Multirate Digital Signal Processing
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Lecture – 38
Review of OFDM with CP

Good evening and welcome, we will start the lecture number 38, lecture 37 we had looked at the several interesting elements, let me just sort of quickly summarize.

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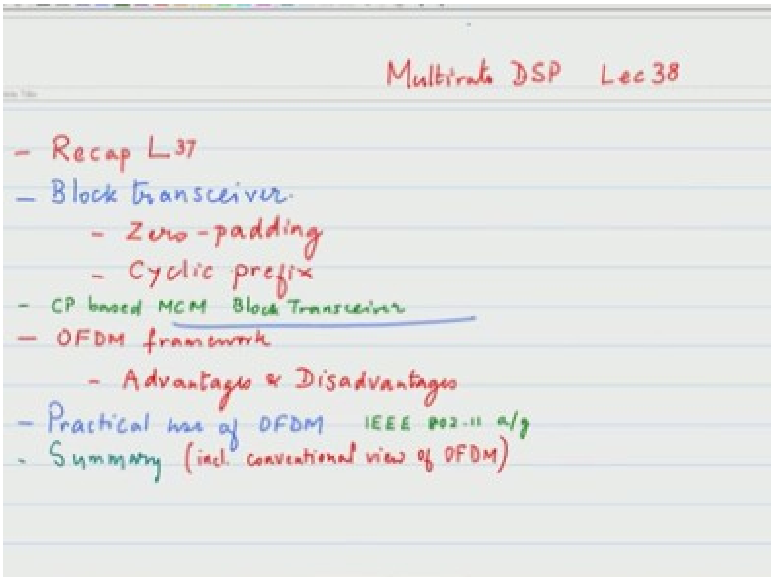

The key aspects was; we looked at cyclic prefix based systems, once you had the cyclic prefix based systems we showed how the multi-carrier modulation with the block transceiver could be constructed and that was the point at which we had completed so basically, in today's lecture we will introduce the OFDM framework and we will also look at some practical applications of OFDM in the technologies that we are familiar with.

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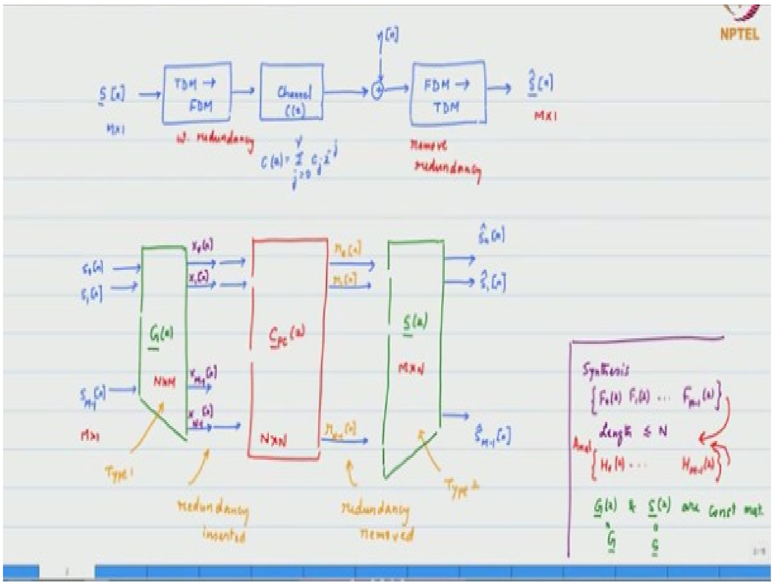

So at a broad level, we are trying to do transmission through a channel that has got time dispersion, we want to send a block of data and be able to recover it in a very efficient manner at the other end, we have to worry about inter block interference, intra block interference and all of these have to be addressed in the system that we are designing, we agreed that there will be the benefits of redundancy, if you say there is no redundancy it becomes a very difficult system to work with, complexity will increase.

So, we say that we will introduce redundancy to the extent that it will simplify the receiver structure, so at the transmitter we will insert the redundancy, we show that by the processing but at the end of the day we are doing time division multiplexing to frequency division multiplexing because that is what will create for us a wideband signal, so the FDM signal is what is the wideband signal which is what we want to transmit through the channel.

And before we transmit we will introduce the redundancy and again will it be part of the modulation process or will it be inserted post the modulation process, what we have found is that we need to introduce it after the modulation process because either 0 insertion or cyclic prefix insertion is not part of the modulation, it is after we have done the modulation so, the insertion of the redundancy, removal of the redundancy eventually getting back the signals from which we can make an estimate of the transmitted signal.

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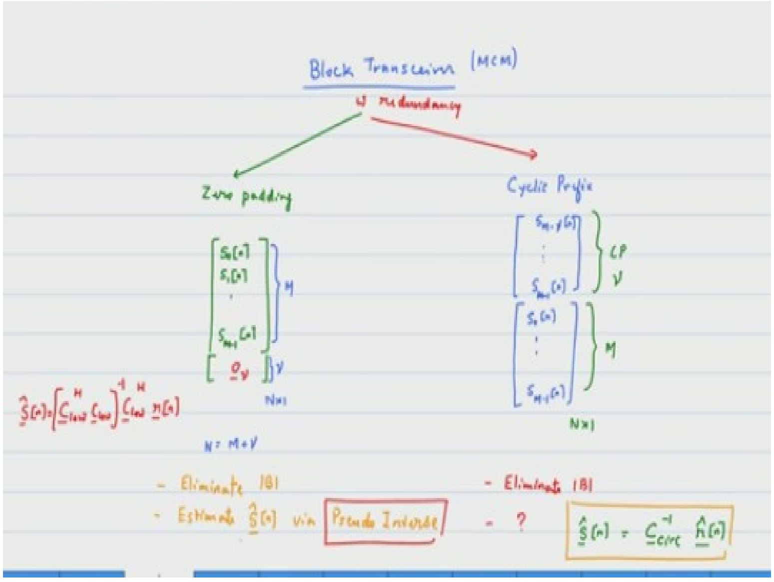

So a good way to visualize the introduction of redundancy in the transceivers that we are working with is; one is zero padding where we add the zeros, the number of zeros must be at least equal to the number of channel taps that you have in your signal, it cannot be less it can be more but at least and because we want the redundancy to be as minimum as small as possible, we want to keep it to the length of the channel time.

In the case of the cyclic prefix, we said that it will be the last new elements that you are transmitting, you reproduce it at the head of the; at the start of the transmission so, effectively the cyclic prefix gets added in both cases it becomes an N cross 1 vector now, in the case of the cyclic, the zero insertion, the final result is obtained via a pseudo inverse operation, let me just write that down.

