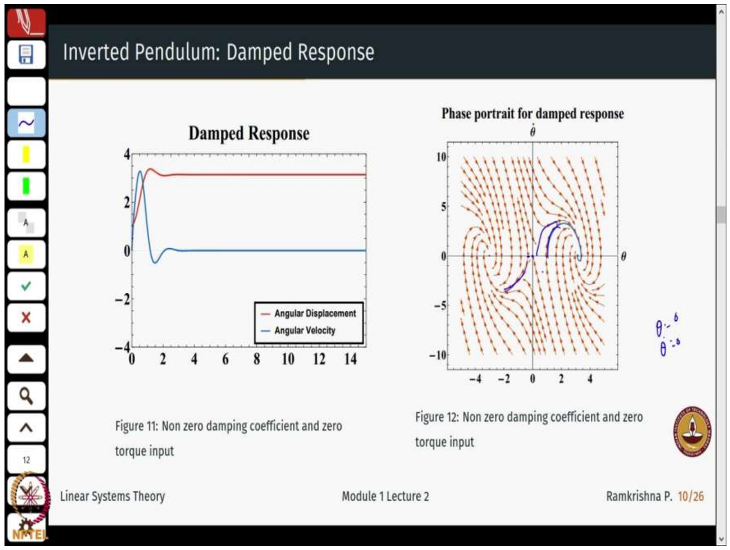

So, what is interesting here is, when I plot the phase portrait right. So, again if I start at θ = 0 and $\dot{\theta}$ = 0 and always be here ok. So, if I maybe start at a point close slightly to the left of the origin, you see that my trajectories are actually diverging right ok.

We will see the global behavior of this later on, but the initial thing would suggest that they are actually diverging from like the origin. So, if I start from here, I might just go here, start from here and might just go from here. That is also true because take a pendulum with upside down, perturb it, you expect it to go down right and where it might converge? Possibly it might converge to the dollar position which is maybe somewhere here.

See this is like starting from θ equal to sorry φ = 0 I am might just go all the way till maybe possibly converge to φ = 180º ok. So, that is what the phase plots say here right. So, if I take a damped response of a system well, this is how it looks like right. So, the; if am at the origin and always be at the origin, if I slightly perturb then my trajectories would diverge from the origin ok.

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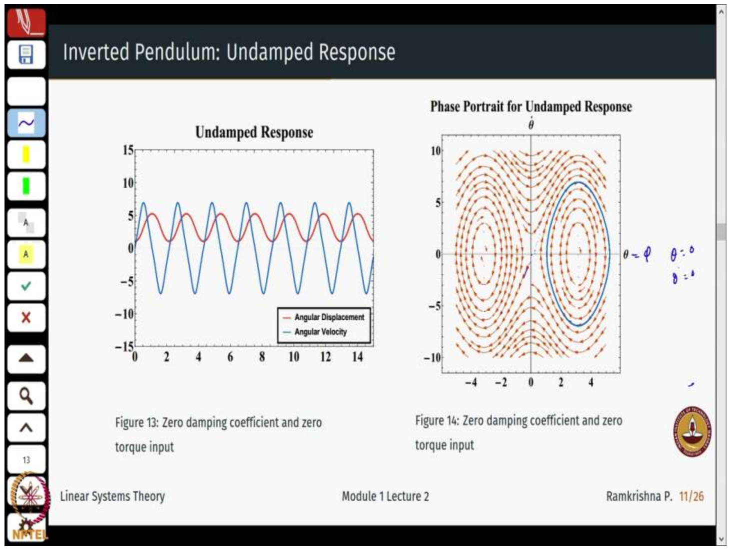

Now, interesting cases what happens in the undamped case, where here we had closed orbits well see what happens in the case of an inverted pendulum when there is no damping in the system. Well, again I start from the origin and I am always at the origin. In the undamped case I start from the origin I will always be at the origin; origin again I mean $\theta = 0$ and $\dot{\theta} = 0$ ok. So, that is a little rotational mistake here this theta and phi are the same. So, I call it θ here, but this actually a φ here. So, they are the same ok.

So, if I start close to the origin well, again my trajectory is like go away right. So, here also I start my trajectories go away, what do they do after they go away is, for example, I start from here I my trajectories go away and maybe they might just be oscillating around the another around the point φ = 180º over here. So, if I just perturb I just go down all the way here and might just keep on oscillating here. So, that is captured also in the phase space over here right; so, over here.

So, as you can see that if the initial condition is somewhere here, then if I or if I perturb till here it might just be oscillating around this orbit here or this orbit here ok. So, what is the observation here? So, first is that if I let me take an underdamped case for a simple pendulum, a slight perturbation around the origin will get it back to its original configuration right. So, over here, slight perturbation I again come back to the origin ok. Now, what is the origin here? So, these are also called equilibrium points of the system or even the steady state conditions ok.

So, here side perturbations will restore the system configuration to the origin whereas, here slight perturbations might take the system away from its original configuration, they might go somewhere else right. So, this kind of behavior with slight perturbations of from the origin make the trajectories diverge are usually called as an unstable behavior whereas, if I go back to this case. So, this is actually called stable behavior ok.

