Multirate Digital Signal Processing
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Lecture – 31 (Part-3)
Capacity of Wireless Channels - Time-invariant Frequency Selective Channel

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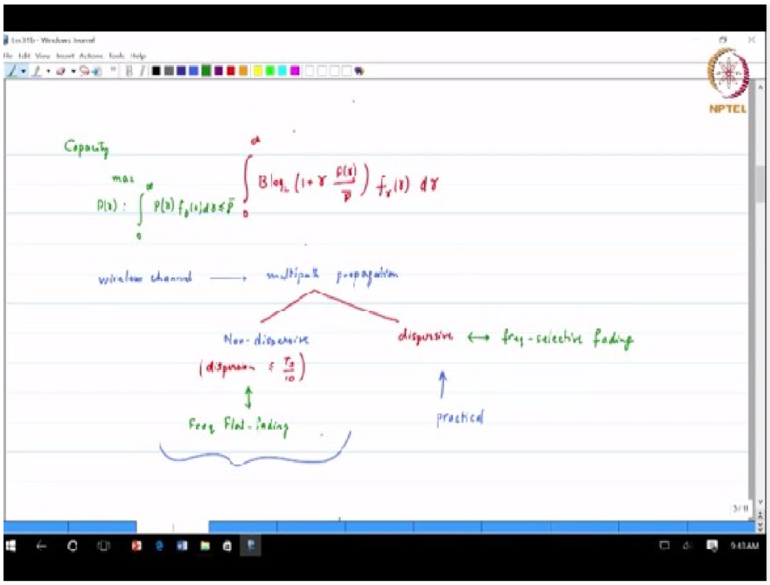

Pay close attention to the following aspect. Now here is the scenario. What we have said so far is if I have a flat fading channel with different SNRs, the way to get the maximum out of it is to do water filling, okay. That is the result so far. So this is what we have completely solved now. Now I want to move over from here to the right side where you have dispersive channels where most of the practical channels that we encounter are of that category.

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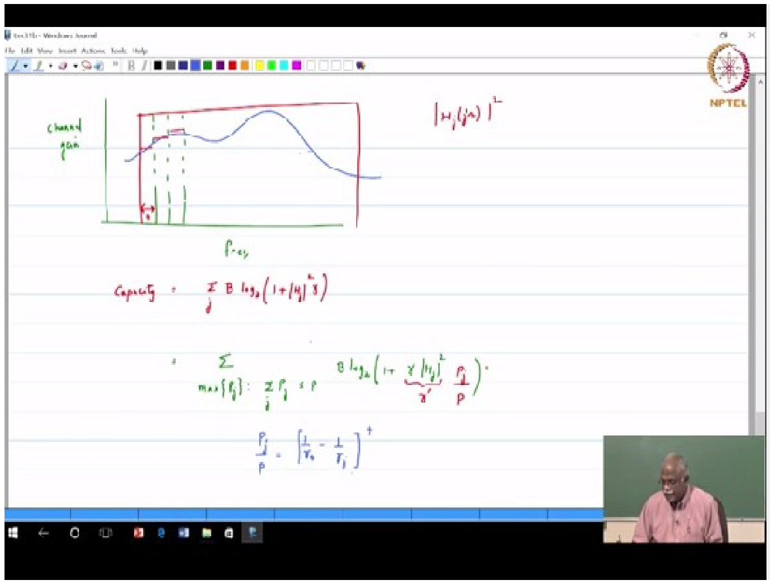

So the visualization of such a channel, this is frequency, okay. So you think of the channel frequency not as being constant. This you can think of the channel gain. A frequency selective fading channel basically says that there is some variation in the gain across the frequencies, okay. Now if I had a scenario where my signal was this wide, right. That is when I see the frequency selective fading.

All the frequencies in my signal are not getting the same channel gain. Now I have a problem. Now if I tell you, now please tell me what power to use. You see well I really cannot tell you anything because the result that we have is only if the channel gain is flat, right. If the channel gain was flat and I could have told you, you know what, currently the channel condition is bad, use the low modulation scheme, once the channel condition got good.

Now given this sort of scenario, the obvious sort of intuition says okay, can you break this wide band signal into signals which are narrow in bandwidth, okay. So that is a visualization. So basically I am going to say that okay, this is the gain for this portion of the bandwidth. This is the gain for this. And now I ask you okay, now I have got a wide band signal where the gains are all different.

I am going to break them up into some number of subbands, okay. So the total, each of these, let me say that the bandwidth is B. It is much smaller than the total bandwidth that you have, okay. I want to now ask the question; can you tell me what is the capacity of this wide band channel. I could not have given you an answer if it was a single wide band carrier but now if I break them up into subbands, actually I can.

So I will tell you that the capacity of this wide band channel is the following. It is B, which is the bandwidth of each of those, summation over j, whatever number of such channels that we have, logarithm base 2 one plus, very important what is the channel gain that you see or what is the SNR that you will see? The gamma which is Ps/Pn and whatever is your channel gain, magnitude H of j, it should be H of j j omega, okay, continuous time frequency, magnitude squared, that is the gain that you are seeing.

