Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology - Madras

Lecture – 03 (Part-2)
Signal Reconstruction - Part 2

Okay, now I would also like to, just as we are getting familiar with the whole process of sampling, the spectral representation, I would like to spend little bit of time on the spectral part. But before we go there, let me ask you a question that I hope will be an interesting one for you to explore.

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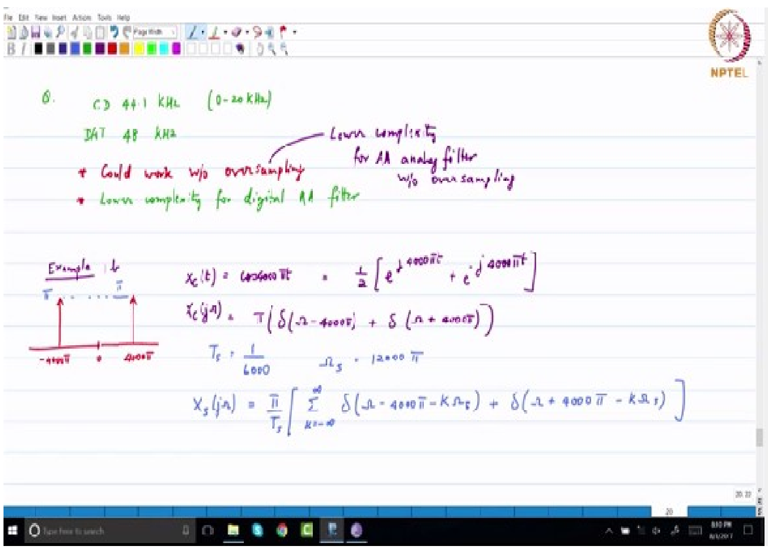

So you have understood how the continuous time music signal is recorded on to a CD format, okay. You apply an anti-aliasing filter, you sample it, in fact oversample it, apply a digital filter and then down sample, okay. So you have got the overall feel for it, okay. Now this is an applied question. Compact disc was trying to sample at 44.1 KHz, when the range of interest was 0 to 20 KHz.

Now digital audio tape we mentioned, is a format that is not very commonly used but it has been used quite extensively in the past, 48 KHz. Just on the basis of what you have study or what we have seen so far. Does that have any advantage over the CD? Just on the basis of, both eventually going to produce a 16 bit or 22 bit representation of the signals. Is there any advantage that that would have over the CD or if you were designer, would you have any reason to increase the sampling rate?

Anti-aliasing filter, okay, that, or you may think why do I bother with oversampling. May be I can do with normal sampling. I will just sample it at 48 KHz. My analog filter is now has got a little bit more room to be designed. It is not as sharp as what you would have for a CD player. So I may not need to go to. Again, now what would be the trade off? Why you would want to do that when going to oversampling and coming down would be, but it is an interesting question.

But it does have the following advantages, that you could have, you could work without oversampling, okay. That is one possibility. The second one of course was, as rightly pointed out, would be lower complexity for your digital filter, lower complexity for the digital anti-aliasing filter, okay. And basically what this one is implying is that there is a lower complexity for the anti-aliasing analog filter without oversampling, okay, compared to the CD case, okay.

So ultimately it is our ability to understand the constraints and be very clear about what are the concepts, what helps me, what hurts me, and then take advantage of that, okay. Very quickly, let us move into some of the things that we wanted to cover in today's class, okay. Back to the example. Let me call it as example 1b. It is the same example as before but as I mentioned we are trying to bring out newer elements.

So Xc of t is cosine 4000pi t. So I am interested in getting Xc of j Omega. So we write this as 1/2 e power j4000pi t+e power -j4000pi t, okay. So this one basically using the Fourier transform of a complex exponential. This would be pi*Dirac delta of Omega-4000pi+delta of Omega+4000pi, okay. Again, these are all probably very basic things for you but it is important for us so that we can derive the key results that we are looking for.

So in the spectral domain, so I will have 2 Dirac deltas for the continuous time frequency. So this is 0, 1 Dirac delta at 4000pi radians per second, the other one at -4000pi radians per second. Do not forget to mention the height of the impulses, that scale factor is pi, that will be important for us so that we do not lose out on the scaling aspects, okay. So the height is pi, okay. Now I have sampled it at Ts=1/6000 which means my sampling frequency is 12,000pi and the sampled spectrum, Xs of j Omega is pi/Ts.

