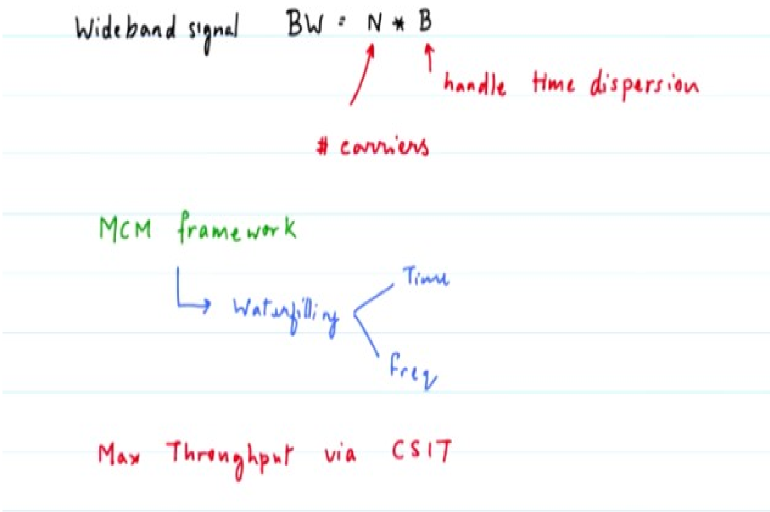
Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture – 35 (Part-1)
Pseudo Circulant Structure (Part 1)

Good morning let us begin last time we had 2 lectures back to back 33 and 34 quite a few new concepts were covered. So today it will be a little bit longer and more detailed review so that we can pick up from there. We are talking broadly about the topic of Multicarrier modulation.

(Refer Slide Time: 00:35)



And the motivation for Multicarrier modulation is stems from the fact that we want to deal with wideband signals so that is primarily our interest. A wideband signal has several challenges you get frequency selective fading and the complexity of equalizer is quite a challenge. So we said that the wideband signal we will treat as N narrow band carriers N*B. So this is where the Multicarrier modulation concept is coming from.

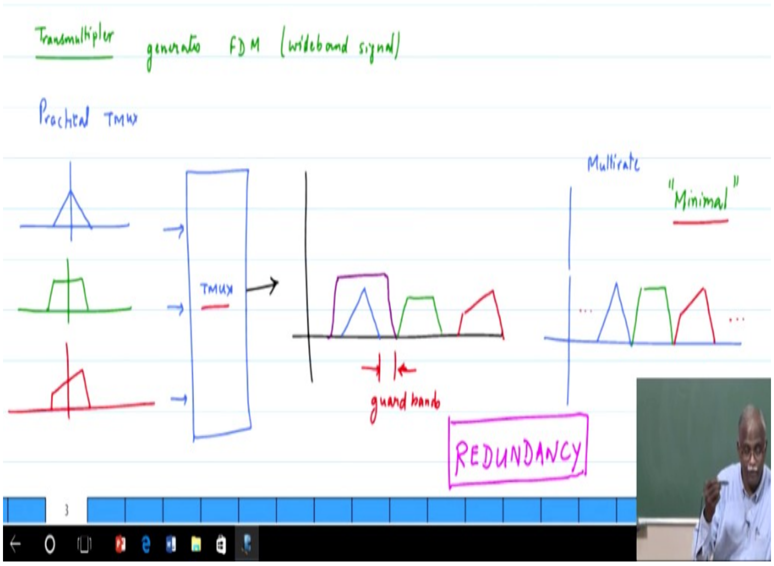
So B the bandwidth of each of the sub carrier or each of the carriers is chosen to handle dispersion. We said that the bandwidth is related to the baud rate. Baud rate determines how sensitive you are to dispersion. So basically I will say that B is chosen you have to choose it to handle time dispersion. Once you have done that then the number of channels gets fixed. N denotes the number of carriers and the number of carriers you need to achieve a certain bandwidth.

