Linear Systems Theory
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Module - 01
Lecture - 01
Introduction to Linear Systems

Hello everybody,welcome to this course onLinear SystemsTheory.This would be for kind of an introduction to any advance level courses in systems and control theory.Of course, there would be several others which could go in parallel, butto start off I thinklinear systems theory is a nice gateway to expose yourself tocourses on say hybrid systems or non-linear systems, geometric control optimal control and so on. So, one of one of the basic things in engineering is to equip ourselves with the right kind of tools,tools for analysis and design purposes.

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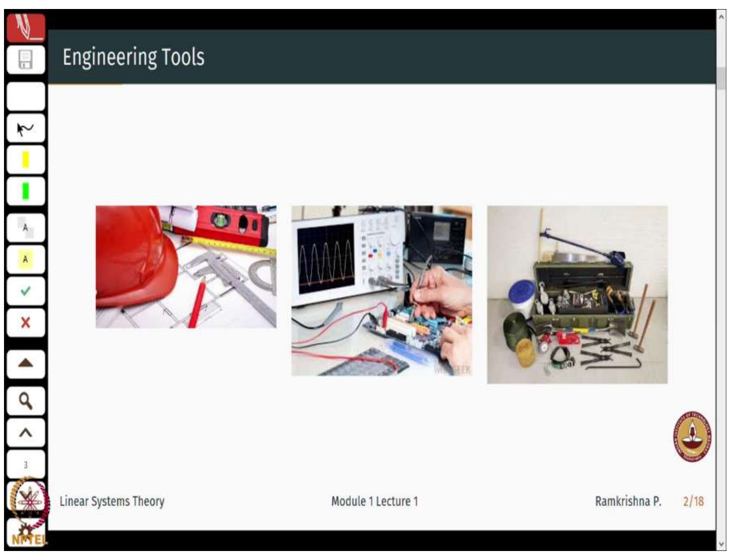

So, if I just talk ofengineering tools something that would outright come to your mind is something like this rightacross domains and that is what we are trained to beas engineers.So, that was, what was in our mind when we were as kids aspiring to be engineers. So, one of the important tools whichwe donot really think of or which we are introduced too much earlier in life is onthe tools of mathematics.

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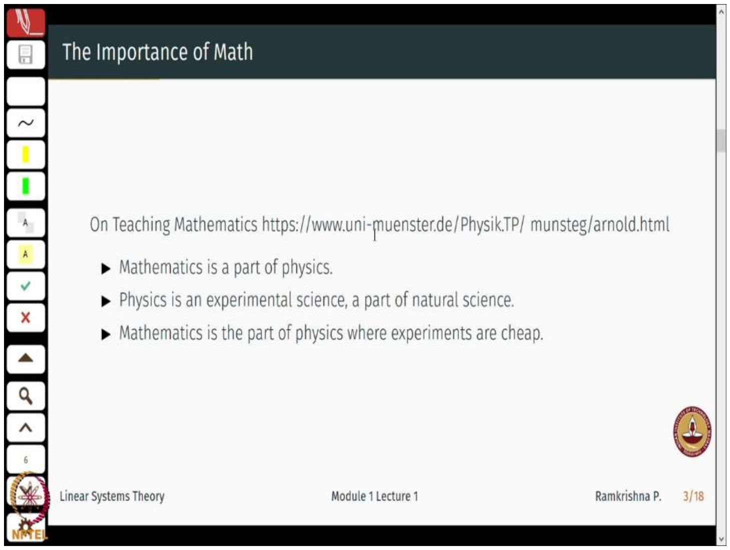

So, just to begin with I will just quote something from a very nice article which you can just read from this linkis on teaching mathematicsof. So, this is by one of the very famous mathematical physicists of our timescalled V IArnold and what he says isthat maths is a part of physics right and physics is an experimental science a part of natural science.

So, if you look at allromanticized versions of inventionsor discovery which we learn through our school training could be the Newton's apple or the Archimedes principle or somebody discovering something via an accidental experiment in a garage was all results of some experiments or some observations from nature right. So, therefore, he says physics is an experimental scienceand a part of natural sciencewhere does mathematics fit into all these things right? So, what he says later on is that, mathematics is a part of physics where experiments are cheapok. We have done lot of these experiments without realizing that we are using mathematics as a tool of experiment.

So, if you are in electrical engineering. So, in one of the earlier courses on machines you would do for example, an equivalent circuit of a transformer,why is that important right.We never ask ourselves that question, but why is, but we are trained to solveproblemsgo to the examhalls solve those problems finish our course and go back.But the equivalent circuit of a transformer is powerful in itself right. So, once I get the equivalent circuit parameters, the circuit modelthen I can run whatever experiments just by using standard circuit theory for example, what could be the efficiency of a transformerwhich is operating on a load of a power factor point 6 and blah blah blah right.

So, all those things I donot really need to buy a transformer run through experiments and get my result right, then I have to do a million experiments and it is very likely that I might end up damaging the equipment. So, the power of math here or what I am trying to introduce youthrough this course at least in the initial part is the power of mathematics inunderstanding dynamics of a system and also to develop toolsfor analysis and design purposesok. So, just to begin with I will just say that one of our ideas would be to just get the physics right and rest is all mathematics.

So, these the tools from maths are as important as what you see in the slides say for example here or maybe they arein some sense more powerful ok

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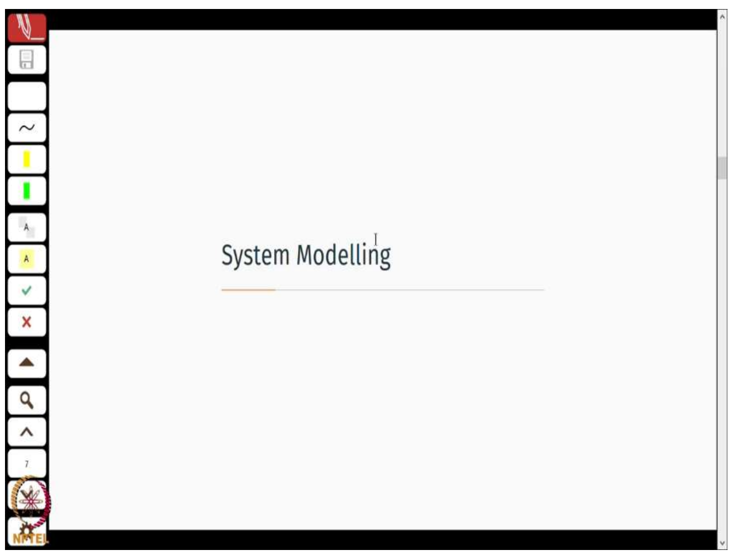

So, before we go into designing any system or designing since we are in the control course. So, we think that most of the times the objective is to give me a plant and I will design a controller say, rocket launcher or automotive device or a process plant and much of that, but before we do any of those techniques of control we first need to understand what the system is about right.

