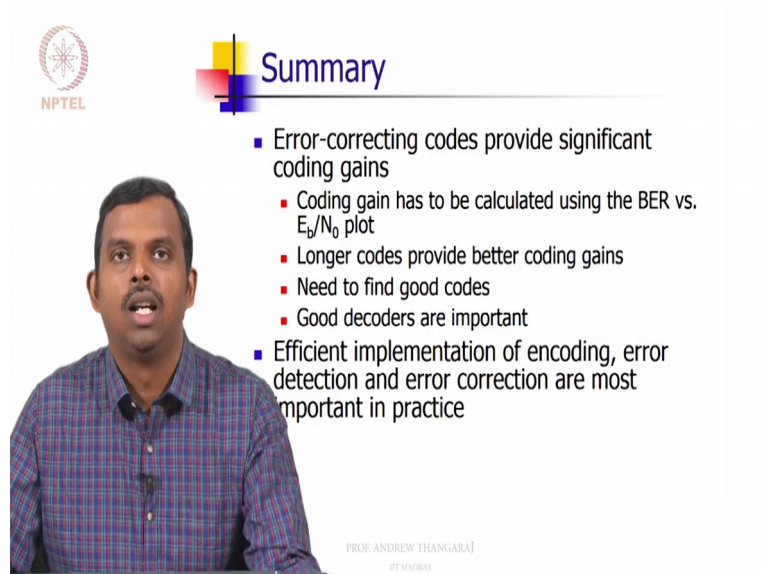


LDPC and Polar Codes in 5G Standard
Professor Andrew Thangaraj
Department of Electrical Engineering
Indian Institute of Technology Madras
A Brief Introduction to Linear Block Codes

(Refer Slide Time 00:16)



Summary

- Error-correcting codes provide significant coding gains
 - Coding gain has to be calculated using the BER vs. E_b/N_0 plot
 - Longer codes provide better coding gains
 - Need to find good codes
 - Good decoders are important
- Efficient implementation of encoding, error detection and error correction are most important in practice

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Ok. Welcome to this lecture. We are going to proceed talking about error control codes. In particular we will study linear error control code generator matrices, parity check matrices etc as the main topic in this lecture, Ok.

So I would like to begin with the summary of what we did in the previous lecture. We saw through some simple examples then that error control codes can provide coding gains, the coding gain which to be computed in the B E R versus E_b/N_0 plot and longer codes provide good coding gains but we need to find good codes, not every big code is going to be good.

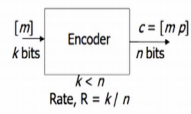
Also once we have a code, we need to be able to decode it well. We need, in particular soft decision decoders which are very important, Ok. So overall I think finding a good code and efficiently implementing and coding, decoding etc. is the main challenge and this has been overcome today and we will describe that in the rest of this course, Ok.

So let us proceed with today's lecture,

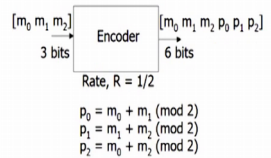
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



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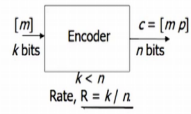
A Brief Introduction to Linear Block Codes

Ok. So today's lecture is on linear codes. We will begin by describing encoders for linear codes. So if you see here, this is a rate k by n code.

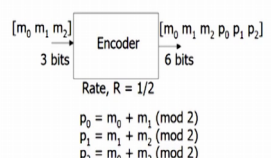
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



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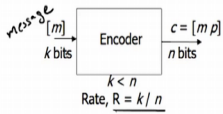
A Brief Introduction to Linear Block Codes

Ok, there are k message bits coming in. This is the message m . There are k message bits

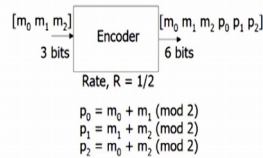
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



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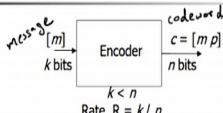
and this is the codeword c .

And you can see

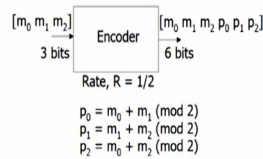
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



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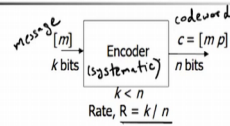
A Brief Introduction to Linear Block Codes

that I have assumed that the encoder is systematic in the sense that the message appears as a part of the codeword.

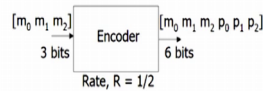
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$$p_0 = m_0 + m_1 \pmod{2}$$

$$p_1 = m_1 + m_2 \pmod{2}$$

$$p_2 = m_0 + m_2 \pmod{2}$$

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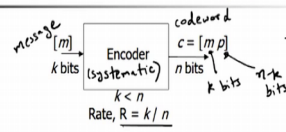
A Brief Introduction to Linear Block Codes

Ok, message will appear in the part of the codeword. I have chosen n bits for the codeword, first k bits is the message and n minus k bits is the p

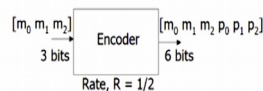
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$$p_0 = m_0 + m_1 \pmod{2}$$

$$p_1 = m_1 + m_2 \pmod{2}$$

$$p_2 = m_0 + m_2 \pmod{2}$$

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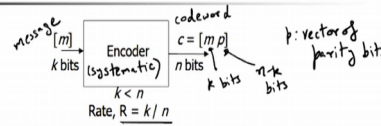
part and this p vector is the set vector of parity bits, Ok.

So this is

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Basics of linear codes: Encoder

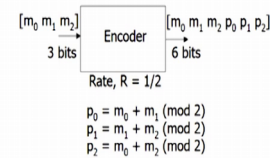


message $[m]$ k bits → Encoder (systematic) → codeword $c = [m \ p]$ n bits

$k < n$
Rate, $R = k / n$

p : vector of parity bits

Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$[m_0 \ m_1 \ m_2]$ 3 bits → Encoder → $[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2]$ 6 bits

Rate, $R = 1/2$

$p_0 = m_0 + m_1 \pmod{2}$
 $p_1 = m_1 + m_2 \pmod{2}$
 $p_2 = m_0 + m_2 \pmod{2}$

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A Brief Introduction to Linear Block Codes

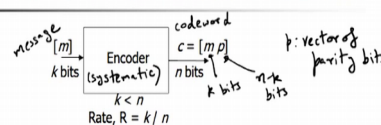
the most common picture of error control code. You have a message. You send that message. Along with that message, to protect it from errors and provide coding gain we add parity bits and sent that as a codeword. Ok

So the rate is k by n , k is less than n , this is the picture. So here is a simple example. So it is always good to have a simple representative example and then understand the whole thing. And here is an example of how a linear block code would work. You have a rate half, 3 bits for the

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Basics of linear codes: Encoder

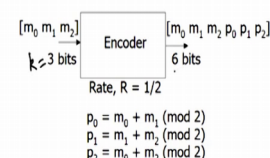


message $[m]$ k bits → Encoder (systematic) → codeword $c = [m \ p]$ n bits

$k < n$
Rate, $R = k / n$

p : vector of parity bits

Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$[m_0 \ m_1 \ m_2]$ $k=3$ bits → Encoder → $[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2]$ 6 bits

Rate, $R = 1/2$

$p_0 = m_0 + m_1 \pmod{2}$
 $p_1 = m_1 + m_2 \pmod{2}$
 $p_2 = m_0 + m_2 \pmod{2}$

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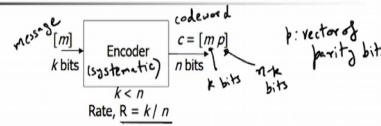
A Brief Introduction to Linear Block Codes

message and 6 bits for the

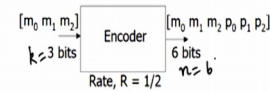
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$p_0 = m_0 + m_1 \pmod{2}$
 $p_1 = m_1 + m_2 \pmod{2}$
 $p_2 = m_0 + m_2 \pmod{2}$

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A Brief Introduction to Linear Block Codes

codeword, Ok.