So, another thing about non-linear systems is, it is about equilibrium right. So, you can have multiple equilibrium points. So, we will talk about that slowly when we talk when we learn about linearization of non-linear systems. But for the moment so, the idea was just to show you two different versions of pendulum, how the models change and how do we even analyze the behavior of the model via the phase space.

So, more in the modern context said we all talk of networks; social networks and everything emerging that the maybe IoT is also a large network system in the world things originating from cyber physical systems and so on. So, let us see what can we a very simple example at which a bit of electrical engineers would know and also maybe this is of interest to other domains.

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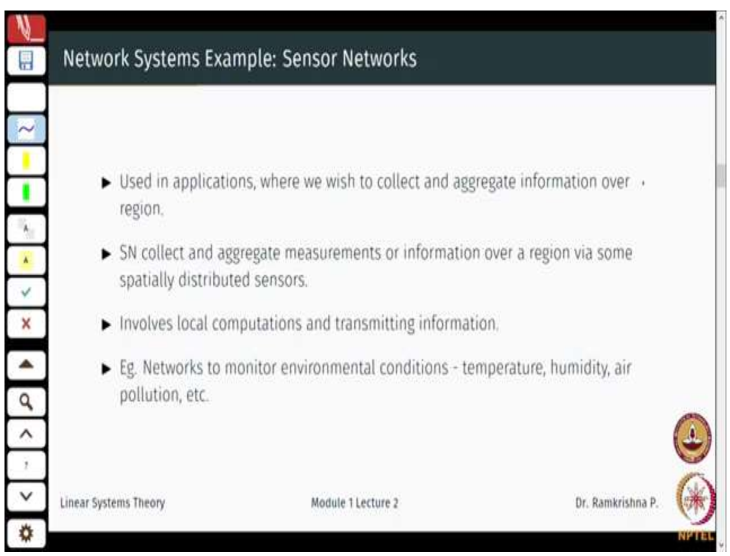

For example say pollution monitoring which is neither an electrical domain nor recessive kind of a multidisciplinary problem right. So, let us talk of this simple example on sensor networks or more like, wireless sensor networks. Not do the details of this, but just to give a little introduction to what this sensor networks are supposed to do. So, they are these are used in applications where we wish to collect and aggregate information over a region.

So, what do this sensors or sensor networks do they collect information with they compute and then aggregate all the measurements with some special distributed sensors and this say have their own computing and transmitting capabilities right. So, a typical example would be if I just want to monitor temperature of a region. So, I just put a temperature meter over here may be the thing might be slightly different 2 kilometres on the line and so on.

So, how do I evolve at a certain numbers saying that the temperature around this region is say 32 degrees or the humidity is 50%, air pollution carbon monoxide levels and so on ok.

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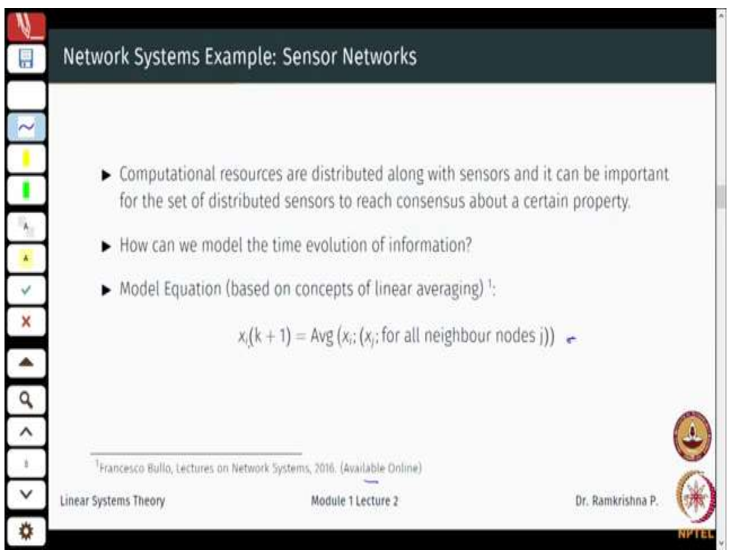

So, let us see well how do I model this right. So, before even I build a sensor network, I may be equipped with all the computation and communication aspects. So, even before I do that what do I need to be careful off, right. So, what is important is that if I put this say 5 temperature meters across the region. So, do they reach consensus about the temperature that the temperature of this region is say 32 degrees.

Now, how can we model the time evolution of this information? That I just put at t = 0. Different things might give me little different initial temperatures maybe 32.1, 31.8, somebody might give me 32.5 and so on. Can we how can you model the evolution of this or to reach a consensus for a temperature or humidity in an area ok.

One model equation or one thing is based on the concept of linear averaging. So, I just took this example from this beautiful book on network systems which is available online by Francesco Bullo ok.

