Multirate Digital Signal Processing
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Lecture – 32 (Part-1)
Capacity of Wireless Channels - Time Varying Frequency Selective Channels

Good morning. We begin the lecture 32 and in today's lecture, we will be covering some very interesting material, the link between what we have been looking at in terms of the capacity of a wireless channel and the concept of OFDM.

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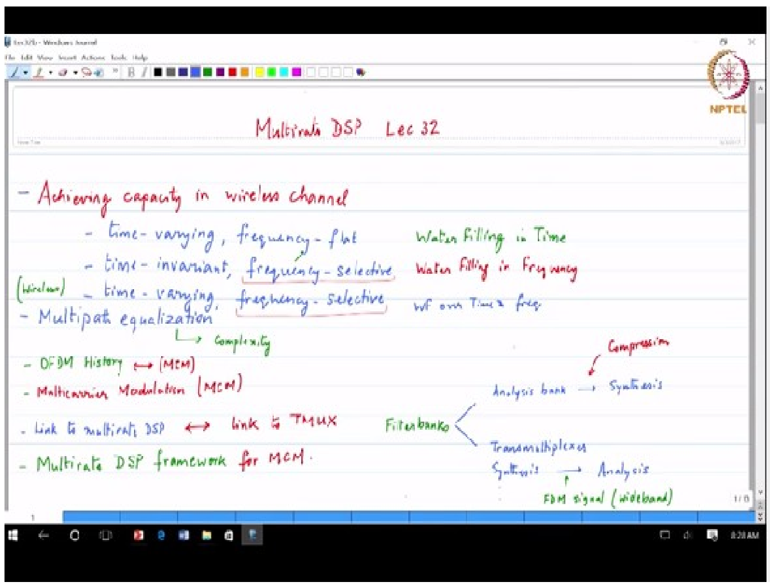

In wireless channels, we have talked about the achieving capacity and we have talked about one very specific type of wireless channel. It is a time varying channel, that means the gain is not constant in time, but it is flat, frequency flat.

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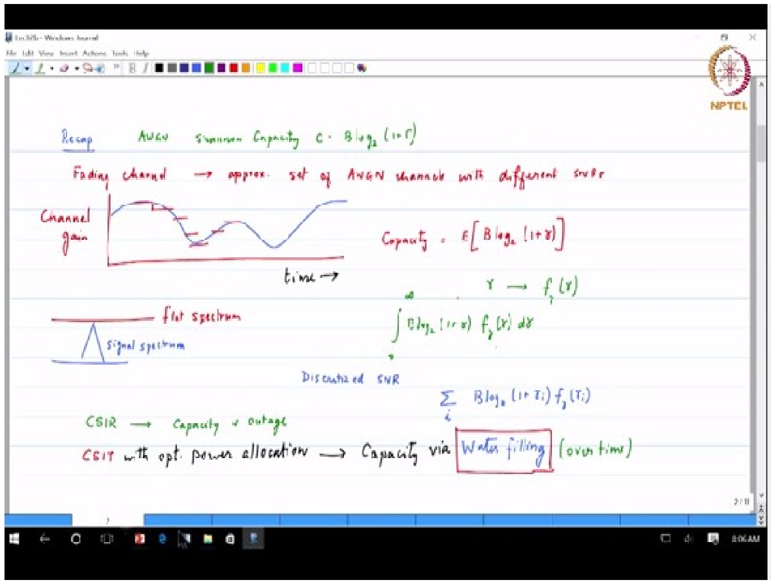

So in other words, the bandwidth of the channel is such that the gain of over the entire bandwidth can be considered flat. But this level is not constant. It is fluctuating. So that is the case that we have studied so far. And for this the optimum method we said to achieve capacity is to do water filling over time. So over a time, whenever the channel changes, you adjust the power level to achieve the capacity.

So there is a stochastic element in the, probabilistic element that is there in the water filling because the water filling, the channel gain is not something that we know ahead of time. Based on the channel conditions, we would have to do the water filling. Now we said that there are 2 more variations that we have to talk about. And both of them have to deal with the type of channel where the frequency response is not flat anymore. It is not flat, but it is frequency selective.

