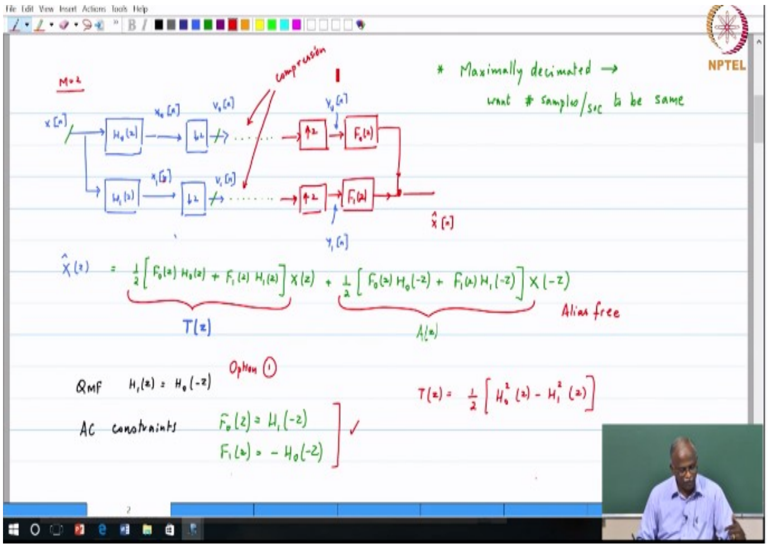
Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture – 28 (Part-1)
First Part Name: Perfect Reconstruction Final Overview. Second Part Name: Introduction to OFDM - Motivation – Part 1

Good morning. We will begin lecture 28. Today, we will conclude our discussion on the two-channel filterbank and begin a new section on OFDM. So there were a few questions in the; after the lecture yesterday, so I thought I would just clarify. We are looking at a two-channel maximally decimated filterbank. The question the first question is why maximally decimated?

(Refer Slide Time: 00:38)



Maximally decimated basically means that whatever is your input sample rate, number of samples per second will be the net sample rate across these different subchannels. If you did not do the downsampling by a factor of 2, you can find that if you split it into 2 channels, the number of samples per second will go, so maximally decimated, one of the key reasons is because we want the sample rate to be the same.

We want the number of samples per second, samples per second to be constant or be the same right; otherwise we would end up having to store more information. So this is one of the key reasons why we have maximally decimated. The second element is what is typically the processing that will happen with these subchannels. The processing that happens with the subchannels invariably is in the form of compression.

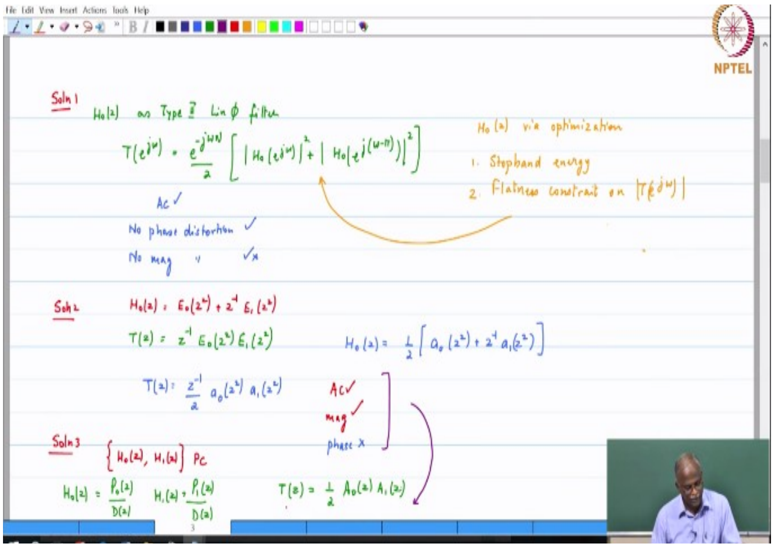
At this point, it is usually a compression, compression for storage, compression for transmission. Then, why do we need the reconstruction, at the other end you would have to recover the original signal. So typically these signals which have been compressed have to be uncompressed and then recombine to form the original signal. So how do you test whether your recombining section is working properly?