So, s hat of n, the detected signal will be equal to the matrix C low that is derived from the using the first M columns of the matrix, C low hermitian, C low inverse C low hermitian times the received signal r of n okay, r of n has the; has more observations than the number of unknowns that we are trying to detect so, it becomes a least square solution, least squares problem for which we do the pseudo inverse.

In the case of the cyclic prefix case, we showed that we can obtain the final result in terms of a matrix inversion not a pseudo inverse but a matrix inversion where we have the inversion of a circulant matrix and we made the arguments, why it would become a circular matrix. So since that is the key focus that we are interested in let me, just sort of quickly give you the framework so then, we can complete today's discussion.

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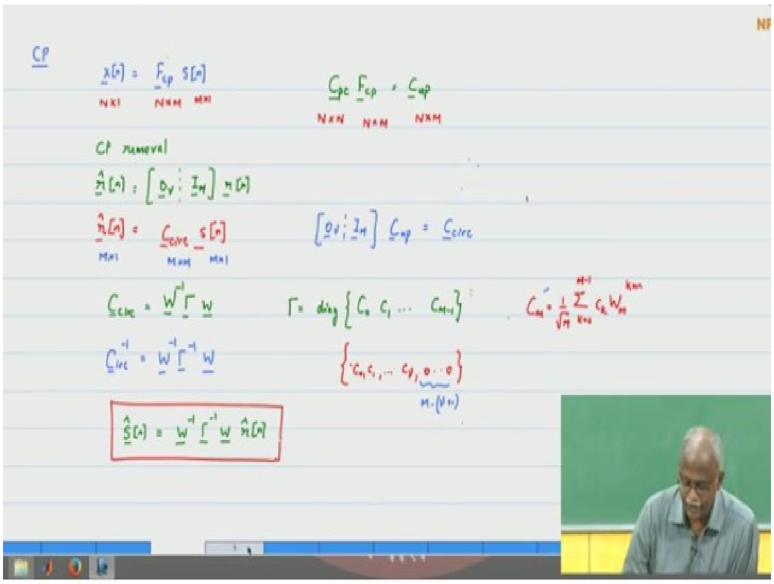

So, the case where we have cyclic prefix that is the one that is of interest to us so, the first step is we create the signal to be transmitted and that is done through the insertion of the cyclic prefix so, you take the input vector s of n delay and this case I just want to write down all of the matrix dimensions, it is important that we are able to capture that this is a N cross 1 vector, this is a N cross M matrix again, it has only 1's and 0's, this is a M cross 1.

And that will give you the vector that we are interested in and we also have another result which says that you will get a pseudo circulant matrix as part of the transmission through the channel and that we have seen even in the case of the, when we did the numerical examples, the pseudo circulant matrix times Fcp, this is an important one, though Fcp was actually associated with the transmitted signal in the analysis we are associating it with the channel.

So this basically, gives me a matrix Cup basically, it takes off and once you have this let me just make sure that we got the dimensions correct, Cpc is a N cross N matrix, this is a N cross M matrix, this is a N cross M matrix okay, and after this if we do the cyclic prefix removal next step is the CP removal; CP changes the dimensions of this matrix so basically, we do the cyclic prefix removal by nulling out some of the unwanted elements.

And that is shown by means of a matrix with 0's, M rows and new columns followed by an identity matrix times r of n and this will basically cause this vector to interact with the Cup and the product of that gives us the resulting equation that we want to work with, r hat of n becomes equal to the C circulant times s of n okay, so again one let us just write down the dimensions, this is a M cross 1 matrix, the circulant matrix is M cross M and s of n is M cross 1.

So, basically we have removed the redundancy and then we finally get to working with this so, the observation is that this vector 0 Nu times Im times Cup actually is the one that gives us the circulant matrix, what is remaining is the circulant matrix and that is what we are working with, okay so, the product; the property of the circulant matrix we will just quote from last time, we have shown that any circulant matrix can be written in terms of the normalized DFT matrix inverse gamma times W.

Gamma is a diagonal matrix of the following elements uppercase C0, uppercase C1, uppercase C m – 1, these are not the channel coefficients, they are the DFT of the channel coefficients so, the Cm equation = summation, there will be a 1 over root m, is that there or not, yes basically, 1 over root m, C subscript k Wm km, yes and k = 0 to m – 1, basically the computation of the DFT okay.

So, once you get the DFT but the coefficients C case are not m in number, what you have is C0, C1 up to C Nu and then you add 0's remaining, how many 0's did we add; we added m – Nu + 1 that many 0's you added and then computed the DFT and then you obtained it and this tells us that the circulant matrix can always be inverted with some small error which is negligible from our point of view.

And that will be equal to W inverse gamma inverse times W and basically that is done by perturbing the; if any of these channel coefficients is very close to 0, you replace it with some constant epsilon and then do that, it is equivalent to the MMSE approach rather than doing zero forcing, you find the solution but the reason we say that it does not affect us is that is that will happen on a particular sub channel that has got low SNR.

And you would not have done any transmission on that channel to begin with so, really what happens there is not going to affect us okay, so the key equation for us is the fact that we can obtain the computation of the transmitted signal s hat of n by means of the following equation, W inverse lambda inverse W times r hat of n that is the equation that is where we reached and that is where we would like to pick up from in today's class okay.

So, I hope this part of it is clear and that you are comfortable with this information, let me just show you some of the slides from last time that with that we draw okay.

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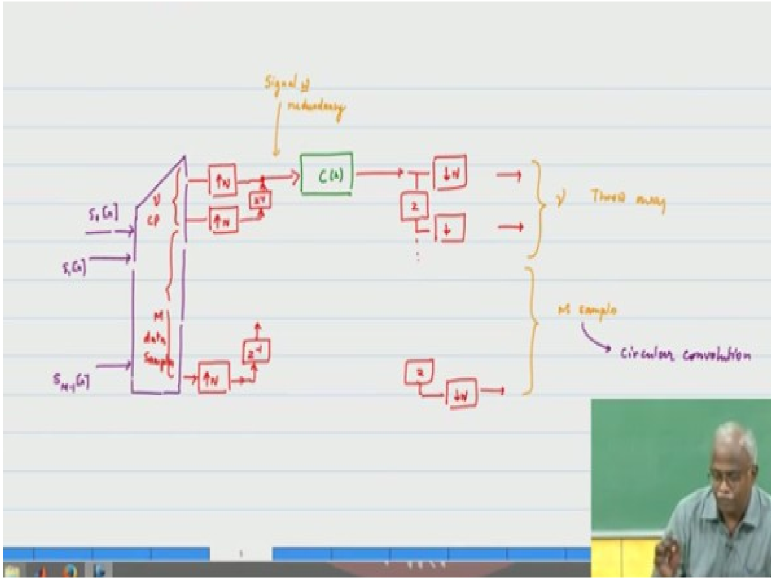

The insertion okay, the insertion of the cyclic prefix, so that happens through the multiplication with the Fcp matrix, so we can see that is happening by adding some new entries at the top then you make it in from parallel to serial that is becomes a signal with redundancy passing through the channel, channel will do linear convolution that is always the case.

