Multirate Digital Signal Processing

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Lecture - 27
Study of Two-Channel Filter Bank with Perfect Reconstruction

Good morning. Let us begin today lecture 27. We will do a run-through of lecture 25 and 26 because those were both covered yesterday and we had a number of new concepts.

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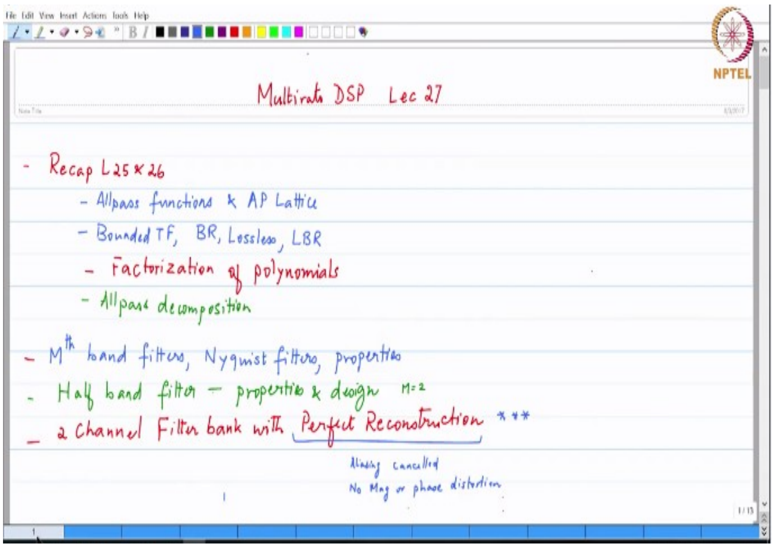

Again, the idea is to put all these pieces together like the jigsaw puzzle and then see how they fit together. So in the last lecture, if you had to sort of quickly run-through the concepts, we talked about all-pass functions, properties, all-pass lattice, talked about certain qualities or characteristics of functions called boundedness, losslessness and the combination. We talked about factorization of polynomials.

When you want to divide the zeroes of a polynomial into two sets maybe in those zeroes inside the unit circle, those outside the unit circle, that is one type of factorization that we would like to do. Then, also the allpass decomposition theorem and somewhere along the line we talked about M-band filters, Mth band filters, what are their properties. We also talked about half-band filter.

And then we said that these pieces together will help us reach perfect reconstruction which we did not reach yesterday but we will reach today okay. So by way of recap, here are some key points may be a good idea if you want to note them down, that way it will be revision and also a reinforcement of what we have discussed in the last lectures.

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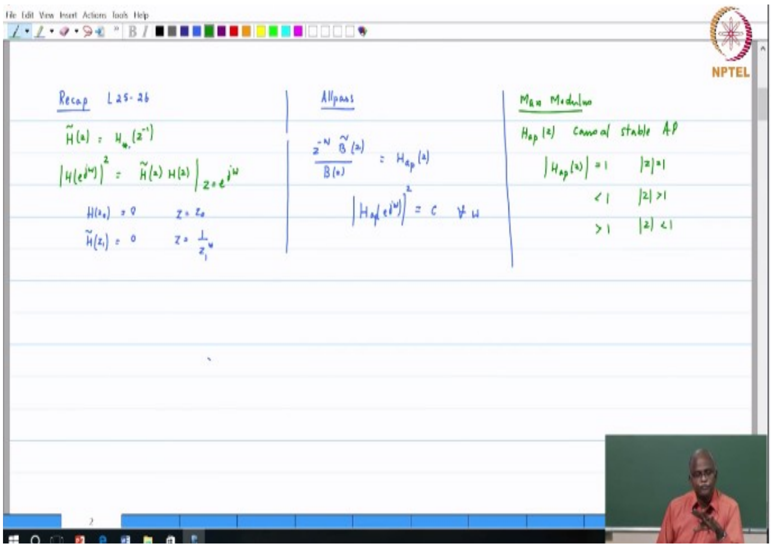

So these are concepts covered in lectures 25, 26 not necessarily in the same order but all the points are mentioned. One of the key elements or key tools that we are going to be using is the para conjugation, where you conjugate the coefficients and replace z with z inverse and this is a very good definition that we want to keep for us. Now the reason para conjugation is very beneficial is because that is the analytic extension of conjugation on the unit circle.

So this is something we will use quite extensively especially in today's lecture. This can be viewed as H tilde of z, H of z on the unit circle okay. That is an important element. Now moving from this, this also tells us that the zeros are at reciprocal conjugate locations. So if H of z0=0, H tilde this means that there is a z is=z0, H tilde of z0=0 means that you have a 0 at 1 over z conjugate okay.

So basically the reciprocal conjugate location is where the 0 will lie, so that is where the 0 will lie. So given this, we also said that the structure of an all-pass function can be written in the form B of z/z power-N B tilde of z. The z power –N is only to give you a causal transfer function. This basically means that you will have an allpass because you are guaranteed that H ap e of j omega magnitude squared is=a constant for all values of omega because the numerator is a conjugate of the denominator.

We also have the maximum modulus theorem, so this is the structure of all-pass functions and the maximum modulus theorem also plays a role whenever we work with, so basically if H ap of z is a causal, stable allpass, is a causal, stable all-pass function then we can say the following : magnitude H ap of z=1 on the unit circle, it is<1 for points outside the unit circle, >1 for points inside the unit circle.

Again, this we saw it actually helped play a role. So these are some concepts that we have worked with.

