Multirate Digital Signal Processing
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Lecture – 31 (Part-2)
Capacity of Wireless Channels - Formulation of Capacity Calculation - Part 3

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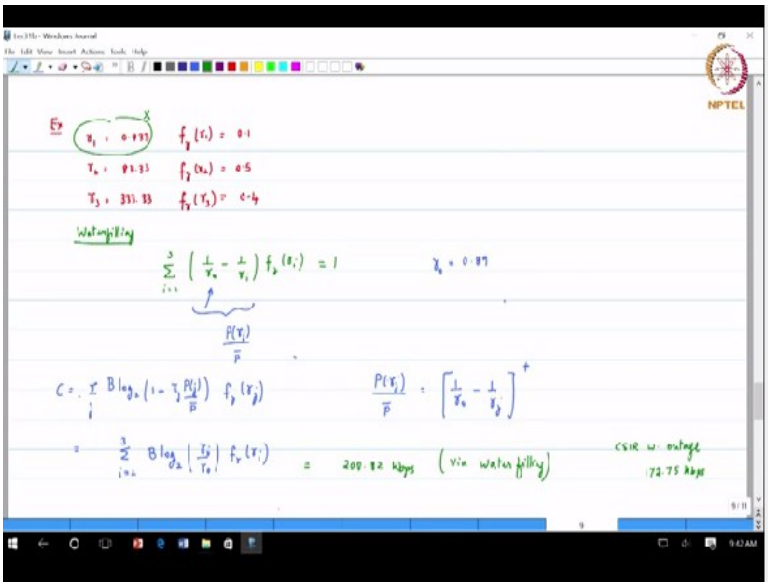

Let us try to gather with an example because that is the best way to do it. Example is actually a continuation of the previous example. We had a channel with 3 SNRs. We go back to the same channel. It has an SNR of 0.833 with a probability f gamma of gamma 1 being 0.1 and SNR gamma 2 being 83.33, f gamma of gamma 2=0.5. Gamma 3 is 333.33, f gamma of gamma 3 is 0.4, we have already computed their capacities.

Right now that is not the task before us. I want to apply the concept of water filling, okay. So the concept of water filling basically says that some SNRs I would like to exclude. So looking through this, this looks like a bad SNR. So maybe I want to exclude this. So what we are saying is, can I do water filling for the remaining 2 channel conditions. So i=2 to 3, okay, 1/gamma 0 minus 1/gamma i f gamma of gamma i, can we set it = 1.

Gamma is fixed. f of gamma is fixed. Only thing that is variable is gamma 0. This is exactly what we need to find out because that is the water filling level. So water filling is coming into the

play by saying how do you do the power allocation such that, basically this is power allocation because what we are doing here is nothing but P of gamma j/P bar, okay, from the previous.

So basically we have to solve this equation to get the value of gamma 0. So it is a numerical exercise. So we calculate gamma 0=0.89, okay. That means your water filling level is at 0.89. Now is it higher than the SNR state that we were trying to exclude? Yes. The SNR state that we are trying to exclude is 0.83. So that SNR state got left out.

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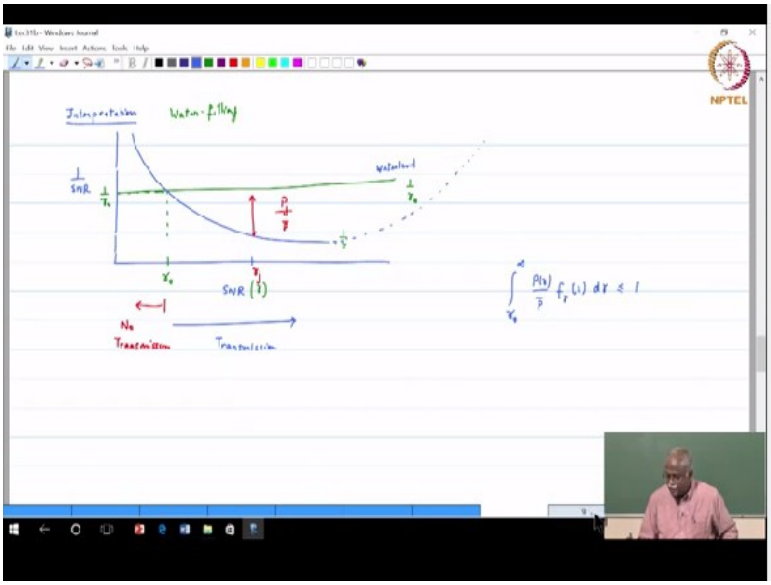

So if you look at the water filling, this gamma 0 is 0.89, 0.83 will fall to the left. So that is excluded. Basically the water filling happens in the rest of the portion. So what exactly is the computation that we have to do? So now let us look at the expression for the capacity? Capacity expression is B log of log base 2 of 1+gamma j P gamma j/P bar*, this would be summation over j f gamma of gamma j, right.

That is what we are trying to do. So basically this portion of Pj/P bar if I substitute the expression for it which is, so I use the following, P of gamma j/P bar is given by 1/gamma 0-1/gamma j, with a + sign. If I use that, basically there are 2 states which will; so this will simplify to summation i=2 to 3 B*logarithm base 2. If you notice when I substitute for this equation, there will be a -1 which will come.

So basically the 1 and -1 will cancel leaving us with the following expression, gamma j gamma i/gamma 0*f gamma of gamma i, okay. So that is the expression that we have for the capacity. Please do substitute the different levels that we have. All of the quantities are known. Gamma i is known, gamma 0 is known. The probabilities are known. The bandwidth is known. You can do the computation.

This comes out to be 200.82 kbps via water filling, okay. Where did the additional gain come from? Because you boosted gamma i/gamma 0. You divided by gamma 0 and gamma 0 is less than 1. So that is effectively like boosting gamma i. That is what you did. Rather than transmitting power also in the gamma 1 state, you excluded the SNR 1 state and then you boosted the power.

And we have also made sure that in the process, we did not end up transmitting more power than allowed because that is also a constraint that we have to satisfy. Now compare this with the case, if you did not do water filling and you had only CSIR, what was the throughput that you could achieve? **"Professor - student conversation starts"** 191. 191.94 if you had CSIT. But if you, otherwise, what, you will basically will have outage which would mean that you would be transmitting around 172.75, I think, okay. **"Professor - student conversation ends."**

So CSIR with outage, the capacity or the throughput that we would have achieved is around 172.75, I think, just verify that, kbps, okay. So because it is sort of a constructed example, we know maybe the gain is not that dramatic but usually what we find is that water filling is a very good strategy. It actually causes a significant advantage. The other advantage is, you did not waste time transmitting power when the channel condition was bad.

Whereas in the case of the CSIR, you actually did end up transmitting power during time when it was not so, the channel conditions were not so good as well.