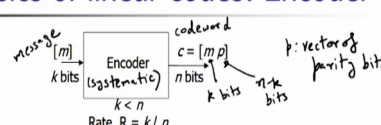
And m_0 , m_1 and m_2 are the three message bits and the codeword is composed of m_0 , m_1 , m_2 first and then 3 parity bits, p_0 , p_1 , p_2 . And how are these parity bits computed? They are computed by linear operations, linear operations like addition etc. but there is this modulo 2.

So what is this

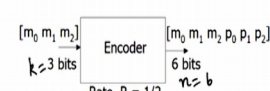
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Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$p_0 = m_0 + m_1 \pmod{2}$
 $p_1 = m_1 + m_2 \pmod{2}$
 $p_2 = m_0 + m_2 \pmod{2}$

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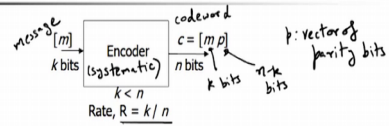
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modulo 2 addition? Ok so this is quite

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Basics of linear codes: Encoder

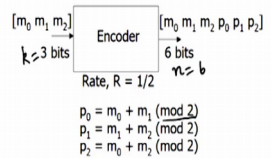


message $[m]$ k bits → Encoder (systematic) → codeword $c = [m \ p]$ n bits

$k < n$
Rate, $R = k/n$

p : vector of parity bits

Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$[m_0 \ m_1 \ m_2]$ $k=3$ bits → Encoder → $[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2]$ $n=6$ bits

Rate, $R = 1/2$

$p_0 = m_0 + m_1 \pmod{2}$
 $p_1 = m_1 + m_2 \pmod{2}$
 $p_2 = m_0 + m_2 \pmod{2}$

(binary XOR)
mod 2 addition

inputs	output
0 0	0
0 1	1
1 0	1
1 1	0

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A Brief Introduction to Linear Block Codes

a very important bread and butter operation for us.

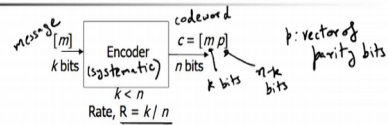
You should be comfortable with XOR, given 2 bits you should be able to quickly XOR it. XOR is exclusive OR. The bits are equal means the XOR becomes zero, the bits are not equal, 0 1 or 1 0, the XOR becomes 1, Ok. So that is the operation, Ok.

So that is what we are going to do here. So how are we doing this? So you can see p_0 , p_1 , p_2 are obtained as XORs or modulo 2 addition of a subset of message bits. So this is a generally true statement for

(Refer Slide Time 04:44)



Basics of linear codes: Encoder

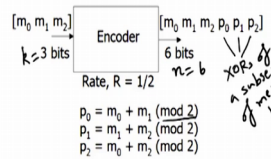


message $[m]$ k bits → Encoder (systematic) → codeword $c = [m \ p]$ n bits

$k < n$
Rate, $R = k/n$

p : vector of parity bits

Encoder: forms codeword c by adding $n - k$ parity bits p to the message m



$[m_0 \ m_1 \ m_2]$ $k=3$ bits → Encoder → $[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2]$ $n=6$ bits

Rate, $R = 1/2$

$p_0 = m_0 + m_1 \pmod{2}$
 $p_1 = m_1 + m_2 \pmod{2}$
 $p_2 = m_0 + m_2 \pmod{2}$

(binary XOR)
mod 2 addition

inputs	output
0 0	0
0 1	1
1 0	1
1 1	0

XOR of a subset of message bits

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A Brief Introduction to Linear Block Codes

any linear error control code. You have message bits appearing in the systematic encoder and then each parity bit is obtained as the XOR of subset of message bits, Ok.

So remember I have just taken two at a time here. But supposing even if you have 3 bits you can make an XOR of all of them. You make the XOR of first two and then you make the XOR of the result with the third one, you can make an XOR of 3 bits as well, or 4 bits or how many ever bits you have, Ok.

So in this case we are just taking two at a time, for instance p_0 is the XOR of m_0 and m_1 , p_1 is the XOR of m_1 and m_2 and p_2 is the XOR of m_0 and m_2 , Ok. So all linear error control codes, when they have systematic encoding is described in this fashion. For instance even in the 5 G standard the LDPC codes are specified in this fashion. The encoder works like this.

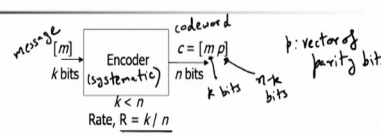
So you have a message vector and then parity bits are computed as XORs of subsets of the message bits. You might have efficient ways of implementing these XORs particularly when k and n are large. But essentially the description is exactly the same, Ok. So this is an example of a linear code. I will use one more notation for linear code.

Such codes I will call them 6 comma 3 code, Ok.

(Refer Slide Time 06:01)



Basics of linear codes: Encoder



Encoder: forms codeword c by adding $n - k$ parity bits p to the message m

$[m_0 \ m_1 \ m_2]$
 $k=3$ bits

Encoder

Rate, $R = 1/2$

$n=6$ bits

$p_0 = m_0 + m_1 \pmod{2}$
 $p_1 = m_1 + m_2 \pmod{2}$
 $p_2 = m_0 + m_2 \pmod{2}$

(b,3) code.

$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2]$
 $n=6$ bits

XORs of a subset of message bits

(binary XOR mod 2 addition)

inputs	output
0	0
0	1
1	0
1	1

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So 6 comma 3 means n is equal to 6, k is equal to 3. But this is not the only 6 comma 3 code. There are some other 6 comma 3 codes but this is reasonable one.

This specific 6 comma 3 code with this kind of an encoder we will use repeatedly in this course as a, as a representative of a generic linear block code. We will talk about decoding, encoding and all that with respect to this code, Ok.

So hopefully the picture is clear to you and also you can imagine even if k becomes very large, even if n becomes very large, k could be like 1000, n could be like 2000 or something like that, this operation is not too difficult to perform, Ok. You just need to know a subset of the message bits, which subset to pick and then XOR them together.

Ok so XORing is a reasonably simple operation to implement. One can do it, Ok so one can implement this encoder quite efficiently in practice. It could work pretty well, Ok. So this is the description of the linear code from an encoding point of view, Ok.

(Refer Slide Time 07:05)



Matrix description

All operations are mod 2. Note $-1=+1$.

$$[p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$


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A Brief Introduction to Linear Block Codes

Now it turns out there is an important and useful matrix description for linear block codes and that is described here, Ok. So the three operations we formed to find the 3 parity bits, p_0 , p_1 and p_2 , p_0 was what? p_0 was m_0 plus m_1 .

(Refer Slide Time 07:26)



Matrix description

■ All operations are mod 2. Note $-1=+1$.

$\overset{m_0+m_1}{\downarrow} \quad \overset{m_1+m_2}{\downarrow}$
 $[p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$


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A Brief Introduction to Linear Block Codes

p_1 was m_1 plus m_2 .

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Matrix description

■ All operations are mod 2. Note $-1=+1$.

$\overset{m_0+m_1}{\downarrow} \quad \overset{m_1+m_2}{\downarrow}$
 $[p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$


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A Brief Introduction to Linear Block Codes

And p_2 was m_1 plus, I am sorry m_0 plus m_2 , right?

