Multirate Digital Signal Processing
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Lecture – 28 (Part-2)
First Part Name: Perfect Reconstruction Final Overview. Second Part Name: Introduction to OFDM - Motivation – Part 2

So we now move to OFDM and always very happy to discuss this topic because this is where we see coming together of the whatever we study in DSP with communications in a very nice way. So there are several ways of that you can introduce OFDM. I would like to do it in the following sequence.

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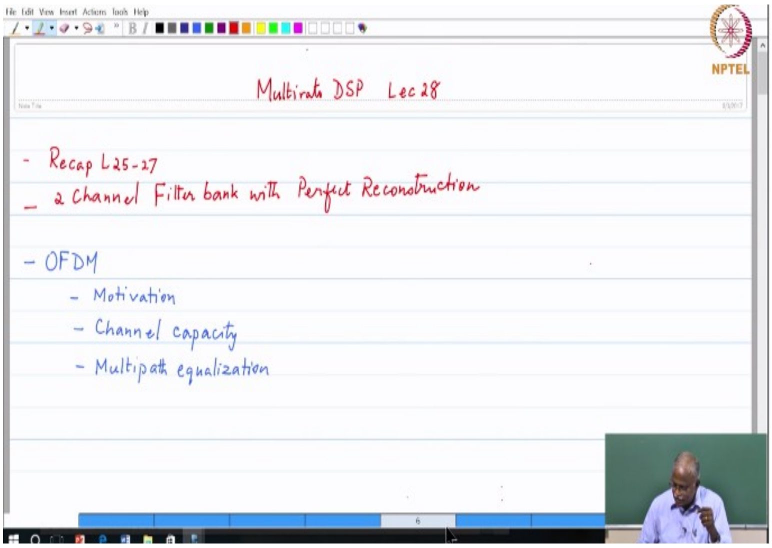

First motivate, why OFDM and then actually present OFDM as the option. So focusing on motivation, I would like to begin by just refreshing your memory on channel capacity okay. If I have an AWGN channel with a certain signal to noise ratio, so basically let me just write it down here.

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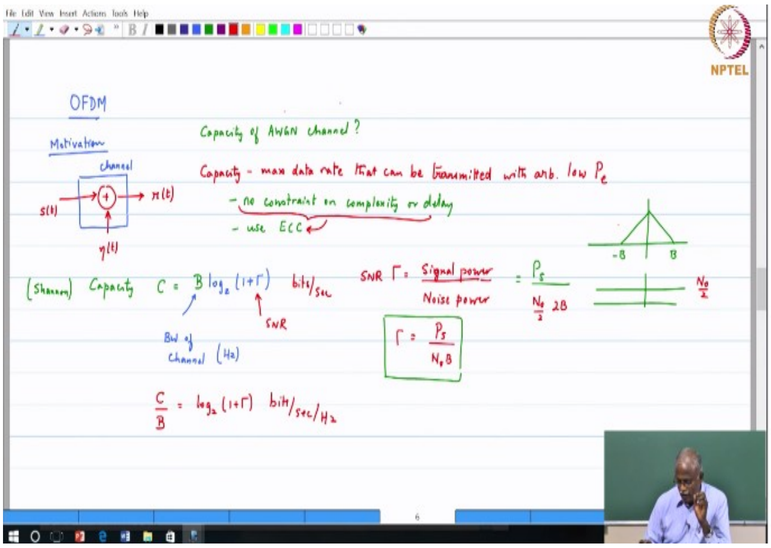

We are entering OFDM, this is a new unit, first part will be the motivation. Why OFDM in the first place? Why not CDMA, why not something? Why not something else? Okay, so the first question that I want to pose to you is I have a signal which is s of t which is going through a channel where additive Gaussian noise, white Gaussian noise is added. Let me call that as eta of t and this is what I receive at the other end r of t.

So this is effectively my channel. This is my channel. Now the question that was posed by Shannon and also answered by him is what is the capacity of this AWGN channel. Capacity of the AWGN, additive white Gaussian noise is the only impairment. What is the capacity of this channel? But first we must define capacity, so capacity is the maximum rate data can be transmitted.

So it pertains to the maximum data rate, so which means the answer will be something in the form of bits per second that can be transmitted over this channel with arbitrarily low probability of error. No point transmitting if you are going to have errors that can be transmitted. I am going to skip some of the words, transmitted with arbitrarily low probability of error, low probability of error PE okay.