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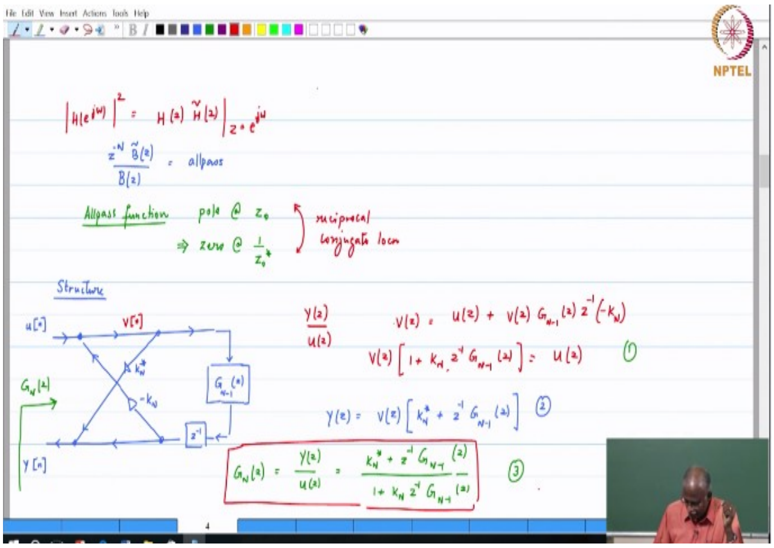

Combine it with the all-pass lattice, where we have the input-output relationship given by equation 3, that is an important relationship and we showed several interesting and important results from this. One of them was that if you choose KN appropriately then you get G N-1 as a lower order all-pass function, if GN of z is a causal stable allpass and G N-1 is a lower order also causal and stable, so those are that is a useful result.

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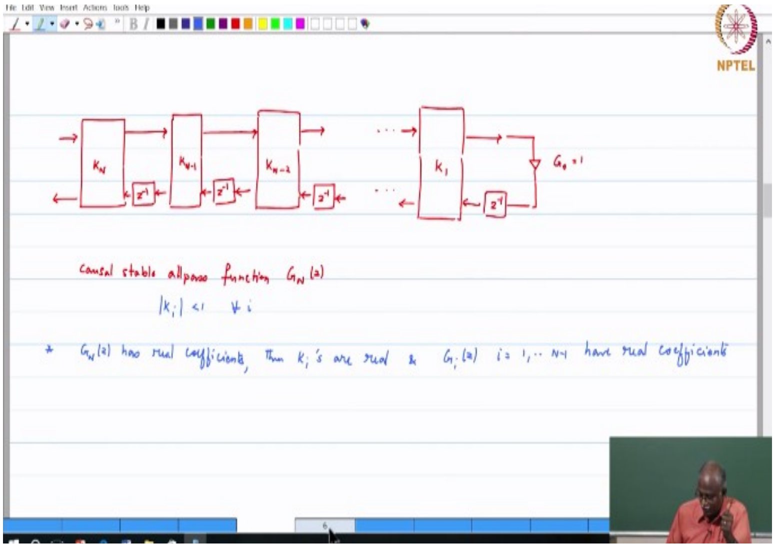

The other one is that you can actually carry this forward for the entire length of the, basically for the based on the order of the filter you will get that many stages and you will basically get these coefficients KN's, you can call them as coefficients of the all-pass lattice, sometimes they refer to as reflection coefficients, again depends on which background you are coming from, Ki's, the important thing to note is that if all the Ki's are upper bounded by 1.

That means they are strictly <1, then you are guaranteed that the transfer function that you have synthesized is a causal stable all-pass function okay and again this is a useful result in the context of allpass, this also tells us that what are the properties in all-pass function should satisfy okay. Before we get to the allpass decomposition, let me just review a couple of other points.

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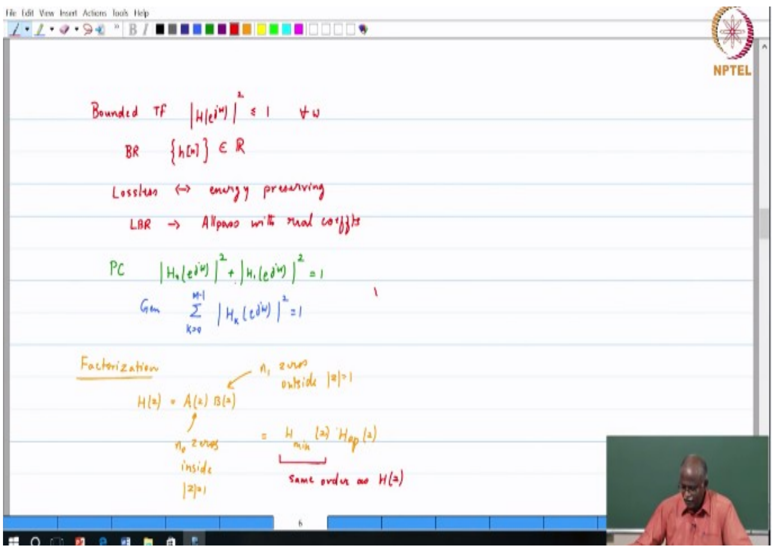

The notion of a bounded transfer function, we are going to be talking about boundedness, is a very important concept. Basically, that means that you do not have the frequency response suddenly shooting to some large value, it is bounded and we also look at transfer functions where the bound is 1, so which means that you know it is like a filter response which maximum response is upper bounded by 1.

So a bounded transfer function : H e of j omega magnitude squared is<or=1 for all values of omega, that would be a good way to define the bound for a filter. Now it becomes bounded real if all these coefficients which are the impulse response, they are elements of the real valued set. Basically, if you have those as not as complex numbers but as real valued, and of course you can have the lossless property which is the same as energy preserving between the input and the output and that allpass is one of those class of functions.

And therefore if you have LBR that basically points to an allpass with real coefficients okay. That is an important observation. We also have another class of functions called power complementary transfer functions. This says that if I take the magnitude squares of these functions or filters, then they add up to 1 okay. This is a very useful class as you will see in today's lecture because this is what actually solves the final problem of perfect reconstruction.

Now it is not necessarily limited to the two-channel case. The general case says ; summation K=0 to M-1 mod HK e of j omega magnitude squared=1, you have a set of transfer functions which and we made the following observation that if you have, okay when we come to that we will discuss. Basically, Mth band filters give us a way to satisfy these types of conditions but we will come to that in a minute.

The other element that is very useful for us in today's discussion is the factorization. So the factorization, how do you visualize the zeroes of a polynomial and how do you separate them into two sets? So basically, yesterday we showed that if you had a transfer function H of z which can be written as A of z times B of z, this has n0 zeroes inside the unit circle, inside mod z=1. We are taking the case where there are no zeros on the unit circle.

This has n1 zeros outside, outside mod z=1 and we have said that this can be written as H allpass, sorry H min of z times H allpass of z okay, minimum is minimum phase, allpass is a causal stable allpass. This one has got the same order as the original transfer function okay. This one has got same order, or in other words same number of zeros, same order as H of z. This is a very convenient factorization.

