Multirate Digital Signal Processing
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Lecture - 34 (Part-2)
M-Channel Multicarrier Transceiver (Part-2)

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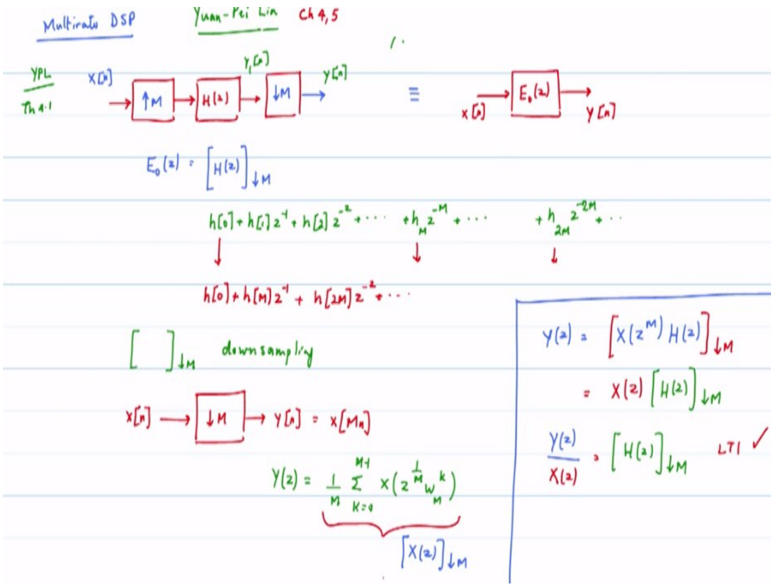

So back to Multirate DSP - all the tools that we have developed we will now re-bring them back into the results that we are going to have. So the first one by the way all of the material that we are doing for this section is from a book by author by name Yuan-Pei Lin and we will upload this chapter in Moodle. So basically this is from the book of Yuan-Pei Lin. So I will use the notation YPL to refer to Yuan-Pei Lin.

In that book there is a theorem 4.1 and the theorem 4.1 is actually a result that we know. So we do not actually have to call it as a theorem, but let us just write it down. So that you will know when you read the notes where this one links in. So basically we are basically going to look at chapter 4 and chapter 5 of Yuan-Pei Lin. So chapter 4, 5 this is in theorem 4.1. The theorem 4.1 in this book says a well-known result to us, but let us pick it up with the notation.

So if I have an input X of n which I pass through an up sampler by M followed by a filter H of z followed by a down sampler now this you will be able to tell me what the answer is. So if I call this as Y1 of n this as Y of n. We know that this structure is equivalent to the following. This is actually an LTI system. We have derived this result or shown this result. This is X of n this is E0 of Z the 0th polyphase component of H of Z and the output will be Y of n this is actually an LTI system.

So basically the theorem 4.1 says that this is an LTI system. This is an LTI system that is given by this expression okay. So the book also uses the following notation so when you read the book so that there will be no confusion it uses the following notation again it is helpful for us. E0 of Z is the 0th polyphase component is actually represented in this fashion. H of Z down sampled by a factor of M.

So what does this actually mean? If I write down the impulse response of h of z this would be h0+ h1 Z inverse+ h2 Z-2 and so on and hM Z power-M…what we are doing with down sampling is keeping h0 keeping hM keeping h of 2M Z power -2 M.. we are retaining these coefficients and we are writing it as a polynomial in Z. So basically E0 of Z is h0+ hM Z-1 h2M Z-2 and so on okay that E0 of Z.

There is a compact notation for that when you say that it is h of Z actually represents a sequence and I am down sampling it. Now can you see if we can just use this notation so that we can get comfortable with that. Okay so basically what we have introduced is this block with a down sampling basically this is nothing, but the down sampling that we are familiar with.

So if I now go back to the original representation we would say X of n passing through a down sampler is M. This would be Y of n this is= X of Mn this is the notation that we are used to this is how we would represent the down sampling. Now here is where if you wrote it in Z domain this would have been 1/M summation k=0 to M-1 X of z power 1/M W MK and wherever there is a down sampler you have to write this equation there is no way to get around it.

Now actually this is a cumbersome form right because you have to write the summation there is all these powers. Now this is where it is very convenient to introduce a new notation this notation is nothing but X of z down sample by M very compactly you can write it down and you will see in a moment why this is so useful for us because this is very helpful for us. Okay now as always apply, apply the result that you have just now obtained.

Now if I were to write down the expression for Y of n is Y1 of n down sampled. So let us see if we can see write this expression. Okay Y of z=X has been up sampled Z raise to the power M followed by H of z then down sampled. Under normal circumstances you would have the summation and all of those things coming up. Now I am going to introduce the notation down sampled by a factor of M.