So that is a key difference between the channels that we have considered up till now and the channels that we are going to consider today. Now in this frequency selective fading channel, there are 2 subclasses. One is where it is not changing as a function of time. That means it is a time invariant frequency selective fading channel. That means there is dispersion but the dispersion components are not changing in time.

So that is what we call as a time invariant frequency selective. Now that is not a very common type of channel but like for example if you had a telephone channel and you were to look at long distance communications, long distance transmission over twisted pair cable, then you could say that the channel very rarely changes. Because the channel is more or less static but it has got dispersion so it is frequency selective.

So time invariant frequency selective channels are present but not the ones that we would study in wireless. So what we would study in wireless, would be primarily the time varying and frequency selective. So this would be the one that we would be looking at most importantly. So that is what we would focus on partly in today's lecture. So this is what we would encounter for wireless.

Now in addition, yesterday we said that when there is a dispersive channel, you have to worry about equalization and we call it multipath equalization because in the context of the wireless channel, the intersymbol interference is caused by multipath. So of course, equalization is to compensate for intersymbol interference. But that intersymbol interference originated because of multipath and we refer to it as multipath equalization.

So how are we going to address multipath equalization? This is an important issue primarily in terms of complexity. Yesterday, we said that if we tried the traditional method of doing a maximum likelihood sequence estimation for a wide band signal, the complexity would be very high. Once you have studied both of these, we will go back and look at what is the history of OFDM.

As I mentioned, OFDM was not always known as OFDM. It was also called as multicarrier modulation. So multicarrier modulation and OFDM, we have started to call it OFDM but in the literature, you would actually refer, see it. So we will go back and look at what is the basic definitions of multicarrier modulation and how does it and where does it link to the multirate signal processing that we are studying.

So in this context, we will also go back and refresh one of concepts that we have already studied in filter banks. So one of the applications in filter banks, so a filter bank basically means analysis filters, downsampling, upsampling followed by synthesis filters. So there was, the traditional way is you have the analysis first, followed by the synthesis filters, right. That would be subband coding or what you would need for doing signal compression.

So analysis bank followed by synthesis bank, okay. So this is typically what you would need for compression. So this is the point at which you would be doing the compression. And we said that there is a second application of filter banks which also is common in telecom which is the transmultiplexer. The transmultiplexer has the 2 banks in the reverse order because this has the synthesis bank first followed by the analysis bank, okay.

And the reason at this point what we have is a frequency division multiplexed signal. So basically it is a wide band signal because you are taking many narrow band signals and you are making them into a frequency. So this would be a wide band signal, okay. Very important that we have it. So basically we will go back. We are not going to go back to compression. We are going to look at transmultiplexers and just refresh saying okay what were some of the results that we encountered in transmultiplexers, how do we link them to multirate DSP.

And then make the link between multirate DSP, transmultiplexers and multicarrier modulations. So basically it is a sequence of gradual steps but I hope you will see the link as we progress, okay. So the first statement is that to review or to just refresh is that we are talking about, when we talk about a flat fading channel, but that is time varying, we said that the best that we can do is capacity via water filling.

And maybe now that you know the subtleties, you can also say that this is not just water filling, it is water filling over time, okay. And that is what we have indicated even here. So water filling over time is the correct specific, correct way of saying that you are achieving capacity for a time varying frequency flat channel. Now we then move and of course what does it require? It requires us to have CSIT and doing the optimum power allocation, okay. Move on.