So basically the key decision in the design of a Multicarrier system one is the choice of the bandwidth B because that is what is going to tell you how robust your system is going to be and once you have a Multicarrier system of this type we said that we will say that now we have a MCM framework, the wideband signal is now transmitted as multiple narrow band carriers and in order to achieve capacity in a MCM framework we will do water filling and that is where the interest in terms of capacity comes from.

And we said that we will do water filling over time and over frequency. Over frequency to achieve the capacity at every time instant and then at each time instant the channel changes and then you adapt accordingly. So all of this is done to maximize the throughput of your system via CSIT, you need to have channel state information at the transmitter so which means that there is a feedback channel. So this is the framework of the Multicarrier system.

Now where does Multirate DSP play a role and what is the benefits that we can get is what we are going to be what we focused on.

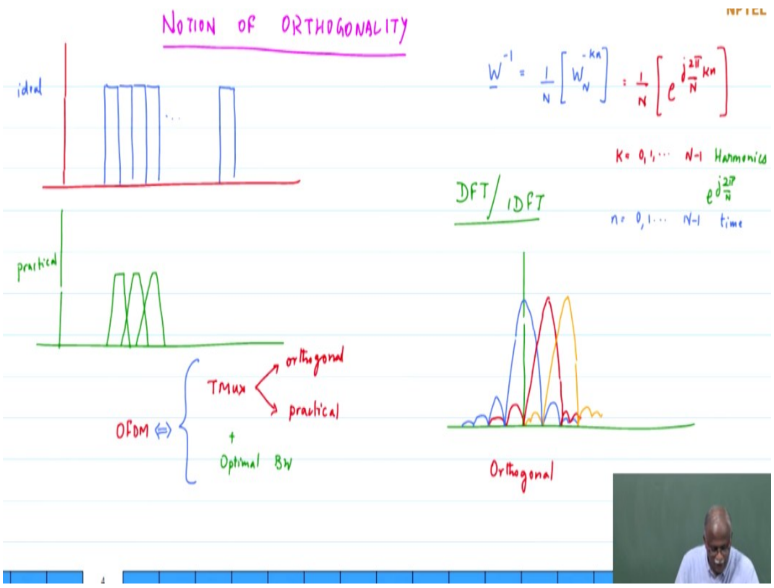
(Refer Slide Time: 03:04)



So we have this notion of a transmultiplexer. Transmultiplexer is a device that converts from narrow band signals to a wideband signal. Now when we did the design of the transmultiplexer what we talked about in the ideal case where the signals were side by side without any gaps that is what we refer to as a minimal system. It occupies the minimal bandwidth that is required.

On the other hand, if you want to have a practical filters we would have to introduce some guard bands so that you can have the filtering and this is where the notion of redundancy has come in and we were leverage this in the last lecture.

(Refer Slide Time: 03:46)



Now redundancy along with that comes the notion of orthogonality basically we want to be able to establish these signals independent of each other these narrow band signals. So we are looking for ways of separating them. Of course separating them in frequency is one way and there are other way by which you can say that okay as long as they do not overlap there is a notion of orthogonality in frequency I should be able to separate.

But from a practical standpoint we said okay that is not possible you would have to allow some overlap because you cannot have perfect filters. The challenge is how do we design with overlap still maintaining orthogonality and maintaining something that is close to a minimal system. We made an observation that the IDFT DFT gives us one such framework maybe just the mathematical bag or the thing that go with this figure.