But more importantly just for you to think about, that if I have zeros and I want to separate them out, the process of separating zeros is the process of segregating factors okay. Factorization basically says find the zeros but once you have found the zeros how do you separate them; aggregate them so that that is a useful process okay.

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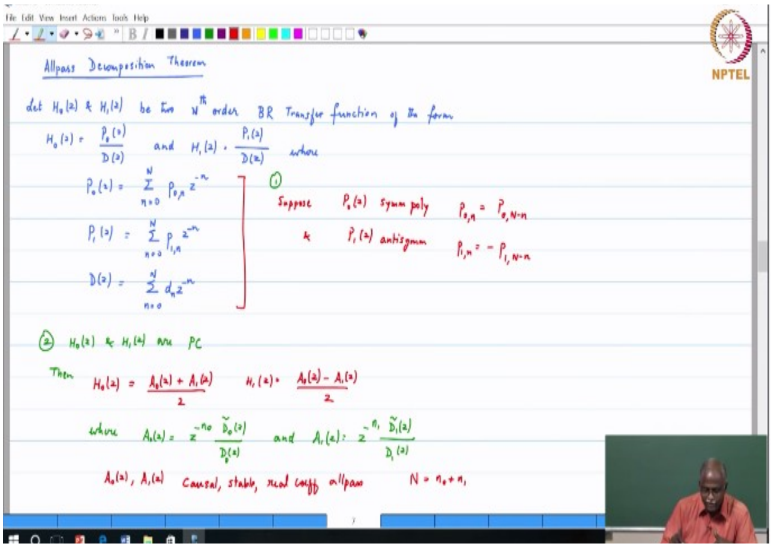

Now allpass decomposition theorem, again want to go through the details. Two Nth order bounded real transfer functions, Nth order bounded real transfer functions of the form P0 z/D of z, P1 of z/D of z. P0 has got even symmetry, the coefficients have got even symmetry. P1 of z have got antisymmetry. So therefore you have this relationship. We showed that if H0 and H1 are power complementary, then we can factorize them into this form okay.

So I thought it would be helpful for us to go back and apply this to the two-channel case. So if you remember the two-channel case is this structure, please apply the allpass decomposition into that framework. So let us just spend a minute to do that.

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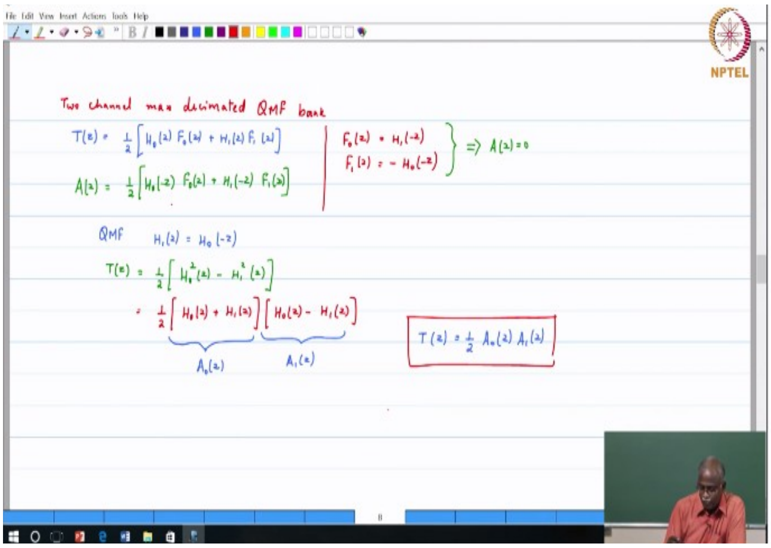

So the two-channel maximally decimated filter bank, maximally decimated QMF bank okay, so keep that structure in mind, so what we have is that we have T of z which is given by 1/2 of H0 of z F0 of z+H1 of z F1 of z and we have the A of z which is the aliasing term, transfer function of the aliasing term. It is 1/2 of H0 of –z F0 of z+H1 of –z F1 of z okay. Again, this is nothing new basically from the earlier result.

We then if we impose the aliasing cancellation constraint F0 of z is=H1 of -z F1 of z is=-H0 of –z okay. This is enough for us to guarantee that A of z=0, so without loss of generality we can get rid of aliasing in a two-channel case. Now the other condition that we imposed was the QMF condition, QMF condition if you recall was that the two filters are not independent but actually shifted versions of each other, H0 of –z.

And if we substitute this into the transfer function along with the aliasing cancellation constraint, then we get T of z=1/2 H0 squared of z-H1 squared of z right. If you go through the substitutions, basically substitute the aliasing cancellation constraint and the QMF constraint this is what we get and this, we can write as 1/2 H0 of z+H1 of z*H0 of z-H1 of z. If you go back and look at your allpass decomposition add, H0 and H1, you will get A0 of z.

So this comes out to be A0 of z, this one comes out to be A1 of z, so the overall transfer function T of z is=1/2 to A0 of z A1 of z and this is a case where we have eliminated the aliasing, eliminated the magnitude distortion, because the transfer function but there is phase distortion which we have not done anything about okay. Is that ok? So this is different from when you did the polyphase decomposition and force the polyphase components to be all-pass function.

This is a different case; this is a case where we did allpass decomposition of the filters themselves. Basically, you had a pair of power complementary filters. In the earlier decomposition, there was no power complementary part. You took the filter, polyphase components and we tried to force them to be allpass components. Here is a case where you did the allpass decomposition.

