Electromagnetic principles
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Module 2.2
Electromagnetic principles - Uniform plane waves

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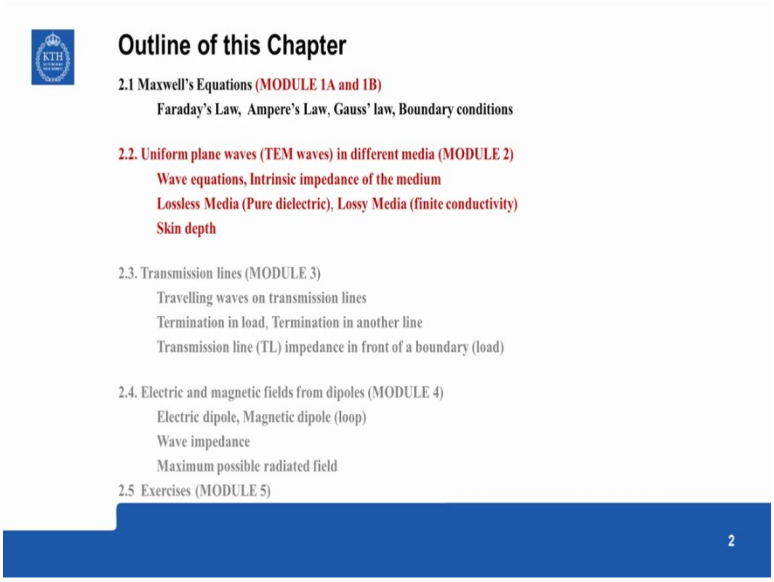

Review of electromagnetic principles, we will continue with that chapter. Previously we have seen details of Maxwell's equation module 1A and 1B. In this module that is module number 2, we will look into uniform plane waves in different media.

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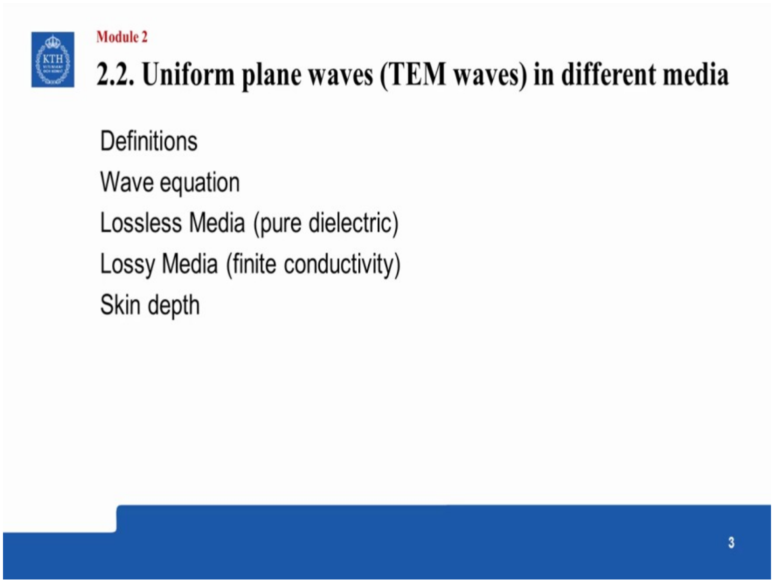

First we will define what is meant by uniform plane waves and what is meant by transverse electromagnetic waves or TEM waves. We will derive the wave equation, then after that we will look into wave propagation in pure dielectric that is lossless media, then in a lossy media which has got a finite conductivity, we will also introduce the concept of skin depth which will be used several times in the remaining chapters.

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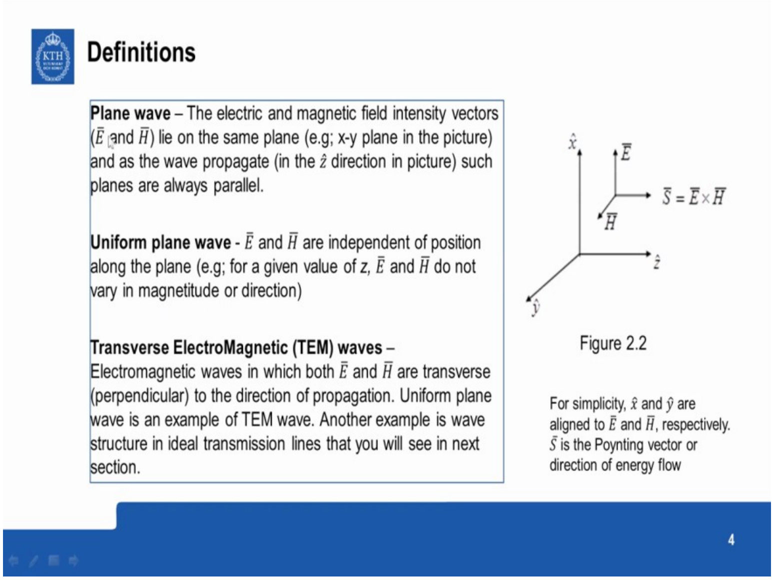

What is meant by plane waves? For this look at the sketch over here in Cartesian coordinates. Imagine that you have a wave that is travelling in the positive z-axis direction for convenience. Then a plane perpendicular to this direction lies in the XY plane. Imagine a sheet lying along XY plane and if all your electric field vectors and magnetic field vectors are lying in this plane then we call it as a plane wave. So in a plane wave, electric and magnetic field vectors are in a plane perpendicular to the direction of propagation so in this case in the XY plane.