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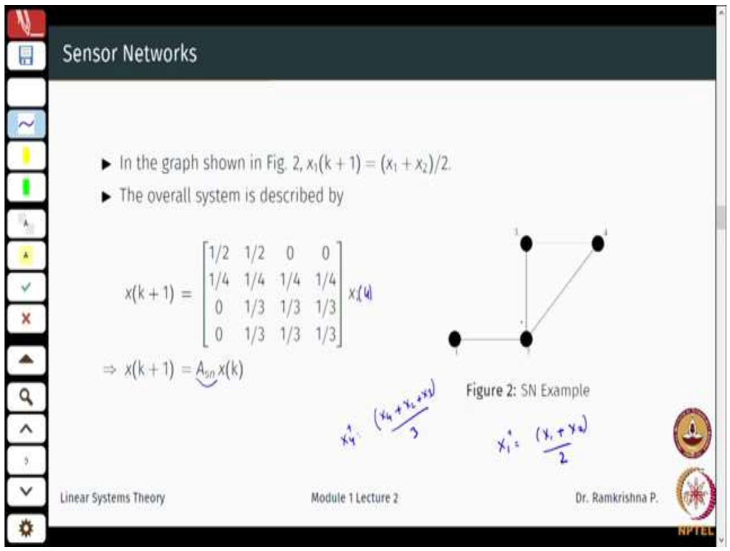

So, let us come to this expression a little later just look at a very simple example say I have this four networks or this four nodes which constitute a network, the graph of which looks like this that 1 is connected to 2, 2 to is connected to all others, 3 is connected to 2 and 4, 4 is connected this way and so on. There is no direct connection between 4 and 1, 3, and 1 so on ok. So, if I were to look at see the evolution of the measurements here, I just say look.

So, what does this algorithm mean that x_i which is some state that I am measuring is the average over x_i which is the individual measurement and x_j for all the neighbouring nodes ok. Let us see what this means in this example. Say take this x_1 for example. So, if I look at the average of x_i that is x_1 itself plus all its neighbours. So, what are the neighbours of one? That is x_2. So, I will just say $\frac{x_1+x_2}{2}$ ok. That will be how the evolution at this is x_1 plus then the for the next time I said.

Say look at say for example, x_4, x_4 would be well x_4 itself and its neighbors. What are this neighbors? Its neighbors are x_2 and x_3 and divide by 3. This will be the next at the next time instant right so, how the evolution depends.

So, I just write this averaging this algorithm what is call the linear averaging algorithm for all the nodes and I will say get a model like this at x(k+1) it should be a k here will depend. So, x_1 will be like this $\frac{1}{2}x_1$, $\frac{1}{2}x_2$, similarly this is connected to all the nodes so, it will have 1 by 4, 1 by 4 etc. So see this is this will be the model of it and again at discrete time model where I call this as a again x(k + 1) as Ax(k) just for notational purposes I called it the A of this sensor network in this example ok.

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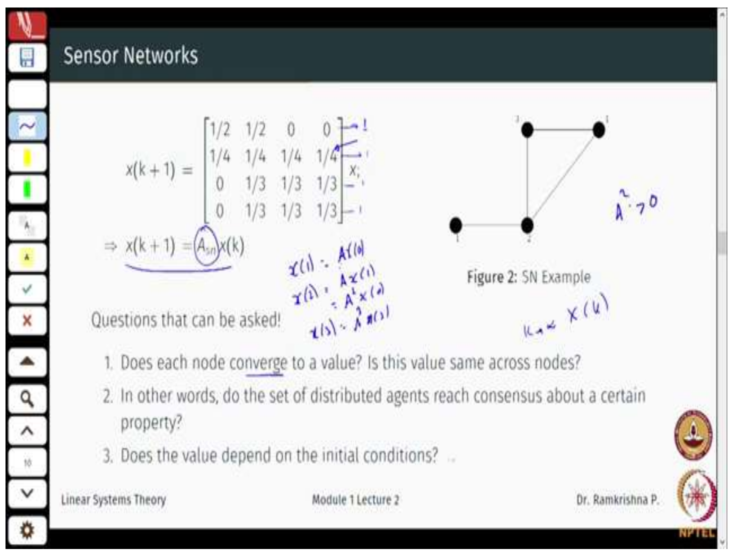

Now, what after this? So, what I am interested in? Does each node converge to a value? Right. So, is the value same across all nodes, what does that mean? So, do a set of agents reach a consensus about a certain property about the that about a temperature or humidity or may be in some senses even in social dynamics say some opinion building.

Now, does the value depend on the initial conditions? Well, can I just look at it and say or in. So, what does it mean about convergence? Does it converge to a value? So, I just want to say what happens say as x(k) plus x(k) as the k goes to infinity. Starting from initial condition give me x(0), I can compute x(1), now give me x(1) I can compute x(2) and so on. With this just using expression right give me any x, I can compute the next one. What happens for larger values of time? Do does this matrix have any hidden information?

So, what one way is to look at look at this matrix and just keep on computing right, until I reach a maybe I can just say powerful computer and say let us compute A^{100}, say A^{200} for example, and say well does it reach to consensus value or not. Do you want to do at all the time? Well, the answer is, no. Now, can we evolve at some methods based on this structure of this A that will tell us the all the hidden information that we all the questions that we wish to answer ok.