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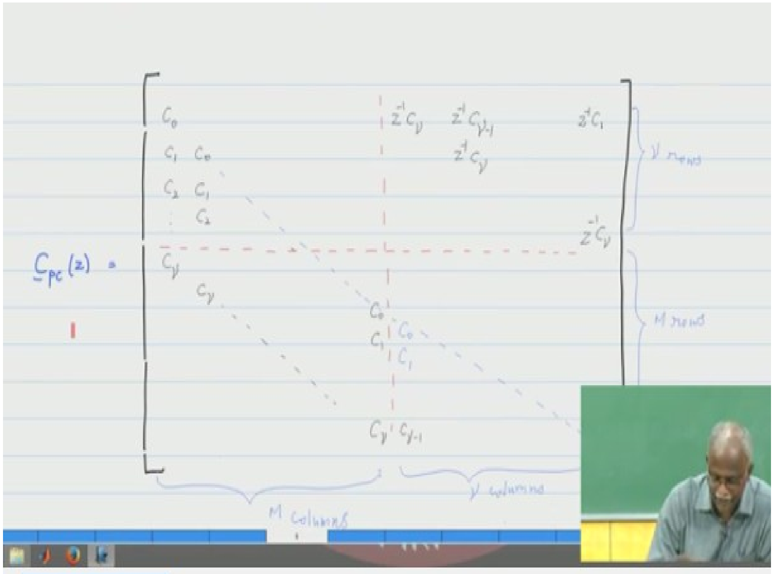

But we showed that as we did the with the matrix, this is the overall matrix.

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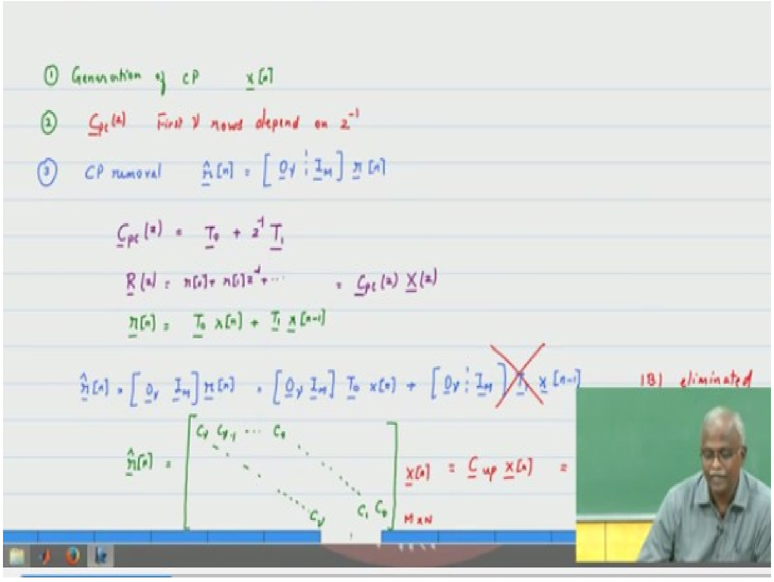

And once we did the CP removal, we showed that what was left is this matrix which is Cup and then multiplied by Fcp becomes a circulant matrix, so that was the process that we had followed.

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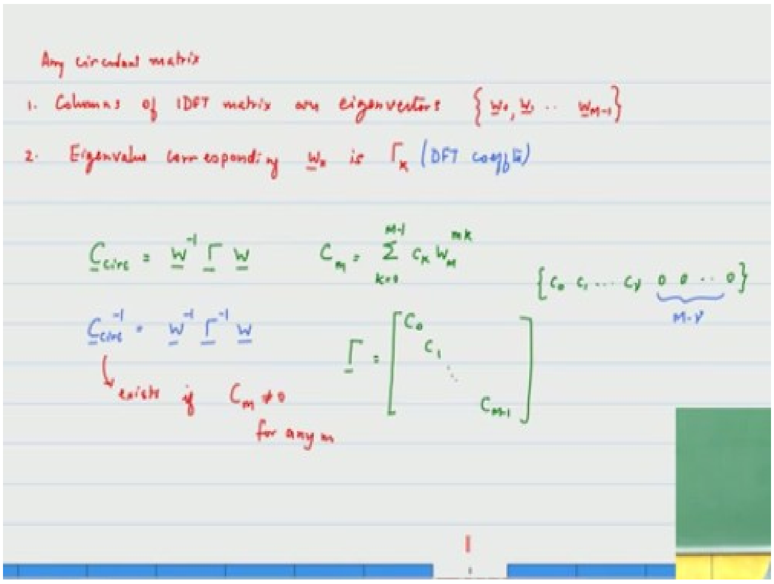

And that is when we came to this conclusion.

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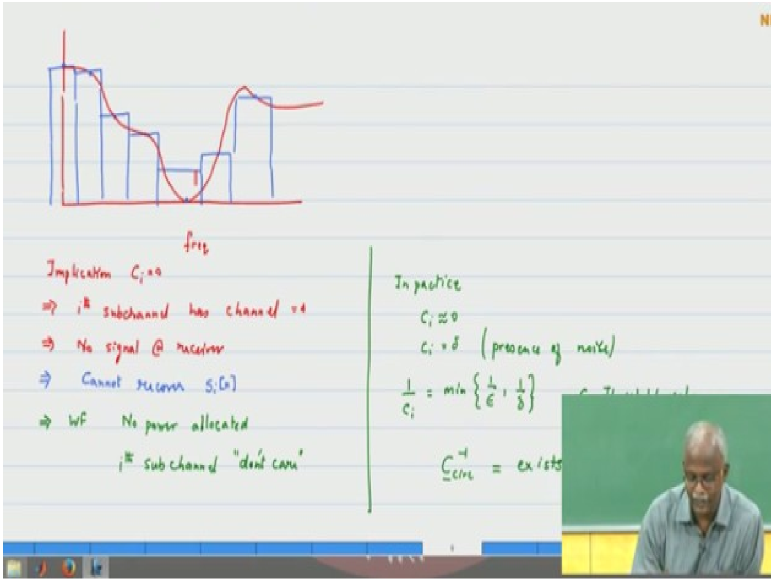

So, as I mentioned these channels where are likely to cause you problems in the inversion of these circulant matrix are not a problem for us because there is no transmission happening in this channel to begin with so, therefore we are able to use this without any difficulty.