Now what is meant by a uniform plane wave? Imagine that the E vector and the H vector has the same value independent of the position in its XY plane that is everywhere along the XY plane, then we call it as uniform plane wave so it is uniform all along this plane. Now what is meant by transverse electromagnetic waves? So uniform plane wave is a special case of waves called transverse electromagnetic waves. It only means that E vector and H vector lies in a plane that is perpendicular to the direction of propagation. Now another example of transverse electromagnetic waves is a transmission line that is two parallel wires or one wire above ground plane, we will consider transmission lines in later modules.

Now just from this figure we can derive some properties for this electric and magnetic field vectors say for example, E and H vector do not have any component in the direction of propagation and they are denoted with respect to X and Y because it is uniform. Now we write these properties in terms of equation in the next slide.

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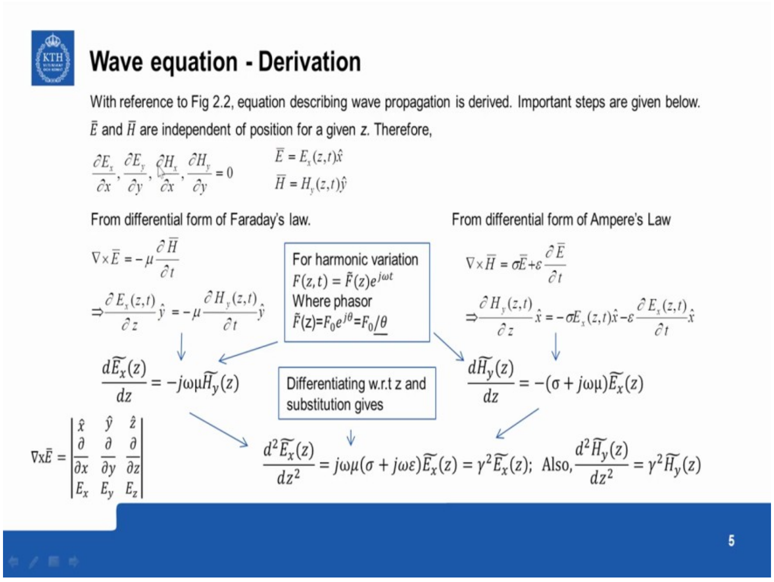

From the above figure we can say that rate of change of E with respect to X is 0, being uniform field, and E is aligned to the X direction by choice without any loss of generality and H is aligned to Y direction without any loss of generality, so the other components are all 0. Now, let us take the curl of the vector E and applying the Faraday's law that we have seen before. We take the differential form or the point form of the Faraday's law. If you now perform your vector algebra, the curl of E is defined by this matrix, here there are the unit vectors with differential symbols corresponding to X, Y and Z and these are the components of E vector.

$$\frac{\partial E_x}{\partial x}, \frac{\partial E_y}{\partial y}, \frac{\partial H_x}{\partial x}, \frac{\partial H_y}{\partial y} = 0 \qquad \begin{aligned} \bar{E} &= E_x(z,t)\hat{x} \\ \bar{H} &= H_y(z,t)\hat{y} \end{aligned}$$

So since by our choice of coordinate system and without any loss of generality, E is aligned with X direction, E_Y and E_Z are 0. Similarly, rate of change of E vector with respect to X and with respect to Y are also 0, so this curl can be simplified by this equation over here that is curl of E equals rate of change of magnetic flux density with respect to time is simplified like this. When S vector is moving forward in the z-direction, vector E can change.

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \Rightarrow \frac{\partial E_x(z,t)}{\partial z} \hat{y} = -\mu \frac{\partial H_y(z,t)}{\partial t} \hat{y}$$

Now let us consider the time harmonic variation, so this is the function of space and time so if the time variation is in the harmonic manner, in the sinusoidal manner, which in frequency domain is represented by E to the power J Omega t, where Omega is the angular frequency and F with tilde on top this is the phaser and that phaser can be written as magnitude of the phasor and this is angle with respect to your reference so we can write it like this also F_0 angle Theta. Now if you substitute this into this equation and similarly same equation can be written for magnetic field also in phaser form then E to the power J Omega t in both sides will cancel each other and what is remaining is this equation, rate of change of E with respect to Z equal to - J Omega, J Omega is coming from the differentiation of this, J Omega Mu H so this is one of the equations.