Please remember do not miss out the pi. This is part of the spectrum of the input signal. So pi/Ts summation k=-infinity to infinity delta of Omega-4000pi+replicas which are spaced at omega S. We can do + or -k, it does not matter. -k Omega S+delta of Omega+4000pi-k Omega S. Basically the original spectrum shifted.

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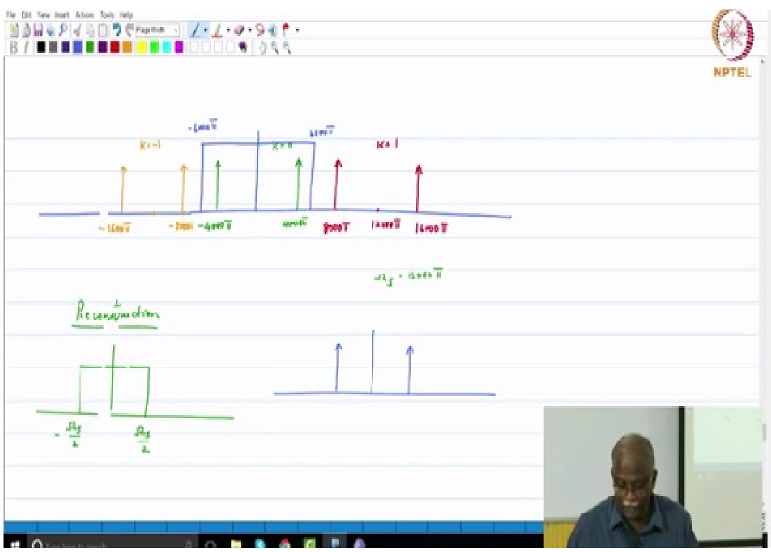

So what I would like to do is sketch this spectrum again, just for the purposes of illustrating the key concepts. So the first copy of the signal corresponding to k=0, I have at 4000pi and at -4000pi. This corresponds to, green corresponds to k=0. Then k=1 will be centred around 12,000pi, right. That is the first shift and it will produce 2 copies of the signal; one sitting at 8000pi, let me just mark these, the green ones are 4000pi -4000pi. Red one is 8000pi.

It is +-4000pi from 12,000pi. So the other one will be sitting at 16,000pi, okay. I am sure this is fairly easy to follow. K=1 and this for completeness, let us do k=-1 and that will be at -12,000pi and producing impulses at -8000 and -12,000. So this is k=-1 and likewise you would basically fill up the entire space. So this is -8000pi, -16,000pi, okay. Now 2 reasons for drawing this. One is to first of all visualize the replication of spectrum but more importantly, the process of reconstruction.

So if I want to go from the sampled signal back to the continuous time signal, then I must remove these unwanted images and we said in the last class that the reconstruction process, basically will retain everything from -Omega S/2 to Omega S/2, okay. What is Omega S for us here? Omega S=12,000pi. So Omega S/2 to -Omega S/2, so let us just mark that portion, that will be from -6000 to +6000, that is the reconstruction process.

So that is -6000pi to 6000pi. Everything that is preserved inside is what will be retained. All other images removed which basically says that, when you do the reconstruction process using our standard definition of what is going to be retained by the reconstruction filter, what is going to be removed, what you get after reconstruction, is exactly what went into your sampling process, okay.

Now do not forget the scale factor. This one had a scale factor of pi/Ts. So each of these will be pi/Ts. Your reconstruction filter will have an amplitude Ts. If you recall, we specified that. So when I multiply the 2, this will come out to be pi, that is the input spectrum, okay, which is what we started off with, okay. So far everything is clean, everything is clear.