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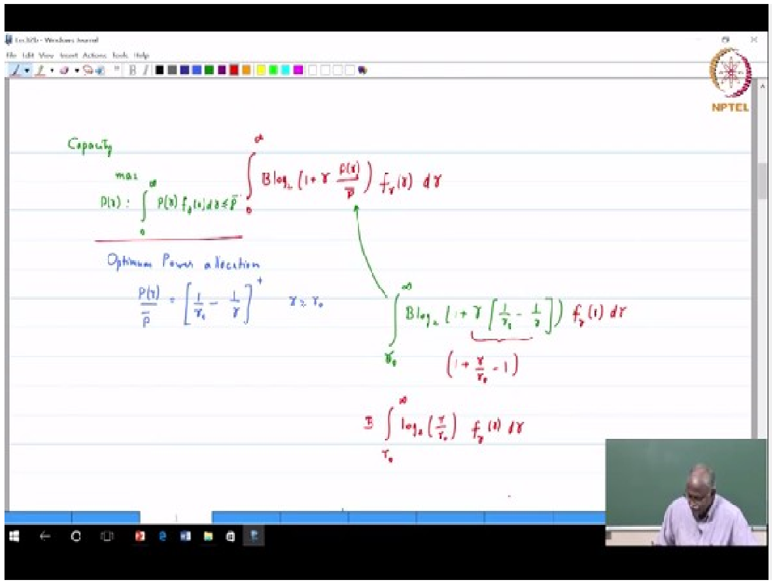

The capacity formulation, again we will not repeat that. You are maximizing over all possible power allocation schemes that are available to us, depending upon the distribution of the SNR and accordingly you then achieve or find that power allocation scheme which gives us the maximum and we said that through the process of the optimization, Lagrangian optimization, we came up with the following result.

So the optimum power allocation is a water filling and what exactly is water filling? We will just write that down. Optimum power allocation, says whatever SNR is there already in the channel, boost it by a factor and the factor is given by P gamma/P bar and this value is equal to 1/gamma 0-1/gamma with a + sign, that means it is strictly non-negative. And this is for gamma greater than or equal to gamma 0. So this is the water filling algorithm.

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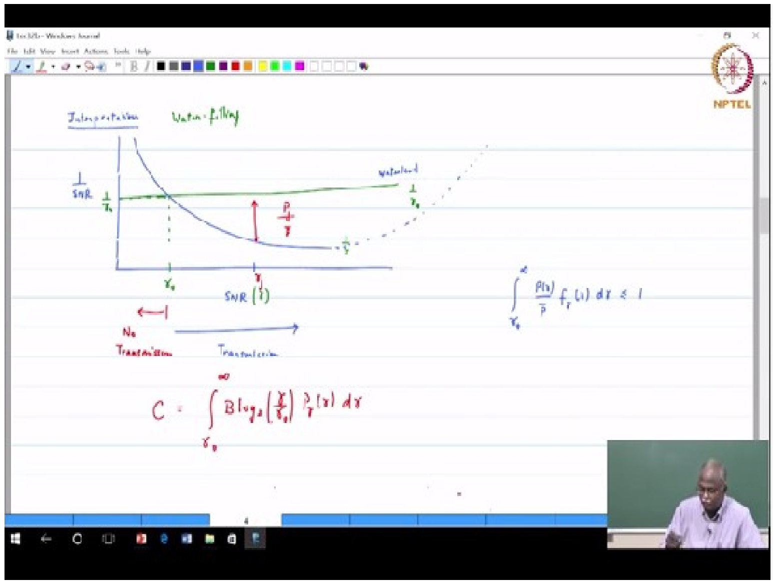

And we had a very good pictorial representation of that saying think of it as a vessel where you are trying to fill the levels. Now after you do water filling, what is the capacity expression? So the capacity expression originally was given by B*logarithm base 2 1+gamma P gamma/P bar. Now we have an expression for P gamma/P bar. So that integrand will become logarithm base 2 1+gamma*P gamma/P bar is 1/gamma 0-1/gamma, okay.

And if I take the integral between gamma 0 to infinity that means I am strictly in the range where this value 1/gamma 0-1/gamma is going to be positive, I can drop the +sign and then I have, close the bracket, change the colour to capture the f gamma of gamma d gamma, okay. So now of course, you can simplify this. It will come out to be (1+gamma/gamma 0-1), so therefore, the simplification of this will be gamma 0 to infinity, take the B out, it is a constant, logarithm base 2 gamma/gamma 0*f gamma gamma d gamma, okay.

That is the new expression for capacity, okay. Now an intuitive interpretation of this is your capacity depended on log of gamma, right, log of 1+gamma with log of gamma especially if you are looking at large SNRs. Now what is it depend on after water filling? It is gamma/gamma 0, okay. And gamma 0 is a number less than 1. For example, yesterday, in the example I think we got 0.89 or something like that.