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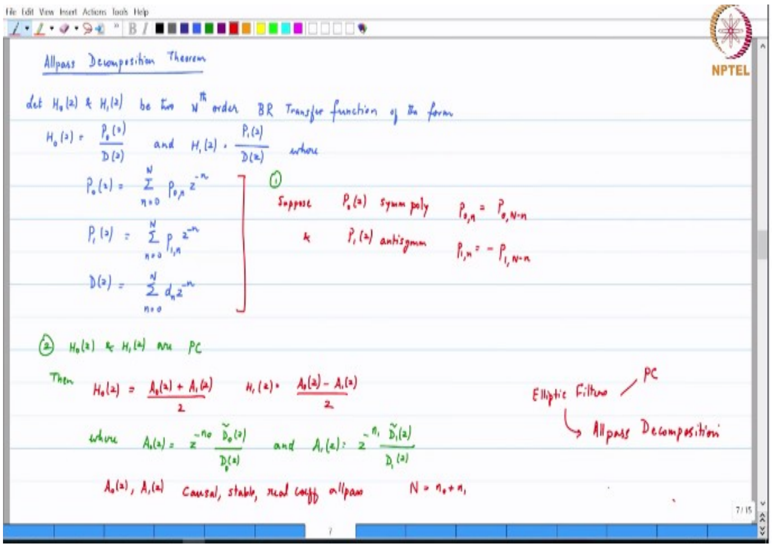

And as we mentioned, there is a class of filters that can be designed primarily in the context of elliptic filters, where you can enforce the power complementary property and because you can enforce the order and the power complementary property, you can also get the allpass decomposition. So this is not just a theoretical exercise, this is quite a practical system that can be designed and implemented without too much of difficulty okay.

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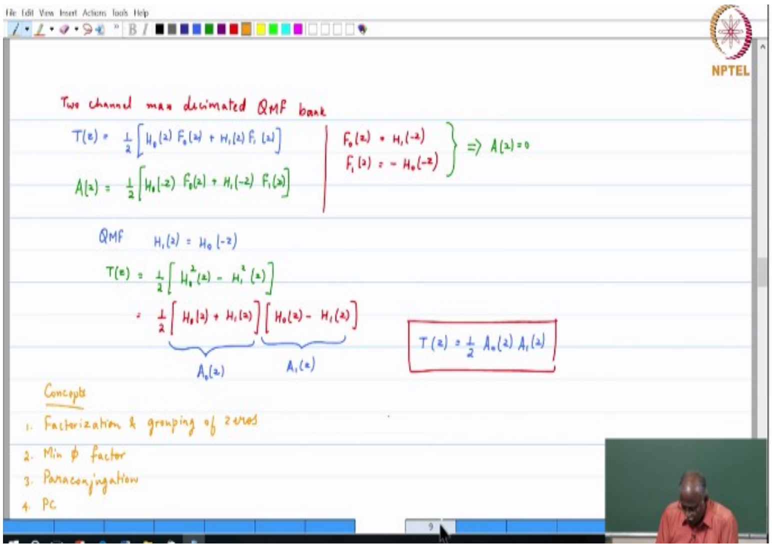

But here is the task I want you to you to tell me now. So, so far what are some of the concepts that we have seen in lectures 25 and 26? So if I were to write them down, so the concepts, again keep it like a jigsaw puzzle, the different pieces that, the first one that we have seen is factorization and grouping of zeroes.

We saw that in the design of the allpass decomposition part, so basically you took the zeroes of P0+P1, the zeroes inside the unit circle, outside the unit circle and then factored it and said this one goes into D of z, the other one goes into D tilde of z. So factorization and grouping of zeroes, that is an important tool that we have leveraged in this section. Second, we have leveraged the notion of a minimum phase factor.

That is one grouping where all the zeros are inside the unit circle. We have also leveraged para conjugation. Para conjugation says a zero inside the unit circle and its reciprocal conjugate; both will produce the same magnitude response. Para conjugation property also a useful part of this and fourth concept that played a role in the allpass decomposition was the power complementary property okay.

So these are the points to keep in mind as we move forward. Now comes the next element where we talked about the Mth band filter or a Nyquist filter okay.

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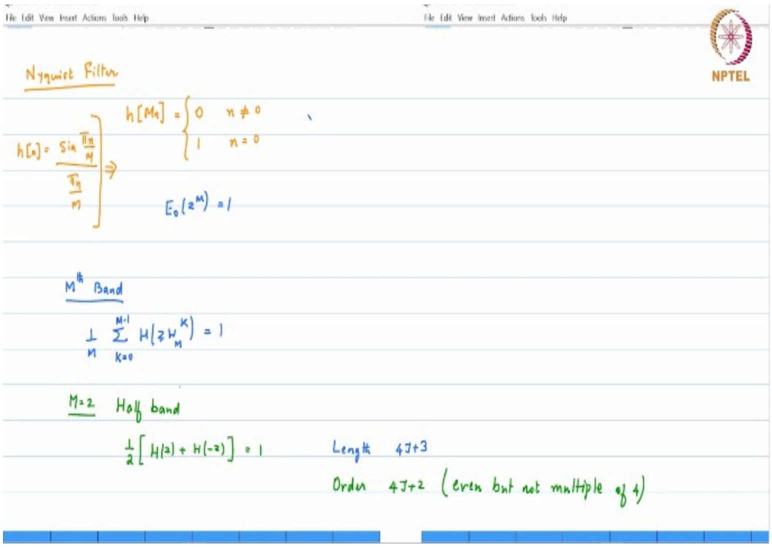

So Nyquist filter again we will not repeat the earlier discussion but basically what we have said was if I look at the interpolation filter for a, interpolation after upsampling by a factor of M then the impulse response h of n comes out to be sin pi n/M/pi n/M and using this we get the following result that h of Mn=0 if n != 0, = 1 if n=0 okay. So basically from the expression of the sinc function.

So what we found was that only one nonzero sample was there for one of the polyphase components. So this is something that we leveraged, so this basically said that if I took the polyphase component E0 z power Mm that as 1, then we have the following property of an Mth band filter. An Mth band filter by definition must satisfy the following condition 1/M summation K=0 to M-1 H of z WM raised to the power K must be=1.

And it is we showed that a Nyquist filter will satisfy this property okay. So but Mth band is more general, it takes into account these types of filters that have this property and we also mentioned that a special case is when M=2, we call it a half-band filter, half band, this is going to play a very important role today. This is 1/2 H of z+H of –z=a constant=1. We looked at a half-band filter.

Basically, took the Nyquist filter with M=2 and we came up with the interesting observation. The observation was that the length of the half-band filter, because of the symmetry and the and every alternate sample being 0, was 4J+3 which means that if I shifted it to the right, made it causal then the filter order will be 4J+2 which is even but not multiple of 4. Now you ask that significant actually turns out to be significant, let us just look at it even but not multiple of 4 okay.