For harmonic variation

$$F(z,t) = \widetilde{F}(z) e^{j\omega t}$$

Where phasor $\widetilde{F}$ (z)=

$$\frac{d\widetilde{E_x}(z)}{dz} = -j\omega\mu \widetilde{H_y}(z)$$

Now similarly, from the differential form of ampere's law states that curl of magnetic field intensity vector is equal to the sum of conduction current and displacement current. So this can be simplified taking the curl here, instead of E it will be H in this form and in phaser notation we can simplify it in this way, where d by dt of E vector will yield J Omega here. Now, from this equation and this equation combined that is differentiating with respect to Z and substituting in each other we get the wave equation total like this, so 2^{nd} derivative of E with aspect to Z equal J Omega Mu Sigma + J Omega Epsilon multiplied by E.

$$\nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \Rightarrow \frac{\partial H_y(z,t)}{\partial z} \hat{x} = -\sigma E_x(z,t) \hat{x} - \varepsilon \frac{\partial E_x(z,t)}{\partial t} \hat{x}$$

So this value J Omega Mu times Sigma + J Omega Epsilon, so this is the function of material property in terms of conductivity and electric permittivity and the frequency Omega, so this factor is called as propagation constant square so Gamma square, so similarly with the H field also we can write a wave equation.

$$\frac{d\widetilde{H_y}(z)}{dz} = -(\sigma + j\omega\mu)\widetilde{E_x}(z)$$

$$\frac{d^2\widetilde{E_x}(z)}{dz^2} = j\omega\mu(\sigma + j\omega\varepsilon)\widetilde{E_x}(z) = \gamma^2\widetilde{E_x}(z); Also, \frac{d^2\widetilde{H_y}(z)}{dz^2} = \gamma^2\widetilde{H_y}(z)$$

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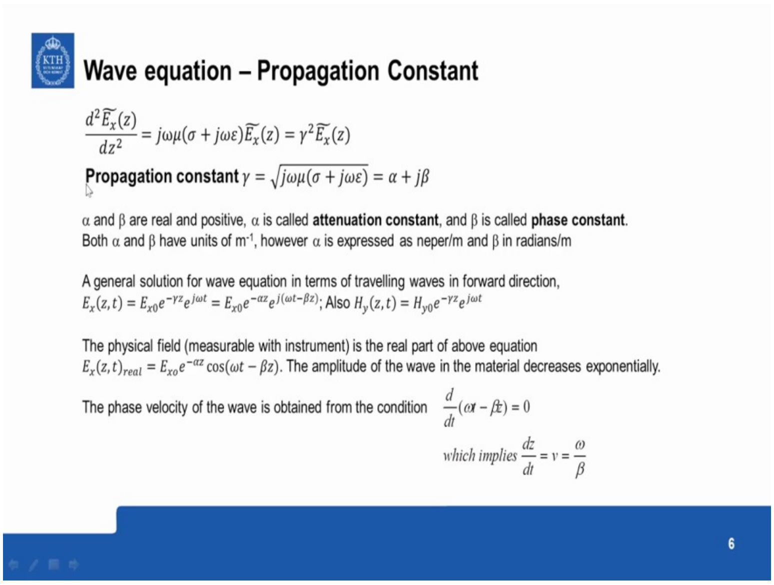

$$\frac{d^2\widetilde{E_x}(z)}{dz^2} = j\omega\mu(\sigma + j\omega\varepsilon)\widetilde{E_x}(z) = \gamma^2\widetilde{E_x}(z)$$

Propagation constant $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$ Mu (Sigma + J Omega Epsilon) that is, it is the property of frequency and the property of the material through which it is propagating. Now this can be separated into real and imaginary parts, and let us say Alpha is real part and Beta is imaginary part. Now the real part is called that attenuation constant and this imaginary part Beta is called the phase constant. Now both Alpha and Beta has unit per meter however, just to distinguish between imaginary and real part we call Alpha that is attenuation part as neper per meter and Beta as radians per meter. Now a general solution for wave equation in terms of travelling waves in forward direction is given by E as a function of Z and t equals some magnitude value E to the power - Gamma Z times E to the power J Omega t.

A general solution for wave equation in terms of travelling waves in forward direction,

$E_x(z,t)=E_{x0}e^{-\gamma z}e^{j\omega t}=E_{x0}e^{-\alpha z}e^{j(\omega t-\beta z)}$; Also $H_y(z,t)=H_{y0}e^{-\gamma z}e^{j\omega t}$

So Gamma can be expressed in terms of Alpha and Beta so if you expand it, this part is the attenuation part E raised to - Alpha Z then you have a phase part also similarly, H can also be written in this manner. Now since it has got the real part and the imaginary part, if you try to measure the field with an instrument, what we measure is the real part of the field that is the physical real part of the field and that part is given by $E_x(z,t)_{real}=E_{xo}e^{-\alpha z}\cos(\omega t-\beta z)$.

So that is the real part of the field. You can see that amplitude is varying, not only that there is a periodicity in the amplitude, not only that wave is going through the medium, it is also decreasing exponentially by this factor.

Now, the wave has a velocity into the medium and that velocity we can derive by tracking the points where $(\omega t-\beta z)$ becomes constant, or you can say that okay you pick up a fix point along the time varying and attenuating wave, so that condition is rate of change of $(\omega t-\beta z)$ with respect to time is equal to 0 and from this we get dz/dt if you differentiate both terms with respect to time, so that is the phase velocity or *v*. From this we get it as ω/β , beta is the phase constant. So this may not be speed of light because it depends upon the property of the medium, and in vacuum or air of course will be the speed of light.