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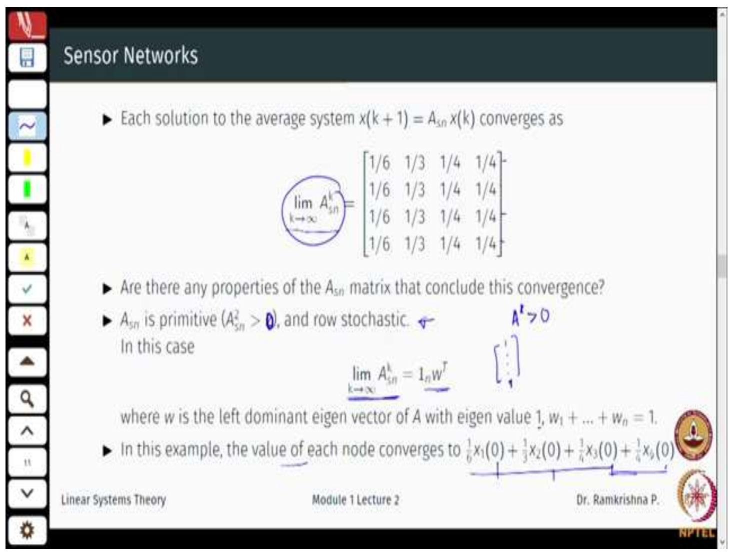

So, we turns out that if I compute this A, A^k. So, why does this power k come here is simply because of this right. So, if I say what is x(1) = Ax(0), x(2) = Ax(1), this is A^2x(0). Similarly x(3) would be A^3x(0) and so on. So, therefore, so, if I look at the time evolution and this is a interested in computing the powers of A, A^2, A^3 and so on. So, convergence will add infinity and I am just interested in computing this ok, but I do not I do not really know how to compute this right even with any powerful computing tool.

But then are there any properties of this matrix right, that I can make use of it ok. We will not do the proofs or anything of this first let us read down this statements, ok. If this A is primitive which means this should be means greater than 0 not 2, if A is primitive which means does there exist a number k for which A^k is positive which means all entries are positive ok.

So, if this happens and if the if the matrix is row stochastic, what does it mean by row stochastic? If you sum all any of this rows, the sum of this rows would be 1, you add these four numbers it will be 1, add these three numbers it will be 1, add these three numbers it will be 1, right.

If so, if these two properties hold again we will not do the proof of this. So, I am just we give an introduction to what kind of properties at we are looking at exploiting. We are just not interested in writing $\dot{x}$ = A x, there is much more to that than just rewriting a transfer function into a state space ok.

So, in this case what happens is, there is a very nice theorem which is an extension of the Perron Frobenius theorem, its all beautifully explained in this in this in this book. If you are interested in the details it says the following that if these two things are satisfied you can see that in this example you compute A^2 and it will be greater than 0, we can just do this as an as a very trivial exercise.

So, this is satisfied and it is row stochastic which means all the rows add up to 1. In this case the limit or the convergence algorithm says something like this that as k goes to infinity, A^k converges to this number where 1_n is simply something like this so, $\begin{bmatrix} 1 \\ 1 \\ \vdots \end{bmatrix}$, n times ok. Now what is this w? The w is the left dominant eigenvector of A with eigen value 1 not really we are not interested in how to compute this what is the dominant eigenvalue or something.

So, the idea here is to show you that there are some hidden properties of this matrices which we can exploit to solve the problem at hand right. So, in this example it converges or it happens that the value of each node just converges to this one you see this. So, all x x_1 will be $\frac{1}{6}$x(0) and $x_1(0)$ and so on and say this it will be same through all the all the four nodes. So, node 1, node 2, node 3 and node 4. So, much of the aim of this course would be to learn equip ourselves with tools which will help us solve problems petty quickly.

So, what is that tool here that I exploited? The tool here is that is just this statement here right which comes from linear algebra that if this matrix is primitive and row stochastic, then I do not need to worry about anything, I can just blindly copy that tool into my application and say yeah yes their actually is convergence of the algorithm and each node converges to a particular value given by this. And let us say it has a very beautiful dependence on the initial conditions ok.

So, slowly we learned tools like this. So, the idea here is to just give you an introduction to what are the the kind of tools that we need to learn here. They may not look even out may be by them of so, shelf on a (Refer Time: 24:52) but we have to imported that from some literature on linear algebra ok.

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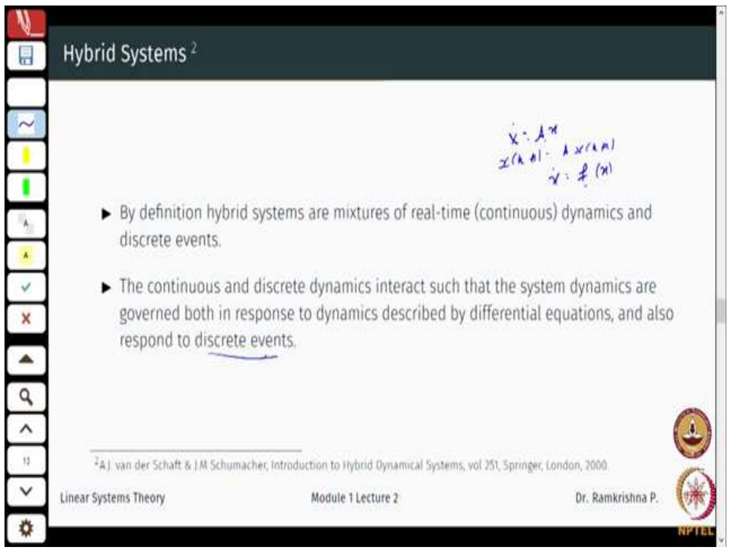

So, another interesting class of systems are what we call as hybrid systems and as the name itself suggest that these are mixture of some kind of continuous time dynamics and discrete events ok. So, it so, what are continuous and dynamics so for we learn there could be like $\dot{x}$ = Ax in the discreate case you will have x(k + 1) = Ax(k), you can have in the non-linear case $\dot{x}$ = f(x) and so on ok.