(Refer Slide Time 07:36)



Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0 & m_1 & m_2 & p_0 & p_1 & p_2 \\ \downarrow & \downarrow & \downarrow & & & \\ [m_0 & m_1 & m_2] & = & [m_0 & m_1 & m_2] \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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So these three operations can be conveniently represented in this matrix form. So you take m_0 , m_1 and m_2 , then multiply it on the right with this matrix, $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Ok right, $m_0 \ m_1 \ m_2$ multiplied by $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ will give you m_0 plus m_1 . And $m_0 \ m_1 \ m_2$ multiplied by $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ will give you m_1 plus m_2 . And $m_0 \ m_1 \ m_2$ multiplied by $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ will give you m_0 plus m_2 .

Remember all operations are modulo 2, so I won't keep repeating this modulo 2

(Refer Slide Time 08:04)



Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0 & m_1 & m_2 & p_0 & p_1 & p_2 \\ \downarrow & \downarrow & \downarrow & & & \\ [m_0 & m_1 & m_2] & = & [m_0 & m_1 & m_2] \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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A Brief Introduction to Linear Block Codes

again and again and so you have to assume that whenever I multiply matrices and all that I always do a modulo 2. So 2 becomes 0, so that is something important to know, Ok.

So the first thing one can do is to make a generator matrix Ok which will produce the codeword

(Refer Slide Time 08:30)



Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0 & m_1 & m_2 & p_0 & p_1 & p_2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ [p_0 & p_1 & p_2] = [m_0 & m_1 & m_2] \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Generator matrix

$$\underbrace{[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2]}_{\text{code word } c} = \underbrace{[m_0 \ m_1 \ m_2]}_{\text{message } m} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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as the product of the message

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Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0 & m_1 & m_2 & p_0 & p_1 & p_2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ [p_0 & p_1 & p_2] = [m_0 & m_1 & m_2] \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Generator matrix

$$\underbrace{[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2]}_{\text{code word } c} = \underbrace{[m_0 \ m_1 \ m_2]}_{\text{message } m} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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and this matrix, this matrix is called the generator matrix and denoted G,

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Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0+m_1 & m_1+m_2 & m_2+m_0 \\ \downarrow & \downarrow & \downarrow \\ p_0 & p_1 & p_2 \end{matrix} = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Generator matrix G

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Ok. So you can see where this comes from.

The first 3 bits of the codeword are simply rep (()) the, just the message bits themselves. So you see you have the identity matrix here. So this part is the identity part, right, identity. I will denote this as $I_{\text{sub } 3}$ to denote that this is a 3 by 3

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Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0+m_1 & m_1+m_2 & m_2+m_0 \\ \downarrow & \downarrow & \downarrow \\ p_0 & p_1 & p_2 \end{matrix} = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Generator matrix G

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identity matrix.

And then I have the parity part, Ok which is the same as what I wrote before. This is the parity part usually denoted P

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Matrix description

All operations are mod 2. Note $-1=+1$.

m_0, m_1, m_2
 $\downarrow \quad \downarrow \quad \downarrow$
 $[p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2]$

Generator matrix

$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2]$
 Code word c message m

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ identity I_3
 $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ Generator matrix G
 parity part P

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Ok. So this generator matrix G is I and then P . So this will be a

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Matrix description

All operations are mod 2. Note $-1=+1$.

m_0, m_1, m_2
 $\downarrow \quad \downarrow \quad \downarrow$
 $[p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2]$

Generator matrix

$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2]$
 Code word c message m

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ identity I_3
 $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ Generator matrix $G = [I \ P]$
 parity part P

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generic structure as we go through as well. So you will have identity part and then a parity part. The parity part is obtained by various XORs.

Ok so you can see clearly that this is the same as the previous operation and one can do it, Ok.

So another way to write this expression $p_0 \ p_1 \ p_2$ is message into this matrix that you have here is to take both parts to one side and equate it to zero, Ok. So this is also equally valid. If you do that, if you take it to this side, what will happen is you will get an identity here, Ok.

So you will get an identity here, Ok. Identity can come here

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Matrix description

All operations are mod 2. Note $-1=+1$.

Handwritten notes: m_0, m_1, m_2 are message bits. p_0, p_1, p_2 are parity bits. $[p_0, p_1, p_2] = [m_0, m_1, m_2]$ with an identity matrix I_3 indicated.

Generator matrix

$$[m_0, m_1, m_2, p_0, p_1, p_2] = [m_0, m_1, m_2, 1, 0, 1]$$

Code word c is the first three bits, message m is the last three bits.

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Generator matrix $G = [I_3 \ P]$

Handwritten notes: I_3 is the identity matrix, P is the parity part.

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and then you will get this parity check matrix, Ok. So you can see this is very much equivalent to that. I have taken both to one side and I have put identity for the p part and I have got equal to zero zero zero, Ok.

So this is the matrix here, so you have noticed you have the identity here I_3 and that

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Matrix description

All operations are mod 2. Note $-1=+1$.

Handwritten notes: m_0, m_1, m_2 are message bits. p_0, p_1, p_2 are parity bits. $[p_0, p_1, p_2] = [m_0, m_1, m_2]$ with an identity matrix I_3 indicated.

Generator matrix

$$[m_0, m_1, m_2, p_0, p_1, p_2] = [m_0, m_1, m_2, 1, 0, 1]$$

Code word c is the first three bits, message m is the last three bits.

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Generator matrix $G = [I_3 \ P]$

Handwritten notes: I_3 is the identity matrix, P is the parity part.

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part multiplies the P, Ok. And then m_0, m_1, m_2 , this is the P. So actually if you look at it very closely, this is actually P transpose, Ok.

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Matrix description

All operations are mod 2. Note $-1=+1$.

Handwritten notes: m_0, m_1, m_2 above $[p_0 \ p_1 \ p_2]$; m_0, m_1, m_2 above $[m_0 \ m_1 \ m_2]$; "identity" circled around $[m_0 \ m_1 \ m_2]$.

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Code word c

message m

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Generator matrix $G = [I \ P]$

Handwritten notes: "identity I_3 " and "parity part P" pointing to the right matrix in the generator matrix equation.

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So because I have shifted the left multiplication to right multiplication, Ok so the identity comes on the left, the matrix comes on the left and everything becomes a column, Ok so I have sort of taken transpose on both sides to get columns. And so this becomes P transpose and this is the identity matrix and then together you have this, Ok.

So this once again is c transpose, transpose of the codeword.

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Matrix description

All operations are mod 2. Note $-1=+1$.

Handwritten notes: m_0, m_1, m_2 above $[p_0 \ p_1 \ p_2]$; m_0, m_1, m_2 above $[m_0 \ m_1 \ m_2]$; "identity" circled around $[m_0 \ m_1 \ m_2]$.

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Code word c

message m

Parity-check matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Generator matrix $G = [I \ P]$

Handwritten notes: "identity I_3 " and "parity part P" pointing to the right matrix in the generator matrix equation.

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This entire matrix is called the parity check matrix H equals p transpose I. Ok

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Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0 & m_1 & m_2 & p_0 & p_1 & p_2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ [p_0 & p_1 & p_2] = [m_0 & m_1 & m_2] \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{matrix} \text{identity } I_3 \\ \text{parity part } P \end{matrix}$$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$H = [P^T \ I]$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{matrix} p_0 \\ p_1 \\ p_2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} m_0 \\ m_1 \\ m_2 \end{matrix}$$

$$G = [I \ P]$$

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and once you have the parity check matrix what you have is this relationship, H times C transpose equals zero.