Under the assumptions that there is no constraint on complexity or delay, you can put in the most complex transmit to the most complex receiver that you want and you can take as much time as you want to. No constraint on complexity or delay. Now where would complexity or delay come? It will not come in the modulation and the demodulation. It will come in the error correction that you will apply.

So basically what we are saying is you may use error correction coding ECC or forward error correction FEC if you are used to that. You may use error correction coding and it can be as complex as you want. So no constraint on complexity, basically refers to, pertains to this particular aspect. No constraint on complexity or delay primarily says you can use very complex encoding schemes and decoding schemes okay.

Now Shannon answered, gave the answer to this and the answer was the capacity of this channel, we also refer to it as the Shannon capacity. Shannon capacity was given to be capacity denoted as C is=B is the bandwidth of the signal, bandwidth of the channel rather bandwidth of the channel which is in hertz, multiplied by logarithm base 2, log base 2 of 1+gamma, where gamma is the signal-to-noise ratio.

I think SNR is a fairly common definition but often there is a confusion with respect to the definition of signal-to-noise ratio. I will just write it for the purposes of clarity. So SNR gamma is defined as the signal power/the noise power. Signal power we can measure, we can specify, we can control. Typically, we say that okay that is Ps. Now noise is something we do not control.

It is in the electronics of the system that we are using. The assumption is that it is white, that means it has got a flat spectrum. So what is the portion of the noise that will come in? Basically, whatever is your signal bandwidth that is the, your receive filter will allow that noise to come in and so if you have a signal that is of the following form; –B to B, complex baseband.

So you would have to allow a filter of bandwidth –B to B. So how much noise goes in? Your noise spectral density times –B to B. Noise spectral density is often denoted as N0/2 watts per hertz. So what is the bandwidth that goes in? It is N0/2*2B, that is the same as Ps/N0*B. This is my definition or underlying definition of the SNR okay. There is a definition of SNR, hopefully this is consistent with the notation and the conventions that you have been following in communications.

But basically this is a quick definition of the SNR. Now if you have been able to estimate the SNR of the channel, Shannon's theorem states that you can come up with some complex coding scheme that will achieve a capacity or basically the capacity of that system will be B times logarithm base 2 1+gamma and the units is as I mentioned, will be bits per second. Now you can also sometimes, you talk about a normalized capacity.

In that case, it will be logarithm base 2 1+gamma, it does not depend on the bandwidth. So it is a per unit bandwidth. So it will be bits per second per hertz, both of which are okay but what we are interested is to understand and utilize the expression for the capacity okay. So basic definition of capacity everyone is comfortable with that? Okay.

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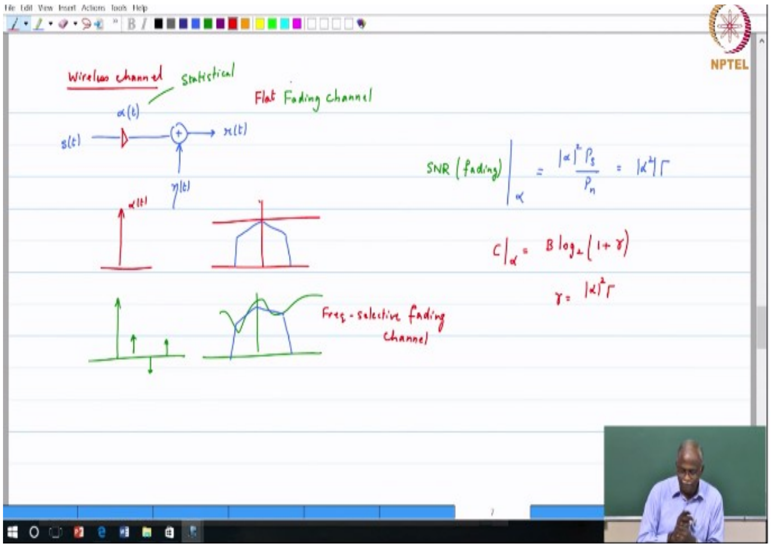

Now here comes the interesting variation. Now if I have a wireless channel then the model that we would have to use at a simplest level would be s of t. There is noise in the electronics, so I would still get eta of t, this would be my received signal r of t. A big difference will be this term which is like a multiplier which is alpha which is a function of time okay. Your signal does not maintain a constant level, but fluctuates okay.

