Multirate Digital Signal Processing
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Lecture – 37 (Part-1)
Orthogonal Frequency Division Multiplexing - Part 1

Okay good morning. We begin the last part of the OFDM discussion. So in the last lecture, we have been talking; we introduced the notion of using cyclic prefix as a way to introduce redundancy.

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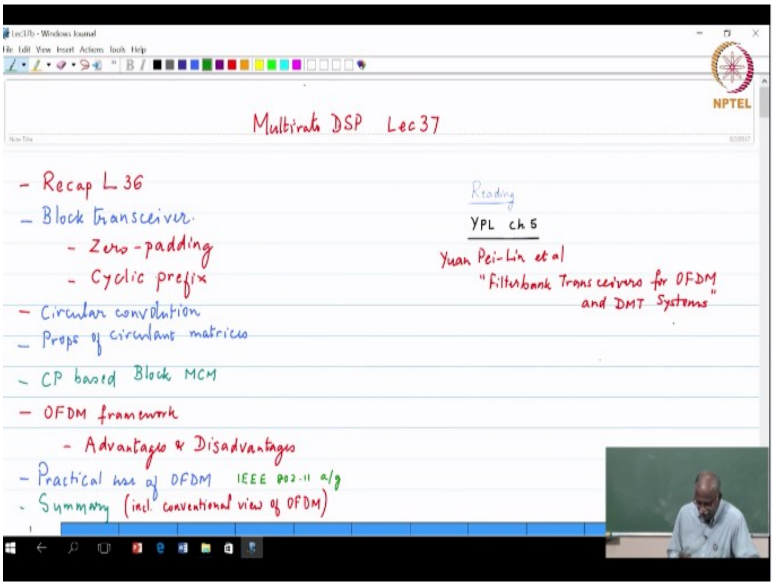

In and the earlier one, we had looked at zero padding. So the purpose of today, the focus of today's lecture will be to look at what happens when we introduce cyclic prefix, yesterday we came to the point of saying that the convolution between the data and the channel becomes a circular convolution. So we pick it up from there and then move forward to understand what happens with how do you process when the convolution is a circular convolution and then how the OFDM framework emerges.

We will also discuss briefly advantages and disadvantages of OFDM. The practical case, the Wi-Fi system that you use has got an OFDM version 802.11a if you use a or g if you use B is a spread spectrum system OFDM is used in a and g. So we will look at IEE 802.11 a/g both of them are based on the same OFDM system. So all that we have studied actually has got a very practical relevance, so we will do that and then sort of take a panoramic view, where we started off was trying to optimize or send maximum throughput through a wideband channel.

And then we went through a series of observations and results and then finally we will end with OFDM. Now what if you did not have the multirate perspective at all? We are not studying it in the context of multirate. There is a conventional view of OFDM. I think it is very important to also have that perspective saying that okay if you do not come at it from multirate signal processing.

But actually just come at it from the communication side, what is your, what is the way. You would come to the same conclusion, cannot be a different one but it is very insightful to see then what is it that the multirate perspective adds over and above the conventional one. So that is so again Yuan-Pei Lin's book chapter 5 is the anchor for our material that we are covering today okay.

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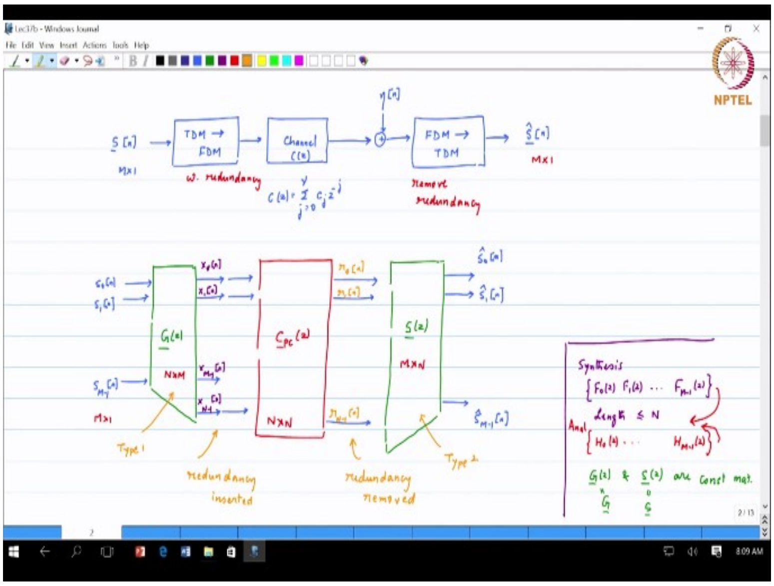