Notice that if I have X of Z power M down sampled by M that means that is noble identity it will become X of z. So this can be simplified as X of z and what is left is H of z down sample by M. H of z down sample by M will be E of z. Now this is why we get an LTI system because you can get Y of z/X of z= some LTI system does not really matter for us it is represented by this okay good.

So this is LTI and this notation actually comes in very handy in the analysis that we are going to be doing so hopefully you will see the advantage. So let me ask a related question. Now this is more of just an application of this.

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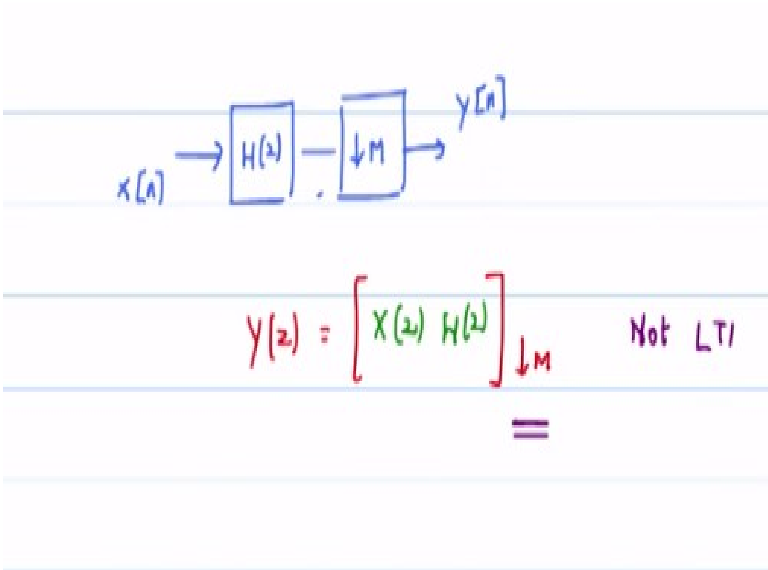

If I fed in X of n to H of z followed by a down sampler and this is Y of n I say oh good I already know how to analyze this, this should be Y of z= X of z, H of z down sampled by M, but I am not able to pull X of z out. So this is not LTI because when I down sample X of z I will get all the shifted version. So this is not LTI because I cannot take out X of z. So again notation actually comes in a very handy and useful manner, but actually when you start using it actually becomes more beneficial.

So here is where we will now go back and build on the framework that we have developed.

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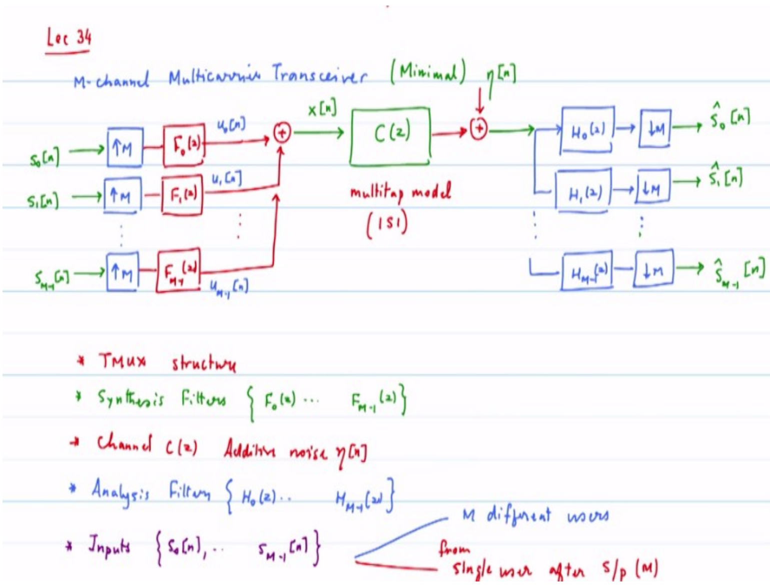

Okay so back to the reference figure I will have to keep switching back and forth. This is my reference figure okay. Reference figure from here this is the synthesis filter, pass through the channel I mean ignoring the noise for the moment, passing through the analysis filter. Now given the result that we have just now studied can you tell you what is the transfer function between S0 of n and S0 hat of n.

There is an up sampler followed by F0 followed by channels C followed by H0 followed by down sampler okay. If you actually wrote down the equation it is actually going to be quite messy, but if you now apply the result that we now have.