And therefore we will spent a good amount of time during this lecture just looking at some basic models not really advanced models like rocket launcheror a Segway scooter or any things like that.We will just keep things simple that we could slowly have a transition from what we learnt in our undergrad course or even frombits and pieces from high school physics to develop a general theory or a general framework for understanding systems. Soand much of these things which we in control borrow or a tools from theoretical physics especially related to dynamical systems.

So, a bunch of textbooks or literature is available on the theory of dynamical systems ranging from ordinary differential equations to chaotic theory and so, onwhich are many many of them are now also some kind of popular books say for example, the book on Chaos theory for example, right.

So, we start by just looking atof understanding or what is the basic question of modeling like physicists spend a lot of time just understandingsystems or and developing models say it dates back to as long or as back as say Kepler's laws which weremodels built by lots of observations just to understand say planetary motion.And everything inphysics or much of the things in physics were just models built by observing stuffok.So, in our context; so, what are we interested in right?

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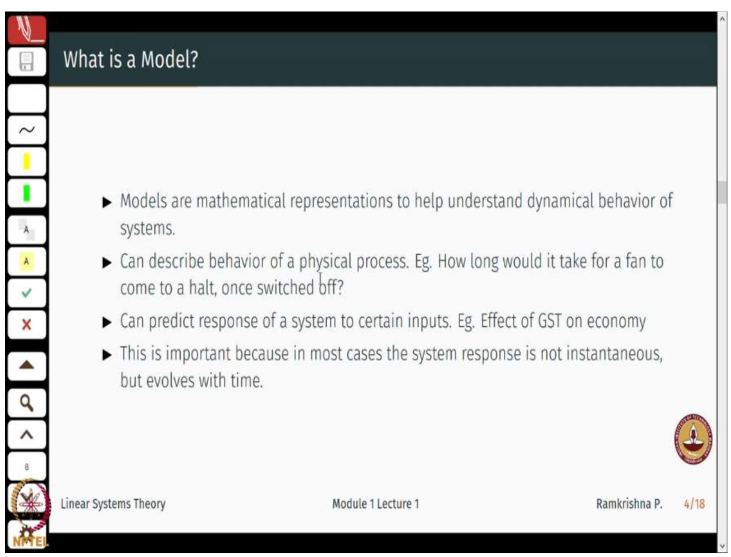

So, first thing is what is a model?Model could be several things right. So, in our day to day life we spend a lot of our effort doing models of systems.Say,I am interviewing you for a particular position academically or otherwise. So, what am essentially trying to do by asking you a set of questions say you are intoaspiring to get a job in PSU, you write your gate examget a score,you go to the interview hall they dump on you bunch of questions and then they decide whether to offer you a job or not. So, the gate exam is tells me something about you like that you are actually apretty well trained engineer who could crack a national level exam.The questions will help me decide on several other factors may be your communication skills, your speed of thinking and so on.

So, based on this test signals which I give you right from the gate exam till yourmaybe a technical interview or an HR interview, I just built a model of yourselfand this model will help me judge whether or not you can handle problems which are in the real world while you actually go to work.Or I can I do you have enough training or enough tools or are you equipped or capable enough that I can train you on handling those tools of the industry.

So, that is a very basic modeling thing which happens. So, and that happens you knowmaybe if I want to buy a cell phone, then you have constraints on the price, you have constraints on maybe the brand and so, on. So, all we do is we arrive at a conclusion by throwing up certain questions and getting certain answers right.

So, modeling is something which we do every time before we even start the actual objectivein this course maybe to design a controller so to speak ok. So, in our course what will these models look like? So, these models are mathematical representations to help us understand dynamical behavior of the system right. So, say things if I can recallthe Newton’s laws for example, iswell the rate of change of momentum is forceor in the planetary motionthey talk ofthe orbits spanning equal areas in equal intervals of time and so, on ok.

So, again;so, this model should be able to describethe behavior of a physical process, a very good example is I just switch offthe fan and how long would it take before it comes to a complete halt.Not only that so, can I predict response of a system to certain inputs like in the case of an interview can I based on the model, can I predict will you be able to handle a certain job in the real plantor for example, the effect of GST oneconomy.It is a little controversial we still do not know we still may have to wait to see what are the long termimplications of that.

In some cases it could also happen that in the initial phase or when youinject an input to the system, you might actually see a negative effect.We will also talk about that;these are essentially related to what we call as right half zeros or non minimal phase behavior. So, the effects are not instantaneous so therefore it is important for usalso to look at the initial response of the system till the steady state how the evolution actually happens rightok.

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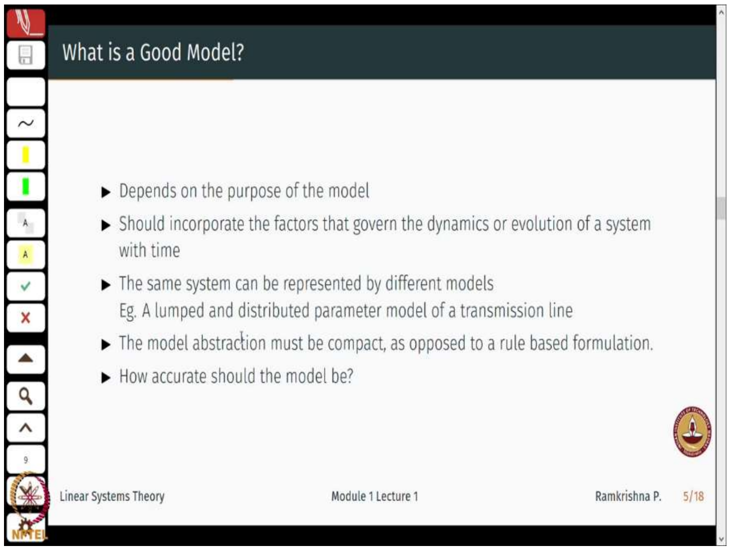

What is a good model, right;again a good model depends a lot on what we want or what are the questions or what we want to do with the modelright. So, what is the purpose? So, if am interviewing for you for a job in say BHEL I donot really need to know maybe who your favorite cricketer is or what your favorite restaurant is right.It does not matter those questions might matter in some other modeling maybe on a dating website possibly those things would matter rightok. So,the model should incorporate all the factors, that govern the evolution of a system.