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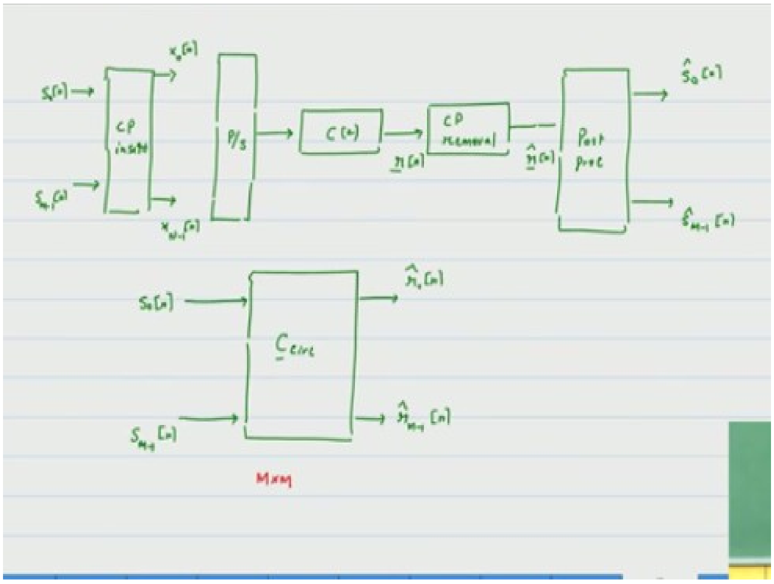

So, the overall block diagram which should have included all of these portions which basically, started with s of n, the modulation or the conversion to an FDM signal, insertion of the CP parallel to the serial conversion passing through the channel, CP removal, the removal of the conversion to TDM and then the separate signals, simplified beautifully into a M cross M problem, this is a M cross M matrix where there are M inputs, M outputs and variable to do it by a matrix that is guaranteed to be invertible, okay.

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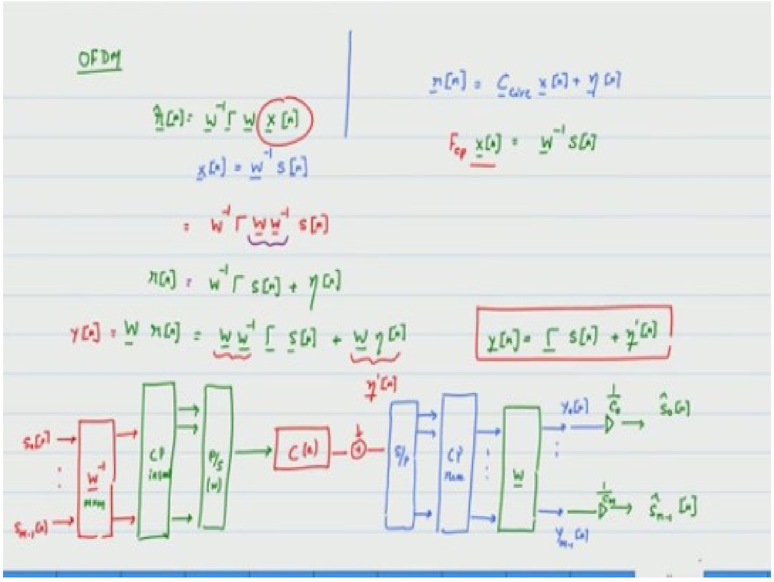

So, as we mentioned this is a very, very crucial point for us to where we start so, the extension to OFDM comes actually very, very naturally OFDM, so let us look once more at the equation, it says the transmitted signal r hat of n = W inverse lambda times W times s of n okay, is it s of n or s hat of n; s of n that is the transmitted signal, so the observation that comes from and supposed to come very naturally once you look at this is how do you get s of n?

S of n is supposed to be the modulated signal, how did you; how to generate, there are different ways of generating it and one of the ways in which we can generate a signal; a modulated signal is by using the DFT matrix, so can we generate x of n through the following process; W inverse times s of n okay, in fact I think I have made a mistake, let me just write down the equation, this is the after we do the cyclic prefix removal, before removing the cyclic prefix, this is the following, this is the equation.

R of n = C circulant times x of n + eta of n; eta of n is the noise term, okay so is this correct, or it should be Cup, i believe it should be; okay, after I remove the; after we pass it through the channel and then remove this thing, it should be the yeah, r of n is C circulant times x of n, I think so, let if you if you catch something wrong just let me know so basically, what we want to do is; x of n will be = the matrix W inverse times s of n okay.

So, how do I get instead of this one, I should be writing there x of n, the transmitted signal so, if I transmit W inverse times s of n then, it becomes W inverse lambda times W W inverse times s of n, okay yeah so, s of n is the input I pass it through the IDFT matrix, I get x of n, I pass this through Fcp and basically get the vector with the cyclic prefix, pass it through the channel and at the other end, I have removed the cyclic prefix.

So, basically the equation that we get is the expression as follows, so these two will cancel each other and what we are left with is the equation r hat; r of n rather, r of n = W inverse lambda times s of n + eta of n, if I take the DFT of this, W times r of n that will become W times W inverse gamma times s of n + W times eta of n that is basically some other noise vector, so I will just label this as eta prime of n, these two will cancel.

So, if this becomes equal to my output y of n, yeah, **"Professor – student conversation starts"** yes that is my modulation step, I am defining it as; here, yes so this is actually not this is x, right, its x, okay, is that okay, okay. **"Professor – student conversation ends".** So, effectively what the overall system becomes equal to is y of n = lambda times s of n + eta prime of n now, this may seem like just yet another equation but it is actually very, very profound.

Let us see what exactly what is it telling us, rather than redraw this the whole picture, let me just sort of take you back to the equation or what is needed for us in terms of the structure, so what is happening here is that I have to do the modulation and then insert the cyclic prefix, the modulation is being done by means of the IDFT process and then the cyclic prefix is getting inserted, so that is the transmission side.

So, if I do that and follow it at the receiver by doing the removal of the cyclic prefix and the DFT, this is what it looks like so, let me just draw a very, very quick block diagram so, this is s0 of n, all the way to s m - 1 of n, I am passing it through the inverse DFT, normalized inverse DFT, output will be M, this is a M cross M, this is a M cross M transformation now basically, we will do the CP insertion, okay.

So, CP insertion and followed by the parallel to serial conversion, parallel to serial, this is of dimension N okay that means you must up sample followed by the delays, you will get a single signal which passes through the signal C of z, noise also gets added, so there is a linear convolution with the noise getting included then, at the other end I have to remove the CP, okay so basically let us do the serial to parallel first, then the removal so, we will get these N outputs.