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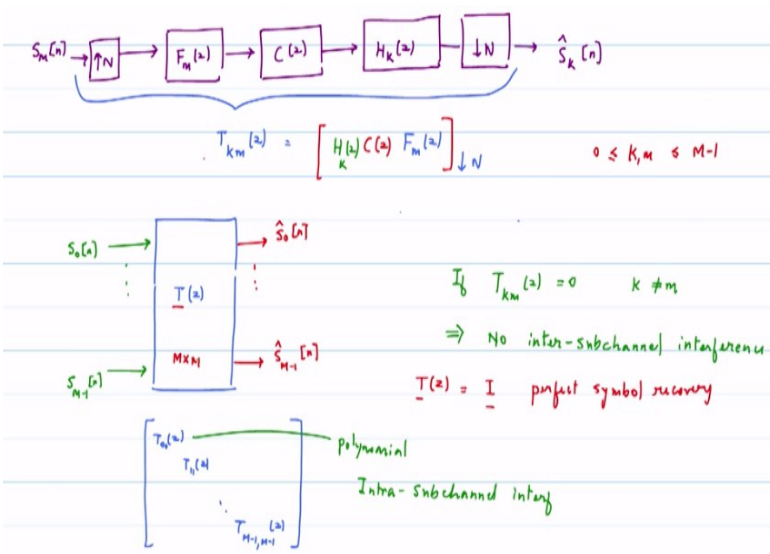

So basically I am looking at a specific transfer function from Sm of n that means it will go through F m of z followed by wait it has to be first up sampled sorry about that let me erase this. There is Sm of n up sampled by M or N does not matter we will just write it with N because that is what we eventually want to do then followed by the channel C of z followed by H k of z down sampled by N and this one comes out to be S k hat of n okay. So Mth input Kth output that is the input, output relationship that we are looking for.

Now if I called this combination of these 3 filters as T km of z. Mth input Kth output that is why I got T subscript k,m; T km of z. Let us write it in the sequence in which the filtering occurs the right most is F m of z that is the first filter that will operate on the input followed by the channel C of z followed by the analysis filter H k of z. So please remember that the sequence of operations is F filters first followed by C followed by H again these can be switched around does not matter.

But the important thing to note is that the transfer function between the input and the output wait one second I wrote the blue line wrong. Okay the transfer function between input and output actually has to include these samplers as well. So this T k of z is actually these 3 filters down sampled by N this is the result that we have just now derived. So this is down sampled by and you can check, you can multiply these filters out and check whether you get what sort of response.

Now the range for k, m < or =0 > or=M-1 there is no N in this part because there are M inputs, there are M outputs. I have transfer functions between them. This matrix T k,m will be M * M matrix (()) (13:25) M inputs can be related to the M outputs. So if I were to capitalize on that I can write an equivalent form in this fashion. This is T of z this is matrix and this is a M*M matrix.

The input to these matrix are these inputs S0 of n all the way to S M-1 of n and the outputs are the S0 hat of n…S M-1 hat of n. Input output relationship is given captured in terms of a matrix transfer function can think of it as a MIMO systems between these. The reason this is very important and why notation is very important is because otherwise when you start writing the equation you get also it is a messy expression, but if you have a very compact notation like this you can visualize.

At the end of the day this is going to be the 0th polyphase component of H k of z, C of z F m of z no problem. Some filter is there I take its polyphase decomposition take the 0th polyphase component and that become my transfer function. So if you are comfortable with this representation then please follow along and say whether you are agreeable to this one. What does it mean if I write down the following conditions if T, k m of z=0 for k0=m.

That means I get a diagonal matrix. Diagonal matrix means S0 hat depends only on S0 Sm depends only Sm and so on. So what have I eliminated inter subchannel interference eliminated. So I have to design my filter such that T of z comes out to be diagonal matrix. So if this condition is satisfied then there is no inter subchannel interference. Now this is why this representation is useful for us.

Because the whole system can be simplified into few very easily visualized conditions no inter subchannel interference okay you agree with that. Okay no problem I will show that in a minute. So when I have up sampling by N followed by filtering followed by down sampling that is what this theorem 4.1 of Yuan-Pei Lin 0th polyphase component. **"Professor - student conversation starts"** So T's are polyphase components. **"Professor - student conversation ends"**

No T is a transfer function. So if you take one particular set of input, output relation it is a T k of z that is represented as T k m of z which is given by the 0th polyphase component of H k of z, C of z, F m of z. If you combine basically now you have all the input, all the outputs you combine you will form by M by M matrix all of which are obtained through this operation correct.

But for each of them it is a different H k will change or F m will change, C of z is there for all of them because all of them pass through the same channel. Now let us make a couple of more observations; obviously the ideal case would be if T of z is= identity. If T of z is identity, then actually we get a perfect recovery. If T of z is= identity matrix this would be the perfect transmultiplexer or the perfect multichannel system.