So, in this hybrid dynamics, what happens is that well these are the response of the system well in the continue case just governed by f or here just governed by A or A and so on. So, here the response is governed by not only f, but also how they respond to certain discrete events.

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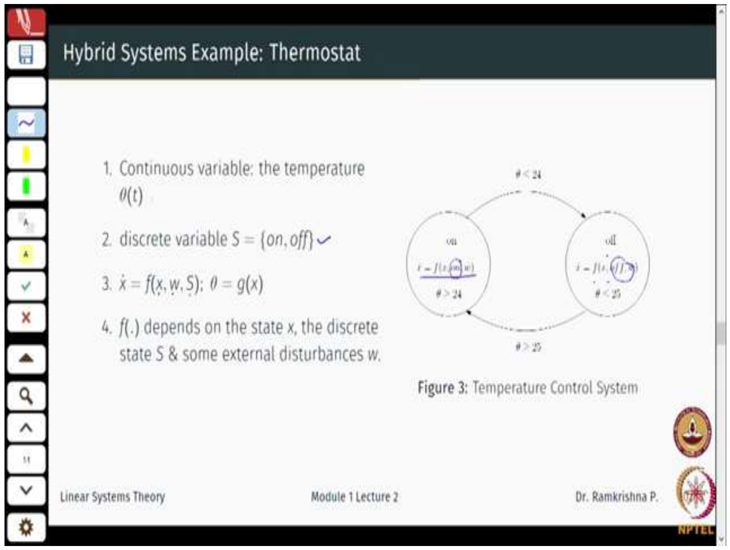

As a simple example I am just looking at a temperature control system if very simple example of a thermostat we learned it in very early trainings of ours of our physics education ok. So, what I am interested in is temperature control of a room that is say I am in this particular room I want to set the temperature to 24 degrees ok. When I enter the room say the temperature is 28, I switch on the air conditioner it will cool it to 20, 24 and once it reaches that 24 setting is at 24 now the it will switch off right, it does not need to cool it.

Further then I use here rise in temperature from say 23.9 again it will go to say some number 24.1, it will again switch on right. So, you have a discrete states here which will tell if the air conditioner should switch on or switch off right. So, that is the discrete event here that or the discrete state or a discrete variable that that governs my system. Then you have of course, a continuous variable $\theta(t)$.

So, my state now evolves according to a non-linear or may be a linear equation also which is defined by x which the current state or the state itself, then you have may be certain external influences like people entering the room or changes in the conditions outside the room and so on and a certain switch. So, if I am in a comfortable so, if I enter the room, I want to switch on the AC to for the temperature 24 my dynamics will take this pattern right. So, I will have $\dot{x}$ = f(x) with the switch in the on state and some presence of disturbances or external influences.

So, if I am on the comfortable region my AC will switch off another dynamics will go according to this one right. So, you have the continuous dynamics are not only depending on themselves, but there is a there is a discrete event that is also guiding them here ok. Again we will not really investigate into details of this there is a infinite amount of literature in within the hybrid systems community.

You can also think of some other examples like a bouncing ball when the ball yes you just release it from a position here it so, the natural evolution of it would be through the acceleration due to gravity, once it goes down it might just again so, while this is upward movement will depend again on the restitution coefficient the friction losses at the impact and so on.

So, you have continuous model when the ball until the ball hits the ground and another continuous model of the reaction of the ball as soon at it as is hits the ground and the discrete event that is hitting the ground ok. And I will not do the example of that, but you can its available in plenty in any literature.

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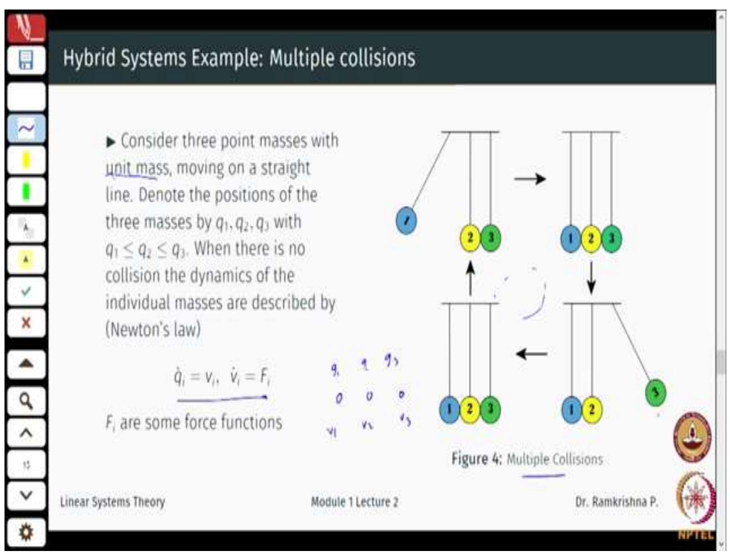

One interesting thing which I thought I would do here is on the system of multiple collisions, this is important because well if you look at much of our jee training or training those basic we will talk of elastic in your elastic collisions and so on ok. Now, can we revisit those in a a different way then what we were taught of in those days ok.