So that is again these are all pieces of the puzzle, then the last part where we stopped was this DFT filter bank.

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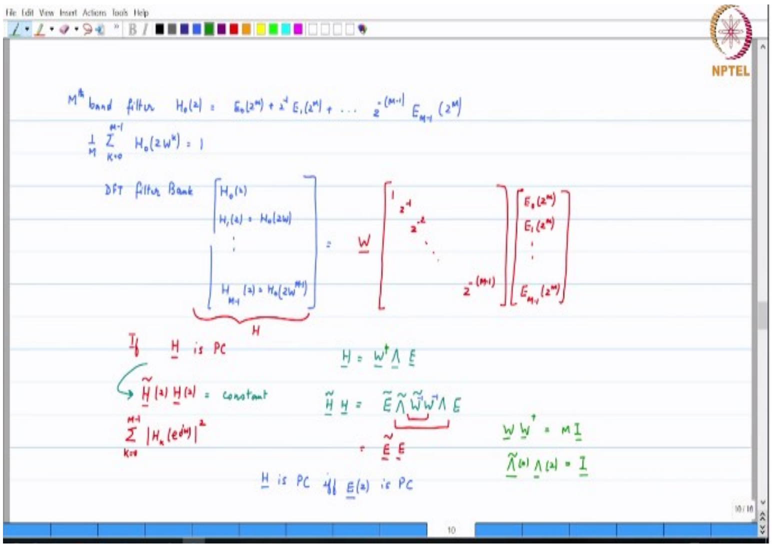

So basically Mth band filter we said satisfied this property, if you had these shifted versions of the basic filter H0 of z and it shifted in this form, we can write this basically this should be W dagger, I think yesterday we made the correction, inverse same as W dagger with the scale factor. So basically we said that the filters, shifted filters the DFT filter bank, they form a power complementary set if the polyphase components form a power complementary set.

Now you may say well I am not even sure where this is going to play a role, but this is an observation that we can make because if you look at what is the computation that we are doing, W dagger times, WW dagger is=M times I, so the inner pair gets cancelled, basically have a diagonal matrix. Gamma tilde of z times gamma of z also is=the identity matrix. So basically that is how the proof works out okay.

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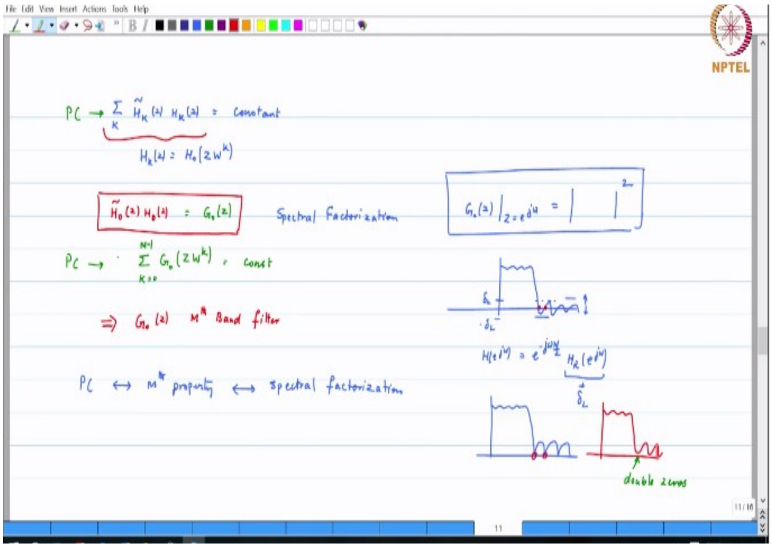

The next statement is probably the one that left people with a few questions but again I want to emphasize that. So the power complementary property says that HK of z, HK tilde HK, if I add them up they should be a constant okay. Now if I could have written G0 of z as H0 of z tilde times H0 okay. If I can write it in this form and G0 satisfied a Mth band property, Mth band property basically says, the shifted versions will add up to a constant.

If I now substitute this factorization, I get a power complementary set okay. There are two assumptions being made, one is G0 is being factorized into this form, second one G0 satisfying the Mth band property. If those two are satisfied, then I get a factorization. Now how do I get G0 to be a factor of this kind, because on the unit circle this means it is magnitude squared that is what we talked about taking the zero phase response.

And then shifting it up so that these individual zeros merge to become double zeros and therefore you have a way to work with this. Let us pick it up from this point. We are interested in a half-band filter. So let us focus in and make some statements about the half band filter.

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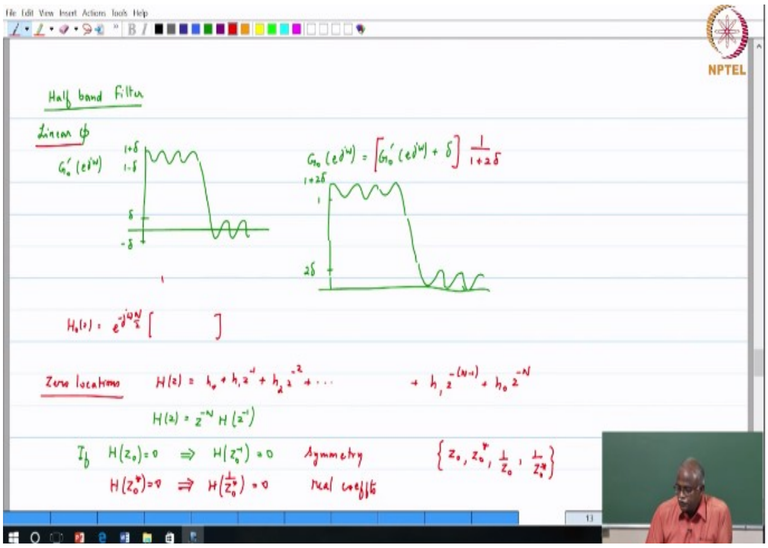

Half band filter, I want to design a linear phase half band filter okay. Now basically we want to say that G0 dash e of j omega is a half band filter, which means its shifted version will add up to and let us say that we have designed it in the following fashion ; that there is Equiripple and the passband and the same magnitude ripple in the stop band also. So the upper limit is 1+delta, 1-delta. This is +delta and –delta.