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Matrix description

All operations are mod 2. Note $-1=+1$.

$$\begin{matrix} m_0 & m_1 & m_2 & p_0 & p_1 & p_2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ [p_0 & p_1 & p_2] = [m_0 & m_1 & m_2] \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{matrix} \text{identity } I_3 \\ \text{parity part } P \end{matrix}$$

Generator matrix

$$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Parity-check matrix

$$H = [P^T \ I]$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{matrix} p_0 \\ p_1 \\ p_2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} m_0 \\ m_1 \\ m_2 \end{matrix}$$

$$G = [I \ P]$$

$$H C^T = 0$$

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Remember zero is a vector zero here, Ok, so there are as many zeros as you need here. So

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Matrix description

All operations are mod 2. Note $-1=+1$.

Handwritten notes: m_0, m_1, m_2 are underlined and labeled as vectors. $[p_0, p_1, p_2] = [m_0, m_1, m_2]$ is shown with an arrow pointing to the identity matrix I_3 .

Generator matrix

Handwritten: $[m_0, m_1, m_2, p_0, p_1, p_2] = [m_0, m_1, m_2]$ (labeled as message m) followed by an identity matrix I_3 (labeled as identity I_3) and a parity part P .

Generator matrix $G = [I \ P]$

Parity-check matrix

Handwritten: $H = [P^T \ I]$

Parity-check matrix H is shown as a 3x6 matrix with columns labeled $m_0, m_1, m_2, p_0, p_1, p_2$. The first three columns are the identity matrix I_3 (labeled as identity I_3) and the last three columns are the parity part P .

Handwritten: $Hc^T = 0$

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I put an underscore to, I mean underline it to denote it is a vector and this is, remember also everything is modulo 2, Ok.

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Matrix description

All operations are mod 2. Note $-1=+1$.

Handwritten notes: m_0, m_1, m_2 are underlined and labeled as vectors. $[p_0, p_1, p_2] = [m_0, m_1, m_2]$ is shown with an arrow pointing to the identity matrix I_3 .

Generator matrix

Handwritten: $[m_0, m_1, m_2, p_0, p_1, p_2] = [m_0, m_1, m_2]$ (labeled as message m) followed by an identity matrix I_3 (labeled as identity I_3) and a parity part P .

Generator matrix $G = [I \ P]$

Parity-check matrix

Handwritten: $H = [P^T \ I]$

Parity-check matrix H is shown as a 3x6 matrix with columns labeled $m_0, m_1, m_2, p_0, p_1, p_2$. The first three columns are the identity matrix I_3 (labeled as identity I_3) and the last three columns are the parity part P .

Handwritten: $Hc^T = 0 \pmod{2}$

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So these 2 matrices play a very, very important role in describing codes, Ok. So typically codes are described using generator matrices and parity check matrices. And they may be more generic than having just this I P sort of structure. May be they are not systematic. May be they have some other sort of form. And we will use both these things when describing code.

So it turns out the polar codes are usually described with the generator matrix and the LDPC codes, the low density parity check codes of course are described using the parity check




Matrix description

■ All operations are mod 2. Note $-1 = +1$.

norm, $m_1 + m_2$ or m_2

$$\begin{bmatrix} \downarrow & \downarrow & \downarrow \\ A_0 & A & A_2 \end{bmatrix} = \begin{bmatrix} m_0 & m & m_2 \end{bmatrix}$$

identity

Generator matrix

$$\begin{bmatrix} m_0 & m & m_2 & A_0 & A & A_2 \end{bmatrix} = \begin{bmatrix} m_0 & m & m_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

code word c

message m

identity I_3

parity part P

Generator matrix

$$G = \begin{bmatrix} I & P \end{bmatrix}$$

Parity-check matrix

$$H = \begin{bmatrix} P^T & I \end{bmatrix}$$

m_0, m, m_2

P^T

I_3

$\begin{bmatrix} m_0 \\ m \\ m_2 \\ A_0 \\ A \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

c

$Hc^T = 0 \pmod{2}$

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the same as p_0 equals m_0 plus m_1 .

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Matrix description

All operations are mod 2. Note $-1=+1$.

$m_0, m_1, m_2, \dots, m_{k-1}$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $[m_0 \ m_1 \ m_2 \ \dots \ m_{k-1}] = [m_0 \ m_1 \ m_2 \ \dots \ m_{k-1}]$
identity

Generator matrix

$[m_0 \ m_1 \ m_2 \ p_0 \ p_1 \ p_2] = [m_0 \ m_1 \ m_2]$
code word c

Parity-check matrix

$H = [P^T \ I]$
 $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$
identity I_3
parity part P

$G = [I \ P]$
 $HC^T = 0 \pmod{2}$



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So hopefully you see how the whole thing is working out, every other parity bit is also conditioned in the same way, Ok.

So you can use the parity check matrix to perform encoding as well. You take the first row. That gives you a parity bit p_0 ; the second row gives you the parity bit p_1 . The third row gives you the parity bit p_2 and so on. So this description is very, very important and in fact all linear codes are described in this way.

Even the 7 4 Hamming code has a description like this. I will show you that soon enough Ok. So this is generator and parity check matrix. It is quite important.

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In general...

- Code: Set of codewords
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$
 - Message m encoded as $c = mG$
 - If G is systematic, $c = [m \ p]$
- Parity-check matrix for same linear code
 - $n - k \times n$ parity-check matrix H (rank $n - k$)
 - Systematic form $H = [P^T \ I_{n-k}]$
 - Codeword c satisfies $Hc^T = 0$



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Ok so let us generalize from what we had before. A code is technically defined as the set of all codewords. You take all the codewords together, you have a, you put them together in a set, Ok how many of codewords you have, that makes a code.

And typically one thinks of a n comma k code, Ok

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In general...

- Code: Set of codewords (n, k) code
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$
 - Message m encoded as $c = mG$
 - If G is systematic, $c = [m \ p]$
- Parity-check matrix for same linear code
 - $n - k \times n$ parity-check matrix H (rank $n - k$)
 - Systematic form $H = [P^T \ I_{n-k}]$
 - Codeword c satisfies $Hc^T = 0$



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so this is k message bits to n codeword bits. That means

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In general...

- Code: Set of codewords (n, k) code
 k message bits
 $\downarrow$
 n codeword bits
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$
 - Message m encoded as $c = m G$
 - If G is systematic, $c = [m \ p]$
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 - Codeword c satisfies $H c^T = 0$



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you have n minus k parity bits. Number of codewords equals 2 power k ,

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In general...

- Code: Set of codewords (n, k) code
 k message bits
 $\downarrow$
 n codeword bits
 $\#$ of Codewords $= 2^k$
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$
 - Message m encoded as $c = m G$
 - If G is systematic, $c = [m \ p]$
- Parity-check matrix for same linear code
 - $n - k \times n$ parity-check matrix H (rank $n - k$)
 - Systematic form $H = [P^T \ I_{n-k}]$
 - Codeword c satisfies $H c^T = 0$



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Ok. So you have 2 power k codewords typically in a $n \ k$ code and all of them make up the code, Ok.

So you have a generator matrix for the linear code which is a k cross n generator matrix of, we usually denote G , it should have rank k . It is the linear algebraic property

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In general...

- Code: Set of codewords (n, k) code
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$
 - Message m encoded as $c = m G$
 - If G is systematic, $c = [m \ p]$
- Parity-check matrix for same linear code
 - $n - k \times n$ parity-check matrix H (rank $n - k$)
 - Systematic form $H = [P^T \ I_{n-k}]$
 - Codeword c satisfies $H c^T = 0$

k message bits
 ↓
 n codeword bits
 # of codewords = 2^k



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over the vector space we are considering here, we do not have to talk too much about it in this class at least.