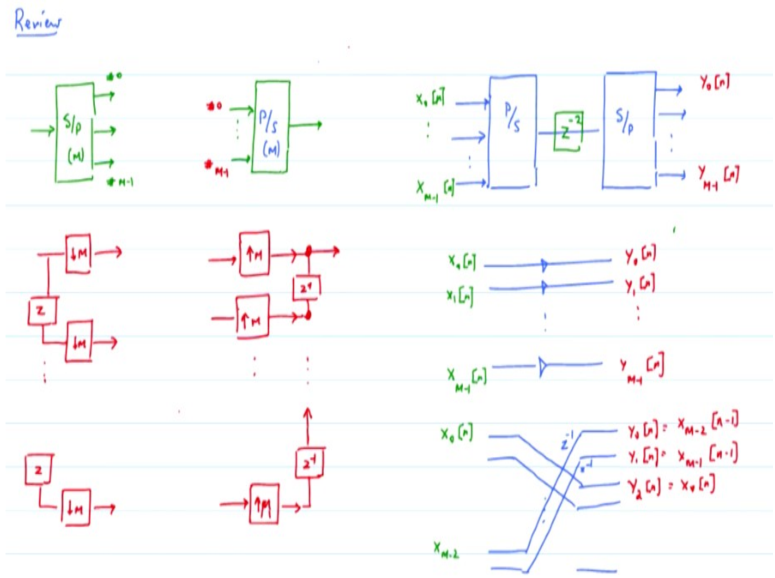
So let us take the IDFT matrix. So basically it is W inverse if you were to ask you to write down the IDFT matrix this would be 1/N times the conjugate of the DFT matrix. The conjugate of the DFT matrix is WN kn this will be WN –kn and the DFT matrix is symmetric you strictly should have written as –NK but it is the same. So what does this actually come out to be.

It is 1/N e power j 2 Pi/N times kn. K represents the harmonics 0,1 to N-1 so you can think of these as the harmonics of the fundamental frequency e power j 2 Pi/N they are the harmonics and you are looking over a time window which has N samples. So you are looking at N samples this is your notion of the times scale. So given this each of the rows are orthogonal though they overlap in frequency.

So that is the figure that we have they are overlapping in frequency nevertheless you have a system that is orthogonal. So the IDFT is a very powerful concept that we bring from DSP and say oh by the way we have a framework that already satisfies whatever system you are looking for you want to have orthogonality yes we have, you want to minimal bandwidth we can do minimal bandwidth.

Overlap is needed yes overlap is present, but we still have orthogonality so that is one piece that we will leverage a lot.

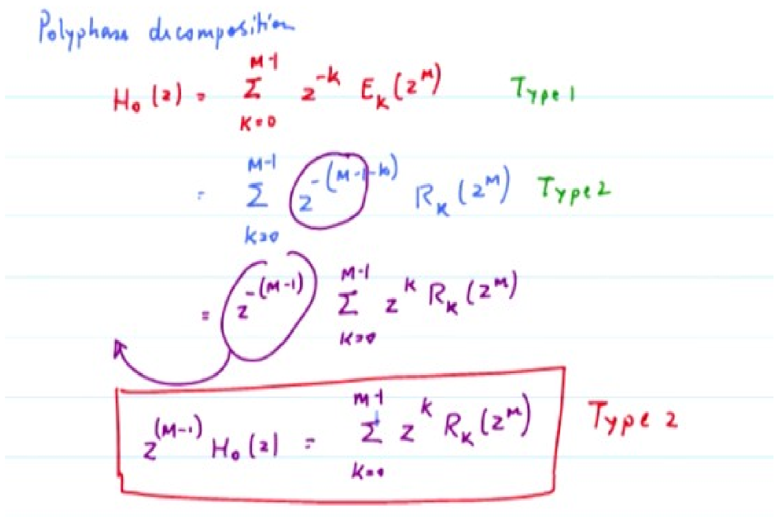
(Refer Slide Time: 06:48)



We also received the blocking and unblocking operations. The blocking operation the creation of blocks happens through a advance chain by down sampling we call this as the blocking, the reverse operation as the unblocking operation with the delay chain. And we said that using the results of up sampling followed by a delay followed by down sampling there are certain simplification that we can achieve if you did not have the Z power -2 these would be straight lines.