The way you do that is assume that there is no compression, can you recover the original signal exactly. If you can then your reconstruction part is working fine, you kind of have to test it under the case of no compression but if you are not going to do any compression then this really you know, why do you split it and then recombine it but whole reason is you are splitting for a very specific purpose like compression or storage.

And then at some point you will want to recover and retrieve the signals. So the input-output relationship, we will not revisit it, basically can be written in terms of the input signal and the alias signal. The constraints that we have used in the past, this is for alias cancellation. This always guarantees that aliasing is canceled. We also wanted to force a relationship between H1 and H0 and one of the ways in which we said was this is option 1.

And this brought us to a point where we could make some trade-offs between phase and magnitude errors but we could not eliminate both of those. So the QMF condition basically says that if you go with this QMF condition, let me just write it down here. With the QMF condition what you will get is T of z is=1/2 of H0 squared of z –H1 squared of z okay and we from this point on, we looked at 3 variations. Let me just quickly remind you.

(Refer Slide Time: 04:00)



This is more of an overview, so the first solution, solution 1 where we have implemented the aliasing cancellation and the QMF condition, you have got T of z of this form. The first solution was; let us design H0 of z as an FIR type 2 linear phase, type 2 linear phase filter okay. Then, what we get is the transfer function T e of j omega is=e power -j omega N/2 mod H0 e of j omega magnitude squared+magnitude H0 e of j omega –pi whole squared okay.

Now how did we design H0? H0 we said would be design H0 via optimization and it will have 2 parts that you will try to optimize. The first one is stopband attenuation because all filters if you want to have a good and typically you would have to have a passband criterion as also but in this case we say the passband criterion need not be optimized separately, we will impose a flatness constraint, flatness constraint on T e of j omega.

The flatness constraint is what is given here and once you ensure that you have good stopband, automatically the other filter will have a good passband. So again this was a case where we could cancel aliasing, no phase distortion achieved, no phase distortion we were able to get, no magnitude distortion via optimization. So this is I cannot put a tick, I cannot put an X, I kind of put a tick and an X okay.

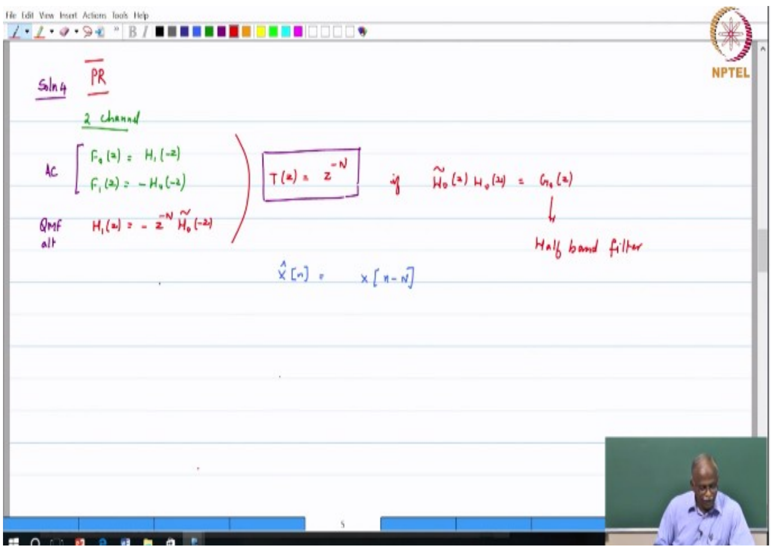
So basically yes I have done it via optimization but it is not fully eliminated okay. So I cannot say no magnitude distortion, some residual may be present. Then, comes solution 2, solution 2 we said we will split H0 of z into its polyphase components, E0 squared+z inverse E1 squared. We also showed that this if you carry it through will give you T of z to be=z inverse E0 of z squared E1 of z squared.

