Multirate Digital Signal Processing
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Lecture – 33 (Part-1)
MCM with Overlapping Spectra

Okay, good morning we today will cover 2 lectures; lectures 33 and 34, picking up from where we left off in lecture 32. In lecture 32, we concluded our discussion on how do we achieve the capacity for throughput basically, there is a number which you think of it has capacity and you want to achieve as close to capacity as possible in other words, we are trying to maximize the throughput of our system and achieve as close to capacity as possible.

So, we are trying to achieve maximum throughput in a wireless channel which is broadband and again the definitions of these come in the context of the dispersion that we come into.

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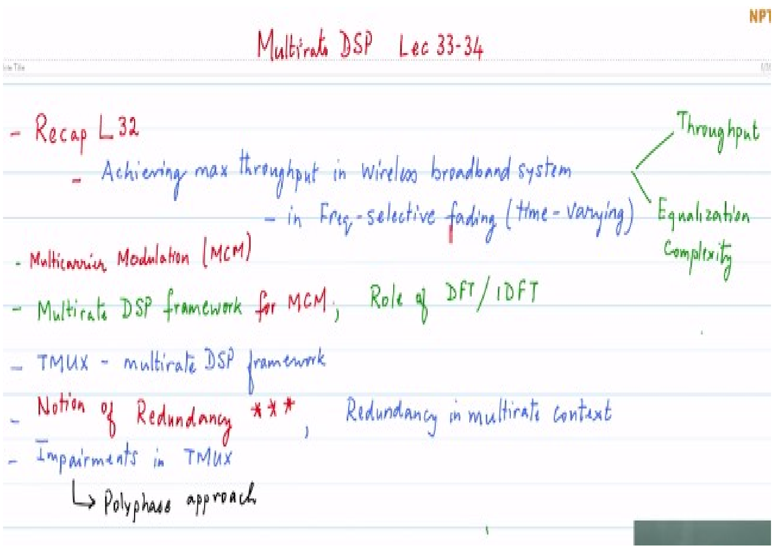

So basically, a broadband signal is one which experiences frequency selective fading that means the multipath components are arriving in such a manner that there is significant amount of inter symbol interference, so that is what we refer to as frequency selective fading, so frequency selective fading, and because it is a wireless system it is also time varying, so that is the context broadband system, frequency selective fading, time varying.

So, the 2 things that we were trying to focus on; one was throughput, you want to maximize the throughput, which means that you must adjust your power, you must adjust your modulation scheme, you must adjust your encoding scheme based on the channel conditions but unfortunately, if you treat it as a single carrier, one portion of the spectrum has got good SNR, other portion has got not so good SNR.

It is very difficult to optimize, so that is where the notion of splitting the wideband channel into a number of narrow channels becomes an attractive feature and we said that equalization complexity also got solved through the process.

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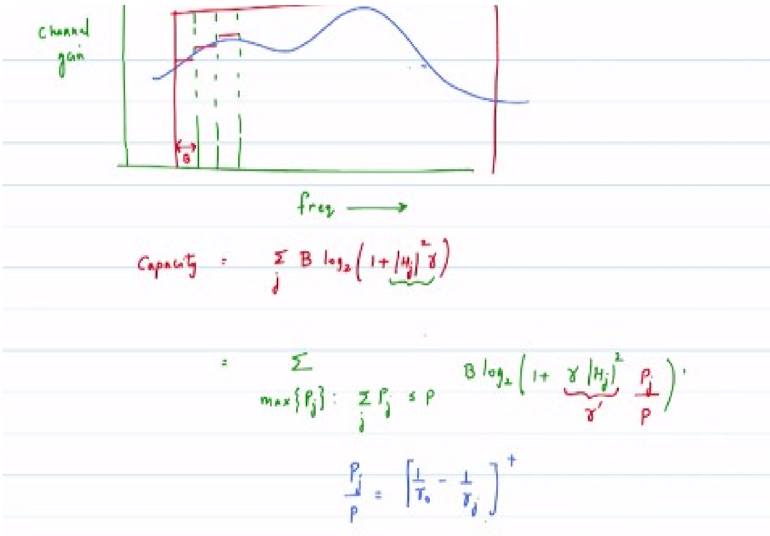

So, let me just quickly refresh your memory, this is a channel gain versus frequency, the fact that it is not flat means, that you have dispersion, frequency selective fading, in order to achieve capacity in the system we said you would do the analog of water filling in time but now you are doing water filling in frequency, that was the observation.