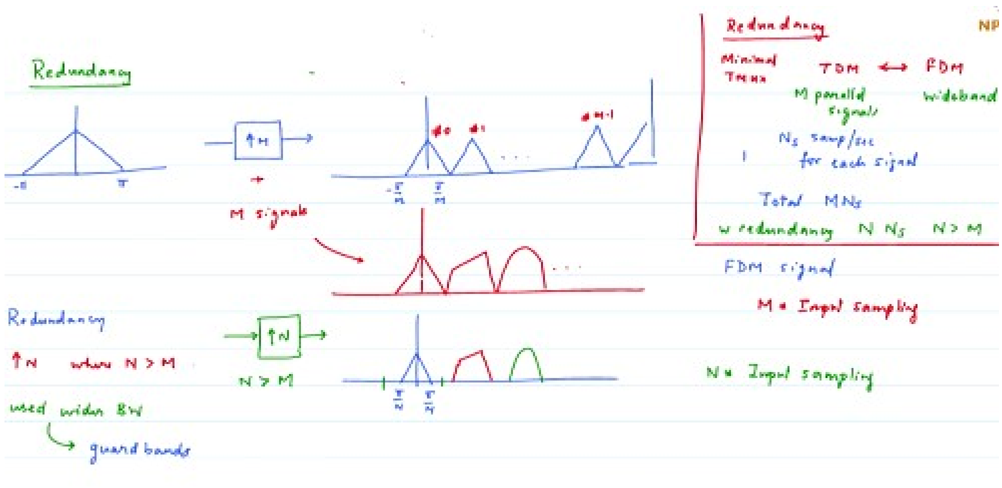
If you have a parallel to serial and serial to parallel converter, but if you had delays then you start to see some changes in the interconnection, but again this is a very simple example, but more important thing is what happens when we have it in the context of a practical channel. So we also mention that we have 2 types of polyphase decomposition. Type 1 associated with when there is a delay chain. Type 2 associated with the structure that has got advance operator.

(Refer Slide Time: 07:54)



So just a quick definition Type 1 you have powers of Z power –k that is a delay chain sequence of delays. The Type 2 has a series of advance operator. So again Type 1, Type 2 was reviewed.

(Refer Slide Time: 08:10)

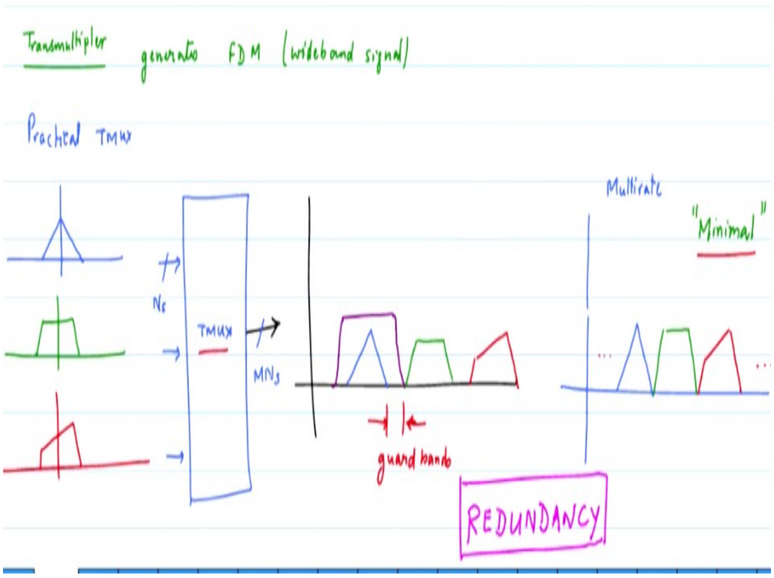


Now notion of redundancy we have already said that a minimal configuration would require them to be sort of packed tightly when you have a little bit more of a gap you would have more spaces. Now how do you visually achieve that or intuitively achieve that. Here in this particular the minimal systems there are M signals by up samples each of the signals by M so the sampling rate is maintained.