So that means you are going to boost this one. Now but after boosting the power, we should not exceed the power limit. So that is where the choice of gamma 0 is not totally free to us. We have to make sure that we still satisfies this constraint, okay. So that is very important so that we satisfies the power constraint and at the same time, achieve capacity. The reason we are able to boost the power is because there are times when you do not transmit.

Because if the channel conditions are worse than certain SNR, we will not transmit and therefore, we take advantage of that, okay. So this is the point at which we were ready to say that yes we now know how to do water filling in time. We have a good intuitive understanding. Now how do I extend it to practical channels.

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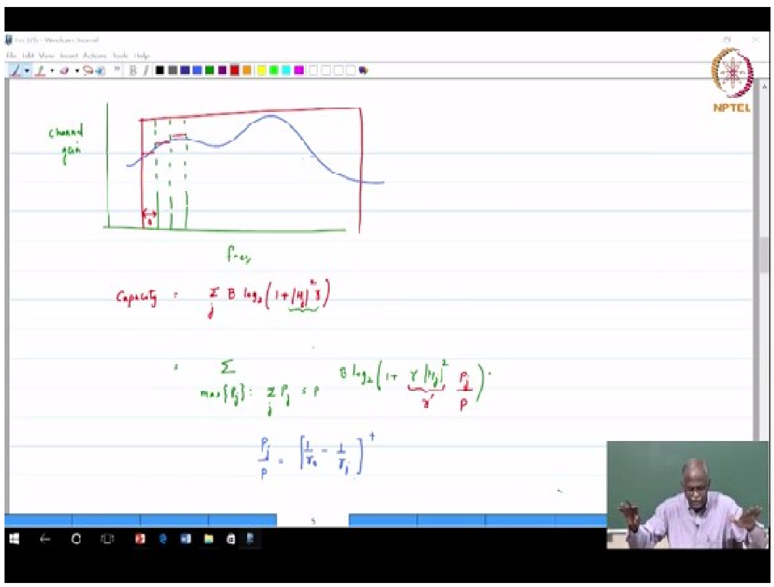

Practical channels will have some sort of a frequency selective channel gain. That means different frequencies will see different channel gains and we said no problem, we will divide it into very narrow sections of spectrum, let us say of bandwidth B and then write the capacity in terms of the summation of the capacity of each of those narrow chunks. We call them subchannels.

Now because each of them have got different gains, we said analogous to water filling over time, we can also do water filling over frequency. So this is not, at this point not time varying. All that we have is that there is a frequency gain variation. So we go back and say that if you have a time invariant channel but it frequency selective, then we do water filling in time, water filling in frequency, okay. That is an important difference. Previously it was water filling over time. Now we have done water filling over frequency.

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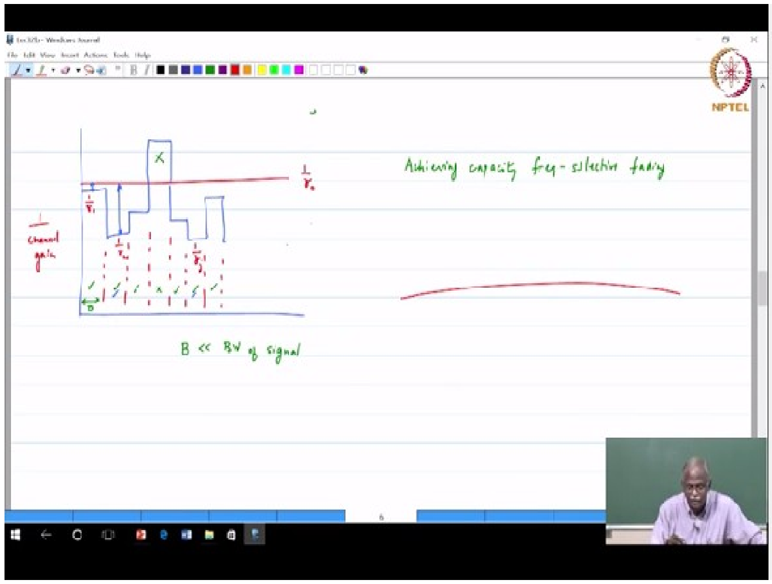