So, let us do this simple example, this is also called a Newton's cradle and the like beautiful videos online which we can watch about this.

So, I just start with three point masses say; q_1, q_2, q_3 say this is our position just q_1, q_2, q_3 and each of them moving with some velocities v_i right since this v_1, v_2 and v_3 and some force functions which possibly are acting on them given by this one. This is again nothing right this just a Newton's second law written in terms of the state space form or in terms of two differential equations that the rate of the position is the velocity rate of the velocity again this is related to the force.

Now, I am assuming that the masses here are just unit masses. So, the m here is this it does not appear, so m = 1 ok. So, a typical motion what you would expect is ok. So, this would hit once they hit then you have a have a motion which is three goes this way and three will come back hit one and then you just follow this process ok. We will see if you can an explanation to why this kind of particular motion happens ok.

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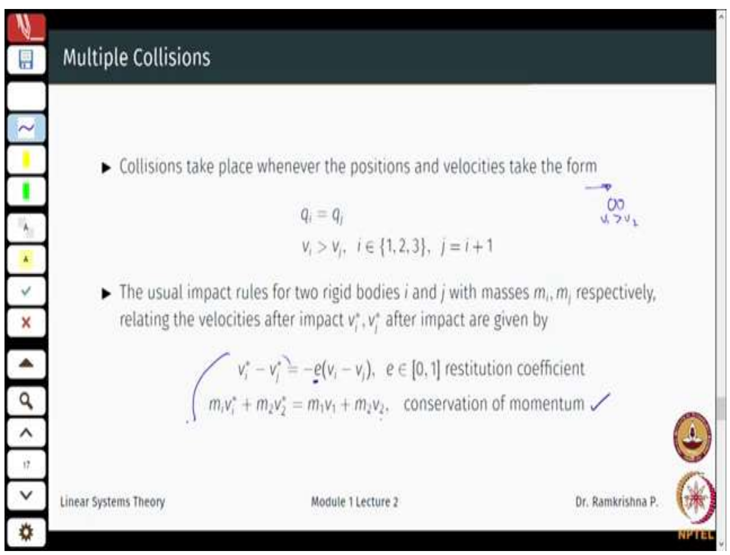

So, under what conditions do collisions takes place? Say if I look at q_1, q_2, q_3 will collision will take place when q_1 and q_2 are just at the same position, q_i = q_j, its point masses so, I can actually quantify this quite nicely. And then the if you the if they are moving in this direction with a velocity of v_1 is greater than velocity of v_2, then collusion will happen ok. Now what do we know from physics?

Well, this is this is very standard thing right conservation of momentum that the total momentum before impact is equal to the total momentum after impact, if I call the velocities before impact as v_1 and v_2 and after impact as again v_1^* and v_2^* ok. So, how do the velocities change will they loss if always be some restitution coefficient which might lose this velocity depending on if the if the collision is purely elastic, purely inelastic something in between and so on right ok.

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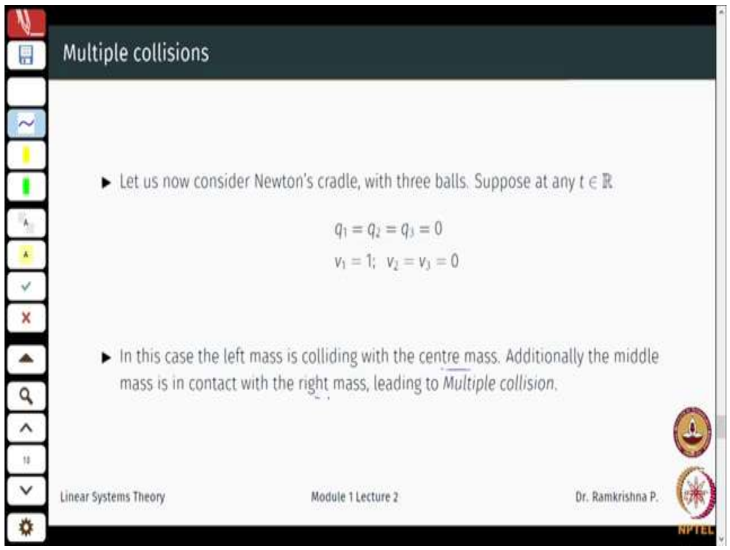

So, let us start do this right so, let us say at time t = 0, I start with this right.