Now maybe we can write this down a little bit more formally. Let me just create a box which we can fill in with some information so the notion of redundancy is what we are trying to capture here. Redundancy says in this case there is additional resources being utilized, but how do I achieve. So if I have a minimal transmultiplexer which means that I will go from a TDM system to a FDM system. FDM is the wideband system.

So this is M parallel signals and this is a wideband signal. So this is the wideband so the minimal system says that if I have a sampling rate Ns samples per second for each signal then I have M of them. So the total is M times Ns okay. Now that is what the parallel branches give you when you have the parallel branches have MM branches.

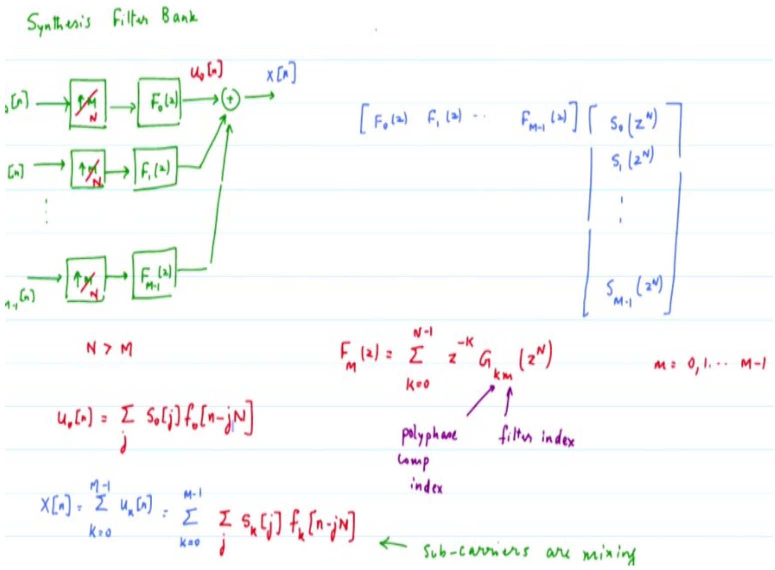
(Refer Slide Time: 10:08)



Now on this side if each of these is Ns and you have M of them and because of the up sampling this becomes M times Ns so that is a minimal transmultiplexer. So minimal transmultiplexer is one where you have, but if you did up sampling by N so with redundancy we are increasing the number of resources. So it is N times Ns where N> M so somewhere the notion of additional resources comes in the minute.

You have higher sampling rate you know that more bandwidth is being used which means that if you still have only M signals obviously there is some gaps which are present. Again may not be exactly as you visualize it, but the notion of redundancy and how it is introduced into the system was an important element from the last lecture.

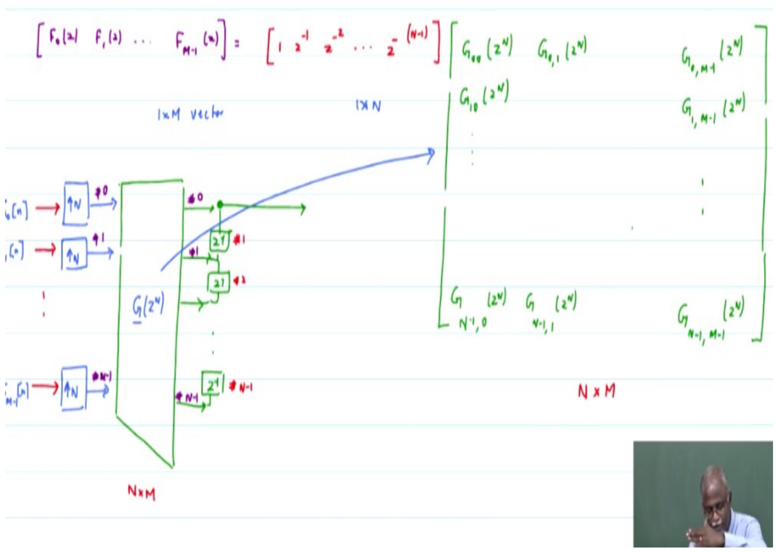
(Refer Slide Time: 11:07)



So with this configuration we said how will the synthesis filter bank look because that is the first stage of the transmultiplexer. Up sampling by M followed by the filter that is a minimal configuration, but we introduce up sampling by N by the same filters, but now these filters have to be slightly differently designed, but that part it will come later. So the vector representation is a scalar F0, F1 up to F M-1 multiplied by these signals which are up sampled.