Those are the limits or where the fluctuations are observed okay. Now this is a zero phase response, I have taken out the phase so this is a real valued function but it can take positive and negative values. Now if I define G0 e of j omega to be=G0 dash e of j omega+delta okay. So then what happens, this amplitude response shifts upwards and now we have the response okay. So the upper limit is 2delta okay and the upper limit here also is 1+2delta.

Because everything got shifted and the lower variation is at 1, everything got shifted. Now in order to compensate for this, we could also scale this by 1/1+2delta. So well I will do it in a minute. So basically what will happen is the peak will become scaled to 1 and everything will fit together yeah. **“Professor - student conversation starts.”** So that is why I said I am taking a zero phase response.

So when you take linear phase functions you can write it as H0 of z as e power -j omega N/2 and then amplitude term. So I have taken off the e power -j omega N/2 and only working with the amplitude term right. So that is why easier to explain it in the context of a okay. **“Professor - student conversation ends.”** Now I want you to quickly follow along with me because we have made the assumption of linear phase, I want to make a statement or just refresh your memory about the zero locations of a linear phase filter, zero locations.

Because of the symmetry, H of z has the following form H0+H1 z inverse+H2 z-2, dot dot dot, +H1 z power –N-1+H0 z power –N. I am looking at the case where there is even symmetry okay. So you can verify that this can be written as H of z can also be written as z power -N H of z inverse. These are real coefficient, so if I have a 0, H of z0=0 if this is true, this automatically implies H of z0 inverse also has to be=0 okay.

This is because of the linear phase symmetry because of the symmetry conditions. Now from the polynomial having real coefficients, this also means that H of z0 conjugate must be=0. There should be a conjugate 0, only then you will get real coefficients. So that is comes from the real coefficients part. So this also, coupled with symmetry, will also say that z0 conjugate 1/z0 conjugate is also a 0.

So what you find is that you get a quad of zeros, you will get if z0 is a 0 and it has real coefficients and linear phase, then you should also have a 0 at z0 star, should have a 0 at 1/z0 and 1/z0 conjugate. This is something you already are familiar with. I just wanted to just refresh your memory on this particular result. So if I were to plot the zeroes of a linear phase polynomial or linear phase filter they would look something of this type okay.

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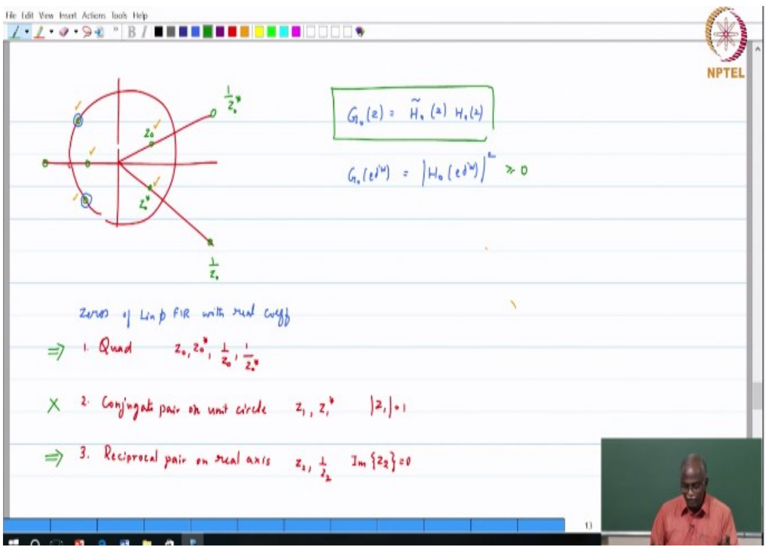

I would have zeros which occur as quads, so that means there is a zero here at the conjugate location, at the reciprocal locations also. So if this is z0, this is z0 conjugate, this is 1/z0, this is 1/z0 conjugate. So you will get quads of this type. This is because of the symmetry. Then, you may also get zeros on the unit circle because it is real coefficients its conjugate should also be present, and you can have zeros on the real axis inside or outside does not matter.

You can have zeros inside or outside but its reciprocal must also be present. So the reciprocal will also be on the real axis okay. This is what has to be the zero configuration or zero locations of a linear phase real coefficient transfer function. Of course, you could have multiple, instead of the one quad, you could have two quads, you could have many, many zeros present but basically you will have zeros of 3 types.

A quad, you will have a pair which is on the real axis and you will have a conjugate pair on the unit circle okay. Everyone is comfortable with this; you agree with that. This is how a, now when I take a linear phase of this type, that is what it will have. When I shift it up to get it to this form, two things happen, one is this G0 e of j omega becomes strictly positive, G0 e of j omega becomes>or=0 that is one.

So therefore it allows me to write it as something magnitude squared that is a step one. The second one from a zero observation, there will not be any more single zeros. They will have to double zeros, because those zeros have merged okay. Now this is very significant, very important for the following reason. Now if this is my zero configuration, what is my zero configuration?

I can have quads, I can have reciprocals on the real axis and I can have double zeros on the unit circle. Now look at this observation, if I now have something of this form G0 of z, can I factorize it of the form H0 of z times H0 tilde of z which means every zero must either go to H of z or H tilde of z and they must give you the same magnitude response. So here is a choice of zeros for spectral factorization.

I will take z0, I will take z0 conjugate, so that means the so that the filter coefficients will be real. I can take the zero inside the unit circle; I will take the blue one here and the blue one here okay. Now what is left for H tilde, everything that does not have a tick mark and you will find that you actually are able to satisfy the para conjugation property where the zeros have to be in reciprocal conjugate locations okay.

So this also affirms that you can get something of the form H0 of z, H0 tilde of z okay. So which also says that I can write G e of j omega, e of j omega as H0 e of j omega magnitude square okay. **"Professor - student conversation starts."** Sure. So the, any linear phase filter with the real coefficients will have the following. So I will write it for the I will write it in this form. The zeros of any linear phase FIR with real coefficients.