So we have dispersive channel which has nu +1 taps, so effectively it is a ISI channel with nu symbols of ISI. We want to introduce redundancy, so at the transmitter we want to introduce redundancy. Therefore, upsampling becomes by a factor of N. So the channel now becomes encapsulated in the form of a pseudo-circulant matrix which dimensions N x N. Then, at the receiver we want to remove the redundancy and then extract out the signals and this is the broad framework okay.

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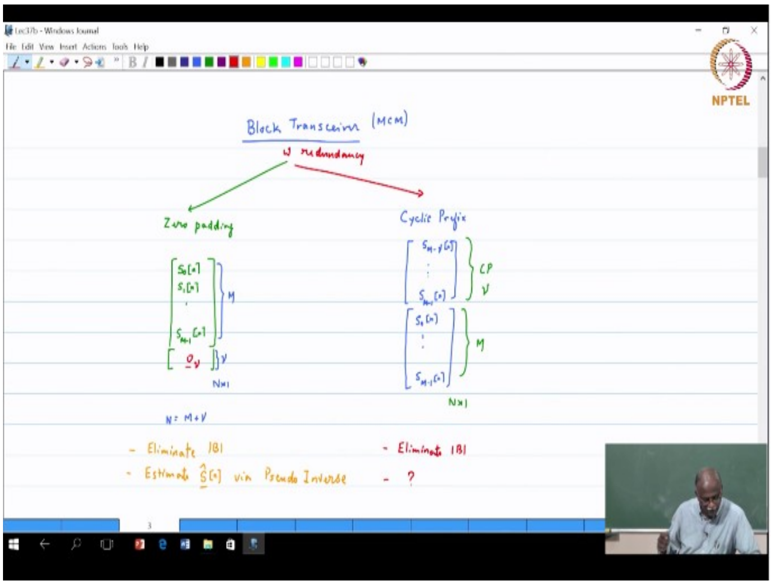

Now what we said was in the context of the block transceivers, so in the framework of the diagram in the previous slide, we call that as a block transceiver okay. It is also intended to be a multicarrier modulation. Basically, our intention is to extract them and it is also with redundancy okay. So this is the broad framework that we are doing. One approach that we have seen is the zero padding approach.

Zero padding approach says take your data block S0, S1 of n one particular time instant, S M-1 of n and to ensure that we are able to handle the, remove the interblock interference, we add an additional zero padding and we add nu, at least nu zeros has to be added. So this is the zero padding portion that was extended. So basically you extend it, so it now becomes a N x 1 vector, so there are M data symbols, there are nu zeros and N is=M+nu okay.

So this was one way by which we could eliminate IBI. We showed that, we also showed that we can recover the transmitted signal through a pseudoinverse operation. So eliminate IBI, estimate transmitted, no estimate the transmitted signal S hat of n via a pseudoinverse operation. Pseudoinverse, I will just write so you can fill in the rest of the information okay. On the other hand, there is another family or another branch where we said that we will we will introduce the zero pad the cyclic prefix.

It is advantageous for us to think of cyclic prefix as preceding the data block. So we have S0 of n all the way to S M-1 of n and the extension for the cyclic prefix is the last nu symbols. So effectively it is S of M-nu of n all the way to S M-1 of n. So this is the cyclic prefix part.

This is the CP part, it has got length nu and of course the data portion has got length M. So the amount of redundancy this is also a N x 1 vector.

We have same amount of overhead on both sides, here also we eliminate IBI. Today's task is to how do we estimate; this is the part that we are going to be addressing today okay. How do we estimate the channel, how much complexity will be involved and the rest of the elements okay? So this is the broad framework. We have got two classes, zero padding fully solved, the cyclic prefix.