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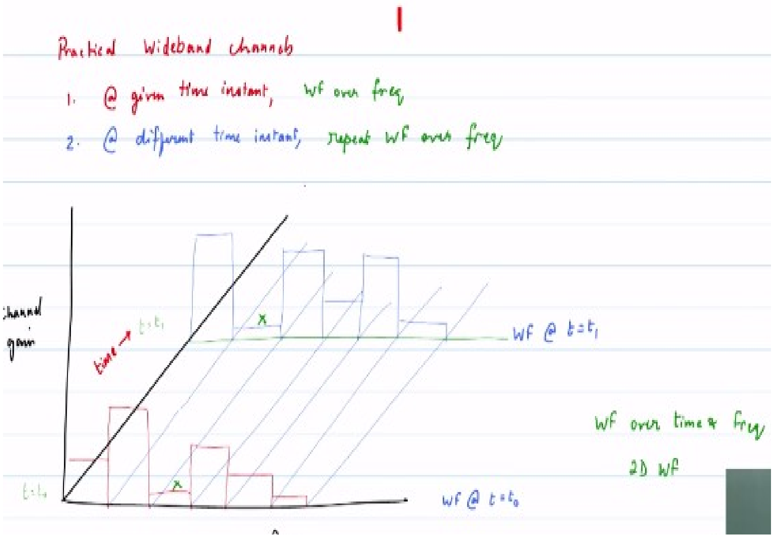

Now, since it is a wideband channel and it is time varying, it is not going to be constant at different instants of time, so if you think of the channel gain on the y axis, frequency on the x axis and then a third axis which represents time, so if you think of this as the time axis okay, the third direction, so time t = t0, t = t1 and you can sort of see, I have just taken shown you 2 slices. The channel conditions are quite different in each of the sub channels.

So, what you would do is; actually water filling for the channel conditions that are currently occurring, so that is the water filling over time and frequency, so that is what achieves the capacity that we are interested in.

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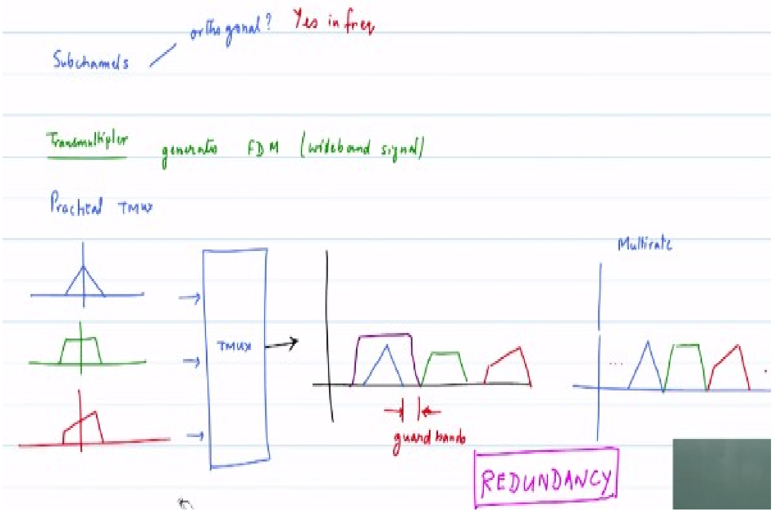

Then we said that okay, we will now move from the notion of capacity to actually the various multi rate framework and also the concepts and the notions that we have, so let me just sort of summarize what we have said so far okay. So, here is a way to visualize what we have indicated.