So this is how the zeros will be distributed. They can occur as a quad that means z0, z0 conjugate, 1/z0 and 1/z0 conjugate okay. They can occur as a conjugate pair on the unit circle. They can occur as a conjugate pair on unit circle okay. So that means z1, z1 conjugate with the condition that mod z1 is=1. It is on the unit circle, mod z1=1 and the third combination that you can get is a reciprocal pair, reciprocal pair on the real axis.

Because this does not need to have conjugate but the reciprocal has to be present in order for the linear phase symmetry to be satisfied. So which means that you will get z2 and 1/z2 where imaginary part of z2 is=0 okay. You will get 3 sets of zeros, now this set can be factorized into two parts, which will one go to H0 of z, one go to H0 tilde of z, the reciprocals are present. **"Professor - student conversation ends."**

This can also be factored into that form, but this cannot be factored into that form because you just have conjugate pairs, you do not have the reciprocal part present. The way you can get that to happen is if these zeros occurred as double zeros, so then what can happen, you can take one from here and one from there and then you basically you have the reciprocal also present, so that is the important part of this discussion okay.

So when does the double zeros occur, is when you actually have lifted the amplitude response, so that there is no negative values. That also says that you can write it in the form H0 e of j omega squared because now it is strictly>or=0 and therefore it also hangs together with the factorization that we are looking for okay. If this is alright, then we are ready to go back to this case, to case.

Two-channel case, please refresh yourself with the configuration. Now we are going to apply it one last time.

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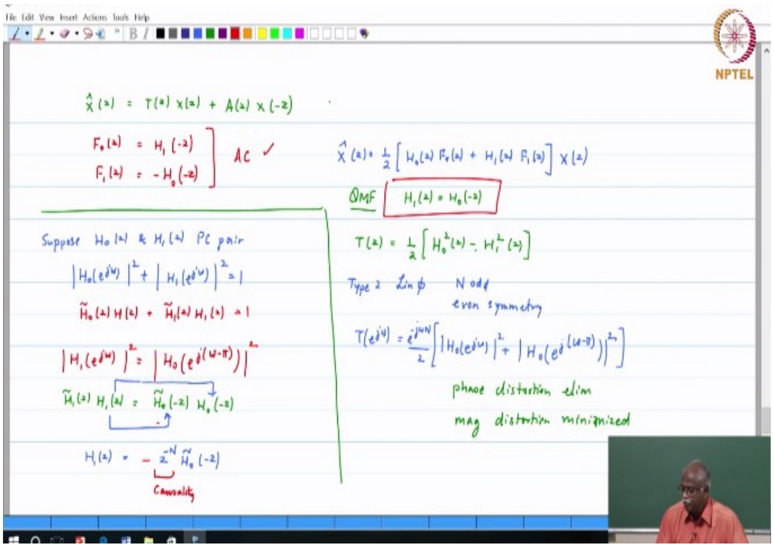

X hat of z is=T of z, X of z+A of z times X of -z, okay that is the input-output relationship. I am going to enforce the aliasing cancellation constraint. Aliasing cancellation; F0 of z=H0 of -z, F1 of z=-H1 of –z wait, this is H1 and this is H0, H1 and this is H0 okay. So this will give me aliasing cancellation. Aliasing cancellation is done. So what I am left with is the remainder of the transfer function X hat of z is 1/2 of H0 of z F0 of z+H1 of z F1 of z times X of z.

Aliasing has been removed and I now have to deal with magnitude and phase distortion. Now recall that we enforced a condition called the QMF constraint. The QMF constraint said that the two filters are related to each other and you can apply the following constraint H1 of z is=H0 of –z. So therefore we got the transfer function T of z to be=1/2 of H0 squared of z-H1 squared of z correct.

We got this expression and the time that we were working with linear phase, we said we would take a type 2 linear phase. Type 2 linear phase means odd order, even symmetry. That is the order is odd, even symmetry that is the type that we are working with and we showed that we can rewrite this of the following form, where T e of j omega comes out to be 1/2 of magnitude H0 e of j omega magnitude squared +H0 e of j omega-pi magnitude squared okay.

And if you recall we even did a design where we would enforce this as one of the design criteria, objective function and then got a pair of filters which were linear phase, so therefore phase distortion was removed and this one was almost=1, you know not exactly=1 but almost=1. I am sorry. **"Professor - student conversation starts."** e power –j omega N you are right, thank you.

So you should have, let me erase this and write it as e power -j omega N/2 correct, absolutely correct. **"Professor - student conversation ends."** So phase distortion eliminated but magnitude distortion minimized. I cannot say it is eliminated. Phase distortion eliminated, magnitude distortion is minimized okay. If you now go back and recall, how did we design these filters?

We said if H0 is a low-pass F0 must also be a low-pass. H1 as a high-pass H0 is low-pass, H1 should be a high-pass and F1 accordingly okay. So here is where a very interesting alternate approach is emerging okay. Suppose H0 and H1 were a power complementary pair that is what we want right, H0 and H1 to be a power complementary pair. Suppose they were a power complementary pair, H0 of z and H1 of z are a power complementary pair okay.

Then, what would you get? You would actually get H0 e of j omega magnitude squared+H1 e of j omega magnitude squared is=1 okay. Keep an eye on the right hand side but look on the left. This can be written using the para conjugate property in the following way, H0 tilde of z H of z+H1 tilde of z H1 of z=1, that is the power complementary property, but why did we get this? Why did we get the right-hand side?

Because we enforced a QMF constraint, QMF constraint said the frequency response of H1 must be similar to the H0, shifted okay. Can I ask if QMF will be satisfied if the following is satisfied? So instead of imposing this constraint, this is QMF constraint right, this is QMF constraint. Can I also impose it in the following way? H1 e of j omega magnitude squared must be=H0 e of j omega -pi magnitude squared.

That is also QMF? That is also what it says, right, it is the magnitude squares have to be shifted. Agree or do not agree? **"Professor - student conversation starts."** Yes, okay. We have to take care of the phase, but in terms of the filter itself because see ultimately what we are going to do is the shape and the phase all are going to get captured but at least in principle the QMF constraint says that, so the QMF constraint can also be visualized in this form. **"Professor - student conversation ends."**

The QMF, this can be written in the following way H1 tilde of z H1 of z, now the phase is going to be present, it has to be satisfied with H0 tilde of z H0 of z okay and wait, I made a mistake, minus, minus there is a minus sign, sorry. H0 of -z H0 of –z. Previously, the old QMF constraint was this option H1 of z=H0 of –z correct. That was what you, it could have very well have been this one right.