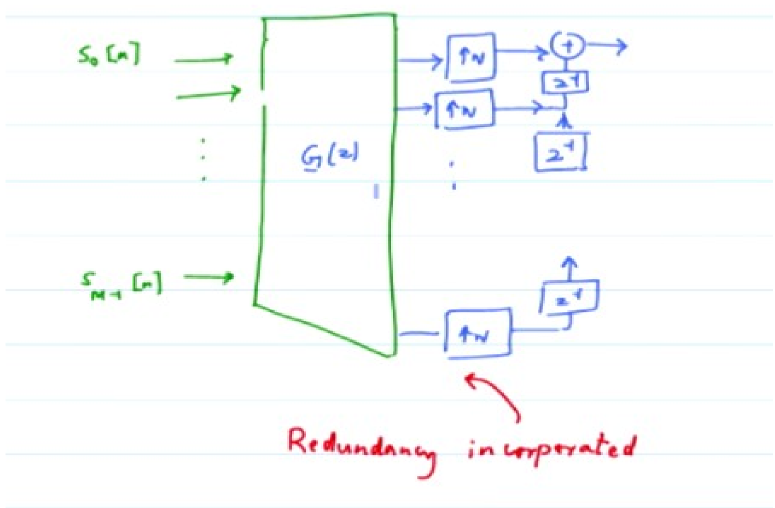
So basically we get a combining of the signal that is why it is called the synthesis filter. We wrote down the time domain expression that is more for completeness, but the one that we wanted to focus on is doing the polyphase decomposition.

(Refer Slide Time: 11:53)



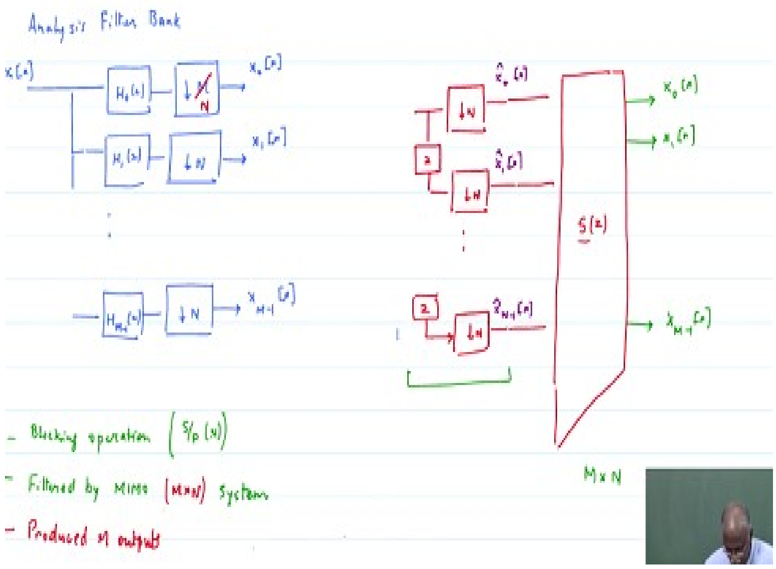
Once you did the polyphase decomposition we showed that the synthesis Filter bank can then be represented in this form where G of Z is a rectangular matrix. You are splitting each filter into N polyphase components there are M of them. So basically the polyphase component of each filter are along the columns and there are M such filters and therefore you get a N*M basically the redundancy part is being shown by a trapezoid instead of a rectangle.

(Refer Slide Time: 12:25)



Similarly, you can introduce so basically you can move using the noble identifies so this is how.