We said now can you find filters where you can have E0 and E1 as delays, not good filters but you can choose them to be all-pass functions. So we did manage to get a, demonstrate that you can do H0 of z to be=1/2 of A0 of z squared+z inverse A1 of z squared. Basically, the polyphase components are allpass which gives us T of z to be=z inverse/2 A0 of z squared A1 of z squared okay.

In this case, there would be aliasing cancellation, magnitude distortion, just going to write it as magnitude and the phase distortion: no, phase distortion has not been satisfied. We also saw a solution 3 which was of the same genre, where we said that we will enforce the constraint H0 of z H1 of z to be power complementary and we also said that these filters are IIR filters with certain specific properties.

P0 of z/D of z H1 of z is P1 of z/D of z, this P0 is symmetric, P1 antisymmetric. We finally showed that T of z comes out to be 1/2 of A0 of z A1 of z okay. Again, it is a same category as before. So whatever we saw here is applicable here as well. No aliasing, magnitude distortion is removed but phase distortion is still present okay. So the same criteria or conditions or observations apply here as well.

(Refer Slide Time: 09:02)



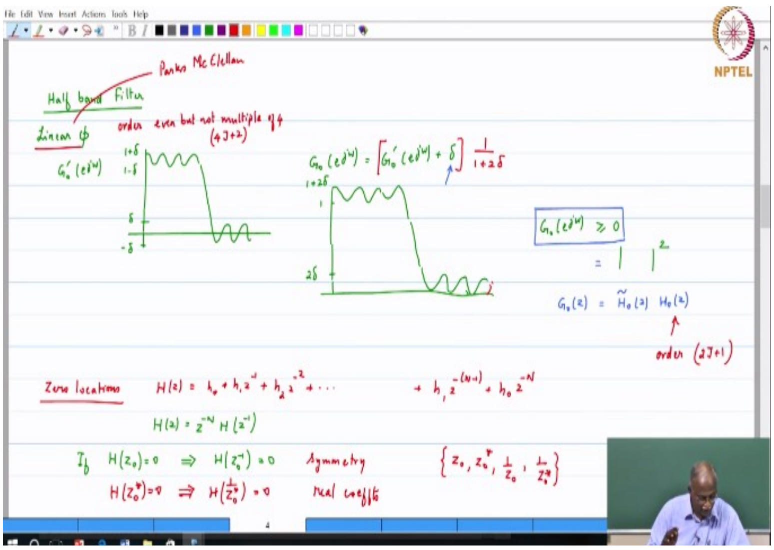
And so the final step that we took yesterday was the two-channel case where we said we will do aliasing cancellation in this manner and instead of choosing the QMF constraint where H1 of z is=H0 of –z, we said we will take its para conjugate with the delay so that it becomes causal. So this is it is alternate to the QMF right. It is a QMF alternate; it is basically saying the same thing.

The magnitude response is the same as H0 of –z but you have taken it in the form of its para of conjugate. Given this choice, go back and substitute and verify that the transfer function comes out to be T of z where T of z is z power –N. I believe yesterday when I wrote in the class I wrote it as 1/2 of z power –N, please do make a correction, it is z power -N okay. So this is the one that actually gives us the perfect reconstruction.

And therefore exactly what we are looking for okay. So this is solution 4 or more importantly the one that gives us perfect reconstruction, probably the most important of the 4 options, it just did take us a while to actually develop the perfect reconstruction solution. On the way, we had to actually bring in the para conjugation because without the notion of para conjugates this would not have been justified, just out of the blue you pull something out.

But the fact that we started to think about para conjugate in the context of all-pass functions and then eventually applied it in the context of the two-channel case, hopefully gives you a complete picture.

(Refer Slide Time: 10:53)



Now just a comment about the design and I think we will then wrap it up with that. I want to design a half band filter. Why half band filter? Because half band filter has got the property that when you shift and add, it will add up to a constant. Now if I factorize the half band filter then I will get a power complementary set. So that is where we are going towards because if you remember we are interested in power complementary property.