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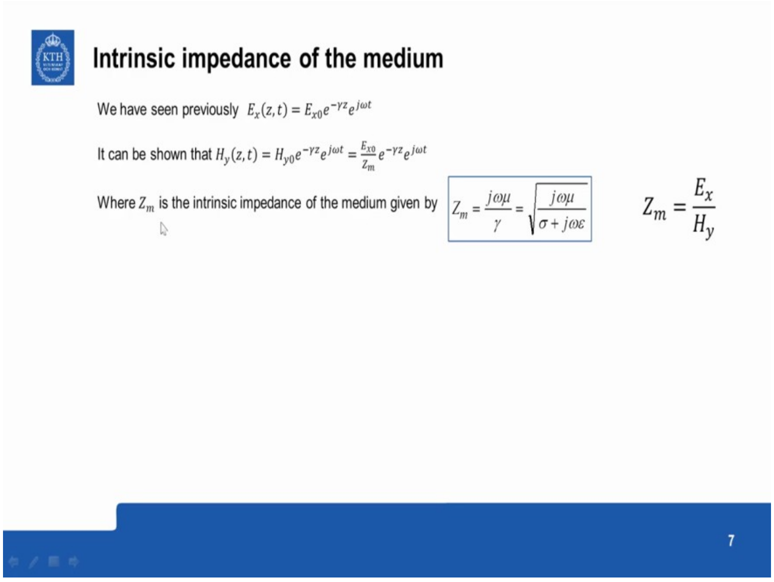

Now we can define another property of the medium, intrinsic impedance of the medium. We have seen previously the expressions for E and H, and in general we know that impedance is expressed as E_x/H_y, but even without using this you can manipulate wave equation and find that H is given by E_x/Z_m and Z_m equal to $\frac{j\omega\mu}{\gamma}$, so that is equal to $\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$. So impedance of the medium is given by the frequency as well as in properties of the medium, there are 3 basic properties for a medium; one is the conductivity Sigma, the electric permittivity Epsilon and the magnetic permeability Mu, so from this we get the impedance of the medium.

$$E_x(z,t) = E_{x0}e^{-\gamma z}e^{j\omega t}$$

$$H_y(z,t) = H_{y0}e^{-\gamma z}e^{j\omega t} = \frac{E_{x0}}{Z_m}e^{-\gamma z}e^{j\omega t}$$

$$Z_m = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

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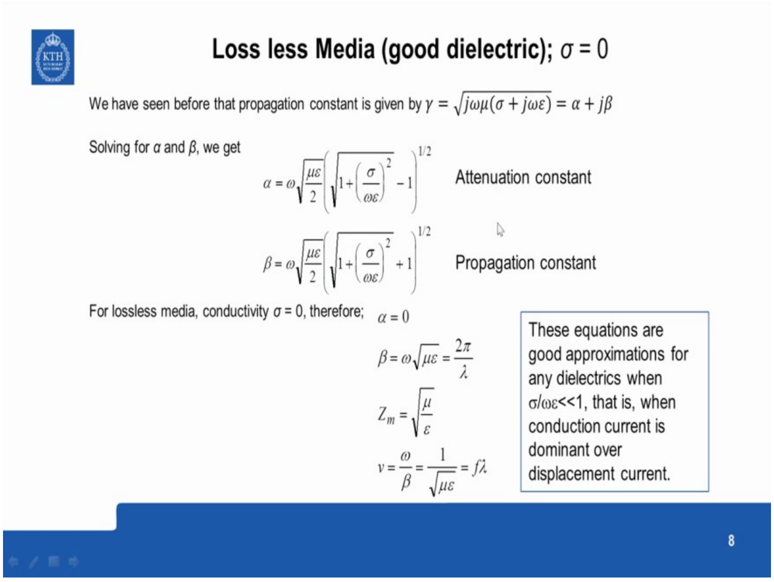

Now let us go to the lossless media that is a good dielectric, where conductivity is equal to 0. Previously we have seen propagation constant γ which can be expressed as $\alpha + j\beta$. From this equation we can solve for α and β, you know you can remove this square root by squaring both sides and then equating all the real terms to the left and right, and equating all the imaginary terms to the left and right, you will get for α this particular expression, that is the attenuation constant and for β you will get this expression that is propagation constant. You can try it at home and see that this is correct. Now α and β differs in this term over here, here it is - and there it is +.

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)^{1/2}$$

Now let us consider the lossless media where conductivity sigma is equal to 0 so we substitute this value into this, immediately we see that this is (alpha) equal to 0 but this (beta) will not be equal to 0 so this will be square root of 1 + 1, so basically 1 and here there is a square root. So this square root of 2 and this cancels and you get Beta equal to Omega square root of Mu Epsilon which is nothing but $2\pi/\lambda$. Now intrinsic impedance becomes square root of Mu by Epsilon that you can see by substituting Sigma = 0 in this special form intrinsic impedance of the medium that we have seen before. Now phase velocity V = Omega by Beta that is equal to substituting for Beta, one by square root of Mu Epsilon which is nothing but frequency times lambda. When $\sigma=0$,

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

$$Z_m = \sqrt{\frac{\mu}{\varepsilon}}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} = f\lambda$$

These equations are strictly true for sigma equal to 0 but we can use these equations with very good approximation for any dielectric when we have $\sigma/\omega\varepsilon << 1$. So when $\sigma/\omega\varepsilon$ is far less than 1, you get something like 0.002 and still these expressions can be valid. So we can say that when conduction current is represented by conductivity σ and $\omega\varepsilon$ is displacement current. So when conduction current is dominant over the displacement current or $\sigma/\omega\varepsilon << 1$, these equations are true we can say like that.