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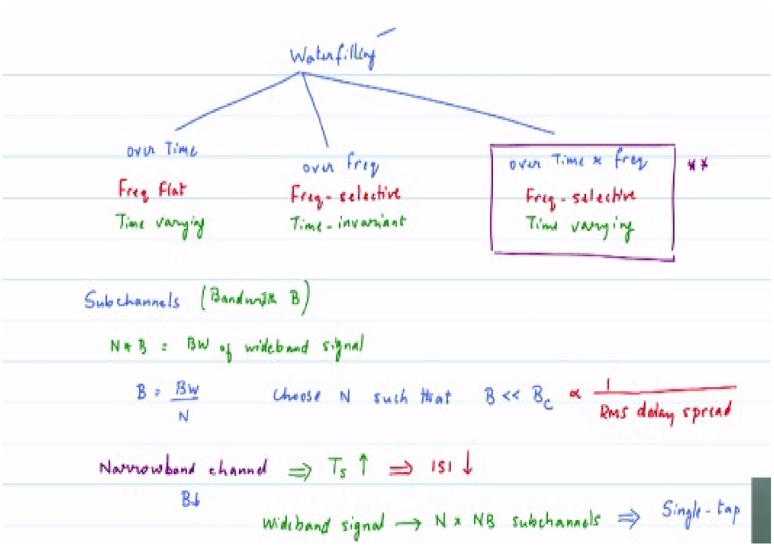

So, if you think of water filling, there are 3 flavours of water filling, there is water filling over time, water filling over time is intended or is the right thing to do if you have a frequency flat channel; channel conditions, that means you this is this invariably means that you are working with a narrowband signal or something that is not very wide band and the reason you are doing a channel; water filling over time, is because it is time varying okay, so that is the notion of water filling over time.

Then, comes water filling over frequency, now this is intended for channels that are frequency selective that means your transmitted bandwidth is much wider than the coherence bandwidth of the signal, so that means you are seeing some channel gain variations however, we can assume that if that if the channel is not varying in time; time invariant, then all you need to do is water filling over frequency and then the third flavour which is what we would like is over the time and frequency.

And the reason we do that is because we are working in a frequency selective fading environment and it is time varying, okay so that is the flavour that we are interested in, in the context of the wireless systems which are high data rate systems so basically, which means that you must be working with broadband systems, so this would be the double star what we are interested in.

So, the strategy that we have taken is that we will divide into sub channels again, these are important for today's lecture, so I just thought it will be good to write it down, so the bandwidth of the signal that is, so if each of these has got bandwidth let me write it down, so sub channel with bandwidth equal to B okay, so this is the sub channel and we discussed yesterday how to choose these sub channel bandwidths.

So, if there are N sub channels times B that = the bandwidth of the wideband signal, okay so the choice of N, the bandwidth of the wideband signal is given to us so you get to choose, and the way you choose is by choosing N, so bandwidth divided by N and it is chosen such that you choose N such that the bandwidth B is much smaller than coherence bandwidth and yesterday we defined coherence bandwidth in terms of the dispersiveness of the channel.

So, basically this would be 1, reciprocal of 1/ the RMS of the delay spread, so the amount of dispersiveness may not have defined RMS delay spread but think of it as a measure of the dispersion in the channel okay, so here is the other side of it, the equalizer complexity side of it. The equalizer complexity side says if I have a narrowband carrier; narrow band channel, if I have a narrowband channel, this basically, means that the bandwidth is small.

That is basically, if I reduce the bandwidth, the impact will be seen, in that the symbol period will increase correct, if I reduce the bandwidth that means, the baud rate has reduced that means the symbol duration has increased, so this is going to imply that the effect of ISI; inter symbol interference is actually going to be less, okay so this is going to reduce, so that is the direction in which we have moved to solve the equalization problem.

So, therefore having achieved or making this observation, we basically have come to the conclusion that if I have a wideband signal, this is the previous line is for each of those sub channels, so if I have a wideband signal, then we treat it as N narrowband signals; narrowband channels; sub channels, okay and therefore, the equalization for each of them is not very difficult.

So, in the worst case, I can get by, by doing a single tap frequency domain equalizer FDE; frequency domain equalizer, okay so that is the summary statement okay, so the move now is towards the understanding of the multi-carrier systems which we are going to now say that okay wideband transmission, I want to do it, so we referred very briefly to the history of multi carrier modulation.