We know what it is going to introduce, it is going to cause a circular convolution and basically we would now like to see how to take care of that okay.

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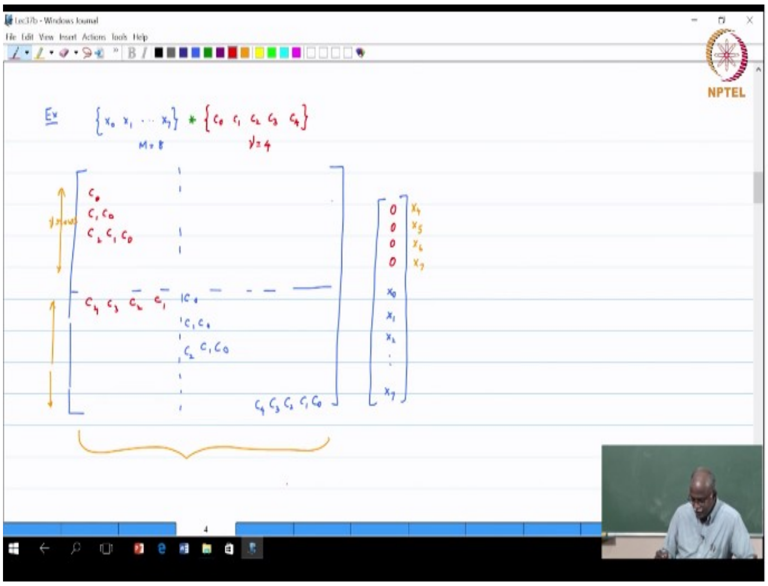

So one just to refresh what we are talking about, linear convolution we did the matrix how do we write down the rows of a matrix. Then, if you replace these zeros with the last 4 symbols of what are transmitted then we showed that what you get is circular convolution.

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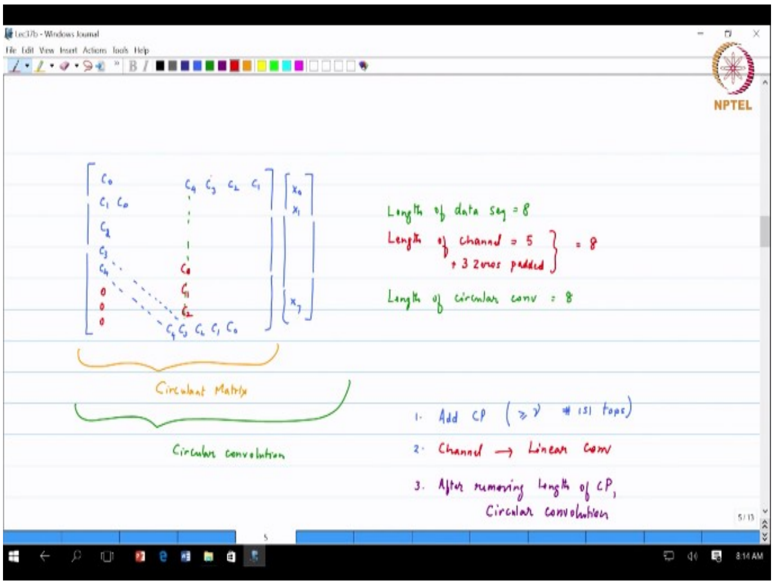

So this is how the circular convolution comes about. So the cyclic prefix is very important for us to be able to generate the circular form and once you get it into this structure, we also recognize that this matrix is circulant and by definition will also be Toeplitz in its form okay and we will take advantage of that okay.