So, take a physical model say I am modeling a car, I need to know lots of things about the carsay for example.The maybe a basic model of $F = ma$ kind of model till themodels of the fields the friction, the engine and much more right of even the fuel injection and so on or if am looking at electric vehicle you know how well I model theelectrical energy to the mechanical energy domainok.

Now is a model uniquewell the each system can be represented by different not each system, but in lot of cases the same system can have different models I will run you through a bit of examples later in the lecture.But,it could start with well a very basic thing of what we do inelectrical sciences from our say one o one course on circuits till electromagnetics.

You look at say for example a transmission line can have a tmodel or a z modelbased on what is the objective of the model over there. It can also be modeledas a distributed parameter system right which you can model delays reflections in lines and so, on if you vaguely remember what happened in those courses.We will touch upon these examples as we go through the courseok.

Now how do these models or what kind of models are we comfortable working with right? So, it should be; it should the abstraction of the model. So, againwhat I will try to write in terms of models are, may not be the true models, but they areok, they are good enough to help me be predict the behavior of the system. So, it's all what we call as an abstraction of the system.

So, a certain differential equationis an abstraction of a system, it is not the exact system just that, I just establish an equivalence betweena certain car and the certain differential equation or a transfer function for example. So, this is much;so, the abstraction should be compact as opposed to a rule based formulation, say maybewhich waspossibly from computer sciencepeople used to use these kinds of models that if this happens in something like this would hold true, something else happenswe cannot really have a set of exhaustive list of rules for amodeling purpose right.

So, we want it to be compact and descriptive enough.Now how accurate the model should be? It again depends on what I really want to do with the system or what is my analysis point, should I really capture all the small components of the system or should I just look at the bigger picture, which will just be enough to tell me the immediate behavior of the system. So, some of these questions we will slowly touch upon and we will not really jump into abstraction as yet, but we will slowly start building this abstraction with the help of certain physical examples which we would have which you would already know ok.

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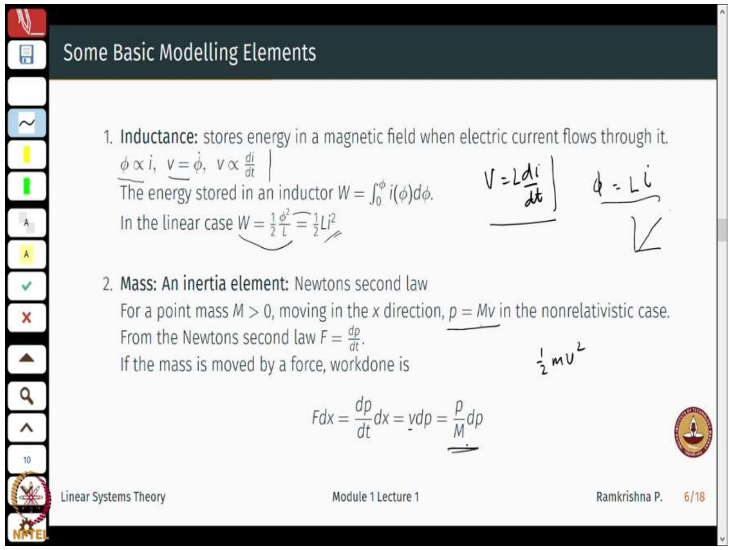

So,if you are an electrical engineer. So, I will just touch you through a couple of elements and the rest will be you will follow right.So, if I look at the inductance right as an element.What do I know about it that it is an energy storing element right, it stores energy in a magnetic field and as a certain current flows through it, it generates some flux and I know thatthe voltage induced is the rate of the flux or voltage is proportional to $\frac{di}{dt}$this is what I know right or which I woulddeduce by someamount of experimentsok.

So, the energy stored that we know thesetwo right. So,what do I learn if I say what is energy stored in inductor?If you remember by heart, we will say oh energy is $\frac{1}{2}Li^2$ or we also know we will just directly writethis expression as V equal to sum $L\frac{di}{dt}$

But is this completely true,well the answer is well it is true in a certain operating region, where the flux and the currentare related through a linear relation right. We are then ignoring the saturation effects which depends on the magnetic material and so on. So, if I just derive by the standard principles I would derive the energy to be $\frac{\varphi^2}{L}$in the linear case.Again φ is my basicparameter or basic quantity here which defines my systemand this has a direct relationbecause of this relation of$\frac{1}{2}Li^2$. So, this is where it comes from right. So, we;so, this is the first principle here is this one and this one. So, what we see as $\frac{1}{2}Li^2$

or $L\frac{di}{dt}$ are very specific cases of that where the flux and the current are like related linearlyok.

So, similarly with capacitors, resistors and so, on;so, all we learn in our circuit scores are things essentially restricted to the linear systems and we know that they are valid over a large range of operating conditions. So, we are happy doing this just this $\frac{1}{2}Li^2$. So, how accurate the models should bewell all by circuit design I am just look happy with this kind ofdescriptionin a linear setting.Similarly right so, if I look at a mass element right.

So, I take a mass m moving in a certain direction. So, what I know is again if I ask you what is the kinetic energy and obvious thing would be to remember $\frac{1}{2}mv^2$,where does this come from? So,in the non-relativistic sense I know that momentum and mass are related this wayright and what does Newton's second law tell me that it is not $F=ma$right.

So, that is what we kind of tend to interpret or say it directly that oh,$F=ma$is Newton's second law, but what did interest me is the rate of change of momentum,$F=ma$ is a special case of that. So, if I look at the work done, then I look at force times displacement high school physics and I can write it this way right,$\frac{dp}{dx}$I know that $\frac{dx}{dt}$is the velocity because of the standard definitions and then velocity and momentum are related this way and I get this expressionok.

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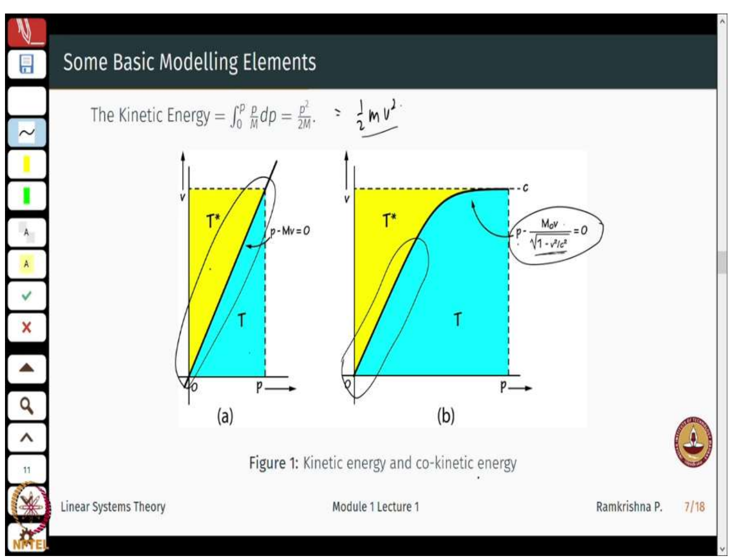

Now, and then I can derive the kinetic energy to be $\frac{P^2}{2m}$right and this is also$\frac{1}{2}\boldsymbol{mv^2}$ of what I learnt, because of the linear relationship between the momentum and the velocityok.