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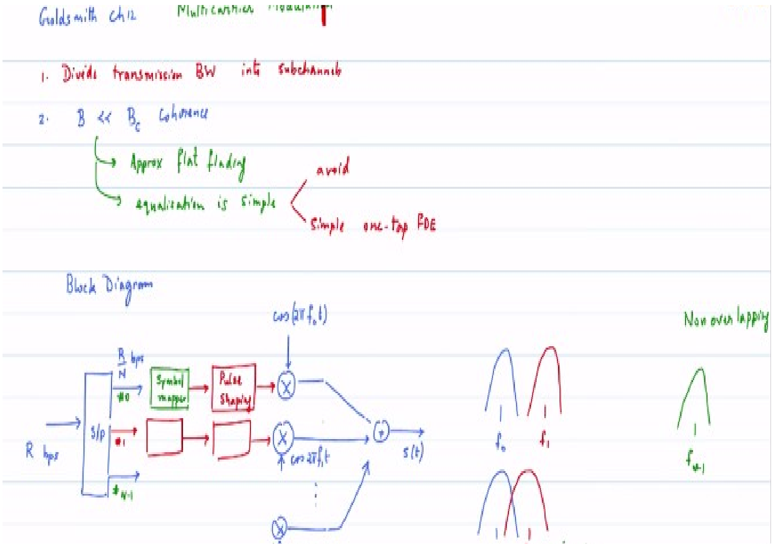

Multi carrier modulation basically took multiple streams, it could be from a single source, right very often it could be from a single source, all you are doing is parallelizing them, it could also be data for N -; N users, each of these strains correspond to a different users, it really does not matter to us from the transmission point of view, so you take the information of each sub channel, you map it into symbols, do pulse shaping and then you shift it up in frequency.

And the multi carrier modulation methods typically, would place these sub channels such that it is easy to retrieve them through filtering okay.

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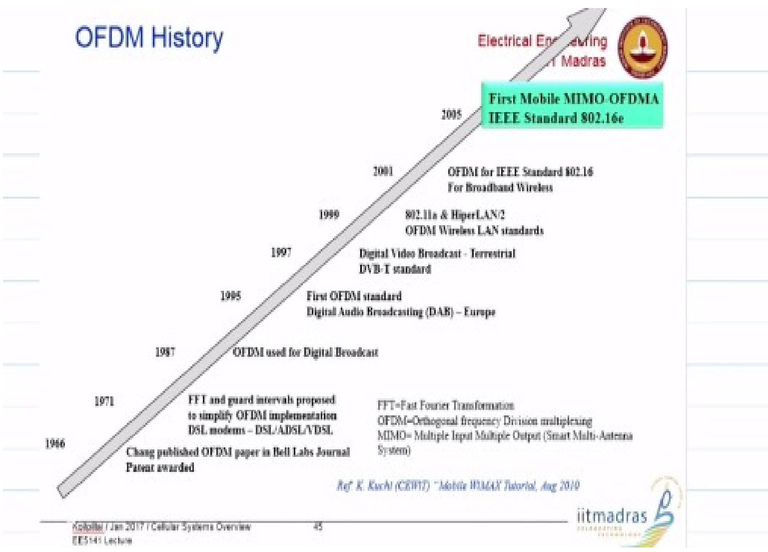

So we said that the very, very early users of multi carrier modulation were the military but from the context of the commercial use of multi carrier modulation, in fact the name OFDM itself came much later it was always called multi carrier modulation and it basically, went back to 1960s where they were trying to achieve the capacity in a telephone channel then eventually, they also did it for the cable television modem.

So, basically the importance of multi carrier modulation was seen or was appreciated for channels that did not have constant channel gain basically, there was variation in channel gain and therefore they wanted to achieve that. So in this context, the representation of the multi carrier modulation was either through non-overlapping channels, this was the traditional view of it, so these are the non-overlapping, as opposed to the scenario where you could have an overlapping situation.

And an overlapping situation would be where you have some amount of overlap but it is a controlled overlap, so that you can still recover the signals in an effective manner okay, so this would be a situation where it is a multi-carrier system but there is overlap is being permitted as part of the design and eventually, you will separate it out in the context of, okay, so given that this is the flavour of the information that we have, by the way, centre frequencies now have shifted.