This is the CP removal and then what I will be left with this a vector of dimension M, I am going to process it through a DFT process W and at the output, I have the observations which are given by y, okay, these are the observations, y0 often; are they equal to the corresponding S's; not quite because what you are observing is scaled by a complex coefficient, you cannot call that as y yet.

So, in order for us to so, this is ym - 1 of n, how do I get the; get to the last stage; I would have to do scale it by the first branch by 1 over C0, the last branch by 1 over Cm and then these would now be effectively s0 hat of n all the way to s m - 1 hat of n, okay so, this is my block transmission scheme where we have introduced the; modulation is done using the inverse DFT and the reason I use the inverse DFT was because I know that my circulant matrix which is effectively what is going to come in between my transmitter and receiver can be simplified.

I mean can be written as in a diagonalized form, I am taking advantage of that and making sure that I get and at the final stage, I just do a 1 over C0, 1 over Cm being transmitted okay, now if this is an equivalent representation of C0, then the simplified representation is absolutely spectacular.

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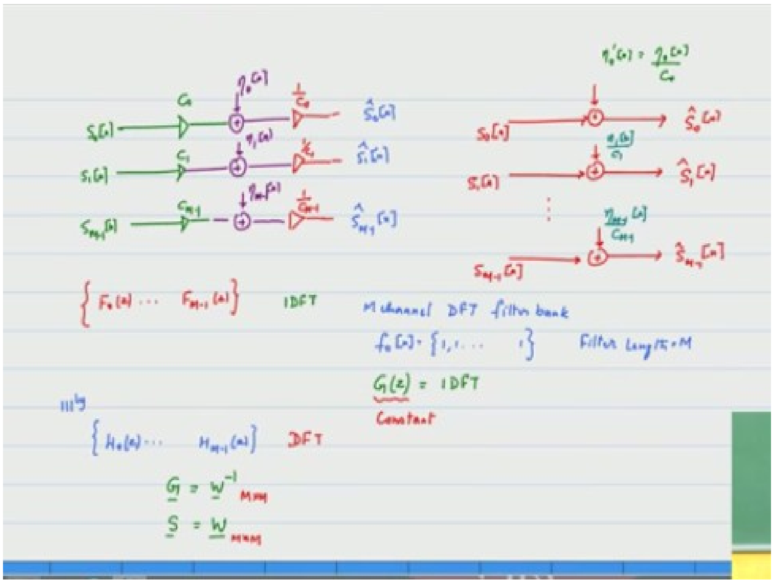

Because now what you find is the equation is s0 of n, s1 of n, s m - 1 of n, it is as if each of these are getting multiplied by corresponding complex coefficients in the channel okay that is all that is happening as far as the channel is concerned, there is a noise term getting added to each of these branches, this is eta0 of n, this is a another addition term, eta 1 of n and then the last one, eta m - 1 of n, these are the noise terms.

And what we do at the other side is 1 over C0, 1 over C1, 1 over C m- 1 that is the one that we do just before we declared the output and this is s0 hat of n, s1 hat of n, sm - 1 hat of n so, the introduction of the DFT as your modulation matrix and the, sorry, IDFT as the modulation matrix and the DFT as the one at the receiver parallelized your channel, okay what was the single wideband signal channel which could have caused you all kinds of problems with the inter block interference, inter sub channel interference and intra sub channel interview, all of that got very neatly, very compactly replaced in this fashion.

And if you want to take it even one step further, then you can actually say that this is how it looks like in the final scheme of things, s0 getting transmitted at one end is showing up at the other end with an effect of a noise term which is eta 0 dash of n which is eta0 of n divided by C0, that is it and this shows up at the other end with s0 hat of n, the only thing is it is like an AWJ in channel where there is nothing, all other impairments have been removed.

And likewise, you can draw the second channel s1 hat of n, this input the noise term is eta 1 of n divided by C1 and that comes out as s1 hat of n dot dot dot, the final channel input noise, this is eta m - 1 of n divided by C m - 1 and this is the final one okay, so very simple very elegant this is what it is all about, if you have done the and this sort of gives us a very good feel for why OFDM is so popular.

Because you can now extend this to any number, any M you want, you can said 1024, 2048 whatever you want depending upon and how many channels you will choose; depends on your channel dispersion and that is something that you can decide, so this is the reason why OFDM has become so popular, there is almost no complexity in this one because computing the DFT at the transmitter easy, you can do a fast algorithm, fast transform at the receiver, another fast transform.

And then just some scaling that happens and now what tells you that you can transmit on channel 0 and not on channel number 1, what is it that will make you decide one way or the other? Value of C0, okay, C0 is very small, what will happen; it looks like that particular channel has got a lot of noise because C1 is the denominator and your power allocation algorithm, what they said hey! do not waste your power transmitting on this channel, remove that from the equation so basically, you are having an advantage over there.

So, this is OFDM, this is in a nutshell, the big advantage with the OFDM and how we look at the complete thing, so let us summarize. So, the summary is that we have at the transmitter, we have to do the modulation and that will happen through the filter bank that does the TDM to FDM conversion, so basically we must have this set of filters at the transmitter, F m -1 of z, there are M filters that we have to do.

And what we have done is; we have taken it to be the IDFT matrix, okay, IDFT matrix if you have to describe it in a mathematical term, it is an M cross M transform but if you have to give it the filter bank interpretation, this is what an M channel DFT filter bank, correct DFT filter bank where the prototype filter is F0 of n is given by 1, 1, 1, it is a rectangular window of dimension M, okay.

So, basically it is a the filter length = M, all the coefficients are = 1 and we basically, get this so, if you did this and then go back to the original diagram, what you will find is that you will find that the G of z matrix which you what which we had is now basically, the IDFT matrix that is all that is left in it, okay and so it is no longer G of z, it is actually a constant matrix, it is not G of z, it is no longer a polynomial, it is only the IDFT matrix, okay.

So, similarly in the same manner at the receiver we have to have a filter bank which is H0 of z, all the way to H m -1 of z and this is we have introduced using; we have introduced using as a DFT filter bank and this case it is the DFT matrix, why did the IDFT matrix become the DFT matrix, why did it become that? Remember we did type 2 polyphase decomposition, there is a type 1, type 2 difference and that is why this will become different compared to that.

So, basically you can verify that once you implemented as a type 2, you will get a DFT matrix so, here is a summary if you go back to the original diagram, you can show that G = W inverse s = DFT matrix and of course, the dimensionality of the transforms very important, it is M cross M, the increase in the dimensionality the redundancy comes through either through cyclic prefix addition, it is not through using a larger transformation.

So, again keep that picture in mind okay, so given this we now have a complete picture of OFDM.