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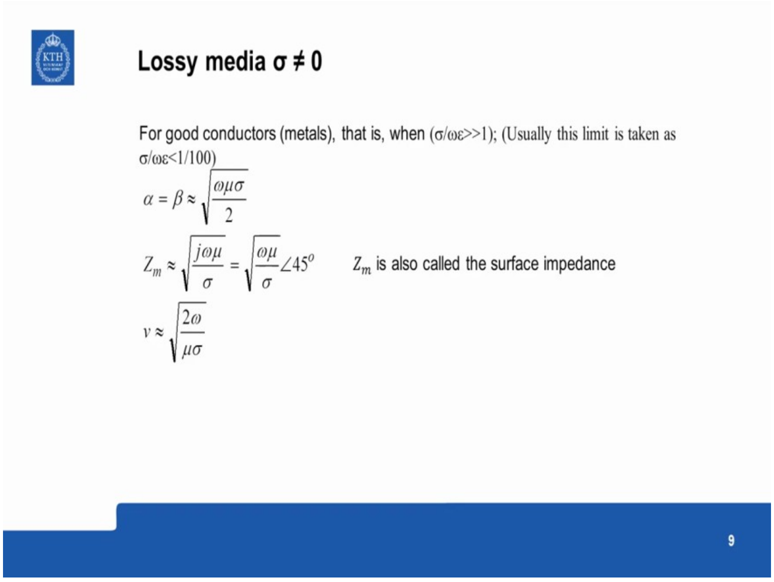

Now consider lossy media where Sigma is not equal to 0, now there is a special case of lossy media and the good conductors like metals like copper, iron or aluminium, where the conduction current is far dominant over the displacement current, displacement current is negligible in those and conduction current is very high so $\sigma/\omega\varepsilon >> 1$. Usually this limit is taken as $\omega\varepsilon/\sigma << 1/100$.

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So in that case if you go to the previous slide (expressions for $\alpha \wedge \beta$¿ this under brackets is far greater than 1, so 1 can be neglected, so you can simplify as $\sigma/\omega\varepsilon$. So in that case you will see that both are Alpha, Beta are equal because whatever is in this bracket is simplified as square root of $\sigma/\omega\varepsilon$.

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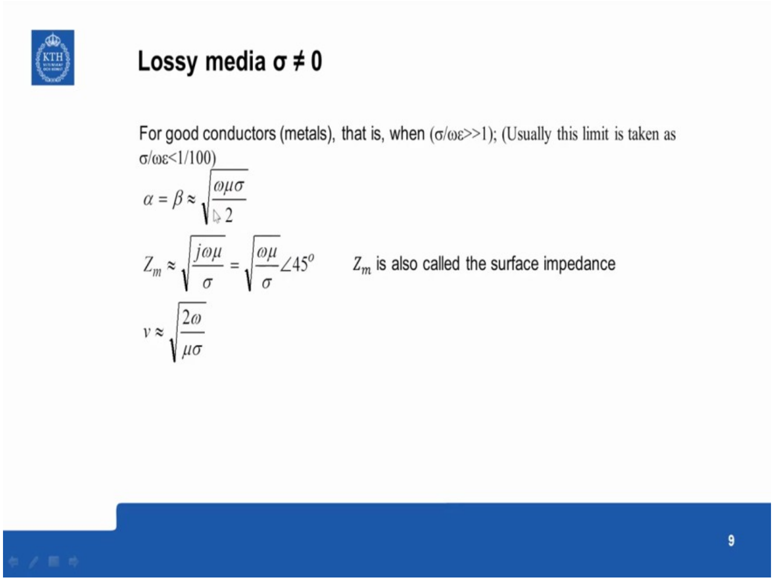

So you see that Alpha equal to Beta and which is supposed to be equal to square root of Omega Mu Sigma by 2. And intrinsic impedance of the medium is given by square root of J Omega Mu by Sigma or Omega Mu by Sigma square root of J, in terms of phaser notation we can take it as angle 45 degree so J represents angle 90 degree, this is called surface impedance in the case of very good conductors or metals because the field is not very much penetrating into the metal, so this is also called surface impedance or intrinsic impedance of the medium. And phase velocity V is equal to square root of 2 Omega by Mu Sigma. You can see that phase velocity is depending upon the frequency, it changes with frequency and it depends upon the magnetic permeability as well as conductivity of the medium, now we can look into the consequences of this, before that we define skin depth.

$$\alpha = \beta \approx \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$Z_m \approx \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$$

$$v \approx \sqrt{\frac{2\omega}{\mu\sigma}}$$

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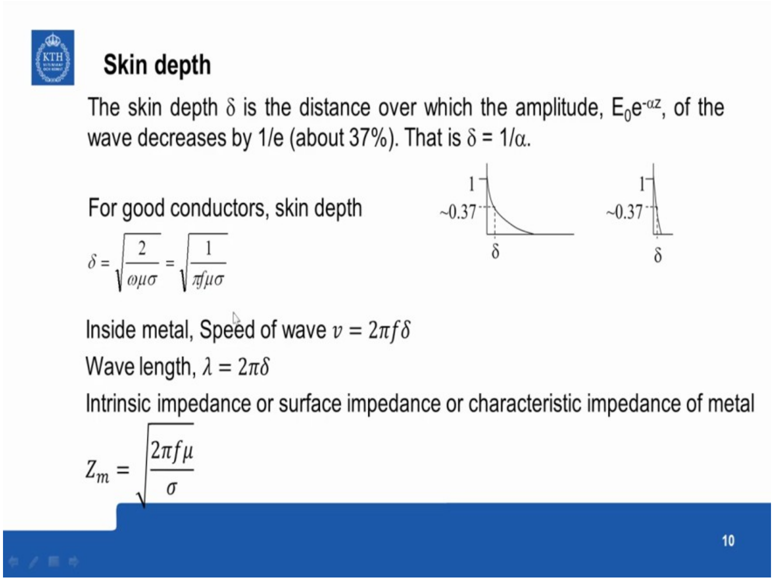