So, what does this mean right. So, when I say well how accurate the model should be. So, much of the case times when I say$\frac{1}{2}\boldsymbol{mv^2}$, I am justapproximating or I am just working in the region which looks like this or in the here maybe I am just operating here right.

But if I see the relation were in a relativistic sense,the relation between the momentum and the velocity wellit takes some other form right it depends on c and much of the realistic velocities which we work are much muchlesser than c and therefore, I am happy with a linear approximation right. So, how accurate the model should be well depends on you know what I;on which region I am operating or what I really want to do with the models right.

So, Idonot need to really memorize this formula, because in any practical case which I would be dealing with as an engineer, I would not go closer to the speed of light. So, I kind of I am happy here therefore, $\frac{1}{2}\boldsymbol{mv^2}$is good enough for what all we learn.And of course, physicist called thisso, thisside of the energy as the kinetic energy and this side as the kinetic co or co energy or the co kinetic energy.We will not go into the details of that, I canlater in the course refer you to a beautiful article which kind of exposes you toall thisbasic energy based frameworksok.

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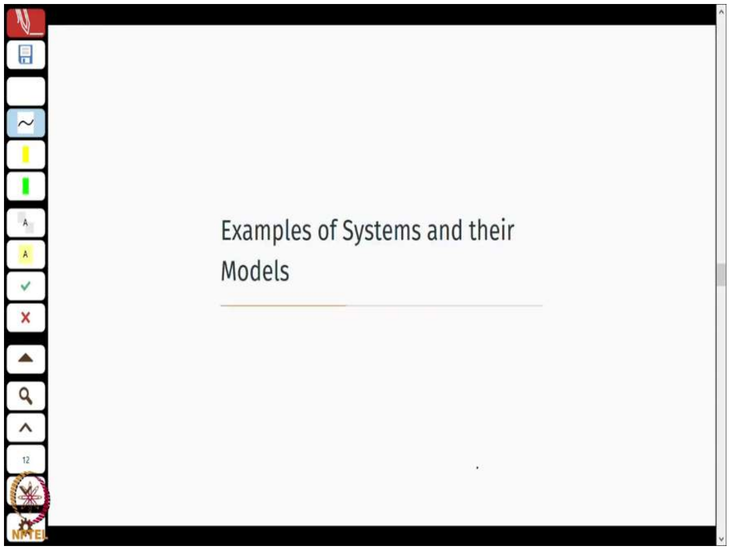

So, let us do some very basic exampleswhich you definitely would have done even in other control related courses.

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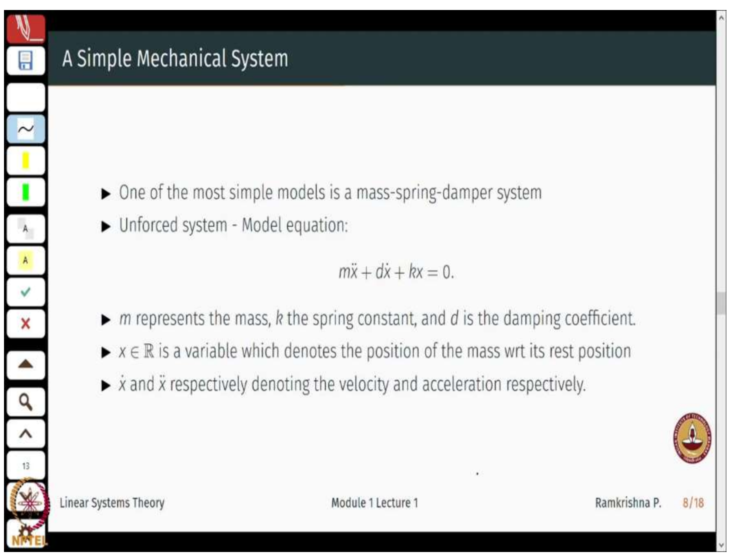

So, one of the simplest example is of any mechanical system which will have a kinetic energy element, which will have a potential energy elements say a spring and some kind of friction in the system and any mechanical system typically can be modeled in this way right. So, m is a certain mass,k is a spring constant, d is damping coefficient and we are not looking at any external forces. I am just looking at a plane autonomous system with x being my position $\dot{x}$ the velocityand $\ddot{x}$ dot the acceleration.And applying basic Newton's laws to tell me thatthese are the dynamics of the system, we can also derive this by from purely physics point of view from Lagrangian equations orthe Hamiltonian equations of motion ok.

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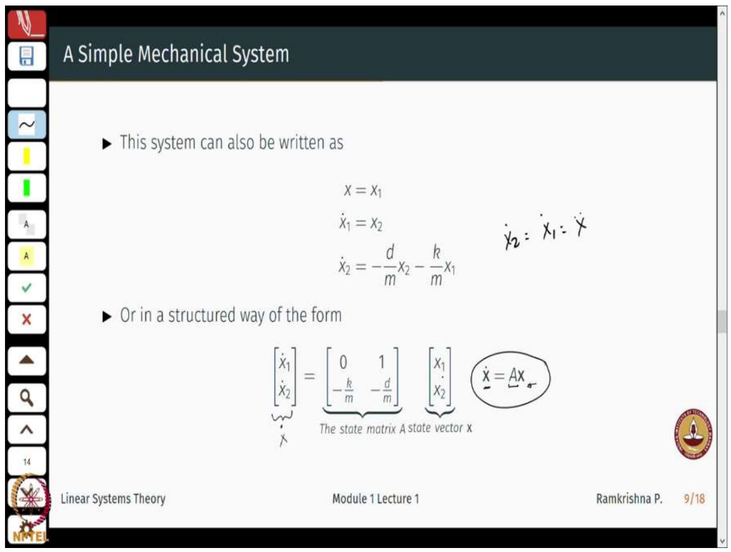

So, I can write this equations also in the following form right.