So in systematic form G can be written as I sub k , the identity part and

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In general...

- Code: Set of codewords (n, k) code
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$
 - Message m encoded as $c = m G$
 - If G is systematic, $c = [m \ p]$
- Parity-check matrix for same linear code
 - $n - k \times n$ parity-check matrix H (rank $n - k$)
 - Systematic form $H = [P^T \ I_{n-k}]$
 - Codeword c satisfies $H c^T = 0$

k message bits
 ↓
 n codeword bits
 # of codewords = 2^k



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the parity part. Ok so the P is a k by n minus k matrix, Ok. So

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In general...

- Code: Set of codewords (n, k) code k message bits
 $\downarrow$
 n codeword bits
 $\# \text{ of codewords} = 2^k$
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$ $P: k \times n-k$ matrix
 - Message m encoded as $c = m G$
 - If G is systematic, $c = [m \ p]$
- Parity-check matrix for same linear code
 - $n - k \times n$ parity-check matrix H (rank $n-k$)
 - Systematic form $H = [P^T \ I_{n-k}]$
 - Codeword c satisfies $H c^T = 0$



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A Brief Introduction to Linear Block Codes

when you multiply m with G you get a codeword, Ok.

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In general...

- Code: Set of codewords (n, k) code k message bits
 $\downarrow$
 n codeword bits
 $\# \text{ of codewords} = 2^k$
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k \ P]$ $P: k \times n-k$ matrix
 - Message m encoded as $c = m G$.
 - If G is systematic, $c = [m \ p]$
- Parity-check matrix for same linear code
 - $n - k \times n$ parity-check matrix H (rank $n-k$)
 - Systematic form $H = [P^T \ I_{n-k}]$
 - Codeword c satisfies $H c^T = 0$



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So when you multiply m on the left side, m will multiply with the identity part to give m itself.

Then m will multiply with the capital P matrix to give you the parity part P , Ok. So this is exactly what we had before. One can generalize.

But now there will be some codes where G will not have this form, will not have $I \ k \ P$ form, it will not be in systematic form but nevertheless that is a valid description of the code. It turns out; you can go from non-systematic forms to systematic form and all that.

But it is not, maybe it is not so important in this class but nevertheless you should know that G can have a more general form. In fact when we talk about polar codes I will describe them using the generator matrix and when I specify the generator matrix for the polar code you will see it is not in systematic form not in the $I_k P$ form, Ok.

But nevertheless you can always form the codeword as m times G , Ok. It won't have, m may be appearing by itself in the codeword but it will produce some other m -bit vector which you can transmit as the codeword, Ok.

So the parity check matrix for the same code is an n minus k cross n matrix usually denoted H . It needs to have rank n minus k and there is one important condition, G times H transpose has to be equal to all zero matrix, Ok.

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In general...

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So every row of G should, if you take a dot product or if you multiply with every row of H you should get zero, Ok.

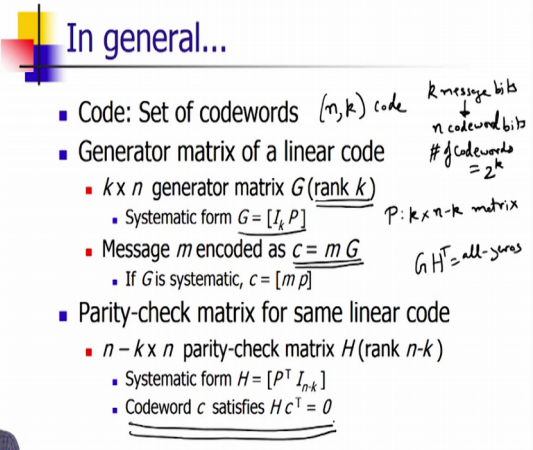
So that is the condition of the parity check matrix. It is said to describe the dual code and all that. In general in this class we will not talk too much about the linear algebraic properties of codes and linear codes. It is not so important to us but nevertheless it is good to do this and read it, read about it.

I will be putting up some additional lectures from prior courses which precisely describe these linear algebraic properties. If you are interested please go through them. But it is sort of additional reading as far as this class is concerned, Ok.

So if you have the generator matrix in systematic form $I_k P$ it turns out, a parity check matrix is really specified. You get H to be P^T I_{n-k} . So it is a little bit of an exercise to check that G specified as $I_k P$ and H specified as $P^T I_{n-k}$, if you take a transpose product of them you will get zero. It is something you can check.

And the most important property is this, Ok. Once you have the parity check matrix

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In general...

- Code: Set of codewords (n, k) code
 - k message bits
 - n codeword bits
 - # of codewords $= 2^k$
- Generator matrix of a linear code
 - $k \times n$ generator matrix G (rank k)
 - Systematic form $G = [I_k P]$ $P: k \times n-k$ matrix
 - Message m encoded as $c = mG$ $G H^T = \text{all-zeros}$
 - If G is systematic, $c = [m p]$
- Parity-check matrix for same linear code
 - $n-k \times n$ parity-check matrix H (rank $n-k$)
 - Systematic form $H = [P^T I_{n-k}]$
 - Codeword c satisfies $H c^T = 0$



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every code word satisfies $H C^T$ equals zero. So just like the generator matrix is useful in encoding, typically one uses the parity check matrix for decoding. You can also use it for encoding. It is sort of the same as the generator matrix but typically one uses it for encoding.

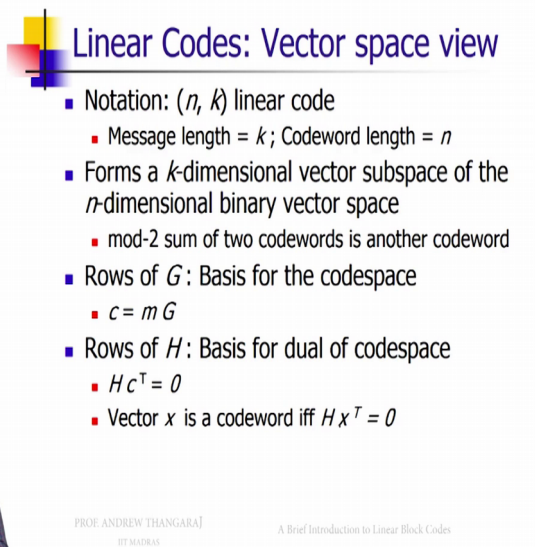
So for instance if I give an n -bit vector, some n -bit vector and ask you whether or not it is a valid codeword, Ok or does it belong to the list of all codewords that you have in the code? Does it belong to the code, if I ask you that question it is easiest to answer if you have the parity check matrix?

Ok, so what can you do, you can take the parity check matrix, multiply that vector transpose on the right and see if you get zero. If you get zero then it belongs to the code. If you do not get zero, it does not belong to the code, Ok.

So you can see that given a vector you can do this and you can use it in decoding also and it is quite efficient. On the other hand with the generator matrix may be you cannot answer that question immediately and quickly, Ok.

So this is a description and when we describe polar codes and LDPC codes I will describe them using generator and parity check matrices, Ok.

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Linear Codes: Vector space view

- Notation: (n, k) linear code
 - Message length = k ; Codeword length = n
- Forms a k -dimensional vector subspace of the n -dimensional binary vector space
 - mod-2 sum of two codewords is another codeword
- Rows of G : Basis for the codespace
 - $c = mG$
- Rows of H : Basis for dual of codespace
 - $Hc^T = 0$
 - Vector x is a codeword iff $Hx^T = 0$

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So like I mentioned there is a vector space view. I will just mention it briefly. There are other lectures that I will upload which will talk about this in more detail.