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Wideband channel $\longrightarrow$ Narrowband channels (M)

$\Downarrow$

M parallel channels each with different SNR

Water filling max. Throughput

OFDM

- Low complexity
- Achieves max capacity

Neg

- Peak-to-Avg Power Ratio (PAPR)

PAPR $\uparrow$ M $\uparrow$

OFDM is a very powerful modulation scheme it is one where we have a wideband channel, we have split into several narrowband channels, channel has been split into narrowband channels, the number of narrowband channels is M, okay and we are making sure that the transmission happens with redundancy and when we use the OFDM based structure, this becomes M parallel channels, each with different SNR okay.

And the minute you know that you have different SNR's, you know that the best thing to do is water filling and once you do water filling, you can get the maximum information through and basically via water filling, you maximize throughput you not only do that you remove those channels which are not good, maximize throughput or you can say that you optimize your transmission, once you have that at the receiver you are able to recover.

So, the beauty of OFDM, so this is OFDM, the beauty of OFDM is low complexity, you have removed the complexity of the equalizer, all you have to do is 1 over C0, 1 over C1 that is a very simple operation, compute the DFT of the channel and then it also achieves or maximizes throughput, achieves maximum throughput via water filling all of that which we have studied okay, so this is why OFDM is so attractive, this is why 4G, 5G, your Wi-Fi everyone wants to use OFDM this is so.

But there is a saying in business term that there is no free lunch, right that means that somewhere you have got to pay, you cannot get something which has got in a low complexity, low, good performance where is the penalty, it turns out that what we are generating when we complete, after we do the inverse DFT is as if we are adding many parallel channels, right basically all of them are modulated.

And the peak to average variation is very large so, the only drawback that we have with OFDM if you want to write down as a negative, is the peak to average power ratio and that will affect your power amplifier efficiency that will affect your overall total power required for your receiver, a peak to average power ratio; PAPR and there are techniques by which we can address the PAPR issue but this is probably the only penalty that you pay.

And then most of the time we say that okay and this becomes more severe as M increases, okay so, PAPR will become more severe as so, more number of channels are largest the DFT size, the more difficult becomes the; so this is advantage that we have to live with and it is not a big disadvantage and there are some techniques that are known to be able to reduce the PAPR.

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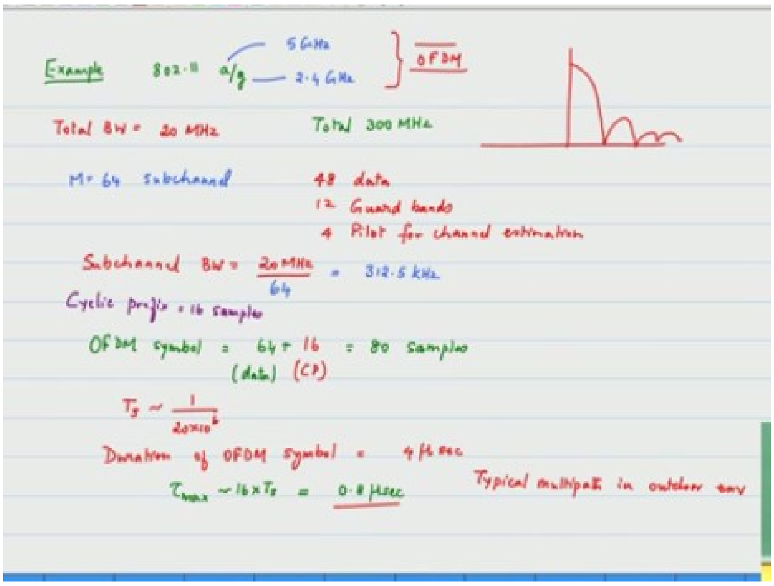

But next important element is where do we use OFDM and look at a practical example, so probably, the technology that all of you are using your phones are using is a 802.11, you create a hotspot, several people connect okay that is using 802.11 technology, it could be 802.11 b or it could be a or g, we are looking at primarily a and g, a is the technology that works in the 5 gigahertz band, it is based on OFDM, g is the one that works in the 2.4 gigahertz band also based on OFDM.

These are the two that we are going to be looking at today and you will see very quickly everything that we have studied actually kind of falls into place, okay, the total bandwidth of the modulated signal, total bandwidth is 20 megahertz, it uses 20 megahertz, there are it is about 300 megahertz is available altogether, so total available is around 300 megahertz, so you can divide 300/ 20 and you can say so many channels are available.

But what is of interest is what happens inside the 20 megahertz that is the transmission part, now rather than transmitting as 20 megahertz as a single carrier, the design of 802.11 does the following, it says I want to design it as a 64 subcarrier system or sub channel system, 64 sub carriers which means that I will use IDFT DFT size of 64 and it also specifies the following things if you go in and read the standard it will say that out of these 64, 48 will be used for data that is user information, 12 of them will be used as what we call as guard bands.

Because your spectrum tends to spill because you are using a rectangular window, it has a basically, the rectangular window has you know has got a very slow roll off, so you do not want to cause interference outside your 20 megahertz, so they use guard bands okay, so 6 in the beginning, 6 at the end are used and then there are 4 channels on which you send pilot information.

Pilot information; the minute you see pilot that means it is known information, it is not unknown, the reason known information is transmitted is so that you can estimate the channel and why do you need to estimate the channel; because you can do the 1 over C0, 1 over C1 at the transmitter, so pilot is used for channel estimation, so pilot for channel estimation, okay. So, this is the structure of the pilot symbols for channel estimation.

So, you get your 64, channels there okay, so now very, very important, how do we decide on the rest of the system okay, so the subcarrier sub channel bandwidth is 20 megahertz divided by 64 that is the number of sub channels that will give you 312.5 kilo hertz, okay and this is a system where we use a cyclic prefix, it is designed to carry a cyclic prefix of 16 samples, how many data samples will be there? 64.

Because you are using a IDFT of size 64, to that you will add 16, so it will become 80 samples, 80 samples will become your block length and that will become the frame or the OFDM symbol that you are transmitting, so one OFDM symbol is going to be 64 + the cyclic prefix of 16, this is the cyclic prefix, this 64 is the data, you can see the redundancy coming in so basically, this will be equal to 80 samples that is what is going to be that is going to be transmitted, okay.

Now, what is the duration of one symbol, this is a very important element because what we are looking at is; what is the rate at which the channel is going to be exercised okay, so basically the bandwidth of the channel is 20 megahertz, so which means that as far as the symbols are the if you were to look at, think of it is a single carrier system, you will be signalling at a baud rate which is with the duration which will be the reciprocal of the bandwidth, 20 * 10 power 6, okay.