We also are interested in the half band property, so we initially say that I want to design a linear phase, we are going to have to specify the order, order; as even but not multiple of 4 okay. That means it would be of the form 4J+2 and the reason is one when you spectral factorize it, it will come out to be an odd order filter which is exactly what we need to get perfect reconstruction.

And anyway if you want to do a half band filter, you would have to specify it to be even but not multiple of 4 that is the only way you can satisfy that condition. So if I have designed my filter to have a ripple of delta in the passband and a ripple of delta in the stopband and I am showing it as Equiripple because these linear phase filters have got a program called the Parks-McClellan program which gives you the, Parks-McClellan, which give you Equiripple filters and Equiripple filters are optimal in terms of order.

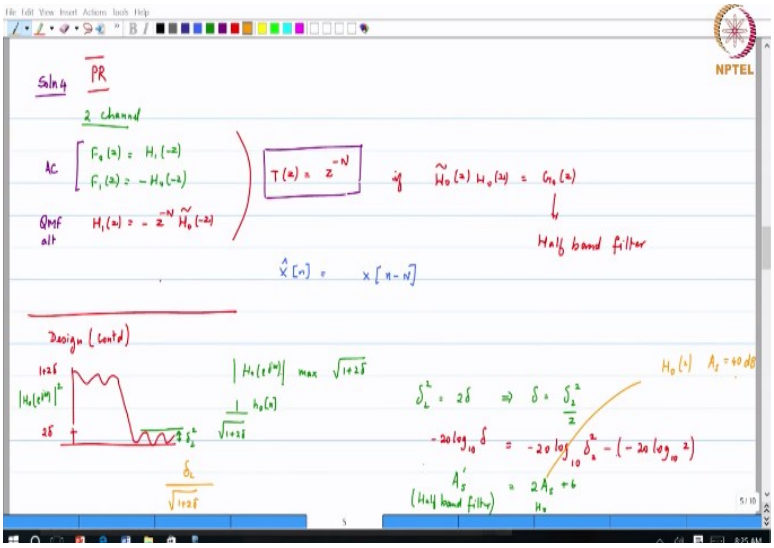
So why not treat it as Equiripple, so again half band but now what else do I have to do to the half band? In this form, it is not factorizable because notice that the amplitude response has got both positive and negative. I would have to lift the magnitude response by delta, so I take the zero phase response and then lift it by a term delta. That gives me G0 e of j omega, so basically the whole thing has moved up.

The peak limit is 1/1+2 delta, the lower limit is 0, the stopband ripple is 2 delta okay. At this point, we can make the following statement that G e of j omega is strictly>or is>or=0. So therefore I can write it as something magnitude squared and that is where we get the statement that G0 of z now can be factorized as H0 of z tilde times H0 of z. Spectral factorization requires you to have that.

Because when you look at it on the unit circle, the right hand side would be something magnitude square, so you cannot get something which has got negative valued functions or negative values on the, on the left hand side okay. Once you have done that, you can verify that you have a filter whose order is 1/2 of the original half band filter. So this filter has got order, order is 2J+1okay.

And of course when you look at the zeros of G0 of z, you can argue that basically they will occur as quads and all of that we can go through. The zeros on the unit circle will be double zeros, so therefore you can factorize one of them to H0, the other one to H0 tilde.

(Refer Slide Time: 14:43)



One last point about the design process, so this is the design continued, more of a closing comment. Now what is your upper limit? What is the peak value that you are seeing here? That is 1+2 delta okay and the stopband is also 2 delta. Those are the two values okay but now if I want to treat this as something magnitude squared, so if I want to visualize this as H0 e of j omega magnitude squared okay, then the first thing that I want to do I want to look at is the maximum values of H0 that I will get for H0 e of j omega, magnitude.

The maximum value will be square root of 1+2 delta okay. So if I want H0 to have a maximum magnitude of 1, then what I would do is take h0 of n and then scale it by 1/1+2 delta, straightforward, to get the scaling and therefore so this would be a normalized value of the filter okay. Now I just want to look at what is the stopband attenuation that we will achieve for the H0 okay.