This is the crux of the story. It turns out a n, k linear code forms a k -dimensional vector subspace of the n -dimensional binary vector space. Basically that means modulo 2 sum of two codewords is another codeword, Ok. So that is quite easy to see in the way described it.

The rows of G are basis for the code space, the basis for the subspace which is the code space and the rows of H form the basis for the dual of the code space, Ok. So that is the whole story from a vector space point of view.

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Examples

- (3,1) Repetition Code = {000, 111}

$$G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$


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So here are the couple of examples. I will provide one more example but this is the two simplest examples, one to two examples we have seen before in this class, Ok. The first one is the 3 comma 1 repetition code. The code itself has just 2 codewords, 0 0 0 and 1 1 1, Ok. The generator matrix is just 1 row, 1 1 1. Ok so you can see it is a 1 cross 3 matrix

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Examples

- (3,1) Repetition Code = {000, 111}

$$G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$


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Ok.

And if you multiply by 0 on the left you get 0 0 0, that is one codeword. You multiply by 1, you get 1 1 1, that is another codeword, Ok. Now the parity check matrix is a 2 cross 3 matrix.

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Examples

- (3,1) Repetition Code = {000, 111}

$$G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$


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Remember it is n minus k by n and you can check that this is the valid parity check matrix.

So you multiply with G trans/transpose, G with H transpose you will get zero and that is the parity check matrix, Ok.

The next 6 comma 3 example code that we saw has this list of codewords. You can see the message part comes here for instance. You look at the message part here. This is the m part, Ok.

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Examples

- (3,1) Repetition Code = {000, 111}

$$G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$


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And then you have the p 0. p 0 being XOR of 1 and 0, p 1 being XOR of 1 and 2 and then p 2 being the XOR of 1 and 3, Ok.

So that is a codeword.

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Examples

- (3,1) Repetition Code = {000, 111}

$$G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

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Likewise all the 8 different codewords have been listed out here and generator matrix is given here, the parity check matrix is given here, Ok so this how one can see examples quickly,

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Minimum Distance

- Hamming distance between two binary vectors is the number of places where they are different.
 - dist(000, 111) = 3
 - dist(1101, 0110) = 3
- Minimum distance of a code: The minimum Hamming distance between any two codewords
 - $d_{\min}$ of (3,1) repetition code = 3
 - $d_{\min}$ of (5,1) repetition code = 5
 - $d_{\min}$ of (6,3) example code = 3 (How?)
- (n,k,d) – code:
 - Block-length = n, Dimension = k, $d_{\min} = d$

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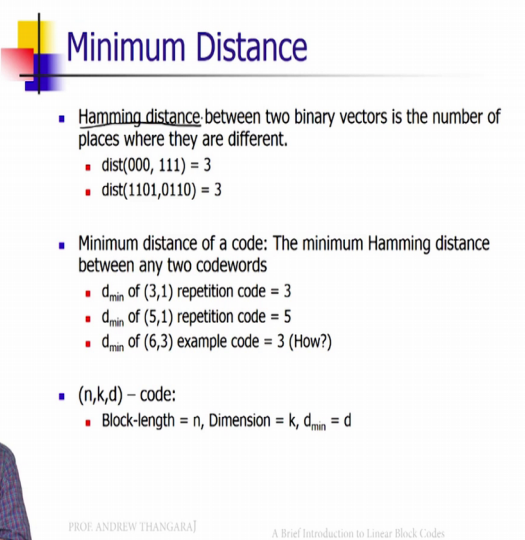
Ok.

So, Ok so the last idea that we will briefly introduce in this lecture is this of minimum distance, Ok. It is, it is not something that is very critical for us in this course. But it is an important design principle. It is very, very useful to know and have some intuition about the minimum distance and the role it plays in encoding, in decoding successfully. Ok.

So this is extremely important. Lot of people who design codes have intuition about minimum distance. It plays a role but as it turns out in modern codes the role is little bit, it is sort of used as a useful design criterion but people do not really try to optimize this as much as they used to do before. But nevertheless it is important to know the definition, Ok.

So the first definition you need to know to understand minimum distance is the definition of Hamming distance.

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Minimum Distance

- Hamming distance between two binary vectors is the number of places where they are different.
 - $\text{dist}(000, 111) = 3$
 - $\text{dist}(1101, 0110) = 3$
- Minimum distance of a code: The minimum Hamming distance between any two codewords
 - $d_{\min}$ of $(3,1)$ repetition code = 3
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 - $d_{\min}$ of $(6,3)$ example code = 3 (How?)
- (n,k,d) – code:
 - Block-length = n , Dimension = k , $d_{\min} = d$

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So what is Hamming distance? If we take two different binary vectors of the same length, Hamming distance is the number of places where they are different, where they differ, Ok.

So here is a couple of examples. You can see 0 0 0 and 1 1 1 clearly differ in 3 places. This one may be is a little bit more difficult to write down but again the difference is in 3 places, Ok, in all these 3 positions they differ. So the Hamming, minimum, the Hamming distance between

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Minimum Distance

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these vectors is 3, Ok.

Now what is the minimum distance of a code? You have lot of codewords in the code, Ok so you have 2^k codewords. You take 2 at a time and find all possible distances, Ok and the least among those distances is the minimum distance of the code.

So one can, means this is not a very easy thing to calculate given a code but for some codes one can easily do it. For instance with the 3 comma 1 repetition code the minimum distance is clearly 3 because there are only 2 codewords 0 0 0 and 1 1 1 but what about the 6 comma 3 example code?

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Examples

- (3,1) Repetition Code = {000, 111}

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- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

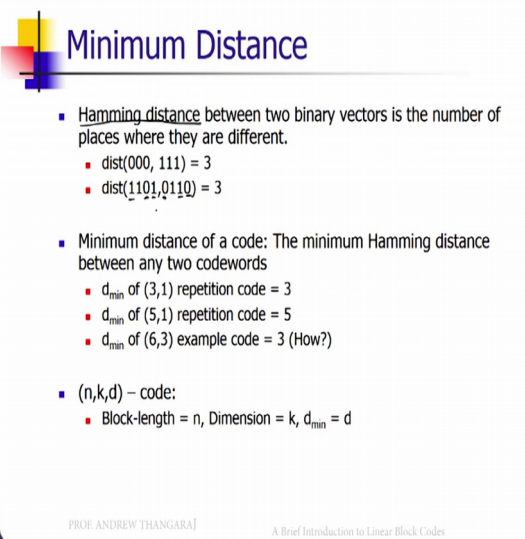
$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$


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You look at this list of codewords here. What is the minimum distance? It is not immediate, right? So you have to go through all possible pairs, Ok. There are lot of pairs here. You can look at distances between any two of them and then find the minimum distance of that code.

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Minimum Distance

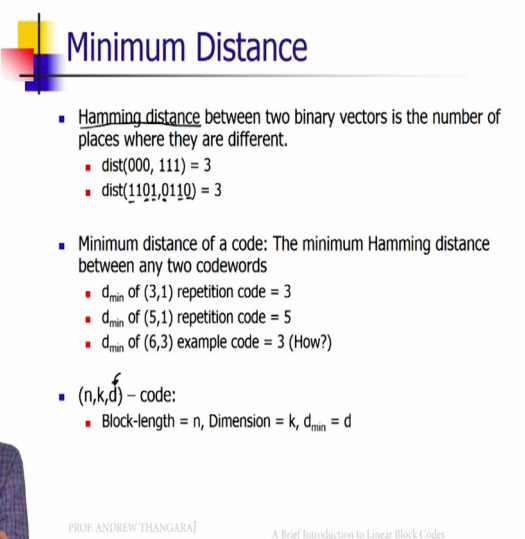
- Hamming distance between two binary vectors is the number of places where they are different.
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It turns out the answer is 3 for that code. Ok it is not very immediate, Ok but nevertheless this minimum distance is an important parameter. So typically people specify that along with

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Minimum Distance

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the n k they will introduce that as a new parameter d , they will say it is a n k d code, block length is n , dimension is k or message length is k and then the minimum distance equals d , Ok.