So, basically what is the duration of the OFDM symbol; OFDM symbol will be 80 times TS, right so duration of OFDM symbol; one OFDM symbol which basically corresponds to 64 of these; 64 of the channel usage this thing, so but basically we are going to use 80 samples are going to be transmitted is going to be roughly, I think what does it come out to be 1 over 20 * 10 power -6 and multiplied by 80, I think this comes out to be around 4 micro seconds, you can just verify that okay.

Now, the key element is what is the multipath, how much multipath protection do you have; I have 16 samples of cyclic prefix so basically, I have protected myself against them so that maximum delay spread that I can withstand, tau max basically, my number of cyclic; my cyclic prefix should be longer than my delay spread, so 16 times TS approximately that is what I have protected myself against.

So, this comes out to be 16, wait this is 16 times this thing, did I get my this 4 microseconds correct, is that correct okay, 16 time TS what will that come out to be; point 8, okay, point 8 microseconds now, we said somewhere I think when we are talking about multipath and other things we said that typical delay spreads is of the order of 5 microseconds okay, typical multipath, typical multipath in outdoor environments, typical multipath delay spread in outdoor environments is approximately around 3, 4, 5 microseconds.

Let us write down as 5 microseconds okay, which means you are not allowed for enough or for your this thing, why did they design it such a way, any idea, OFDM sorry, 802.11was not designed for outdoors, it was designed only for indoors and indoors what do you see a few nanoseconds is what you see so, they have actually designed it very well for indoor now, if you take and deployed this outside, it may or may not work depending upon how much multipath you are seeing.

Sometimes, you may say what is this, it worked very well indoors, it did not work outside why because CP, cyclic prefix was not enough and you ran into trouble okay, so that is one element of it okay.

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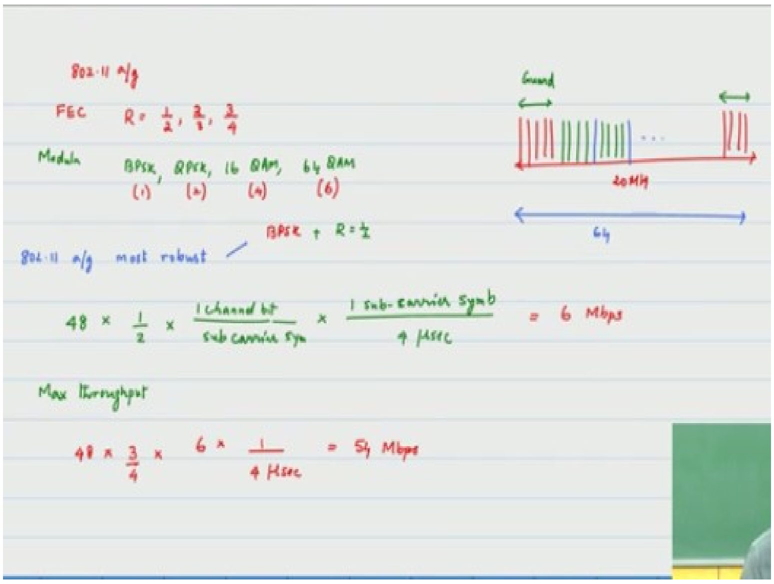

Now if you go back and read the 802.11 standard; 802.11 a bar g standard, it tell you that they have forward error correction or error correcting codes and the rates of these codes can vary between 1/2, 2/3rds and 3/4ths okay and the modulation methods options that you have can go from BPSK which means one bit per symbol, QPSK two bits per symbol, it can you can do 16 qam 4 bits per symbol.

And you can do 64 qam, which is 6 bits per symbol, I will just write the within the bracket what how many bits per symbol I am going, so this is 1, 2, 4, 6 bits per symbol, okay so, what is the minimum rate that you will get on an 802.11 system, minimum rate means most robust so, what is the rather than calling it minimum rate, let us give it a positive flavour, what is the most robust scheme that you can do?

Of course most robust means you will say choose BPSK and you will say combine it with rate 1/2 coding right that is what we would like to do so, here is the calculation for the minimum rate, I have 48 data carriers okay and each time I transmit 1 symbol on the channel, the information that I actually I am carrying because it is a rate 1/2 code, it is actually rate 1/2 so, I have to reduce it by that factor, only half of it is information okay.

And basically, I am carrying one channel bit per subcarrier symbol, each of those 48 sub carriers each of them is carrying a BPSK symbol, so I am getting one channel, out of that one half of it is only information bit okay into and one subcarrier symbol, every 4 micro seconds, divided by 4 micro seconds okay, is my calculation clear, 48 sub carriers, each of those sub carriers is carrying one bit of information.

Because I am using a cube BPSK transmission but only one half of it is information because it is encoded with rate one half and each subcarrier symbol is carried these 48 subcarrier symbols are carried in 4 microseconds okay, if you do this calculation it will come out to be 6 megabits per second that is your most robust scheme, okay. So, what is your maximum throughput scheme?

You can write it down almost by inspection, 48 * 3/4 that is the information rate * 6 channel bits per subcarrier symbols * 1 over 4 microsecond, this we cannot change okay, if you do this 54 megabits per second okay, go and look at a router, it will say rated for 54 megabits per second that is it that is where you get it from, okay and how is it done; you have taken 20 megahertz, the first 6 and the last 6 were guard bands, okay.

I have not drawn enough number but you can these are the guard bands okay, now apart from that there are 48 channels which are carrying information, so there are information carrying and then occasionally, you will find a channel which is carrying pilot information, so basically some number followed by a pilot, followed by another pilot dot dot dot, okay so total of 64 and this is why this is how it is designed and this is how it is developed.

So, again the technology that we have used is absolutely scalable to any bandwidth that you want and achieve the data rates that you are looking at basically, this is all there is to the design of the system and you can also tell whether a system will work well in indoor, outdoor by looking at how many CP if you; now if you wanted to modify this to use it in an outdoor environment what would you do?

You would have to add more cyclic prefix where will you pay the penalty; this 4 microseconds may become 5 microseconds, the same amount of information now with more cyclic prefix what will happen, you will need more information and more time to transmit that information, so which means that you will your throughputs are going to go down, okay but it is a trade-off that you can make to make a system design work well, okay.

So, the last element is that what if you have not studied multi-rate signal processing, all this is fine, right we came up with a very elegant method, so does it mean that people could not would not have you know come up with the OFDM, okay so, let us look at it assume erase multi-rate signal processing, you are not taken multi rate signal, all you have done is basic DSP and you have done communications okay, here it is.