I am just going to write it as magnitude H of j. So magnitude H of j squared*gamma, okay. So what did we do? I know how to get the capacity of a signal with a certain bandwidth B which has got a flat gain. This wide band signal, wide band channel, I broke it up into subbands for each of which I could sort of make the assumption that the gain is flat. Then I use the old formula, okay. So this is the, now in the earlier case, there was a f gamma coming because there was a probability distribution.

Does that need to come here also? No, there is nothing probabilistic here. There are some n channels. So basically there is no PDF actually involved, okay. Now if no PDF is involved, does it still make sense to do power allocation? Does it still make as well? Answer is yes. Why? Because some of these channels are good, some of these are bad. So why do you give equal power.

So the capacity formulation now can actually be enhanced to do the following, okay. So it is summation over the maximum of over the power allocation of these channels such that the summation of these power allocated to each of these subbands is less than or equal to the total power allocation. It is not P bar, it is the total power allocation P, okay. So now the objective function will be B*logarithm base 2 1+gamma*magnitude H of j squared, that is my, this actually now becomes my effective SNR, okay.

This multiplied by Pj/P. This is my optimum power allocation and the constraint would be that sigma j Pj less than or equal to P. So this is a quantity that will be a ratio which we can adjust and the optimum power allocation again I am taking it almost without any proof that Pj/P will have to be of the form 1/gamma 0-1/gamma j with a + sign which is water filling, okay.

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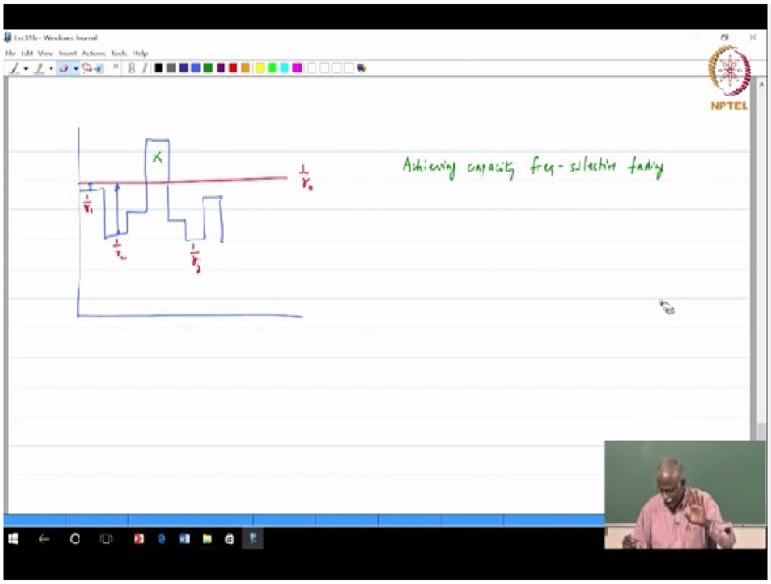

So the key result is that when I have a wide band signal. First of all, I am going to have frequency selective fading. And the process by which we will achieve capacity is I am going to quantize the SNR in each of those bands, okay. And I am also going to do 1/gamma water filling. So this is 1/gamma 0. This is 1/gamma 1, sorry, 1/gamma 1, this is the first subband. This is 1/gamma 2 and 1/gamma j, okay.

What is the power allocated? This one gets very little power. This one gets a lot of power. Because it is a good channel. 1/gamma 2 is small means gamma 2 is large. This one gets no transmission, no power allocation because you are above the water filling line, okay. So very interesting. A wide band signal where I can do some very flexible allocation, I can even choose not to transmit at a certain power level, at a certain frequency band.

So now go back and look at the diagram. So if I want to transmit a lot of data, I have option of doing a single carrier. Single carrier means what you see is the red line. But I cannot do power allocation. I cannot do cannot exclude certain portions of spectrum saying you know what currently the channel conditions are really bad in this portion of the spectrum. I am going to avoid transmission.

I cannot do that. However, if I split it into these subbands, I not only can exclude certain portions from transmission. For the remaining part, I can actually do optimum power allocation and get you the maximum capacity out of this system. So this is a very very key observation. So this is a way of getting your capacity, achieving capacity in a frequency selective fading channel. So this is a mechanism for achieving capacity, achieving capacity in a frequency selective fading channel.

If you did not do this, really there is no way we can optimize for this type of achieving the capacity. You can, you will get some throughput but it will not be anywhere close to the throughput that you will get with this type of system, in a frequency selective fading channel, okay. Now there is another dimension which is very important from the communication side. Let me just sort of highlight it and then we will close there.