But like I said in this course which is focused on LDPC Polar codes, the minimum distance won't make an explicit appearance. I will allude to it may be when I can later on, Ok,

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Minimum Distance: Linear Codes

- Minimum Distance
 - Hamming weight of a binary vector is defined to be the number of 1s in it
 - $\text{dist}(u,v) = \text{weight}(u+v) \pmod{2}$
 - $d_{\min}$ of a linear block code is the minimum weight of a nonzero codeword
 - u, v : distinct codewords; $w = u+v$: nonzero codeword
 - $\min \text{dist}(u,v) = \min \text{weight}(u+v) = \min \text{weight}(w)$



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alright. So it turns out for linear codes, because the sum of 2 codewords is also a codeword one can simplify some of the computation involved in the minimum distance.

So for instance this is an important identity, Ok.

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Minimum Distance: Linear Codes

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So if you want to find the number of places where 2 vectors differ, you can take the XOR of two vectors, Ok and simply count the number of 1s in the XOR, Ok.

So remember Hamming weight is defined

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Minimum Distance: Linear Codes

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as the number of 1s in the vector,

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Minimum Distance: Linear Codes

- Minimum Distance
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 - $\text{dist}(u,v) = \text{weight}(u+v) \pmod{2}$
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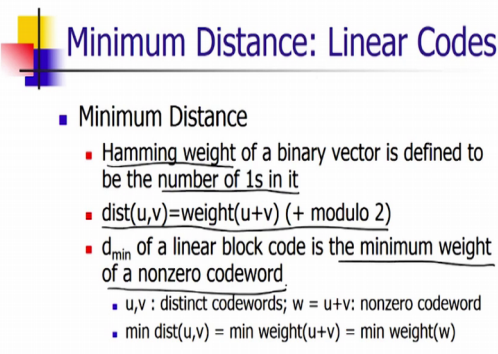
Ok. That is Hamming weight. Remember Hamming distance is between two vectors, number of places in which they differ. If you have 1 vector, Hamming weight is the number of 1s in it, Ok.

Now given 2 vectors u v there is a relationship between the Hamming distance between two u v and the weight of u plus v . So if you take the XOR of u and v , wherever they differ the XOR is going to be 1. Wherever they are the same, XOR is going to be 0. So if you take the

XOR and count the number of 1s you get the Hamming distance. And that is this relationship, Ok.

So if I have a linear block code the minimum distance is actually equal to the minimum weight of a non-zero codeword,

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Minimum Distance: Linear Codes

- Minimum Distance
 - Hamming weight of a binary vector is defined to be the number of 1s in it
 - $\text{dist}(u,v) = \text{weight}(u+v) \pmod{2}$
 - $d_{\min}$ of a linear block code is the minimum weight of a nonzero codeword
 - u, v : distinct codewords; $w = u+v$: nonzero codeword
 - $\min \text{dist}(u,v) = \min \text{weight}(u+v) = \min \text{weight}(w)$



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Ok. It is easy to prove this. I have written a small proof for it here. The, for a linear code the minimum distance is the minimum weight of a non-zero codeword.

So in general if you want to have a large minimum distance you should make sure that there are no codewords of low weight in a linear block code. So if you go back and look

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Examples

- (3,1) Repetition Code = {000, 111}

$$G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$


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in this example, if you want to find all possible pair wise distances in the 6 comma 3 example code, it is a lot of distances you have to compute.

But if you just have to look at the codeword of minimum weight, you have codewords of weight 3, right all this is weight 3,

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Examples

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- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$


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right this is weight 4, right, so this is weight 3,

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Examples

- (3,1) Repetition Code = {000, 111}

$$G_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Handwritten notes: A '3' is written above the first column of G_1 . An arrow points from the text 'weight 3' to the second row of H_2 .

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

Handwritten notes: An arrow points from the text 'weight 4' to the first column of G . A '3' is written above the third column of G . An arrow points from the text 'weight 3' to the third column of H .

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

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this is weight 4, weight 4, weight 3. So

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Examples

- (3,1) Repetition Code = {000, 111}

$$G_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad H_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Handwritten notes: A '3' is written above the first column of G_1 . An arrow points from the text 'weight 3' to the second row of H_2 .

- (6,3) Example Code = {000000, 001011, 010110, 011101, 100101, 101110, 110011, 111000}

Handwritten notes: An arrow points from the text 'weight 4' to the first column of G . A '3' is written above the third column of G . An arrow points from the text 'weight 3' to the third column of H .

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

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you can quickly tell that the minimum weight of a non-zero codeword is 3 and since the code is linear the minimum distance of this code is also 3, Ok.

So that is a nice, little quick little calculation

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Minimum Distance: Linear Codes

- Minimum Distance
 - Hamming weight of a binary vector is defined to be the number of 1s in it
 - $\text{dist}(u,v) = \text{weight}(u+v) \pmod{2}$
 - $d_{\min}$ of a linear block code is the minimum weight of a nonzero codeword
 - u, v : distinct codewords; $w = u+v$: nonzero codeword
 - $\min \text{dist}(u,v) = \min \text{weight}(u+v) = \min \text{weight}(w)$



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that one can do and that comes from this nice relationship between, the special relationship between minimum distance and minimum weight of non-zero codewords in linear codes, Ok. This is a good thing to know.

So in general you do not want to have too many low weight codewords in a code, Ok and that is a good design principle, Ok. So avoid low weight codewords, Ok. This is a good design principle for

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Minimum Distance: Linear Codes

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Avoid low-weight codewords

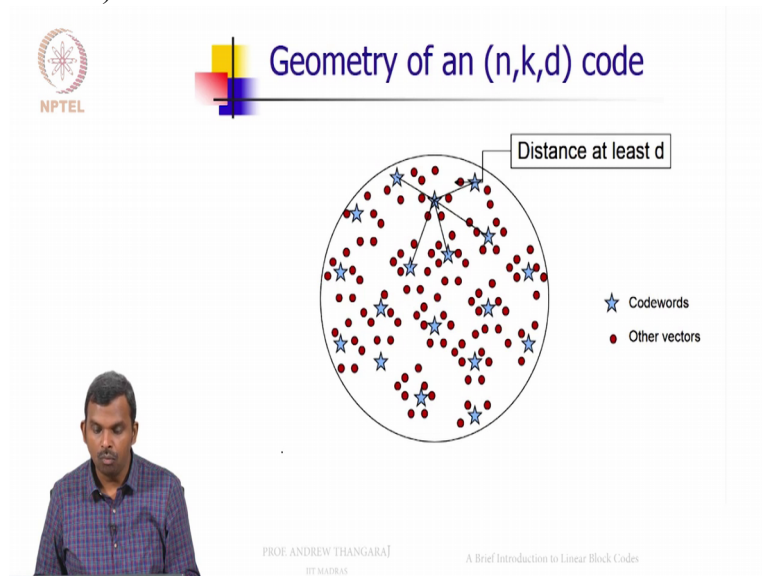


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linear code, you want to design it and come up with a good code, Ok.