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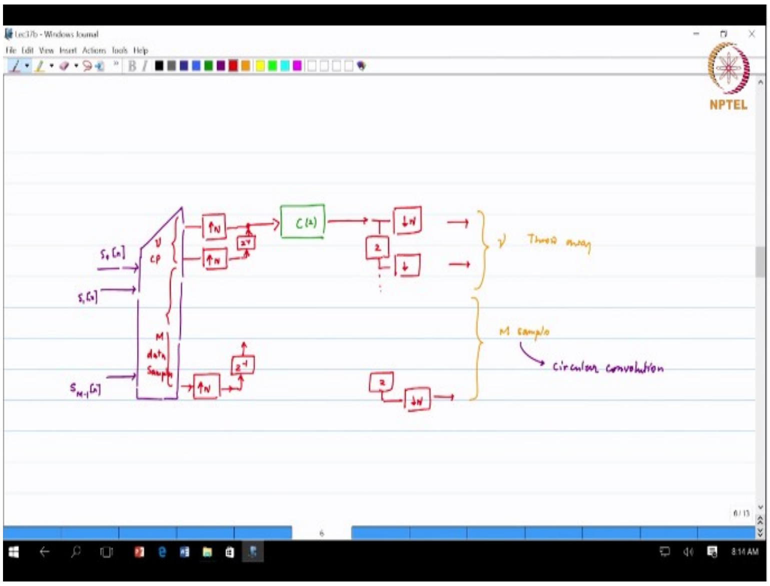

The block diagram for the introduction of any form of redundancy particularly in this case we would like to look at the redundancy in the form of cyclic prefix. We look at some additional data getting added before the data symbols and all of this information getting transmitted. So the cyclic prefix sort of is the information that is transmitted along with the data. At the other end, we recover the information; throw away what we consider as part of the cyclic prefix the first portion of the data and then you take the latter part and then we process that okay.

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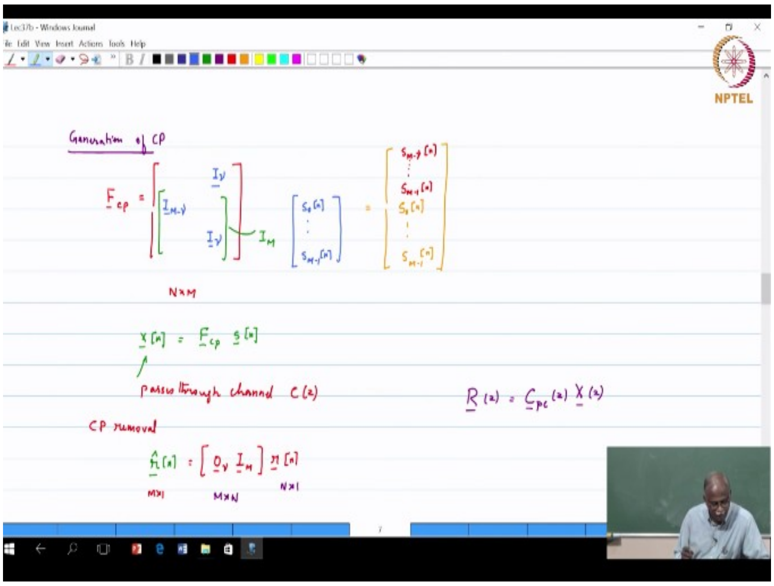

So this is the broad framework, today's work begins at this point. We showed yesterday that the introduction of cyclic prefix can be shown as a matrix operation on this vector. Again, it is one of those things where if you are able to write it in matrix form, there are some advantages, we will show that in a minute. So the signal that is transmitted, signal with redundancy X is the data signal with the cyclic prefix added to it.

And that is what passes through the channel, so the first step is to generate the signal or the vector with the cyclic prefix.

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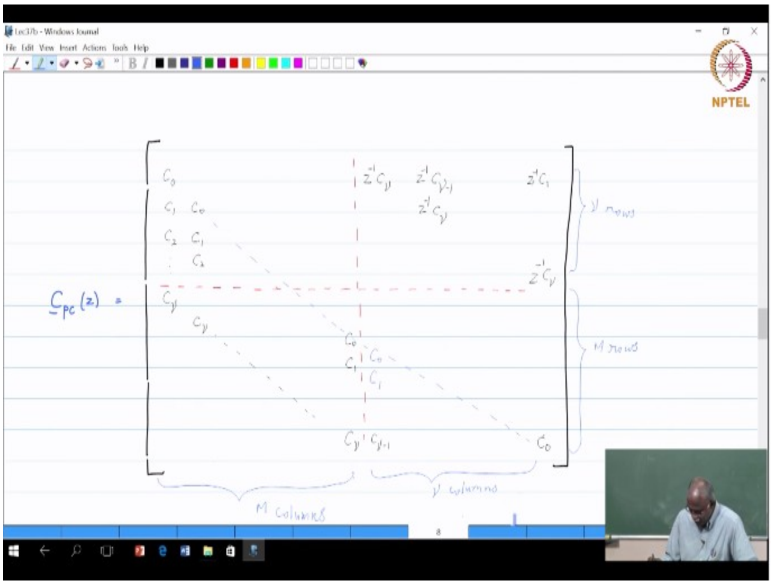