I want to make a few comments about that, again water filling over frequency will basically mean that you look at the channel gains, reciprocal of the channel gains. Again, this is 1/the channel gain is what we are plotting, okay. And depending upon the channel gain and the channel gain directly affects the SNR. Yesterday, we showed that where the affects the SNR, it multiplies the existing SNR and based on that we then do water filling, 1/gamma 0 is my water line.

So anything which falls above the water line, the reciprocal of the channel gain goes above the water line, we say okay, this channel is not good. I am not going to waste power transmitting on this particular channel. So in other words if you were to think of these as your subchannels, okay, in this power allocation scheme, okay, I am just going to indicate something here. This one is carrying information.

This one is carrying information? Yes. No information and this is not carrying any, this is carrying information, carrying information, carrying information, okay. Now if I want to do a little bit more fine representation, which of these is carrying more information? This is carrying more information. This is carrying more information. Why? Because the reciprocal of the channel gain is low which means that they will get a boost in power and therefore they can carry more information.

So you see that even within one signal, I am able to, in a very fine tuned manner, I am able to optimize and that is the reason why we are able to achieve capacity. So this is a time invariant channel but it is frequency selective. We have done water filling over time. So couple of more comments about this. How do I choose this bandwidth B? Okay, any suggestions. How should I choose this bandwidth B?

Depends on how much variation is there in the channel, okay. So basically what you would end up doing typically, you will choose bandwidth much less than the bandwidth of the signal, yes. Because the signal is wide. So bandwidth of the signal. Okay. The other element which is just now pointed out which is a very correct observation. It depends on the rate of the change of the channel. So for example, if I had a channel that is this. Do I need to do a lot of water filling over frequency? No. I just, I would just do, just treat it like a signal carrier.

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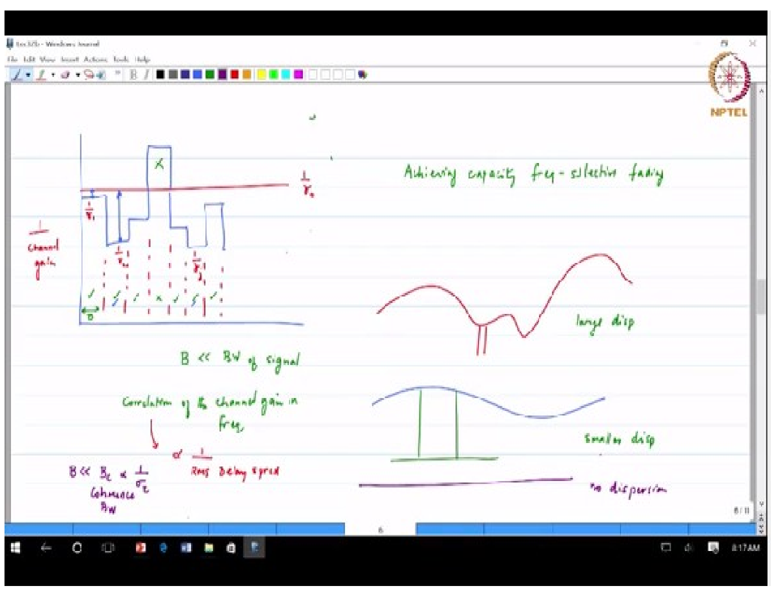

On the other hand, if I had very fast variations, okay. If I had very fast variations in the frequency response, then I would have to choose very narrow band. On the other hand, if I had channel gain that was more or less gradual, then I may be able to take much broader frequencies, okay. Now obviously it is not intuitive how to go about choosing the bandwidth of the channel. So one condition we can in general say that because you are talking about wide band signals, this bandwidth B will be much smaller than the bandwidth of the signal.