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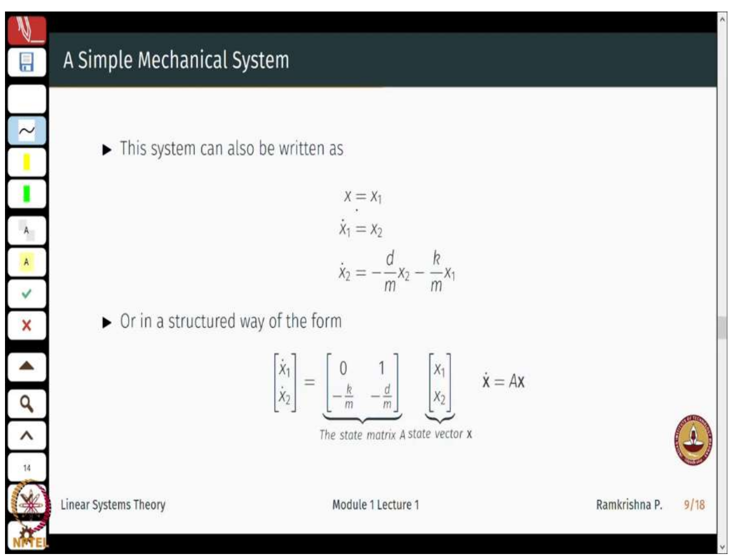

So, I have here a second order differential equation.So, let me define a new variable x 1 which I call as x,$\dot{x}_1$ I can call as$x_2$is a first time derivative, $\dot{x}_2$,so, if I look at here$\dot{x}_2$is essentially$\dot{x_1}$is $\ddot{x}$. So, if I substitute for $\ddot{x}$. from here, what I get is$x_2$is something like this ok.

So, I can write this in a compact form of I have$x_1$and$x_2$as my position and the velocity, I can write well this as the rate of x which I in general will be called $\dot{x}$ with this kind of expression where I have I reducedor I changed or transformed a second order differential equation into two first order differential equation.How does this look like?Well, this looks like on the left hand side,I have $\dot{x}$ the evolution of the rate of the states, on the right hand side I have a constant matrix which I call as the system matrixand then the state vector here right.

So, this isa good looking structure here right. So, which is alsothe first building block of any state space representation of a linear system ok. So, $\dot{x} = Ax$, where x are the states or states are again the variables which are of interest to me, we will elaborate on this a little later.But in this casewhat am I interested in is the position and the velocity; of the evolution of the positionand the velocityok.

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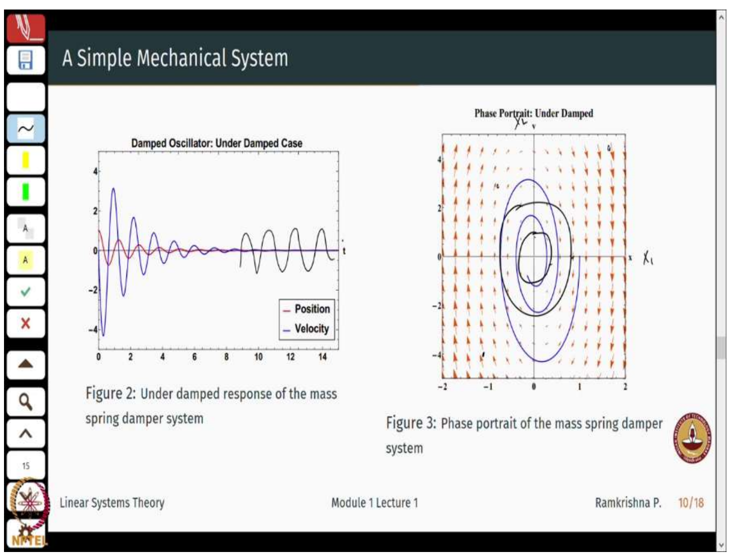

So, if I look at wellwhat does the system tell me. So, if I say subject the initial the systemto a certain initial condition,well I can see that well the system will eventually go to rest because there is no external force acting on the system and also because there is damping in the system.

So, any initial conditionphysically it would mean that you start with a certain kinetic energy, certain potential energy, the energy will dissipate and eventually you will reach to a state of zero energy, and that is what happens even when you switch off a fan right. So, there is no external input. So, eventually your system will come to rest it will dissipate all the energy in terms of heat or whateverok.

So, these are like the typical evolution of the position and the velocity in an underdamped case. So, what is in physics literature also the representedas the phase portrait, I am just looking at the evolution of the state variables with each other. So, here;so, this is my position x 1, the velocity x 2 and then. So, any initial condition will actually go to the origin. So, start from here,you start from here, you start from here this is also typically referred to as the phase portrait.

So, and additionallyif there is no d=0, then I will just see some kind of concentric circles here,circle here, circle here and so, onwe will see the approximations of this the evolution of this little later.And similarly if there is no damping this the position and the velocity will just be oscillating around the equilibrium like that typical oscillator circuit or a simple pendulum which will keep on oscillatingif there is no dissipation.If there is dissipation you will just see that it will stabilize or just come to rest at the equilibrium position which isthe position of say 0kinetic energy like and the 0angle so, to speakok.

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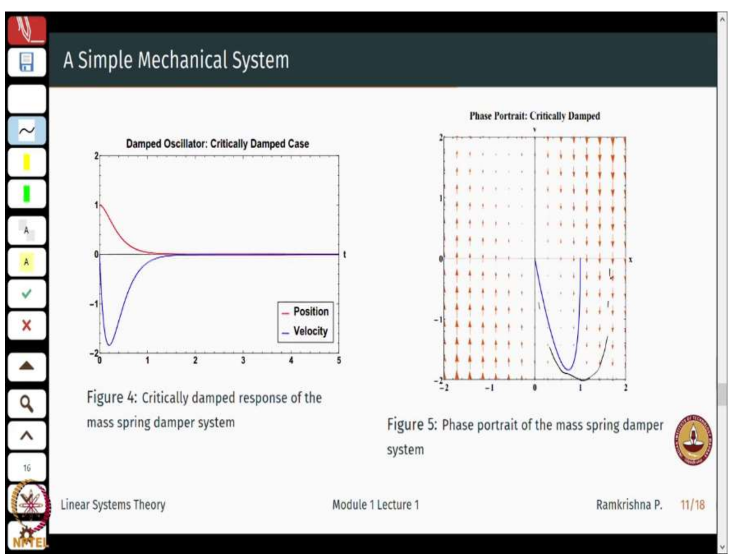

And then you havecategorization, if you remember something from the basic undergrad course of critically damped systemsand over damped system.Critically damped system will have a phase portrait which looks like this and similarly for all initial conditions it will just follow a path in this way right. So, this phase portrait will give me a general behavior for allinitial conditions.