So if you think of this as the stopband of H0 magnitude squared, then I would have to call this as delta 2 squared okay. So delta 2 squared is=2 times delta right. So 2 delta is delta 2 squared. Now if I want to write it in terms of the, did I get that correct, yes, so if I want to, or this is in other words, delta is equal to where delta is ripple of the half band filter, so this would be delta 2 squared/2.

Now if I wanted to write this in dB just so that we complete the discussion, conversion to dB or attenuation will be -20 log base 10 of the stopband ripple, that will be the attenuation for the half band filter, that is related to the stopband ripple of the filter H0 in the following relationship; -20 logarithm base 2 of delta, sorry logarithm base 10, log base 10 of delta squared - -20 log base 10 of 2 okay, I am sorry delta 2 squared correct okay.

So if I were to write it in terms of the attenuations, this would be As prime that is the half band filter is=2 times the attenuation of the filter H0 because it is delta 2 squared. What I should have taken is delta 2 but when I take it, it will be twice that+6 okay. Now why is this even important for us?

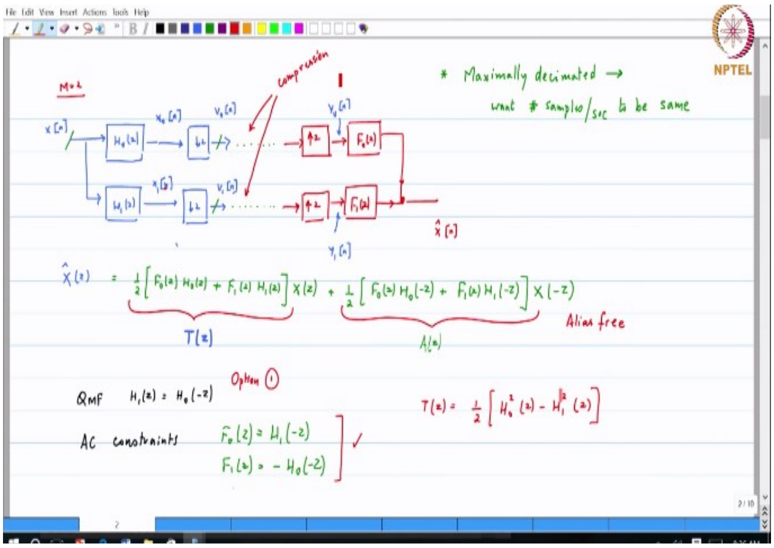
Because if you want to specify a certain attenuation for your H0 right, if you want to specify a certain H0 of z must have attenuation of let us say you want to have attenuation of 40 dB stopband attenuation of 40 dB. How will you achieve it? You have to go back and tell the specification for the half band filter. What will you specify the attenuation of the half band filter as? 86 dB.

Only then after you go through the lifting and the factorization, you will get the correct value. Now if you are going to normalize by 1/square root of 2 delta to get the maximum magnitude to be=1. Then, what will be the magnitude of the stopband ripple? Instead of delta 2, delta 2 will be if you did not normalize. If you normalize, it will become delta 2/1+2 delta okay. So you may want to assume that you are normalizing everything and then rewrite the equation for the attenuation.

But keep in mind that we now have to design a half band filter which is much more stringent than the actual filter we are trying to design. That is always the case when we have to do spectral factorization because the parent filter must have better attenuation, when you then factorize it into two parts, which are para conjugates of each other, they each of them will have poorer performance than the parent filter.

Because it comes with a magnitude squared relationship okay. That completes the discussion on two-channel, so basically to go back and say that we now have a way of designing the analysis filters and the synthesis filters from one single filter.

(Refer Slide Time: 20:44)



Which one is it? It will be a half band filter, which you will design with the appropriate attenuation and scaling. Then, take its spectral factorization that will become H0 and then the mirror image filter, not exactly the QMF version, but you will want to make it the para conjugate becomes H1. Then, correspondingly F0 and F1 are chosen to cancel aliasing. Then, you are guaranteed that the input-output is in the form of a delay LTI system which gives you perfect reconstruction.