(Refer Slide Time: 12:34)

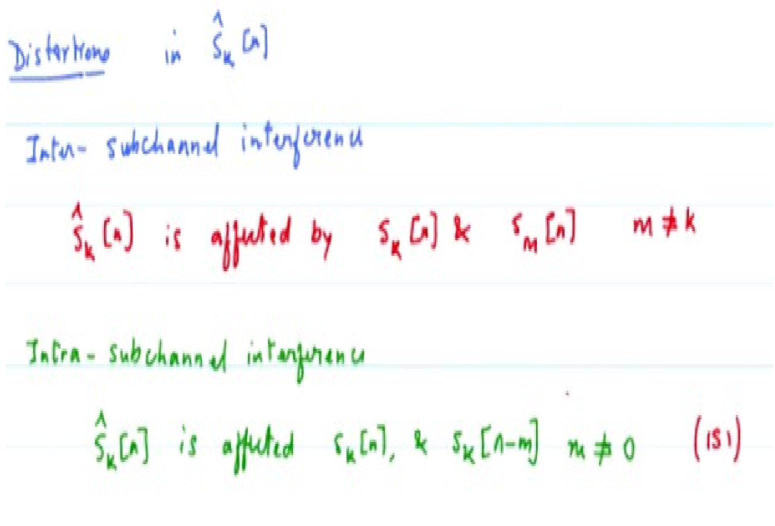


Now analysis filter bank we said we will do a Type 2 polyphase decomposition. So basically this is the structure of the polyphase decomposition of the analysis filter. Since you have up sampled by N which means introduction of redundancy you must down sample by N in order to remove the redundancy because at the end of the day you want to have the original signal bank.

So basically we are doing the down sampling by N following the same procedure doing polyphase decomposition and then the noble identity we get the following structure. So I am assuming this part of it you are able to review and are comfortable with that then we went to putting the entire transmultiplexer together. So the synthesis portion the analysis portion with the channel in between.

Channel has got the dispersive channel as well as the noise we said that the noise we will more or less ignore because that comes as an impairment which we cannot do anything about, but the channel impairment we definitely want to address.

(Refer Slide Time: 13:38)



So given this structure we then went back and said that there is a understanding of what can happen in the channel. You can have mixing of the signals that called inter subchannel interference and then you can inter symbol interference within each subchannel that is called intra subchannel interference. So basically 2 types of distortion both of which need to be eliminated for us to get a uniform detector.

So there is a result that we introduced along with the notion, the result is known notation is new, but we introduce it up sampling by a factor of M followed by a filter, followed by down sampling by a factor of M. We said that this is LTI, important, this is an LTI system and it is transfer function=0th polyphase component. And how did we represent the LTI system we represent that the LTI system in terms of H of z that is the transfer function with the down arrow which basically mean that you will down sample this by a factor of M.

You down sample H of z what will happen you take the impulse response and down sample it you will end up with a polyphase component 0 and all the other things are thrown away so basically what you get is E of Z. So this is an important result please keep this notation reference handy because that is what helps us simplify the figures that we have in the next chart.

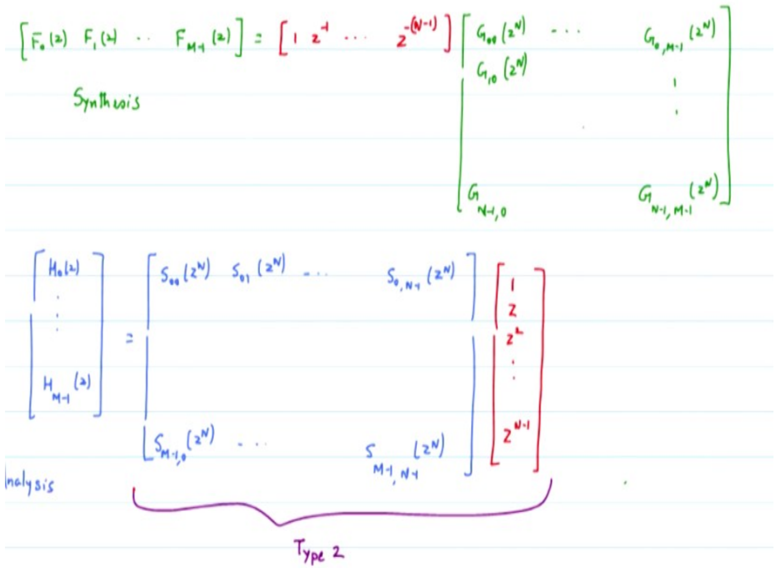
Now if you labeling of the signals very important let me just make sure you have S0 to S M-1 on this side okay. U0, U1, U M-1 on this side and S hat, S1 hat S n-1 hat. So I am looking at the transfer function between S0 and S0 hat. So if you basically look across from one end to the other M can be from any of these and K can be any one of those. There is always going to be up sampling by N a filter the channel.