So we will let us call it as the perfect symbol recovery whatever we transmitted we received without any impairment. This would be the ideal scenario, can we achieve it? But before that let us look at one more element after you have made this diagonal. See supposing you have achieved the diagonal part let us say this is T0 of z, T1 of z, T00 of z, T11 of z the last one is T M-1, M-1 of z okay.

Now is the recovery of S0 hat from S0 is it what is the input, output relationship. It is as if S0 got filtered by T00 of z that is a filter that means there is intra subchannel inference because there is ISI present. So basically if these are polynomials if this is a polynomial then there is no inter subchannel, but there is intra subchannel interference that means ISI is present. Intra subchannel interference okay.

Here is where the power of combining that signal processing and the communications come into play in a very nice way. I am going to now go back to the original formulation assuming that there is no inter or subchannel interference is not the case.

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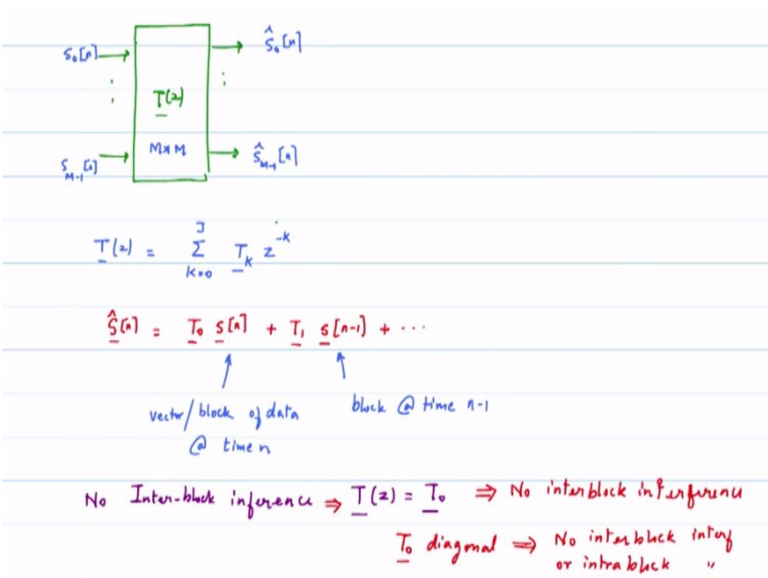

I am going to go back to this particular representation which means that I have a matrix T of z M by M matrix where each of the entries are T00 of z, T01 of z all of those are matrices. I have M inputs I have M outputs this is another interpretation again very importation that you are comfortable with this as well S0 of n all the way to S M-1 of n S0 hat of n, S M-1 hat of n input output relationship indicated by this M *M matrix.

Now T of z is a matrix of polynomials each of the entries of T of z is a polynomial. Now if I take the 0th power that is a constant term from each of those I can write and then group all the Z power -1 terms. I can write this as a polynomial of matrices what do I mean by that summation k=J let us assume J is the highest power of the polynomials that are present in T of z.

I can then write this as T subscript k where T subscript k is a constant coefficient matrix times Z power –k. So basically what did I do took all the 0th power terms of T of z all the Z power -1 term and then group them as matrices. So basically this becomes a polynomial of matrices. So in this sense why is this an important way to represent it because I can now write S hat of n as a convolution very important that you now visualize this convulsion.

It is T0 matrix times S of n the input at this time instant + T subscript 1 that is all the power of the Z inverse that have been combined. Now what will this combine with? It will combine with not the current input vector, but it will correspond to the previous input vector so that would be S of n-1 the previous input vectors that was applied to the system and so on. So basically this way of visualizing is also important.

So if I were to give it some terminology this is the vector, this is the vector or the block of data let me call this as vector/block of data at time n. This is the block at time n-1 okay and so on. So if I now ask you to tell me how to eliminate inter block interference there is another term in Multicarrier system where it says I want to eliminate inter block interference okay. So inter block interference that means S of n should not have any depended of S of n-1 inter block interference.

If I want to ensure that this condition is satisfied it is sufficient that T of z= T0. No inter block interference implies this T of z cannot be a polynomial it has to be a constant matrix because under that condition both of these are by the matrices the T0 is a constant matrix then there is your guarantee that there is no inter block inference. Okay now tell me what is the condition for no inter block interference and no intra block interference.

T0 should be diagonal that is it. T0 should be diagonal constant matrix yeah basically. So that is what we are basically coming down to be. So if T0 is diagonal then we get no inter block or intra block interference okay so both these are interferences okay very good. So this is where it has brought us. So I think we have started to develop some interesting tools, we have started to develop a good framework for what it is. So let me just end it with just the following important results.