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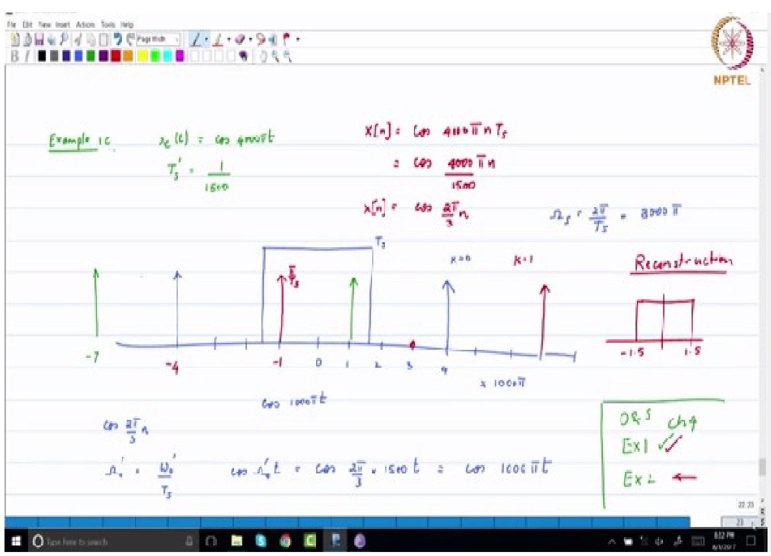

Now here is where it becomes an important example and that is the whole reason of doing this so that we can actually take it to the last step. So this is example 1c, for the third time we are looking at the same example. So Xc of t=cosine 4000pi t, nothing has changed. The sampling period is Ts prime which is now 1/1500. So basically I have gone to a lower sampling frequency. Basically it has gone down to 1500 Hz from 6000 Hz, okay.

First question is what is my discrete time signal. So X of n will be cosine 4000pi*n*Ts. Ts is 1/1500. Cosine 4000pi n/1500. Please simplify this. You will find that what you get is cosine 2pi/3n. The same discrete time signal that is in other words the same expression for the discrete time signal. You are not surprised because you expect that when you under sample, you will get an alias signal.

It so turns out that this alias signal is exactly the same signal that came before, okay. That is effect of aliasing, we are not surprised. Now but please look at it on the spectral side, okay. That is what is important for us. So as before, the original copy of the spectrum is at 4000pi and -4000pi. Just to simplify, with your permission, I am going to do the scale multiplied. Basically whatever we write on the scale as multiplied by 1000pi.

So you do not have to keep writing these big numbers, okay. Whatever is written on the scale is multiplied. So the impulses are at +4 and at -4, okay. Let me just label it correctly so that it will be easy for us to, okay. So if this is 0 1 2 3 4, 1 2 3 4 5 6 7 8, okay. If this is 0, 1 2 3 4, that is where one of the first, the input spectrum; -1 2 3 4, that is where the spectrum is going to be, okay.

And the first copy of the signal, okay, so Omega S=2pi/Ts=3000pi. So my sampling frequency is here, okay. So the first copy of the signal will be, original spectrum shifted by the sampling frequency, k*Omega S where k=1. So the shifted spectrum will be +-4000 of the sampling frequency. So 1 2 3 4, no I have marked it on the wrong one. It should be on 3000. So the first copy of the signal will be at 7000 and it will be at -1000, that is corresponds to k=1.

The blue corresponds to k=0. I presume that is the same notation that we have used in the last case also. Now quickly draw the case for k=-1. K=-1 will place a signal at +1000pi. It will also place a signal at -7000pi, okay. So this is -7 and +1, this is -1, this is -4. So again we get a replication of spectrum except that now the image is seemed to have been interlaced. Whether it is the problem or not, we do not know yet.

Let us figure out, the heights of these. So basically each of these will be at a height of pi/Ts, okay. Now the reconstruction filter says. So let me write down here, reconstruction filter says it goes from -Omega S/2 to Omega S/2, okay. So that basically means that the filter will go from -1.5 to +1.5, when you multiply it on, by 1000pi on the scale. So the reconstruction or the filter that we are going to be working with, this is 1, so the reconstruction filter goes like this with a height of Ts and reconstructs this. So effectively what we have obtained through the reconstruction process, is? What is that frequency? It looks like cosine 1000pi t. I sent in a signal cosine 4000pi t, sampled it at 1500 Hz, obviously aliasing was present, I applied the correct reconstruction filter for my sampling rate and obtained and it looks like, okay.