Now the cyclic, the CPC of z, the transfer matrix we went through and generated it completely. I hope you had a chance to verify. We make the following observation. The last nu rows are the ones that have got the z inverse term, same because of the structure the first nu rows are the ones that have got the z inverse component okay, so that is an important observation. Let us just write it down because that anchors the next step that we are going to write down.

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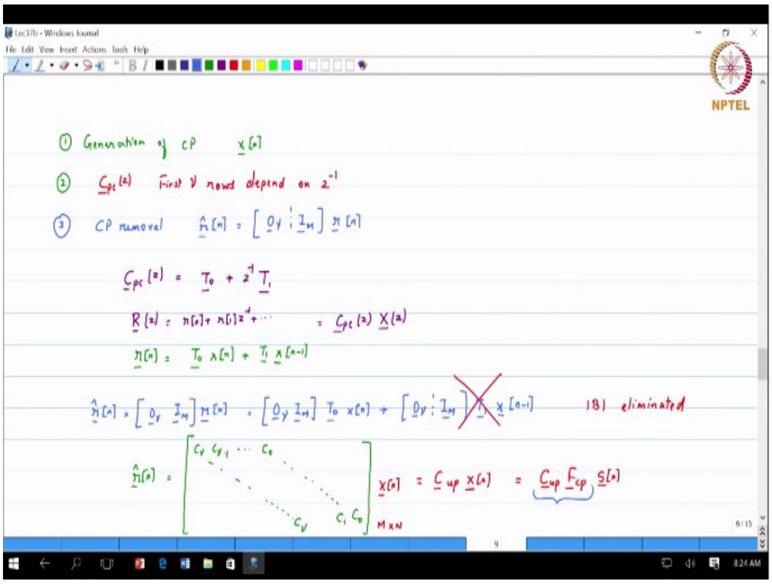

So the first step was generation of CP. Generation of data with CP, so basically that means you have generated X, X of n. The second observation is that in Cpc of z which is the pseudo-circulant matrix. The first nu rows are the ones that contain the z inverse term, rows depend on z inverse, the other rows do not have a z inverse term, so therefore they are a constant matrix okay.

Now the third step, the third step is after it has passed through the channel you will do CP removal and this also we showed through a simple matrix operation. The vector which is back to dimension M is obtained by the following operation where we null out some of the and then retain the rest okay. This times r of n, so basically the first few elements what are the portions corresponding to the cyclic prefix have been nulled out and what is retained is only the latter half of it okay.

So this is where we have left off. Now if you recall in the previous class, we wrote the following form which is helpful for us. Cpc of z you can think of it as a constant matrix T0+z inverse another constant matrix T1 okay and in a minute we will take advantage of this. So R of z which would be r0+r1 z inverse dot that is basically all these vectors at the received side will be=Cpc of z times the transmitted vector.

The transmitted vector was X of z; X was what was obtained after the addition of the cyclic prefix okay. So if you now simplify this expression and then compare like powers of z, we can write down the equation that is going to help us solve r of n is going to be=T0 times X of n+T1 times X of n-1 okay. That is basically there is interblock interference is present and that is what you expect because of the Cpc matrix having z inverse terms.

Now here comes the important step. How do we get rid of the cyclic prefix? r of n requires us to multiply with this matrix zero of dimension nu x nu and the identity matrix IM okay. So basically there are nu columns which are 0, the dimensions of this portion is M rows nu columns and this is an M x M, this multiplied by r of n okay. So this is the step that was going to get rid of it.

Now if you substitute for r of n from the previous equation, what you will get? Just bear with just one additional repetition of this IM times T0 okay times T0 X of n+vector IM T1 X of n-1 okay, just simplifying. Again, you may have already seen what the result is going to come out to be. Now notice that T1 has got only nonzero entries in the first nu rows because that is what correspond to z inverse terms.