The second one that we relate to is how much correlation is there in frequency, okay. So there is a correlation of the frequency, of the channel gain in frequency, okay. I am going to give you a result that is again it is a well known result but again we are going to use it without proof. This correlation of the channel gain in frequency, I am going to tell you is proportional to 1 over the RMS delay spread.

That means if you have a lot of dispersion, RMS of the delay spread is going to increase, so what that says is the more dispersion I have, the correlation is going to be smaller, right. That means this corresponds to a case with large dispersion. This is I would say is smaller dispersion, okay. A sort of qualitatively we are saying, not quantitatively. And if you carry it to the limit, this corresponds to no dispersion.

Am I right? Frequency, the channel gain variation. No dispersion means a signal tap, therefore it is. So this is a reasonable way of looking at it. So the correlation of the channel gain depends on, so this bandwidth that you are choosing for your subchannel, must be much less, much smaller than the region of high correlation. So basically we call it coherence bandwidth. Coherence bandwidth means the bandwidth over which the channel gain is highly correlated.

And this is proportional to 1 over RMS value of the delay spread, sigma. Again these are results, you need not worry about. But just so that you have a feel for how the bandwidth B should be chosen. Depends on the dispersion of the channel. And of course given that it is a wide band signal, this bandwidth B is going to be much less than the bandwidth of the signal. It is also going to be much.

It has to be much less than the bandwidth of the coherence bandwidth of the; now is the bandwidth of the signal greater than the coherence bandwidth? Important question, okay. Let me just open up a new page, because I want you to think about that.

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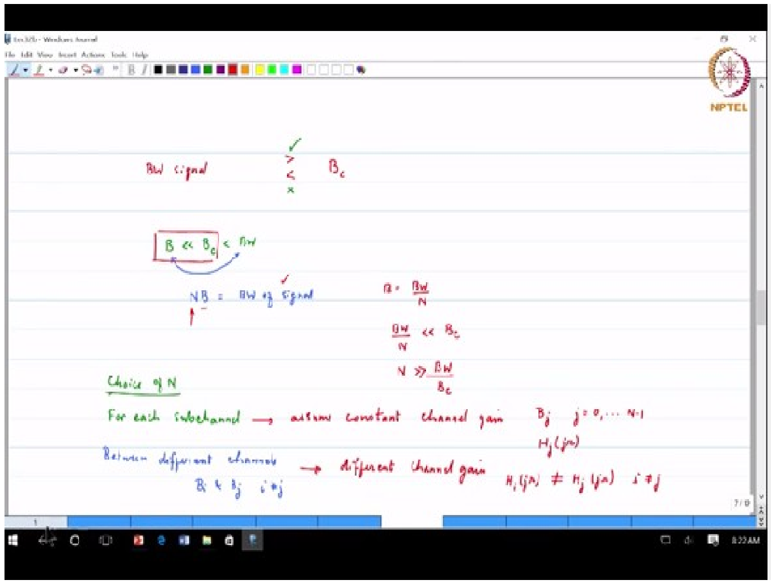

So is the bandwidth of the signal, okay, the wideband signal, is it greater than, less than coherence bandwidth? If it was less than coherence bandwidth, what would be the observation? What type of channel would it be? It will be frequency flat. So then that is not the channel we are talking about, right. So we are talking about, so actually the bandwidth of the signal is actually greater.

It is not, this case is not the one that is interest because if it is less than coherence bandwidth, it will be a flat fading channel, that is not the case that we are looking at. So basically we are looking at the case where the coherence bandwidth is already smaller than bandwidth of the subchannel. So B much less than Bc which is less than the bandwidth of the signal, okay. So that is roughly the relationship that we have.

Now how do I relate the bandwidth of the signal and B. Basically, there are, if I divide it into n subchannels, N*B=the bandwidth of the signal, okay. So in a given transmission system, the things that are fixed are the bandwidth of the signal. You want to transmit at 10 MHz signal. Let us say that is the case. So what is it that you will try to decide? You will try to see what is B. Then you will ask the question, can you tell me for this channel, what is typically the coherence bandwidth, okay?