So what is skin depth? You know that waves are penetrating into the metals but it will be attenuating very fast, so skin depth δ is defined as a distance on which the amplitude

$E_0e^{-\alpha z}$ of the wave is decreasing by 1/e where e is the base of the natural logarithm. So 1 by e is about 0.37 so this is an exponentially changing right, suppose E 0 we take it as 1 here, now this is exponentially decreasing so this is the surface of the metal let us say and this is the depth of the metal. So as we go into the metal, when this wave amplitude becomes 0.37 or 1 over e, we call this depth as 1 skin depth. So you will see that skin depth is equal to square root of 2 by Omega Mu Sigma, you can verify it yourself and that is equal to ω is 2πf so square root of 1 by Pi F Mu Si.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

So this is a function of frequency, as the frequency is increasing you see that skin depth is decreasing and it also depends upon Mu and Sigma. And similarly we can verify that inside the metal speed of the wave V equal to 2 Pi F Sigma. At home you can substitute in your equation and verify all these expressions, so wavelength λ is nothing but 2πδ that is skin depth. So intrinsic impedance or surface impedance or characteristic impedance, you can call by any of these 3 names. So intrinsic impedance of metal is equals $\sqrt{\frac{2\pi f\mu}{\sigma}}$.

$$Z_m = \sqrt{\frac{2\pi f \mu}{\sigma}}$$

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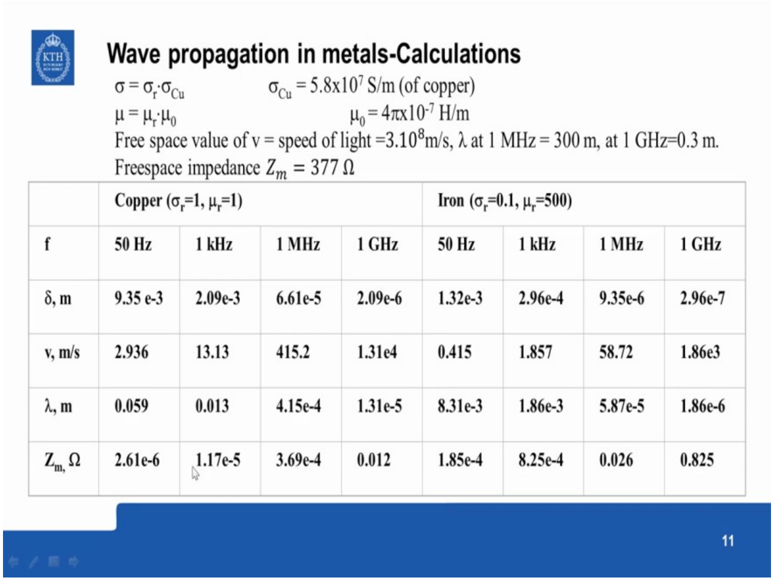

	Copper (σ_r=1, μ_r=1)				Iron (σ_r=0.1, μ_r=500)			
f	50 Hz	1 kHz	1 MHz	1 GHz	50 Hz	1 kHz	1 MHz	1 GHz
δ, m	9.35 e-3	2.09e-3	6.61e-5	2.09e-6	1.32e-3	2.96e-4	9.35e-6	2.96e-7
v, m/s	2.936	13.13	415.2	1.31e4	0.415	1.857	58.72	1.86e3
λ, m	0.059	0.013	4.15e-4	1.31e-5	8.31e-3	1.86e-3	5.87e-5	1.86e-6
Z_m, Ω	2.61e-6	1.17e-5	3.69e-4	0.012	1.85e-4	8.25e-4	0.026	0.825

So let us do some calculation, so I have done all this calculation and represented it in the form of a table here and want to points out some of the properties. Let us first define conductivity, permeability, now conductivity of copper is very high perhaps one of the highest in the common kind of metals that are in use in electrical engineering, this is 5.8 into 10 to the power 7 Siemens per meter. So usually the materials that we use in electrical engineering are copper, aluminium, iron or some combinations of that so among them this has got the highest conductivity so we take it as a reference for easiness. So conductivity can be represented as reference conductivity, times conductivity of copper. And permeability we know of a media is reference permeability or relative permeability multiplied by the permeability of vacuum or air, and permeability of air is defined as $4\pi \times 10^{-7}$ H/m

Free space value of V or speed of light is 3 into 10 to the power 8 meter per second, so we can easily see that the wavelength at 1 megahertz is nothing but 300 meter that is 3 into 10 to the power 8 meters per second divided by frequency 10 to the power of 6, so you get 300 meters so wavelength at 1 gigahertz in free space or air is 0.3 meter. Similarly we can say that free space impedance is equal to 377 ohms. Now you remember these reference values, so in free space you know wavelength is of the order of meters we can say or fraction of a meter and impedance is of the order of 100 of Ohms, 377 Ohms.