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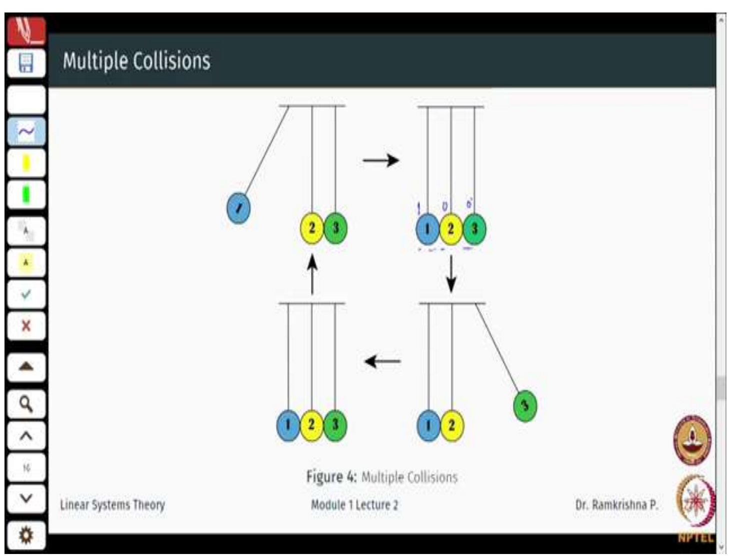

So, I so may be I am right at so, start time equal to 0 and something like this with the velocity here being 1, 0 and 0. If just say maybe just before collusion ok. So, in this case what is this is happening is at the left mass is colliding with the center mass and of course, there is also the mass in the center which is in contact with the right mass. So, this will collide with this, this will collide with this. So, what are the ways of modelling this phenomena? Ok.

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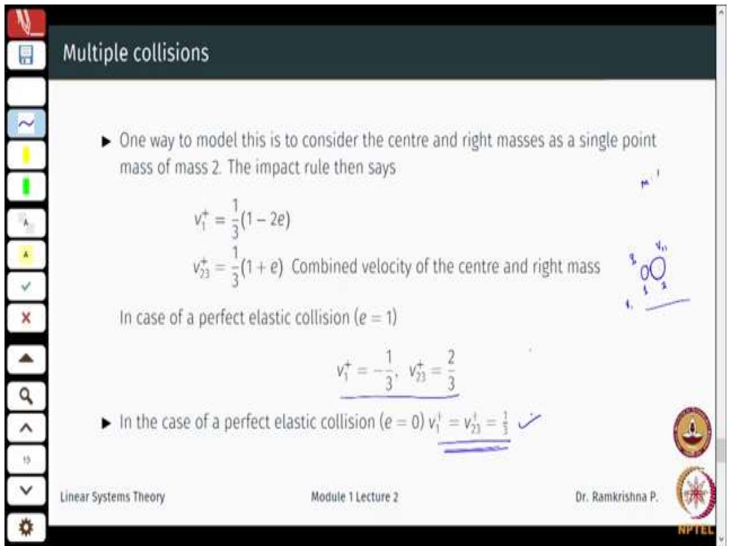

So, one way is that I look at this as say a mass 1 here this is my q_1 1. And I and I take the remaining two masses as something as of mass 2 and let me call this say your velocity e of this is v_1 and this is the combine velocity of 2, 3 right. So, this so, I am just looking at collision of two bodies with a mass 1 and a mass 2 and the initial velocity is v_1. The impact tool you can just simply substitute just use this to derive the rules saying that v_1 plus which is just after collision and v_{23} plus we will take this form right.

So, again all the masses are 1 right so, this expressions are easy to drive. So, in case of a perfect elastic collision well we can expect this right. So, the velocity of this guy changes and this guy becomes 2 by 3 and so on. And similarly in case of perfect elastic collision right when e = 0 ok. So, I think we know bit of this physics, so we will not delve too much into the details of this ok.

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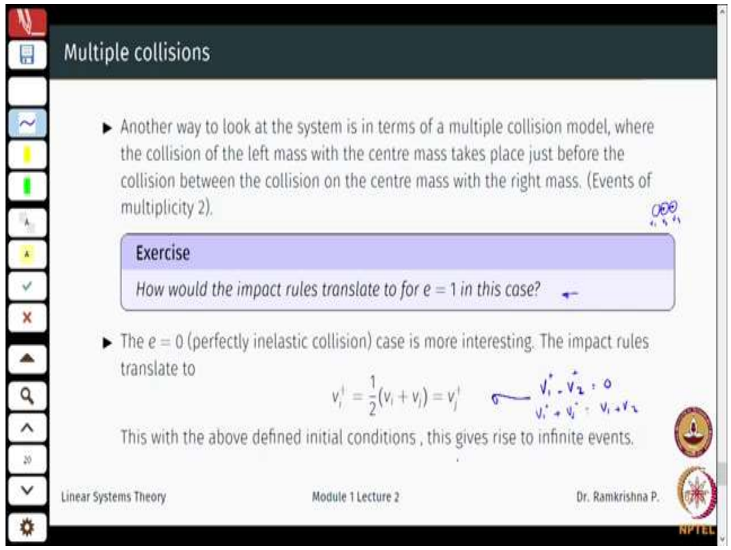

Now, is this the only way to model. I can also look at it as of a multiple collision where the left mass first collides with the center mass and then the center mass then collides with the right mass ok. So, I am looking at again three distinct velocities v_1, v_2 and v_3 just before collision and just after collision ok.

So, I will not do this case this is pretty easy to derive, but what is interesting for us is the case of e = 0. And what happens in the case of multiple collusions or here we have events of multiplicity to. So, one event is this guy colliding with this guy and the next event is this guy colliding with this guy ok. Now, this is this is interesting right so, what will happen in case of a perfect elastic collision then again I rewrite this collision rules.