And what is this going to do is going to kill the first nu rows, so effectively this term gets killed. Basically, there is no contribution from that and what you are left with is a relationship between the input and output where there is no interblock interference. So straightaway we can see that IBI is eliminated, we knew that but we just sort of validating the result that is known to us and now writing down the expression for r hat of n, r hat of n basically is the lower M rows of this matrix.

Notice that it starts from C nu; C nu-1 all the way to C0 and then it is a Toeplitz after that. So let me just write that down, so the structure that was going to be present here is C nu C nu-1 all the way to C0 Toeplitz beyond that point C0 C1 and okay this is C nu. This is the matrix that we have okay. The dimensions of this matrix, we already mentioned that the number of rows is M, the number of columns of the Cpc is N so that is not changed.

So this is an M x N matrix and this matrix we will call as Cup okay. So this is the structure that we have. Now what will happen is to take a closer look at this result okay. So r hat of n is equal to Cup times X of n right, so we should not forget X of n this times X of n, this is times

X of n okay. So I want to take it to the next step. Where did Cup come from? Cup came by multiplying, no. Where did X of n come from?

It is Cup times that matrix F, I think the matrix is not there but we wrote it down yesterday. F Cp, I think maybe we have written it today also F Cp is there. F Cp times S of n okay, so I have just written it down. So finally what it is we inserted cyclic prefix, removed the cyclic prefix, showed that it eliminated interblock interference and in the process left us with a matrix which is a constant coefficient Toeplitz matrix, banded diagonal matrix and X of n itself is you take S of n and introduce the cyclic prefix.

So this is the overall form that we have. Now I want you to just appreciate what this one gives. So let us take one example and it actually gives us a very interesting insight.

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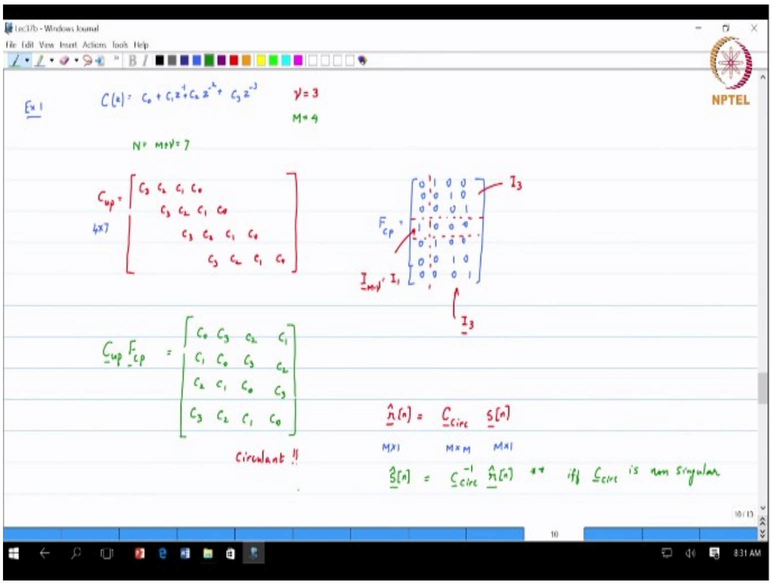

Example 1 for today okay, so I have a channel which is C of z=C0+C1 z inverse+C2 z-2+C3 z-3. So this corresponds to the case where nu=3, 3 symbols of ISI. If I have a data vector of size 4 then N=M+nu will be=7. I would like you to first construct what Cup looks like. Cup must be a 4 x 7 matrix okay. So the entries of it will just make sure we are able to do that. C3 C2 C1 C0 C3 C2 C1 C0 C3 C2 C1 C0 C3 C2 C1 C0 that is it.

That is the Cup 4 x 7 you can verify, so this is a 4 x 7 matrix exactly just so that we are comfortable with this. Now for this particular configuration of data and the channel length, construct what Fcp will look like. Fcp is identity matrix of dimension M-nu, M-nu is 4-3 is 1. So it is basically a constant. I am sorry yeah and then followed by an identity matrix of dimension nu which is 3, so basically it is 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 okay that is the lower part. The upper part is a I3 again so basically 0 1 0 0 0 0 1 0 0 0 0 1 okay.