So, this would be f0, centre frequency now would be f1, so notice that the centre frequencies have been modified, that is how you get overlapping, the bandwidth remains the same you have changed the centre, spacing between the channels okay.

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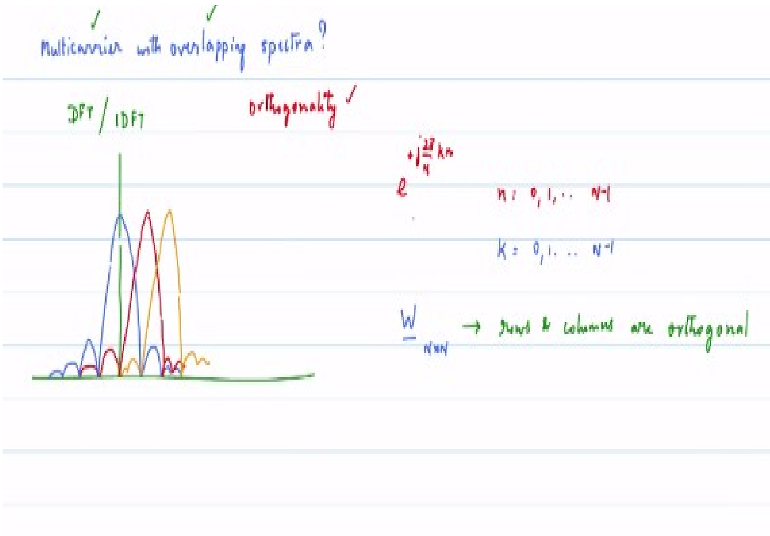

Now, here comes a very interesting element that I want you to think about and respond to so, basically it is a question; have we come across any system of multi carrier with overlapping spectra? Have we studied or have you come across in the course? Yeah, 2 channel case okay that where you tried to split but that was actually, you took a single channel and then you split it into 2.

So, however if you think about the; what we said about the DFT or the IDFT, what were these; these were as if you were passing the signal through; the DFT was passing a signal through a set of filters which was this type right, the rectangular window response, the second channel had a center frequency where this one had it is 2pi over M and then you had this type of a response okay.

I will just draw one more channel, orange okay, DFT; the interpretation was; I take a signal pass it through a filter bank, what is the interpretation for the IDFT? You take the sub channel, if the DFT is taking these and splitting into a signal, the IDFT must be combining them right, IDFT is the synthesis part okay, so interpret for me the IDFT, the IDFT is; takes parallel streams and combines them into a single stream and what are these filters?

These are the filters that are doing this, so in other words, these are effectively paying the role of the multiple carriers, so the IDFT is actually a multi-carrier system, you can visualize just you could as you could visualize the DFT as a filter bank that was splitting channels, you can think of the IDFT as a multi-carrier system where the information that is on the blue and the red and the orange actually gets combined, okay into an single signal.

So, actually the multi carrier is known to us in the OFDM and do these things maintain orthogonality? Yes, do they overlap? Yes, they also maintain orthogonality, so the notion of overlapping carriers maintaining orthogonality and working as a multi-carrier system is actually not new at all especially, if we view this in the context of the multi rate interpretation that we have come up with.

So, okay so orthogonality is also satisfied, so what are the; what is the carrier signal? The carrier signal is e power + j 2pi over N, + or - does not matter, k times n okay, so n = 0, 1 to nN-1 so that is and each of these for the different frequencies, e power j 2pi over N, n times k and you have 0, 1 to N - 1 okay so and these are effectively the entries of the W matrix, which is an N cross N matrix.

And we know that these, the rows and columns you can think of it you think of it in terms of rows or columns, does not matter, you get the same interpretation, the rows and columns are orthogonal; that means, you can separate them without interference between the; so that is essentially the property of orthogonality, so in other words we never think of the IDFT as a multi-carrier system.

But in a sense it is a multi-carrier system with overlapping carriers and which maintains orthogonality, so this is our starting point to sort of link the various pieces that we have together in our portfolio and this is the first of the observation, so IDFT, DFT are going to play a very important role and which is that that is why OFDM comes about and what you see in the OFDM modulator and demodulator are the DFT and IDFT.

In other words, it is a well-known, well-proven multi carrier system that is available to us and that we are familiar with.