Now let us consider this in medium copper and iron; so for copper relative conductivity is 1 and it is a non-magnetic material so μr = 1. In iron in this particular iron let say the conductivity is already 10 percent of copper so 0.1, so the 0.5 10 to the power 7 Siemens per meter iron is magnetic so relative permeability let us say it is 500. So here we have this skin depth velocity in the medium, wavelength in the medium and the impedance in the medium, and here we have frequency 50, 1, 1 megahertz, 1 gigahertz, same thing for iron. Now the skin depth you can see this is 10 to the power – 3 meter, so this is basically in terms of millimetre, at 50Hz the wave will penetrate into iron and within 9.35 millimetre it is reduced in value by e that is 0.37 of the original value so that is my skin depth and it is travelling at a velocity of 2.936 meters per second, so you see how different the velocity wave is in metal.

Here in free space it is 3 into 10 to the power 8 meters per second that is 300 million meters per second and here it just 2.936 meters per second extremely slow. And what about lambda? Lambda is 0.059 meters only, in free space what will be lambda? At 50 hertz it will be several thousands of kilometres, so several thousands of kilometres versus fraction of a meter that is 5.9 centimetres wavelength in metal that is in copper and the intrinsic impedance of the medium here is 2.61 milli Ohms, in free space it is 377 Ohms. So you see that properties of waves in metals are very different from that in air or free space so this is very important in EMC studies, this distinction.

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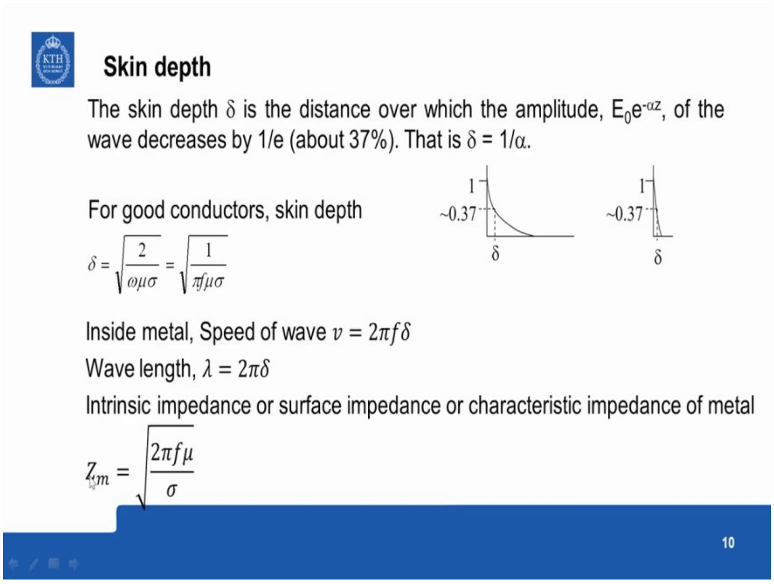

Now as the frequency is increasing, we have seen over here what happened to skin depth, skin depth should be decreasing, impedance is increasing then the velocity is increasing and the wavelength is decreasing, so let us see here kilohertz.

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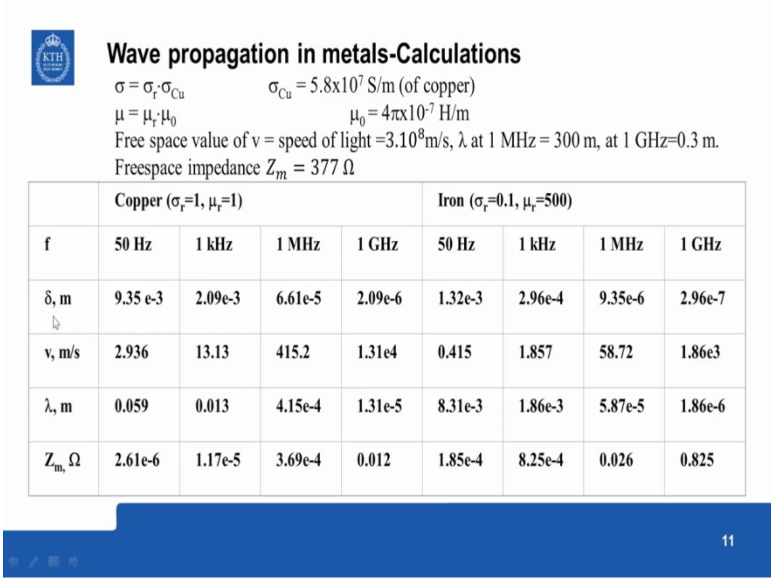

This is decreasing from 50 hertz, velocity is increasing, lambda has decreased further, the impedance has also decreased further, so similarly you can calculate 1 megahertz and 1 gigahertz. Now for iron with 10 percent conductivity of copper, but 500 times more permeability we have calculated the values here, so you can take 1 kilo hertz, you can see that skin depth is even smaller in here which is more difficult to penetrate mainly because of this permeability, we can see how it is happening.