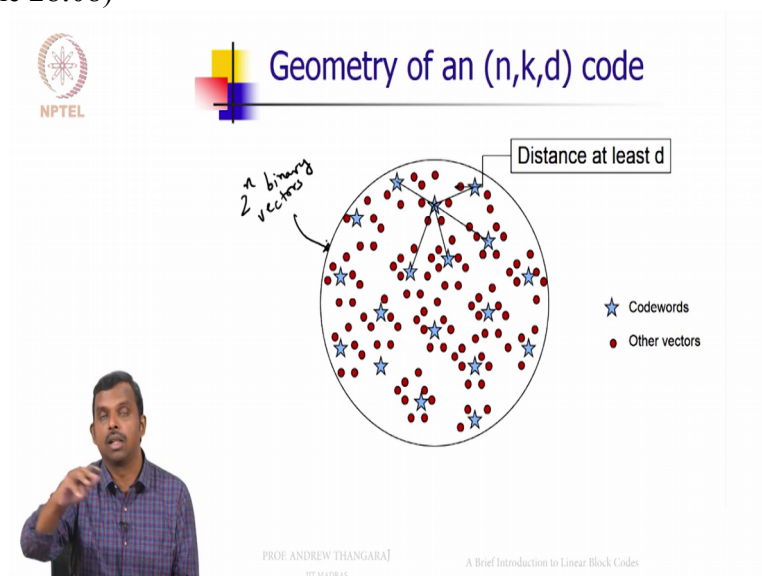
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So here is a picture and I think this picture is very good to have in your mind when you think of codes you should have a picture like this in your mind. And this picture is very, very, it gives you a certain intuition of how codes work, Ok.

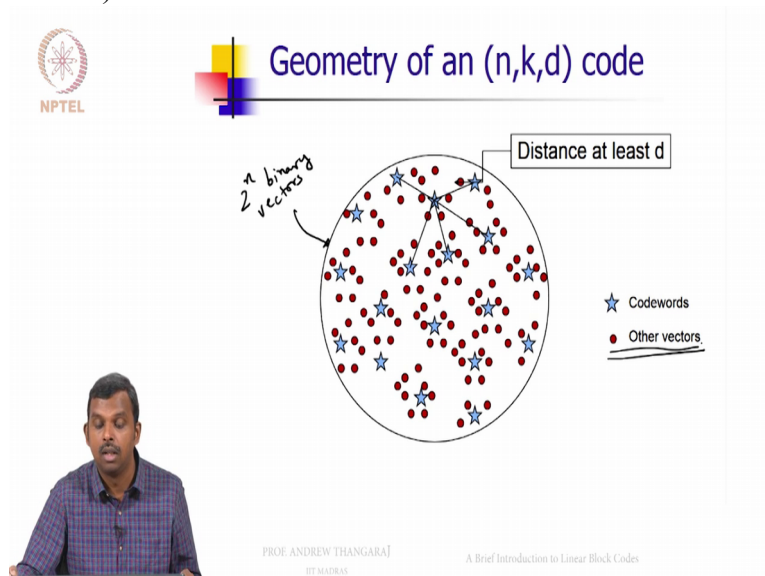
So you can think of the space of all 2^n binary vectors. There are 2^n binary vectors. So you put them all in a circle,

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Ok. So you imagine they are these small dots. All these vectors are

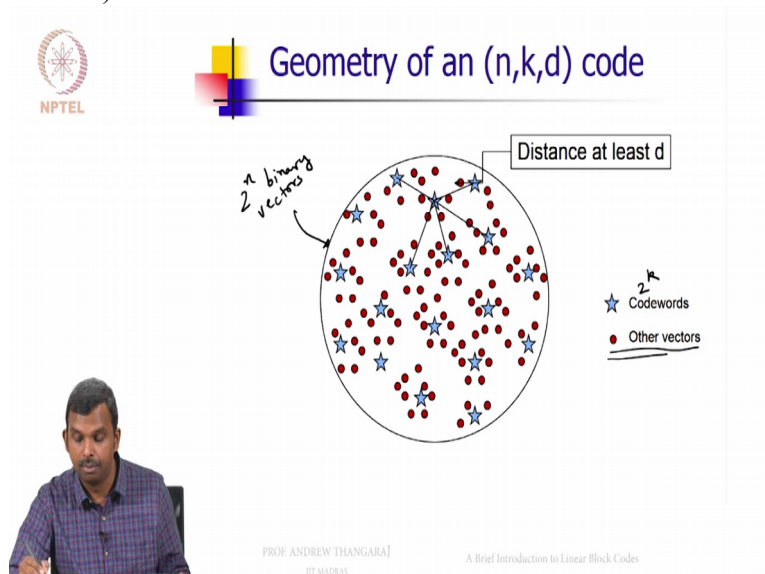
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these are the 2^n binary vectors.

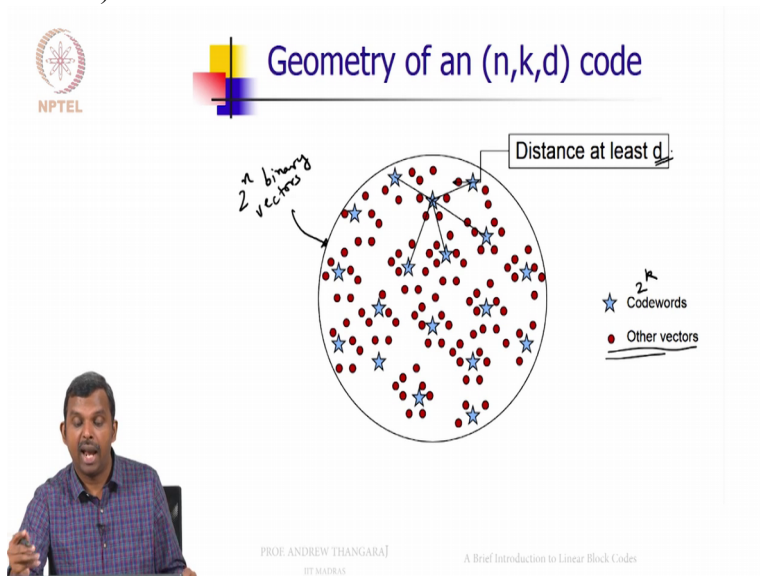
So out of these 2^n binary vectors there are 2^k codewords,

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Ok. Those I am denoting in stars, these blue stars, Ok and what do we know based on the distance d ?

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Any two codewords are at least a distance d apart, Ok.

So that is what the minimum distance d means, Ok. And you can imagine at the transmitter when you are transmitting, you are transmitting one of the stars, right. The codeword is what is transmitted.

But of course noise gets added and what you receive is something else, something else away from the star, Ok. And in general when you want to decode you want, you are looking around from wherever you are you are looking around to see the star.

If there are too many stars you are going to get confused, Ok. So whenever you have a received word and you want to look around and try to find, say the closest star or something nearby you should not have too much confusion. Ok. You should know clearly which direction to go, Ok.

And if all these stars are too close by you are going to be confused. You won't know where to go, Ok. So that is actually a pretty good intuition to have about how modern decoders work.

So modern decoders will have the received word and start looking for these codewords, Ok and they look for them in multiple ways. And remember you have to be efficient also. They use some very clever ideas. We will discuss some of these ideas as we go along to search for the closest possible codeword in some sense, Ok

And if you have too many close possible codewords you get confused. So the code becomes bad. So if you design a good code you won't have typical received vectors. You won't have too many codewords contesting to the closest codeword. And your decoder will succeed, Ok.

So this kind of intuition about how the code works is very crucial and when I describe the decoder for these modern codes, I will urge you to keep this picture in mind. And the modern codes use some very nice clever ideas to search for this code, stars so to speak which are nearby received vectors, Ok.

So we will stop here for now. In the next lecture I am going to be doing some Matlab coding to show you how soft decision maximum likelihood decoder works for the Hamming code and may be the example code that we had here, Ok. So we will do that in the next lecture, thank you.