And its fluctuations dependent on your motion and how fast you move and where you are and again when you study wireless communications, you will see that there are several ways to characterize but the best way is to say that this has got some sort of a statistical behavior and in practice I am sure you have experienced it. If you hold your cell phone in one position and you do not get a good signal what do you do? You move around, you get a good signal.

What is happening? That alpha is getting perturbed and therefore something that was not so good became good and therefore this is something that is not a constant depends on your spatial position and your movement patterns. So it has to be characterized by a statistical quantity okay. So this is a what we would refer to as a fading channel but it is not dispersive, so therefore in terms of the frequency spectrum that means it is flat in terms of frequency spectrum.

So flat represents the spectral characteristic and the fact that it is not changing right and the fact that you know, basically, okay let me just make sure what is flat. So my signal s of t is getting multiplied by something that looks like an impulse right. That is a scale factor. If your impulse response is a single impulse what is your frequency characteristic? Frequency characteristic is flat right.

If this is the time domain, frequency characteristic is flat. Now why is that important? Because if I transmitted a signal that had got a spectrum of this type. The whole spectrum goes through without any alteration, only the amplitude of the signal goes up and down. Now on the other hand, if you had a channel which had this type of a response okay. I am just doing some arbitrary response; this channel will have a frequency response.

And now if you transmitted your signal through this, notice that your signal spectrum is going to get distorted. So channels where there is no dispersion are called flat fading channels because all frequencies will see the same fading pattern. These are called frequency selective fading, frequency selective fading channels okay. So right now we are not talking about frequency selective fading channel, we are talking about flat fading channels.

So the only thing that is affecting is that this alpha may go up or down, so it is still easy enough for me to define a SNR. So SNR in a fading channel, obviously I cannot define it as a constant. It will be a function of the value of alpha right. So for a specified value of alpha, so maybe we can even we can indicate that okay. For a specified value of alpha will turn out to be alpha is this scaling on the amplitude.

So which means the power scaling will become and alpha can be complex because it has got an amplitude and phase. So the signal power scaling becomes magnitude alpha squared times Ps/Pn or it is the same as alpha squared times gamma, whatever was your original SNR is now been modified by this okay. So I can now not talk about a fixed number but I can talk about a number that changes in terms of the capacity.

So the capacity expression, now it becomes capacity as a function of alpha will be given by 1, bandwidth times logarithm base 2 1+gamma where gamma is defined as mod alpha squared times the SNR when the fading was not present okay. So you can see that there is not much of a, not a big change but you still have to keep in mind that once you are talking about fading channels, we cannot talk about a constant number as its capacity.

Now it is always important for us to anchor whatever we are saying with something that is in terms of numbers that you can visualize.

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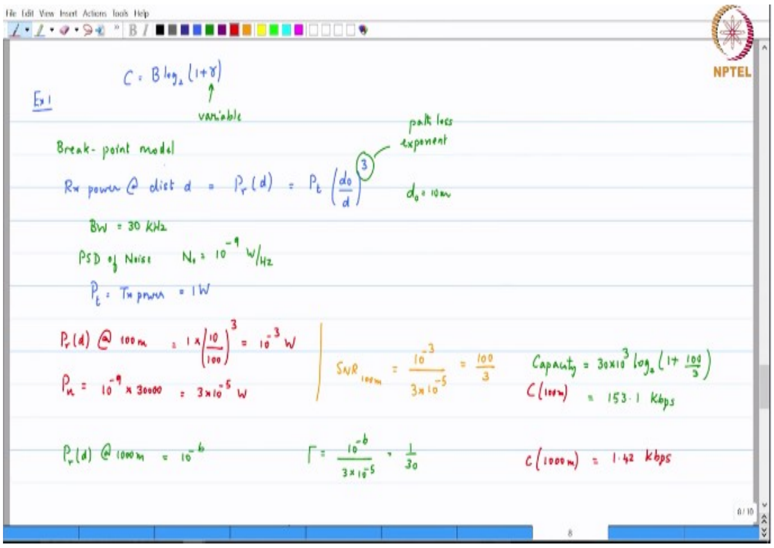

So first is an example which says that okay what happens if my SNR changes? How does my capacity change? And this is what we are going to assess. So keep in mind that the capacity that we are talking about is in a fading channel. This is bandwidth times logarithm base 2 1+gamma and this gamma is not a fixed number okay. Let me call it as it is a variable okay. That is the background.