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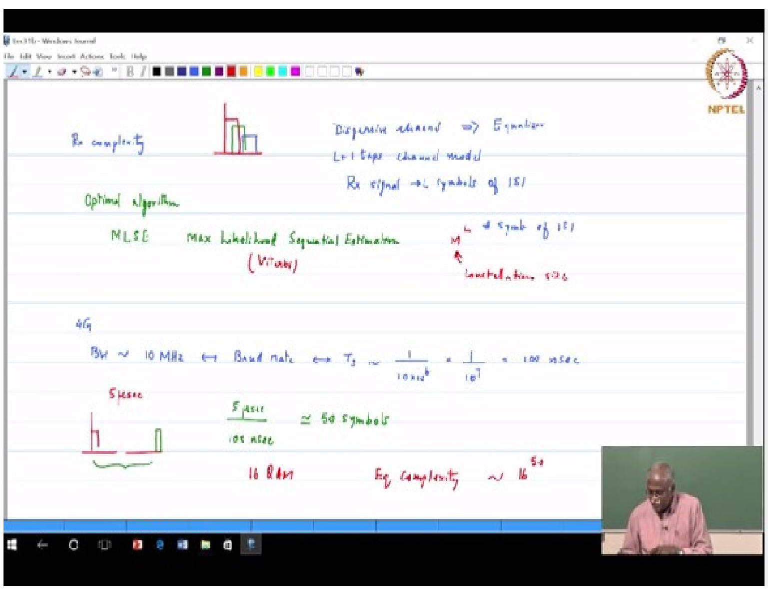

Now this has to do with the complexity in a dispersive channel, complexity of the receiver. So basically we will just introduce this aspect. We will expand upon it in the next class. So receiver complexity, okay. Now a very quick question. This is something which you would know. If I have no dispersion, this is what I transmit, this is what I receive. Now supposing there is dispersion and I get this type of a signal at the receiver.

What do I need to do at the receiver? I need to use an equalizer. So a dispersive channel means equalizer is needed, okay. So dispersive channel means an equalizer is needed. Dispersive channel means equalizer is the only way I can solve this problem, okay. Now if I have L+1 taps in my channel model, that +1 says that that is the signal that I am interested in. The remaining ones are previous symbols that got L+1 tap channel model, okay.

If this is my channel model, my received signal Rx signal contains L symbols of ISI, okay. That is what it means and I am sure you are familiar with this notation, L symbols of ISI and in the equalizer domain of all the algorithms, optimal algorithm for equalizer, can you give me one of the optimal algorithms which you have studied. Different types of equalizers, the one of the optimum ones.

It is called maximum likelihood sequence estimation, MLSE, okay. This is also known as the Viterbi algorithm; in case you are more familiar with that name. So basically this is one of the optimal algorithm. There is one which is called maximum a posteriori algorithm but most of the receivers that we encounter actually implement the MLSE, maximum likelihood sequence estimation.

This is also known as the Viterbi algorithm, okay. The Viterbi algorithm has got a complexity which is proportional to the number of the constellation size, M is your constellation size, raised to the power L, where L is the number of taps. So M is your constellation size. This is constellation size. If it is BPSK, it is 2, QPSK is 4, 16 QAM, constellation size and L is your number of symbols of ISI, okay.

So here is a simple example and we stop with that. I have a 4G system. I have a bandwidth, approximately 10 MHz, that is my bandwidth of my signal. Now bandwidth is related to baud rate, number of symbol, number of times you use the, so this is related to baud rate, okay. It is related to bit rate also but it is related to the bit rate through the baud rate. Baud rate*the number of bits per symbol is your bit rate.

So baud rate is the more fundamental quantity when we talk about the bandwidth. So this is related to the symbol duration, right. 10 MHz means I am sending approximately 10 million symbols in a second which tells me that my symbol duration is approximately 1 over, let me just not say equal to. It is with pulse shaping and other things. It will be slightly, but you can make it a fairly good approximation.

This is 1/10*10 power 6. It is the same as 1/10 power 7 which is approximately 100 nanoseconds. That is your symbol duration, okay. If I have a 10 MHz single carrier system, this is what I will have as the bandwidth, okay. Typical delay spread in a fading, in a multipath channel, is 5 microseconds. So if I transmit a symbol now, it will come after 5 microseconds. A copy will, latest copy will come after 5 microseconds.

This is for sure. Now in this period of time, how many symbols have gone by? I will have to take the symbol delay spread, 5 microseconds divided by my symbol duration. Symbol duration is 100 nanoseconds, okay. So approximately 50 symbols have gone by, okay. And if I was using 16 QAM, my equalizer complexity with 16 QAM, equalizer complexity is going to be 16 raised to the power 50, mind boggling, okay.

So this is the seriousness of the problem that we are encountering. Notwithstanding the fact that if I transmit a single carrier system, I am not going to achieve capacity. I am going to have a tough time because the frequency is not flat. So does multicarrier, splitting the carrier into smaller frequency bands, does it help? Because it helped us achieve capacity, right, did it also solve the equalizer problem? We will answer that in tomorrow's class. Thank you.