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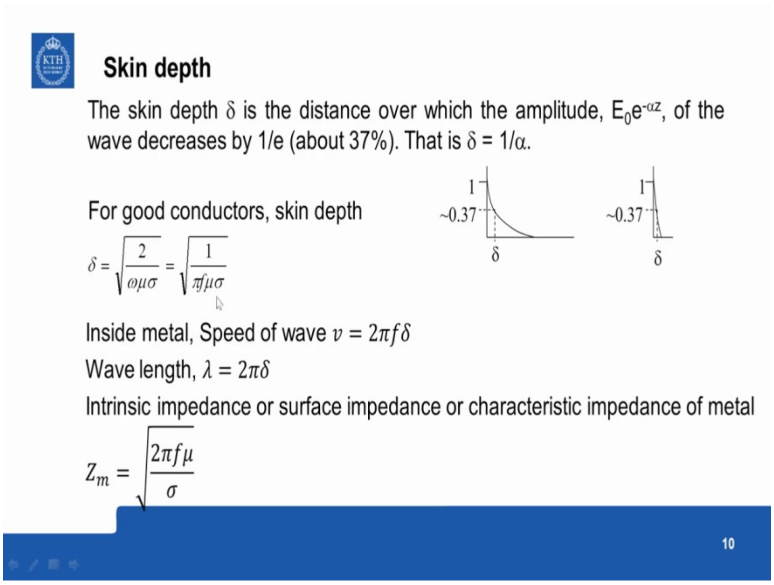

Skin depth, it is the property of product of Mu and Sigma, so 0.1 and 500 so it becomes 50 here whereas for copper it is 1 so that is why you see the difference.

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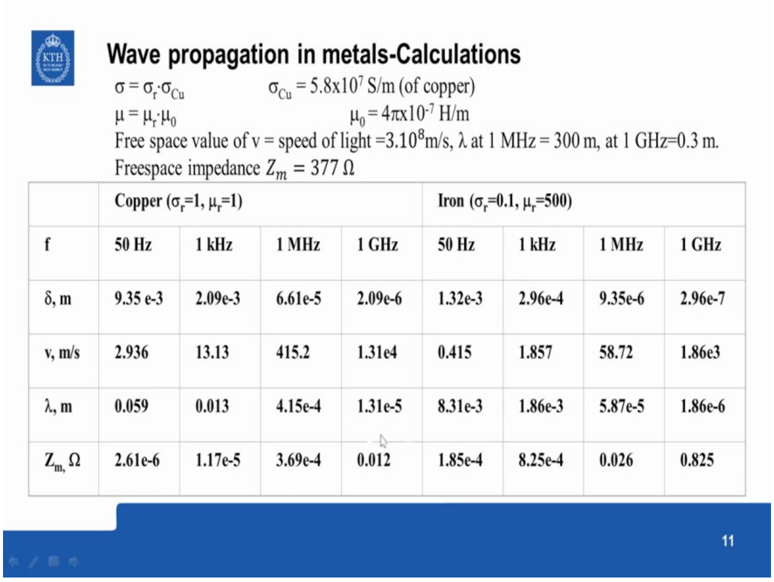

	Copper (σ_r=1, μ_r=1)				Iron (σ_r=0.1, μ_r=500)			
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Z_m, Ω	2.61e-6	1.17e-5	3.69e-4	0.012	1.85e-4	8.25e-4	0.026	0.825

So for iron it is more difficult to penetrate because of the relative permeability, so you get lambda but impedance of the medium is more compared to that in copper so you can do this calculation yourself and verify at home as homework.

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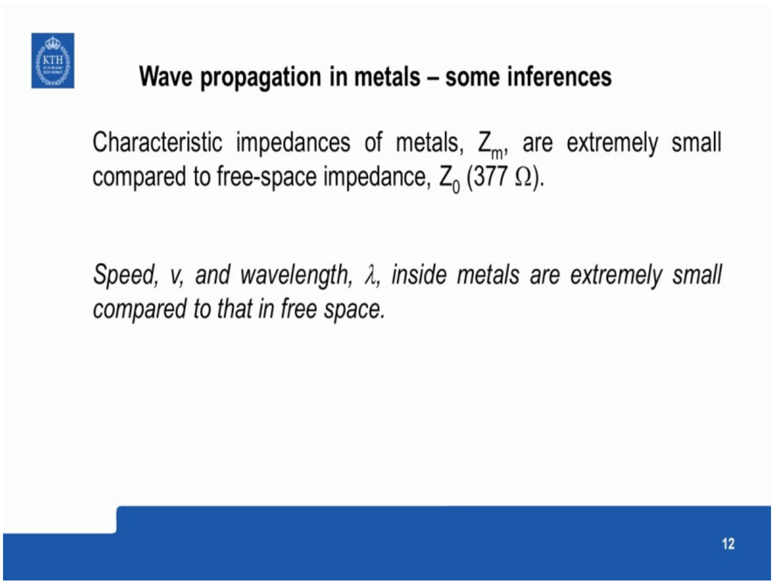

Now some of the inferences that we can take of the table that we have seen, first of all the characteristic impedance of metals are extremely small compared to free space impedance, this is one point that we can take away from this chapter. Second, speed V and wavelength the lambda inside metals are extremely small compared to that in free space, so this is other inference that we can take away and it has got consequences when we talk about how metals can be used to shield electromagnetic waves. Shielding property of metals that you see in hapter 5 derives from this property.