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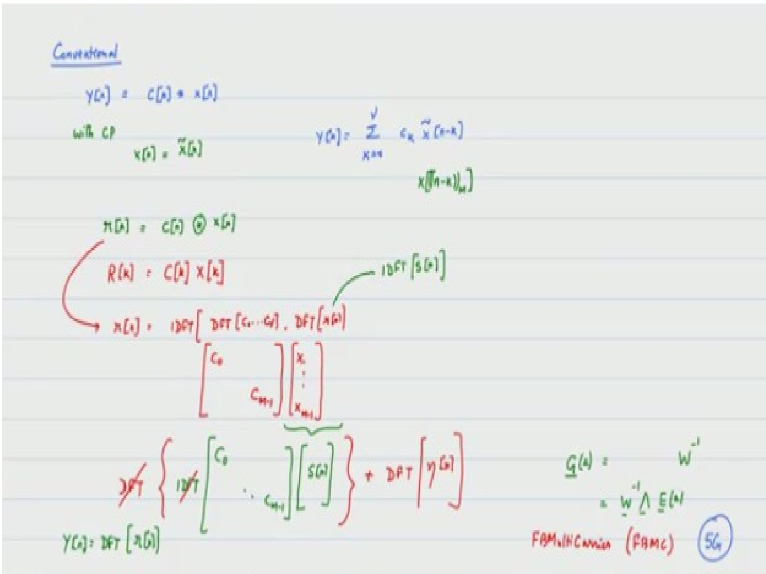

So, this is the conventional thought process, conventional okay, conventional says input-output relationship is always a convolution okay, C of n convolved with x of n, x of n you have put in redundancy whatever it is you want to do, okay now, if I have done cyclic prefix with CP, I hope you will recall that x of n with CP actually becomes looks like a periodic signal, we call it x tilde because you know the data kind of looks like wraparound.

Now, you may say well I did not really extend it beyond a certain point, it is not necessary because you are going to observe the convolution only over a finite duration, so this looks like it is fine, so the input-output relationship now becomes y of n = summation k = 0 through Nu, C subscript k x tilde x of n – k, right so basically and of course, you can verify that this is nothing but x of n - k modulo M okay, so which means that your interpretations will have to be done.

And you can actually show that the linear convolution is actually circular convolution, okay so effectively one frame of the output r of n can be written as C of n is no longer just linear convolution, it is actually circular convolution with one period or just that basic frame x of n okay, so this is the time domain again, I may be a little bit sloppy in my notation but it is a concept that is most important.

Now, can you define, can you express this equation in DFT terms; circular convolution, so which means R of k = C of k times x of k correct, okay so, it is DFT of C that channel matrix C and it is the DFT of the x basically, it is the vector without the cyclic prefix basically, you have taken that portion of it because that is the part that you will have to take the DFT off, multiply the two and then write down the expression.

So basically, what we would write down this equation as r of n would have to be IDFT of DFT of C, C Nu, okay that is that you have to write it as a diagonal matrix, C0 to see C m -1, write it and multiplied by DFT of x, okay, so that would be x, x0, xm - 1 okay, so again I am just quickly sort of getting you giving you the flavour of the conventional approach okay, so here is this is what you would have done to capture the circular convolution.

I have to take the IDFT of it now, very quickly look at the steps what did we say, x of n was going to be? x of n was going to be IDFT of something okay, IDFT of s of n okay, this x of n was going to be IDFT of s of n, okay that is what we were going to do at the transmitter side, so IDFT of s of n so, basically this DFT can be removed because DFT times IDFT will cancel each other, so then this equation this part of it will become s of n okay.

And followed by DFT of the C coefficients, followed by IDFT, so if I were to write here IDFT, C0 through C m - 1 and then I tell you okay, at the output Y of n = DFT of r of n okay, so which means that you will take this whole thing and then multiply it by DFT, this and this will cancel and if there is a noise term there, it will become DFT of the noise term, eta of n and then you tell me, you took so many lectures to explain this, I knew this from traditional communications, right.

I mean nothing complicated, what is it; it is parallel channels, C0, Cm – 1 is not nothing very sophisticated about your derivation, I have to defend against that accusation right, so now you turn down and ask the communication engineer, okay now, what is your spectral leakage; 13 dB very bad very spectral and where did you show, where did it actually show up, remember how many channels you have to put as guard bands in my 802.11.

I did what 6 channels on either side, 12 channels you could have; you have wasted bandwidth, so they say okay, I want to reduce the spectral leakage, communications is looks at it and says, I cannot do anything about it, it is DFT, IDFT, I cannot do anything about it you say change it, what is so scared about DFT, just a; so, wait, wait, I have no way of doing anything because what will happen you change DFT to something else, what will happen?

I do not know what happens to circular convolution, I do not know how to deal with you know, I am basically stuck with the fact that all I can do is this, ask the multi rate signal processing person, what to do with it, they will say well, this is it, G of z is what you were trying to design okay, what did you do in the process, you actually we you made it as W inverse, right, you would made it W inverse but nothing said that it has to be just W inverse in our structure.

You could have very well have done W inverse lambda times G of z which we know is a DFT matrix and then you can design any type of roll off that you want, you can design a system that has got all the benefits, right it has got the benefits because what DFT, IDFT, those will cancel each other now, you have to be a little bit careful on how you deal with e of z, you may have to impose certain constraints but at the end of the day, you now have a flexible framework.

So was it worth going through the multi rate signal processing approach, I hope you will agree the answer is yes because if you were only interested in OFDM the way it is the basic the plain version of OFDM, the communications would have given it to you in 5 minutes basically, you please clean up this notation you will find that the basic information is here on this one slide, you do not have to go through CP, you do not have to go through the circulant matrix, all of that.

But actually, when you have that perspective that is so this is what we now in the 5G system is called a filter bank multi carrier, FBMC okay, it is no longer just OFDM; OFDM is just one variant of a filter bank multi carrier but then you can design it to be a whole family of filters which are called that is what is called by FBMC and this is one of the key topics that is being discussed in the context of 5G.

But again as far as our course is concerned, we conclude the chapter on OFDM saying it is one of the best methods of transmission but you can extend it beyond where it is today, if you come at it from the multi rate approach and then design it using the filter design tools that are available to us, thank you. **“Professor-student conversation starts”** in the conventional derivation, yes because in the thing is in this process, the circular convolution will come only if you have the cyclic prefix inserted.

If you do not have it then you cannot write it in so, yes the cyclic the redundancy comes in the conventional method also, yeah, yes, yes so you will start getting some conditions that will be on E of z, so that you can get proper reconstruction of the things yes, so basically there will be a matrix E on the transmit side, there will be another matrix on the and the product of those must have certain properties.

But it turns out that result comes from M channel perfect reconstruction filter banks already have solved that is a known problem, what those; what should those polyphase component matrices be that is a known result, so again we can just borrow that result from there okay, **“Professor-student conversation ends”.** Thank you.