So here is an example to illustrate what we are saying and hopefully this probably is just a refresher of something that you would have done. So we are going to give a model called the breakpoint model. Now a breakpoint model is one that is used to estimate what is the received signal power at a certain distance from the transmitter. So the breakpoint model that we are going to use a very simple one, the simplest possible.

The received signal power at a distance d from the transmitter, at a distance d is given by denoted as Pr of d, that is the received signal power. It is proportional to the transmit power into d0/d the whole cube. I am just giving you, so this is what is called the path loss exponent. In this particular case, it is specified as 3, you could have channels which have 4, 4.2 you can have, free space would be path loss exponent of 2.

So path loss exponent is in this case it is 3 that is for illustrative purposes and d0 is a constant, it is a reference distance and in this particular example we are taking d0 to be 10 meters. So the first exercise for us to do we are also given a couple of other important points. The bandwidth of the signal is 30 kilohertz, reason bandwidth is given is so that you can calculate the noise power.

The noise spectral density, power spectral density of the noise N0 is given to be 10 power -9 watts per hertz okay and we are given that the transmit power is 1 watt. Pt is transmit power=1 watt okay. Now a typical exercise that you would have encountered in communications is what is the received power Pr of d at a distance of 100 meters okay. That would be a typical example that you would have been asked to look at okay.

So the received signal power is given by 1 watt*d0 is 10 meters, d is 100 meters, I have to raise it to the power 3. So effectively it is 10 power -3 watts okay. What is my noise power? It is always fixed; it is 10 power -9 watts per hertz*30,000 which is my bandwidth of my signal that comes out to be 3*10 power -5 watts. Did I get that correct? Okay -5 watts, so at a distance of, what would be the SNR at 100 meters?

So the SNR at 100 meters would be 10 power -3/3*10 power -5 which is 100/3 and you can convert a dB if you want, but what we are interested in is what is the capacity at this distance. So capacity will be=30 kilohertz 30*10 power 3 logarithm base 2 1+the SNR which would be 100/3 okay. Do a quick calculation on your calculator. It will come out to be 153.1 kilobits per second, very good okay.

What would have been just a power calculation, we carried it one step further and said what is the SNR. Then, we said okay what is the capacity and it is a useful thing for us to have done this calculation okay. Now one more data point which is which I would like you to do is what is the capacity at a distance of 1 kilometer? Okay, so capacity at 1 kilometer, so the first thing that you would do is Pr of d at 1 kilometer.

So which would be 1,000 meters, so this would come out to be 10 power -6 okay, 1/100 raised to the power 3 and you will find that the SNR at a distance of 1 kilometer will be 10 power -6/3*10 power -5. So this comes out to be 1/30 much lower than before and what is most important, this was capacity at 100 meters. The capacity at 1 kilometer or 1000 meters comes out to be 1.42 kbps okay, very important.

What happened? Your capacity went for a toss and why did this thing happen because capacity depends on SNR. So if you have a SNR which is not so good then your capacity is going to take a hit okay. Now why did we even do this exercise, it is for precisely the following aspect.

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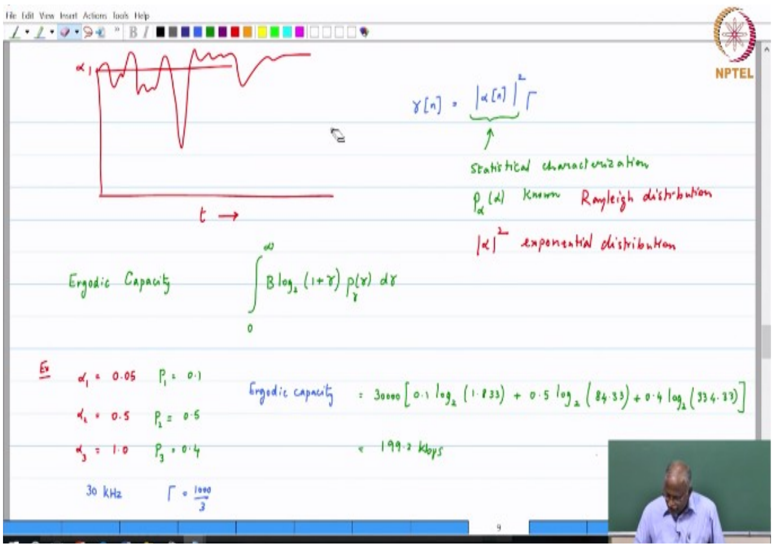

A fading channel if you think of this as time, imagine that you are walking on this corridor, the received signal level that alpha okay nominally should be 1 but occasionally can be more suddenly goes very bad okay. Basically as you are walking it varies, now what is the capacity of the system? It is fluctuating hugely, right and the application that you are doing maybe you are downloading something which requires 1 megabit per second.