So just divide this into segments like this and you can verify that the structure is the same as what we have, so this is I3, this is also I3 and this is I of M-nu which is I1 that is a scalar okay, so that is Fcp. Now very important to look at this structure, Cup that is what is going to happen in the processing. Cup multiplied with Fcp, so if you can just do a multiplication of Cup times Fcp, I would like you to write down the result.

It is easy enough, it is non-trivial that is why I took this particular example but I would like you to just draw. What you will find is the first entry is C0 right, it has got a 1 in the fourth position and then let us do the first column, next entry will be C1 then C2 and then C3 okay. You will find the C0 C1 C2 C3 turn out to be the first column for us okay and just go to the second, move over to the second column.

C0 comes in the second position the diagonal position, C1 C2 come in the next and you will find that C3 comes at the top and of course just quickly complete the rest. It is C0 C1 C2 C3 and then C0 C1 C2 C3 okay. So this is a structure of a circulant matrix okay. Now is it a surprising result? The answer is no definitely not because we already knew that the convolution is going to be a circular convolution.

So if the convolution has to be circular, the matrix part that represented has to be circulant that is when you will get, so this is not a surprising but it is an interesting result, it is consistent with everything, so this Cup times Fcp is actually a circulant matrix. So if you rewrite the equation r hat of n is=C I am going to call it now C circulant 4 x 4, in this case it will, in the general case it will be M x M times S of n.

Let us write down the dimensions, r hat will be M x 1 because we have removed the CP, the circulant matrix will be M x M, this will be M x 1 and I am going to now write the result as S hat of n my best estimate is going to be C circulant inverse times r hat of n and I say that okay we are done let us go home because we have solved the problem. You should have an objection to this.

No objections. **“Professor - student conversation starts.”** Whether it is invertible, very good question okay. **“Professor - student conversation ends.”** So I have to show you that the C circulant is always invertible okay that is non-trivial, so that is the part of the task that we have for us today okay. So this is star star if and only if C circulant is nonsingular right, is nonsingular and we will explore that in today's lecture okay.

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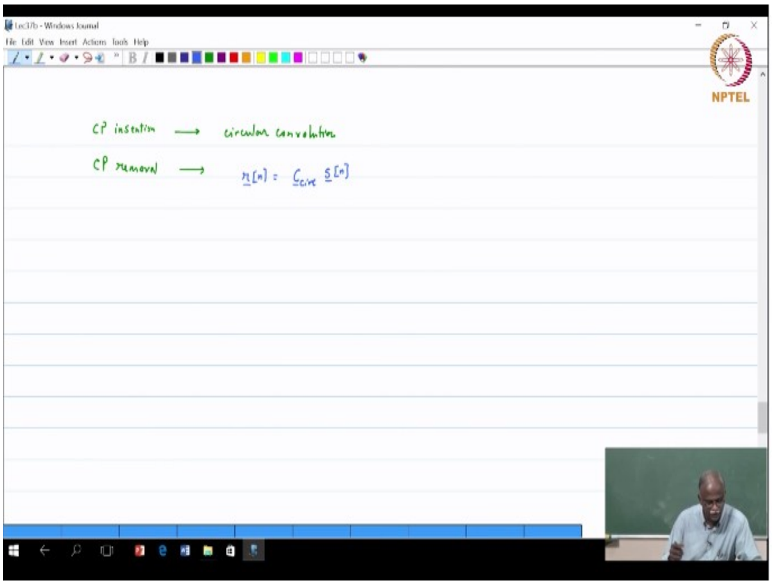

So effectively what we have done or if you were to summarize it, the removal of CP okay. The CP insertion led to circular convolution okay, that was part of the observation and then CP removal basically gave us a input-output relationship which is of the form r of n is equal to a circulant matrix is equal to C circulant times S of n okay. This gave us this result and so basically what should have been linear convolution in the channel became circular convolution.

And that is what we have captured it, when we removed the CP we are left with a equation and we will then try to look at how to address this issue.