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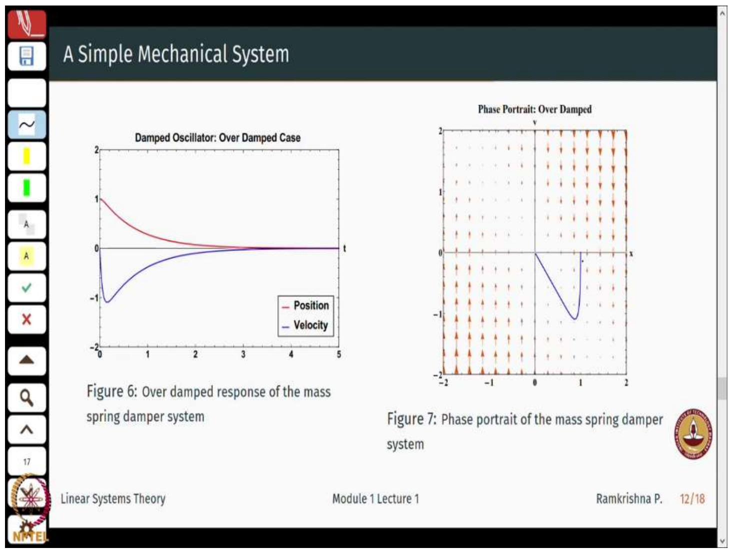

Over damped case will have a slightly slower response depending on how over damped the system at hand is right. So, again the phase portraits will tell methings related to all initial conditions ok. So, again this is nothing new here that you do not know, but the aim of this is to introduce you to a new abstraction in terms of $\dot{x}$equal toAx, earlier we called it gave it the name of a certain transfer function, but here we will talk in mostly in the time domain instead of the Laplacian domain.

We will see how they interrelate to each other the Laplacian domain and the state space domain.Another thing is to look at well of how the evolution of states happens in terms of what we called as a vector field?Who have done a course inelectromagnetics would remember this from the field theory right. So, just brought the vector field I can just guess what its divergence, curl and things like that areso.

Understanding phase space will be a bit important for us towards when we learn about stability and related concepts. So, we will touch upon all these various tools, I am just kind of scratching the surface here I just introducing you to tools that we might be usingin the later part of the course.

So, you might just want to play around with phase portraits, you can also do this for non-linearsystems and they will get a little more exciting too ok.

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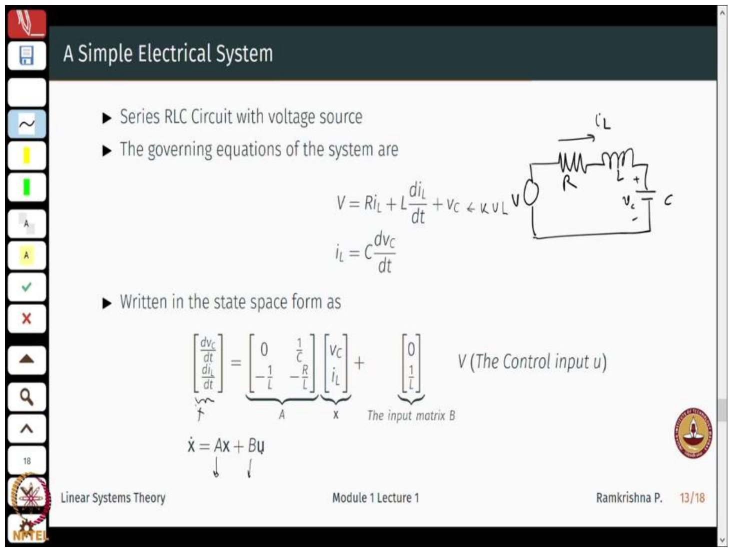

So, what aboutsay a forced system? So, if I just look at a typical electric circuit ok. So, I will just draw it here quickly. So, V,R,L and a capacitor and generating some current iL let me call this RLC and the voltage across here be Vcok. So, I donot need to teach you the first two equations right these are thebasic. So, this is comes from the basic definition of the capacitor again in the linear sense right. So, all here so, far will be linear and then V this comes from the Kirchhoff's voltage law,I can also write this in the previous abstract kind of abstraction.

So, I define my states as the voltage across the capacitor and the current through the inductor hereand the rate of the states again $\dot{x}$would just take this form right. So, I have $\dot{x}$equal to A again this A is always a constant matrix when I am talking of linear systems or linear time invariant systems all the kinds of systemsthat wedealt with like so far.And extra thing here we see is that I have something here,that is because essentially because it is the force system, earlier I just had the right hand sideto be 0.

So, here I have the external voltage, you can call it the control input orwhatever. So, I haveinput matrix B and the control input u. So, here it is just a single input, but you could also have systems which will have more than one input.Againwe will deal with those as and when they come, but this will in general will be a structure of it a system with input.

So, I look at $\dot{x}$is A is as usual the system matrix x is the state vector,B is now the input matrix and a certain input u.And now we know how a certain RLC circuit behaves to an external signal right.

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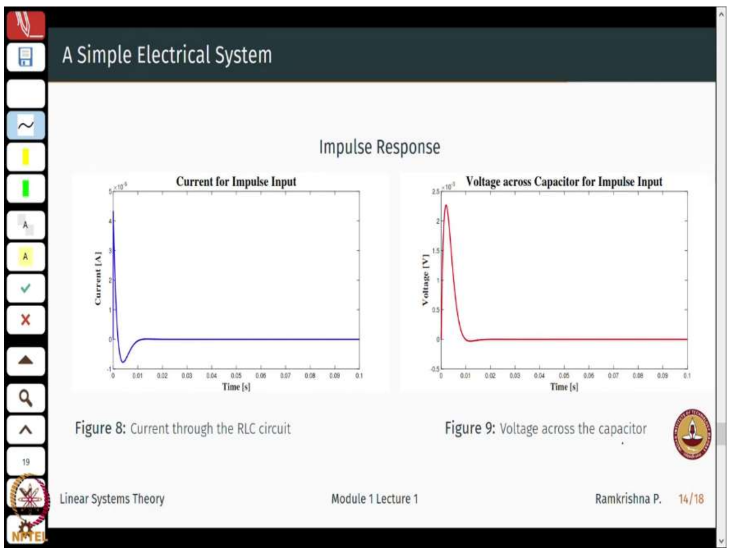

So, if I just look at an impulse that will be its natural response. So, everything will go to 0 because it will all dissipateis as goodas an unforced system in the previous case subject to certain initial conditions. So, the current will go to 0,current across the inductor or in the circuit will go to 0, similarly with the voltage across the capacitor.