Now it may be very easy to do 1 megabit per second when the channel is good but you know what happens when you go into a null. It is, so capacity, when once you start talking about capacity and in the context of wireless channels is a very tricky thing and it is something that has to be really carefully studied and handle and that is why the whole study of wireless communications becomes interesting.

It is not like the standard channels where you work with where it is easy to talk about capacity, easy to talk about this thing because this is a highly dynamic system. So here is what we would like to do okay. So we recognize that the SNR is going to be a function of n okay. So gamma I will write it as a function of n, it is not a constant, it is going to change and so this is going to be a function of alpha of n depends on this squared times gamma.

So there is going to be some variation of this. Now the wireless people have done a fairly extensive work and have obtained a complete statistical characterization. Statistical characterization is available of these wireless channels okay. So in some sense, I know what probability of alpha, of alpha is okay. This is known, it is called a Rayleigh distribution okay. And again it is not important for us.

So I would not spend too much time on it but what we need is alpha squared, so magnitude alpha squared has got an exponential distribution. So again this is also known, again the key point to take away is that the SNR is fluctuating but I know that the statistical characterization of this type of SNR. So now how do I calculate capacity? Okay, I cannot talk about a fixed capacity.

I am going to have to talk about something called an ergodic capacity or a statistical capacity and it is not the Shannon capacity exactly as we know it. It is going to be of the following form. It is going to be B times logarithm base 2 1+gamma but notice this gamma is not a constant, it is a statistical quantity. So I would have to multiply this with the probability distribution.

So probability of gamma, call it p gamma of gamma and I would then have to integrate it over the entire range of the SNR's. SNR is a positive quantity. The worse that you can get is from 0 to infinity. So basically 0 to infinity not, whatever is the range of SNR's that you are likely to encounter. You would then have to weight it by the probability of that particular SNR occurring okay.

So this is the key. This is very important for us to understand okay. Now if I were to ask you to maybe I will leave you this as an exercise for you to try out. So this is an example. I have a channel where there are only 3 states, alpha 1, alpha 2, alpha 3 okay. Alpha 1 is a very bad situation; 0.05, alpha 2 not so bad, alpha 3 is the nominal one. The probability of 1 is 0.1, 10% of the time, probability of alpha 2 is 50% percent of the time, probability of 3 is 40% of the time okay.

Now can you compute for me the ergodic capacity? Ergodic capacity, important for us to verify and ascertain. So basically it will be and take the bandwidth as 30 kilohertz and the nominal SNR, if you were to look at the previous case let us assume that gamma 3 is 1000/3 okay. So basically your gamma is 1 okay. So this is my default value. I would have to let me make sure we have got it correct, 3*10 power -5 yeah correct okay.

So one watt transmitted power yes okay, so compute the ergodic capacity and we will pick it up from there and move forward. So what you would get is 3 terms. It will be logarithm, let me just write this down. So ergodic capacity will be 30 kilohertz 30,000*probability that you get alpha=1, alpha 1 logarithm base 2 of the magnitude of this 0.05 whole squared*this gamma.

I believe it comes out to be 1.833, can you just quickly verify, 1000/3 yeah, please verify. I think this is correct. The second term will be 0.5 probability, logarithm base 2 and it comes out to be 84.33 and the last term is 0.4 alpha=1, so basically I will get logarithm base 2 1000/3 is 333.33+1 will give me 334 33 okay. Verify this; this comes out to be 199.2 kilobits per second, 199.2 okay.

The reason we are doing this is not that we want to understand want to get a whole lot of things into the wireless capacity. It is a motivation for OFDM and we will see it in a minute, in the next class why OFDM comes out as part of the assessment of this. So I hope you have a chance to review what we have talked about, think about the channel, think about capacity, think about a SNR that is varying, think about a capacity that is no longer a constant but something that fluctuates based on the SNR.

Now the key question would be is how do I get the maximum capacity out of a channel that is fluctuating. That is the challenge before us. We will give the answer in the next class. Thank you.