Now please make sure that we are comfortable with this. Cosine 2pi/3n, that is the discrete time representation. We know that Omega 0 prime=Omega 0/Ts, okay. So cosine Omega 0 dash t, that is the reconstructed signal, =cosine the discrete time frequency which is 2pi/3/Ts that will be multiplied by 1500*t. If you look at it, it actually is cosine 1000pi t, okay. So basically you need not have drawn the spectrum, you could have just gone with the relationship between the sampled signal and the sampling frequency.

And you would have gotten the same expression, okay. Now there is one more example that is related to this. By the way, this is an example that is present in Oppenheim and Schafer, chapter 4. What we have discussed is example 1. A closely related example 2 is also present. Again, it is just a minor variation of that. So we have already looked at this. I would like you to suggest that you can take a look at this other case, okay. Now let me just frame the reconstruction problem that, or the task of reconstruction.

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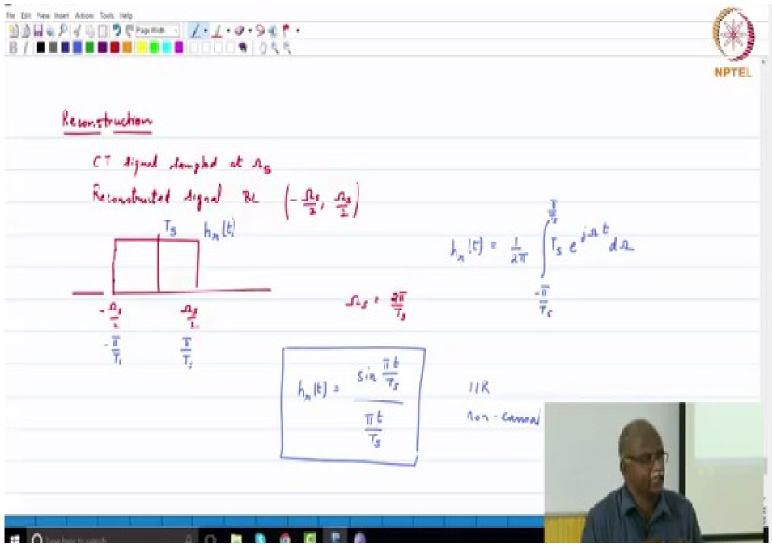

So the task of reconstruction which we will build on the next lecture. So the reconstruction process, we have completely understood the sampling, the effects of under sampling, the effect of aliasing, how the image is formed and how we will reconstruct using the reconstruction filter. Now what we have is a continuous time signal that has been sampled at Omega S. Now whether it was band limited or not, let us say that you have sampled a signal and you have got a sampled signal at Omega S.

Now, when I reconstruct or when you reconstruct this signal, there is no notion of frequencies above Omega S/2 because as far as in our world, if you tell me it is a discrete time signal sampled at Omega S, then to me there is no frequency above that. Because whatever frequency was, information was there, already got aliased down and therefore. So the reconstructed signal, this is an important statement.

The reconstructed signal will be band limited. It cannot reconstruct a non-band limited signal. Because by virtue of the sampling process, you have said that okay sampling can only preserve the information content up to half the Nyquist frequency. So it is a band limited signal from -Omega S/2 to Omega S/2. First and for most that is the key. And of course, the way we draw this filter, we have drawn it a few times already, -Omega S/2 to Omega S/2.

Now can you relate Omega S/2 to the sampling frequency? Yes, we can. Omega S=2pi/Ts. So if you want me to write it down, this will be -pi/Ts to pi/Ts. So I have a simple task for you. If this is my reconstruction filter hr of t, with a height of Ts going from -pi/Ts to pi/Ts. So the reconstruction filter hr of t, inverse Fourier transform, 1/2pi,-pi/Ts to pi/Ts. It is a flat signal with amplitude of Ts, e power j omega t dOmega, okay.

That is my reconstruction filter. Please do verify that what you get is hr of t is of the form of a sync function, sine pi t/Ts/pi t/Ts, okay. Now this is a very important filter, very interesting filter with a unique structure and why is this structure important, how does this structure play a role in the reconstruction process? It is an infinite impulse response filter. It is a non-causal filter but this is what we get as the ideal reconstruction filter.

So how do you make this a realizable practical filter? Those are all very important questions. We will address them in the very next class.