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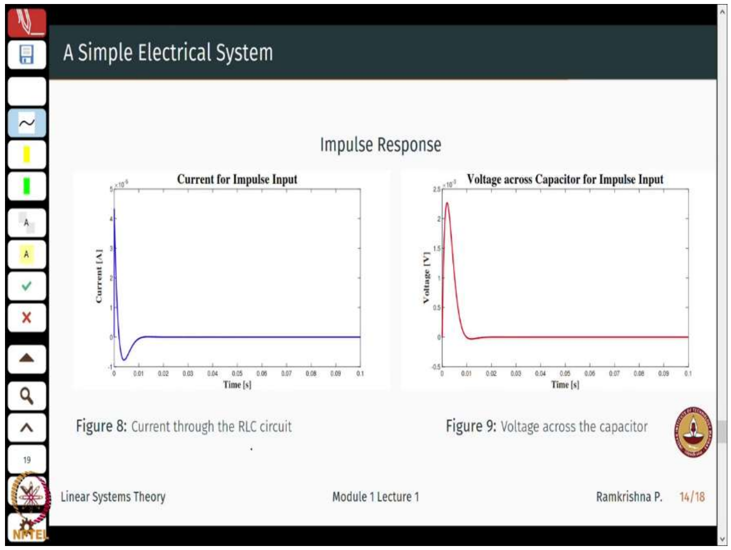

Step response well it is a little interesting that you will have well the current goes to 0, and the voltage builds up to a steady statevalue of 1 volt, this is like a unit stepok.How this happens, why this happens,abit of that we would have done in maybe in an earlier control course or a network course.

But to write it in how to derive it from a general expression of a state space representation that we will learn through this course of how do we make use of this A and B to arrive at these kind of expressions.Again we will reserveall those for later, we are just looking at building up some of the arguments leading to a general theoryok.

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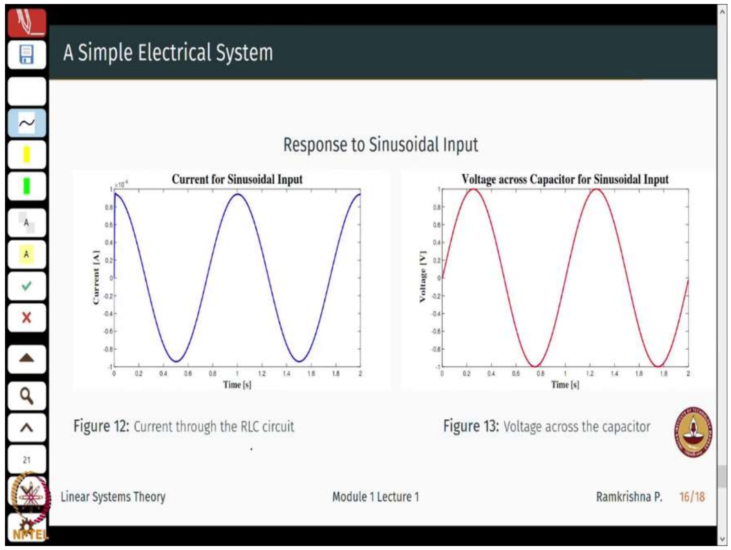

So, I also know that a good or a typical behavior of linear systems is if you; if the input is sinusoidal, the output is also sinusoidal.WhatI do much of thiswe did while we were doing the frequency response; bode plots and nyquist plots and things like that. So, this also we know right that the input to a sinusoidal signal, the output will be a sinusoidal signalwith a little change in magnitude and phase and that will depend on my parameters R L and C of the circuit.

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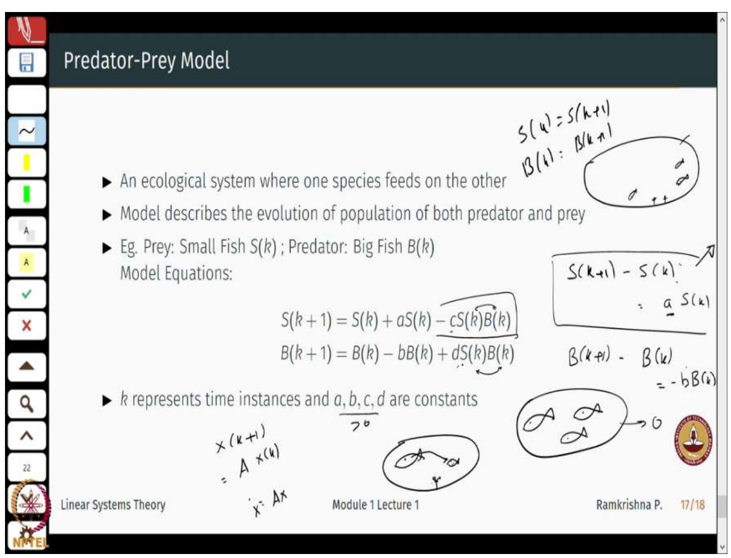

So, this was all about linear time invariant systems, which I could write in this formxdot equal to $Ax + Bu$,this all continuous time right. So, the $\frac{dV_c}{dt}, \frac{di_l}{dt}$ allsignals that are continuous in time.

Lot of times we also talk ofdiscrete systems so, if this wereordinary differential equations we couldwe also knowa bit about difference equations. So, let us talk of a simple example herein our earlier control course we had done this in the continuous case, but we will just have a discrete equivalent of that. So,again this also comes from physics right. So, physicists have spentlot of time doing the predator prey models,they also referred to as the LotkaVolterra modelsand there will be a bunch of literature on thatfrom different domains computational domains, linearization domains,Hamiltonian mechanics point of view that its bi-Hamiltonian system and so, onthat is not the point of interest.

So, let us see what kind of models which we can evolve from this. So, the model is kind of pretty simple or the problem is pretty simple or we are actually working at a very simpleversion of it. So, I have say a pond and also have some kind ofsmall fish,who feed on some weeds here.Andnow I just want to see the evolution of this is when I say evolution, I just want to see wellhow do thisfish evolved with respect to certain initial condition.

So, all the models of interest you start building with some assumptions andthen slowly realize those assumptions.The first assumption I can make here is that well if that this fish they have lot of food to eat and the resources are infinite and they almost never die ok. So, what would I expect even before I write downan equation that, there are say some initial condition say some 1000fish in a pond, they multiply they have infinite resources.