Now based on the coherence bandwidth, you will then choose N, okay. So you will, actually it is not just a random selection, it is something that you are carefully choosing so that you satisfy this condition. You want B to be much less than Bc and B itself is equal to the bandwidth/N. So you will want to make sure that your total bandwidth/N is smaller than, substantially smaller than the coherence bandwidth.

And therefore, you can then say that N has to be greater than bandwidth/coherence bandwidth, probably much larger, okay. So this is the way that we would go about choosing. So N is something that you get to choose. So once you get to choose, what is the condition that you are trying to satisfy? We are trying to satisfy the choice of N, enables us to achieve the following. Choice of N or the selection of N.

Such that for each subchannel, we can assume that the channel gain is constant, okay. Assume constant channel gain. So that is why you have to choose N according to the coherence bandwidth because if the channel is changing quite a lot over frequency, you would have to choose N to be larger. Assume that the constant channel gain, okay. So channel gain. So if you gave a subscript Bj where j could go from 0 to N-1, then the corresponding channel gain we say is Hj of j omega, okay, the 2 j's are different.

What is in the bracket, is the square root of -1 but what is outside is the channel index, okay. Now the other assumption that we are making is that between different subchannels, okay let us we look at 2 subchannels Bi and Bj where i not equal to j. What we can say is in general, the channel gains are different, okay. That is also a reasonable assumption because there is channel variation and I have chosen my channel bandwidth such that it is constant within my channel but outside of it, it could be different.

So different channel gains, typically this is what is the scenario. So in other words, Hi of j omega is not equal to Hj of j omega when I not equal to j. it is also is there. Now this is the reason why I need to do water filling. If all the channel gains were the same, I would not be doing water filling. So go back to the original statement. Water filling over time was a different setting where it was a flat channel time varying. Now I am talking about a time invariant frequency selective fading channel. Now for most practical channels, okay, so last statement, okay.

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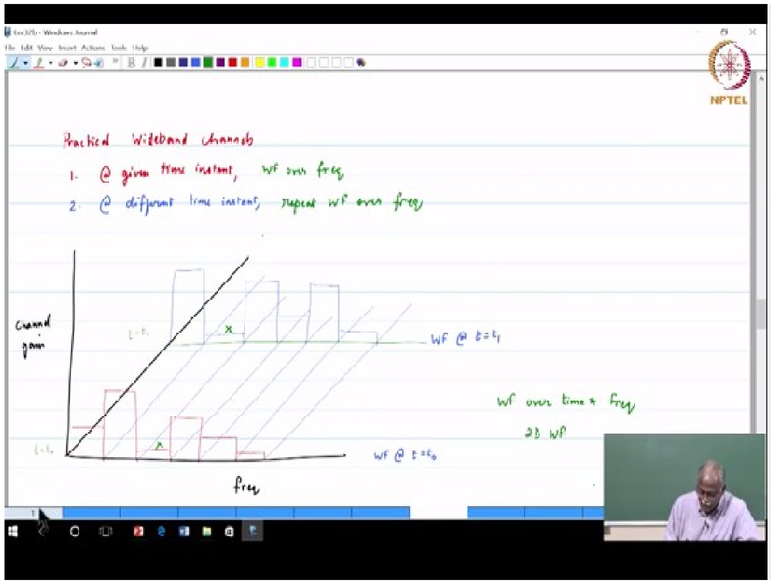

So now for practical wide band channels, this is the scenario that what we can do is you will have to do, because there is frequency variation across, the channel gain variation across frequency. So at a given time instant, what you would have to do to achieve capacity is water filling over frequency, agreed. At a given time, whatever is the channel gain at this time, I have. Now because wide band channels, practical channels are not constant in time, so that means at different time instant, I would have to do a new water filling, okay.

Repeat water filling, because it is no longer the old channel. It is now a new channel. So this is repeat the water filling over frequency, okay. Maybe the best way to capture this is in the form of a simple picture. So think of this as the channel gain and this as frequency, okay. So I am going to draw a third dimension just to help me with. So now I am going to think of, these frequency has been divided into subchannels.