There are some typos here which we will take care of later. So, if I rewrite this collision rules what I get is this one, is an expression like this ok. What does it mean? So, if I so, I just derive it this for the first one right. So, I have v_1, v_2 and v_3, for the first collision if I rewrite this one so, v_i - v_j or if say v_1 - v_2, what is e = 0? So, this will be 0, this is say just after the impact ok, that will be the first two. Second two will tell me $m_1 v_1$ + $m_2 v_2$ is this.

So, I will have m_1 is 1 so, v_1^* + v_2^* is again v_1 + v_2 ok. And then you keep on writing this and then you will just arrive at a general expression like this ok. Now, with this defined condition this will give right rise to infinite events; infinite events would be this we will collide with this, this will collide with this come back collide and then so on. Now, how do a velocities change in this case? This turns out to be pretty interesting ah.

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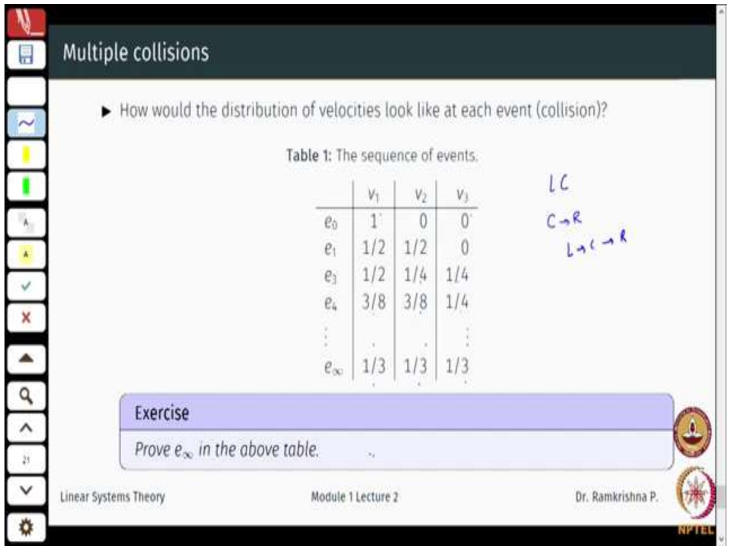

So, at event 0 or when just before the collision I have v_1 to be 1, v_2 is 0, v_3 is 0 ok. Now, in the when the first collision happens when the left and the center collide, then I have this the velocities will be $\frac{1}{2}$, $\frac{1}{2}$ just by this formula here it just derived again from those two impact laws ok. Now, in the second event what happens is that the center and the right collide ok. So, this guy nothing will happen to this velocity, but then I am look looking at events between these two ok. So the velocities will just be half of it. So, it will be $\frac{1}{4}$ and $\frac{1}{4}$ ok.

The next event would be when ok. So, you have left colliding with the center, center colliding with the right and then. Now, next I have transferred now between the so, this guy stays as it is and then v_2 and v_1 collide on the on the other side. So, I will have the velocities again going by this formula and I keep on doing this at infinity I will have velocities converging to $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$ ok. You can actually derive this at it is its you can do this as the as the simple exercise ok. So, what is happening?

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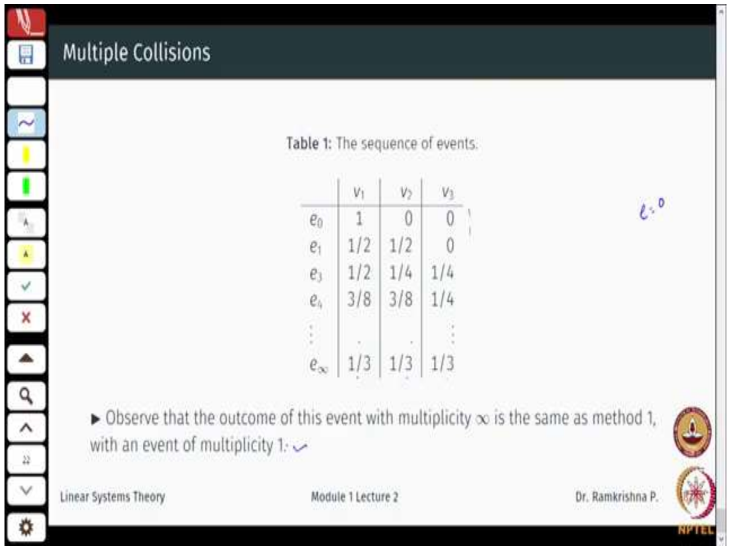

So, this is in the event of multiple collisions, I am looking at e = 0 at steady state if I may call so, that the my velocities is will be $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ ok. Now, let us go back to this situation where we had considered the collisions to be as if that particle of mass m is colliding with a particle of mass 2 whether this two other point masses were likely up together. You see this one right, they are actually the same right ok. Think of it why this happened.

So, the outcome of this event with multiplicity so, you have infinite events in so, of which I call it as a event of multiplicity infinity is the same as method 1 with an event of multiplicity 1 ok. Think of why this happens as we do few more examples in the next lecture.

Thanks for watching.