So, I will expect them to grow to infinity ok.What does that mathematically mean that I canwrite this as a model there? So, if Isay take a difference say. So, this k here would be my time instant or a sampling instant, if I call it in terms of the signals perspective. So, the differenceof the number of fish at time k and a time k+1 it will always be increasing right because I know that they donot die naturally and they have infinite resources to feed on. So, this will be in some proportion to theto the number of fish at the k th instant. The multiplicity will or the multiplying of the population will depend on the initial condition which has started at t=0 if I have 1000, t=1 I may have1002 and so on.

So, if am looking at what will happen at k +1, it will largely depend on what is the condition atsome time instant k ok.And this is some proportionality constant which will depend onwell how much isthe food or constant or it keeps on going or coming. So, my expansion will depend also on maybe if there islot of food sometime, maybe the food shrinks a bit and so, on, but again since I am assuming that its infinite I just say that well this some proportionality constantok. So, let us see another situation where I have another pond, here I have like a lot of big fisha lot of themok, but they have nothing to feed on.

So, what would I expect and I also assume that well if they have nothing to feed on that they will eventually die. So, what would I expect that after a certain time instant, could be a year depending on the population, could be 2 years or so onthat the number of big fish will eventually come to 0ok. So, if I look at compare the number of big fish at an instant kand compare it withk+1,I would imaginethat it will see a decrease in population right. So, well this and the decrease will depend on what was the population at time k and some proportionality constant, which will define the rate of the dk ok. Here in the first case it always increases second case it always decreases ok.

Now, this is how like how we build model based on what our intuition says.Intuition will say that the population here will rise will go unbounded or in mathematics what we call it will go to infinity,here the population will you know cease to be 0 and the species might become extinct so, to speakok. Interesting things will happen if I merge these two; maybe make a pondwhere there arethis big guys, there are smallerguysthere is some food for, the smaller guys and the big guys feed on the smaller guys.

So, the big fish feed on the smaller fishnow interesting things happen right. So, what will you expect to happen is that, since;so, this guys the small fishpopulation was increasing all the time ok. So, now, when there is the big fish feeding on them you will see a certain decrease in population right. So, therefore, I put this c here andall these constants here are greater than 0 by the way.

So, you will see thatthe population increases when they are by themselves,when they are interacting with bigger fish the population decreases that is captured by this term, and the rate of decrease of population will depend on how frequently they interact with each other and in a certain d k term also there.Similarly what will happen to the big fishwhenI have small fish, well these guys will have something to feed on. So, I will expect them to grow right and then the growth again will depend on how frequently they interact with each other and of course, some proportionality constant right. So, this is I cannot write it as any linear system here right, because you see some kind of non-linearity here at the interaction between S and B.

So,I cannot write it as certain see the discrete version of $\dot{x} = Ax$plus $\dot{x} = Ax$ would bethis one $x(k+1)$is Ais $Ax(k)$ this is essentially the discrete version of $\dot{x} = Ax$.I cannot write it because it is some non non-linearity in this; however, what do I;how if I just start with an initial condition,what would I expect?Would I expectsay initial condition not saying that there is there are 2 small fish and say 2000 big fish, then I know that withinthat the entire model will again collapse to 0.Because the two small fish will be eaten almost immediately and then the big fish will have nothing to eat and everything goes to 0also the contrary right.

So, if I havemaybe a lot ofsmall fish say 10000 andjust 2 big fish in the opposite can might happen that one of the species might actuallygo to infinity and so onok. So, assuming those kind of extreme conditions areexcludedwhat do we expect or that the behavior of the small fish and big fish to happen of course, one is that if you start with an equilibrium position then it will always be like that right. So, I just;so, equilibrium is when $S(k)$is $S(k+1)B(k)$ is $B(k+1)$and then I will get certain conditions for $S(k)$ and $B(k)$ right that will depend on the constants a c b and d,we will not derive that that is pretty straightforward here.

So, if I started that initial condition, the population will be similar throughout no matter what happens with the growth and dk right they will still follow the same pattern, but they will just be constant forever.Little around that, that equilibrium points say I just say perturb it a little bit. So, what will happen is the following right.

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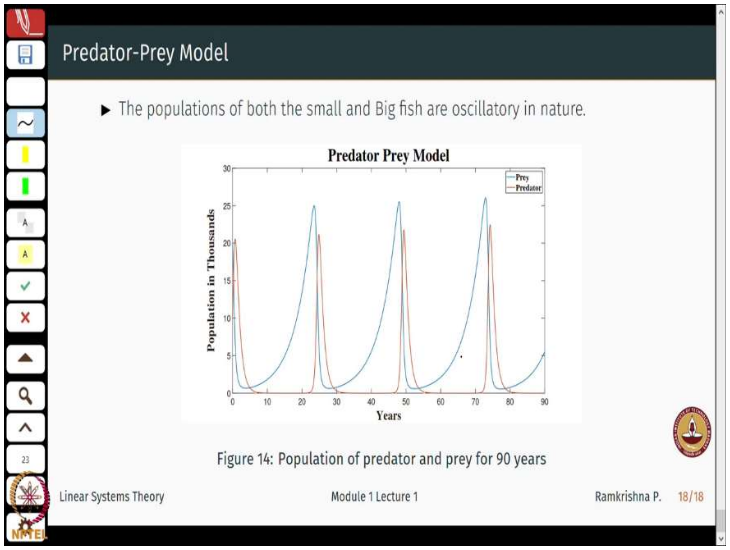

So, at some point of time you see well there will be an increase in the number of small fish or the big fish will have more to feed on,that will decrease in some way the population of the small fish and so on. So, what we see is that the populations have some kind of an oscillatory behavior. If they just plot it for say population in 1000on the x axis and say over a period of say 90 or 100years I see some pattern like this ok. Look at this you can look at this look at like the phase base of the systems and so on.

So, we started off with lineartime invariant system. So, this is a discrete non-linear systemwe will formalize that a little later,which will give some kind of an oscillatory behavior.And in the first two models here I actually knew this equation from physics or as what they call also as first principles right. So, I knowthat the Kirchhoff's voltage laws I know the current laws,Newton made my life easy to model a mass spring damper system and so on, but here the models are well nobody gave me a model here right.

So, I am just building modelsas an abstraction to what I would observe when I can maybe validate those models. So, none;so, here it's partly by intuition and partly by observation also and partly by what I would know abouta bit of evolution theory. So, that concludes the first part of this lecture and we will do some more examples in the next part.

Thank you.