So these frequency subchannels are indicated in the following way. Let me use the thin line, okay. So this corresponds to the frequency subchannel number 1, frequency subchannel number 2, 3, 4, 5, 6, okay. Now at time instant 1, a time t=t0, so time t=t0, this is the channel gains that we are seeing. This is, just as an illustrative. So the channel number 1 has got a certain channel gain.

Channel number 2 has got a higher channel gain. Channel number 3 has got very poor channel gain. Channel number 4, something medium, okay. Illustrative, just an illustrative and channel number 5 has got again something different. Channel number 6 is very poor, okay. So what would you do? You would do water filling over time for this frequency, for this time instant. Now at another time instant, t=t1.

So this will be t=t1, the scenario could be completely different. So channel number 1 could be a very strong channel. Channel number 2 actually has gone down. Channel number 3 has now which was originally very poor, has now become very strong and of course, this one is medium, this one is strong and another channel, okay. So the channel response changes. So now what we are saying is at every time when the channel response has changed, I have to do water filling.

So this is water filling. We go back to medium point. So this is water filling at t=t0. This is, you need to do water filling at t=t1, okay. Water filling means some channels may get eliminated. So for example it is possible that this channel gets eliminated at t=t1. Another channel got eliminated at time t=t0. So again that is the purpose of water filling to achieve capacity. So this is actually referred to as water filling over time and frequency, okay, over frequency first and then over time.

Over time and frequency. This is called 2-dimensional water filling also, okay. So I hope you are able to visualize the, how the channel changes, how each of these channels are, the wide band channel has been divided into subchannels and within, based on the current channel conditions, I would apply the water filling to gain the capacity of the channel for that particular scenario. So the third situation that we talked about is going to be water filling over time and frequency.

That is the third element. So once you have these 3, then we more or less know how to achieve capacity for a wireless channel, narrow band, wide band, time dispersive, time not-dispersive, all combinations are now within your purview of being able to understand and apply. Now another very important observation in our approach to achieving capacity. What did we do? We divided the wide band signal into subchannels. Now are these subchannels orthogonal to each other? Are these subchannels orthogonal to each other, the question that is asked is okay.

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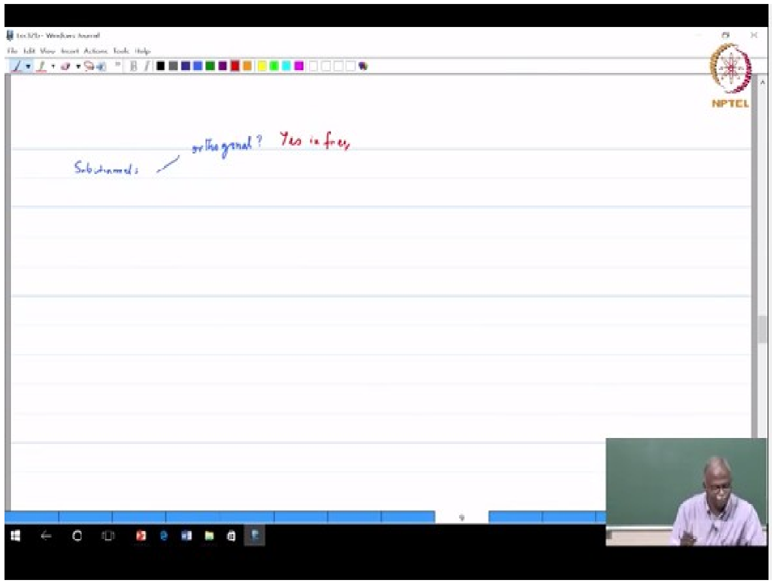

Subchannels are they orthogonal? What is your answer? Orthogonal? The answer is yes. Orthogonal in which dimension? In frequency. Because they are not overlapping in frequency. But is that the only way you know how to achieve orthogonality in frequency? No, even if there is some amount of overlap, we can achieve the; so I think the thought process starts to say that okay I need to start thinking in terms of orthogonality.

Because if I want to parallelize my transmission, I do not want them interfering with each other. Ideally I would like them to be non-interfering. Because our assumption is in this whole process is that these channels do not interfere with each other. That is why I am able to do power allocation and other things. So this notion of orthogonality is very important. So you need to keep that picture in your mind.