Keep in mind I am not ignoring the noise another filter down sampling by N and the output that is what is going to be the chain between input and output. So we know that this configuration can be simplified and we know that is going to give us an LTI system and the transfer function of the LTI system is given by Hk followed by C of z followed by Fm down sample by N.

So given this we could then write down a matrix transfer function between the whole set of input and the whole set of outputs and we related the properties of T of z of the 2 types of interference whether it could be inter block, intra block both of these we described saying okay we get a diagonal there is no inter channel mixing. If you get each of these matrices to be a constant matrix, then there is no inter block interference.

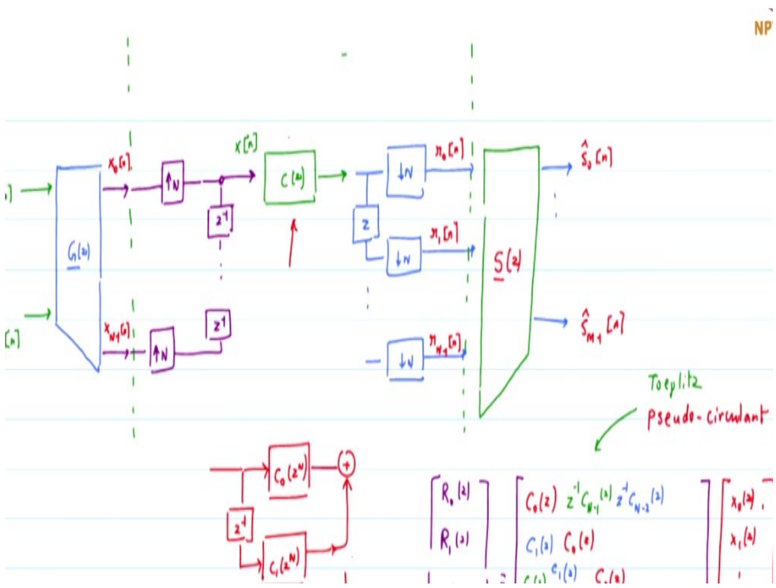
And I think both of them are very important. So basically this is the matrix this is how you could write it is a polynomial of matrices and then we indicated that if this matrix turns out to be constant matrix there is no inter block interference and if it is a diagonal matrix then it becomes no intra block interference. So this is no inter this is intra okay this should be intra block. No it is inter sorry about that sorry this is inter only I am sorry.

(Refer Slide Time: 17:49)



Last review portion of this thing is that we wrote the polyphase implementation the matrix representation of the synthesis filters. These are the synthesis filters followed by the analysis filters very important.

(Refer Slide Time: 18:12)



Once we have this we obtain the representation in terms of the polyphase components of these matrices and we also said that we will write C of z in terms of its own polyphase decomposition. And then we obtain a transfer function between R are the inputs to the S matrix and X0 to N-1 are the output of the G matrix between these 2 matrices between the two sets of green lines we got this transfer function.

And we showed that it was psuedocirculant. So this is a pretty much where we are in terms of the overall understanding and overall representation.