Because these two have the same magnitude response, it very well could have been this but of course you must allow for a make it causal. So let us explore this option. See the whole idea of research is if you have made an assumption and you ended up in some point and you go back and relook at it and say why not try the other option, is it a possibility? So here is an option, can I consider H1 of z to be=z power -N times H0 tilde of -z, okay.

And just take it with the minus sign because that will help simplify things but see this is for causality. This is for causality okay, the minus sign you can come back and fill later if you want but you take it with the minus sign, so without loss of generality.

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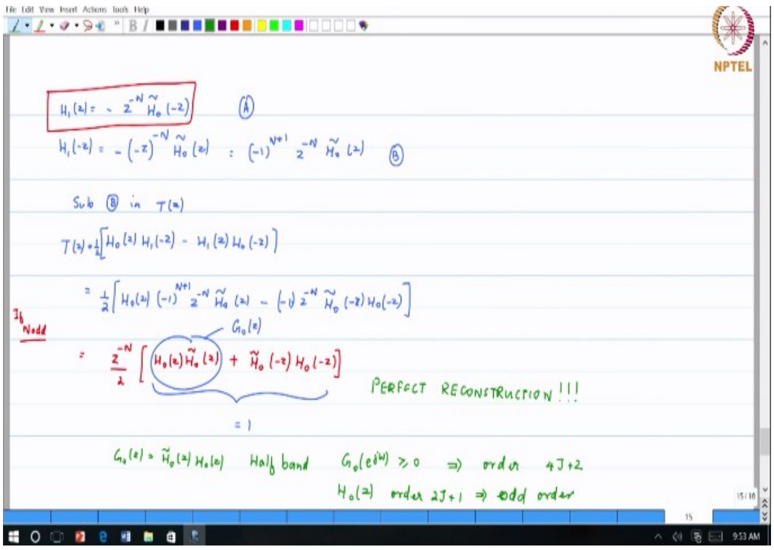

So here is what it is, so we have said that H1 of z is=-z power -N H0 tilde of -z okay. So H1 of -z, which is what we will need in the transfer function, will be – of -1 sorry let me just make sure I get this correct. H0 of -z will be the -1 followed by -z raised to the power of -N H0 tilde of z okay. So this can be written as -1 raised to the power N+1 z power-N H0 tilde of z, so this is we can consider this as equation A.

So if the choice that we have made H1 is not H0 of -z but with its para conjugate, then let me call this as B okay. Now substitute B in the expression for T of z, T of z is H0 of z times H1 of –z - H1 of z, H1 of z times H0 of -z okay. This is T of z, substitute into this expression with a 1/2, so I get 1/2 H0 of z H1 of –z comes from B, it is -1 raised to the power N+1 z power -N H0 tilde of z.

There is a minus sign; H1 of z has got a -1 leading, z power -N H0 tilde of -z and H0 of –z okay. Just regroup the terms, z power -N comes out divide by 2, there is a -1 raised to the power of N, so I am going to make the assumption that if N is odd. If N is odd, then I will get the following expression, N+1 will be even, I will get H0 of z H0 tilde of z, minus and minus becomes a plus sign, z power -N has come out.

H0 tilde of -z H0 of -z okay this is T of z. Now do you know any filter that will satisfy this to be equal to one? If this is a spectral factor of a half band filter, right. So what do I need? I need to design a half band filter, shift it up, spectral factorize it and then use that in the QMF in the two-channel filter bank. Where does that come into play? It comes into play here. I did not take the conventional QMF choice.

But I chose the para conjugate of the QMF, then actually there is no phase distortion, there is no magnitude distortion either. So this is what gives us perfect reconstruction. So this is perfect reconstruction. That is perfect reconstruction, okay but maybe there is a little nagging doubt well you assumed N is odd right, I mean that is arbitrary. What did we say? G0 has to be a half band filter.

What is the order of a half band filter? It is even but not multiple of 4. If I do spectral factorization what will it give me? It will give me an odd filter, odd order filter, so everything hangs together. There is nothing which is fishy about this thing. So the final part is, H0 tilde H0 has to be G0 of z. G0 of z is H0 tilde of z H0 of z. If this, we design it with a half band, as a half band filter with the appropriate lifting such that G0 e of j omega is>or=0.

Then, we are guaranteed that this, the length of this order of this one will be 4J+2, H0 of z then will have order 2J+1, which means that it is odd order and therefore what you will get is the correct condition required for perfect reconstruction okay. So just write it down one last time.

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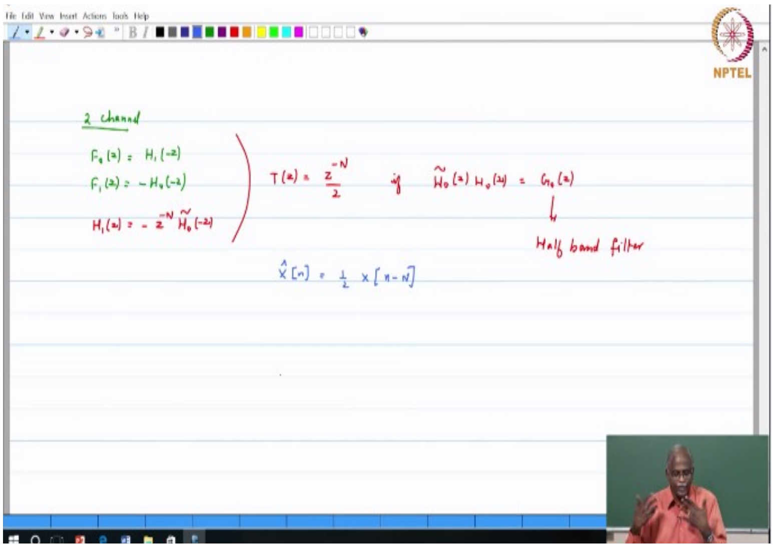

You have a two-channel case; we are going to impose aliasing cancellation just like before, F0 of z is=H1 of -z F1 of z=-H0 of –z. The only difference is that we are now going to do a new choice for the aliasing cancellation for the magnitude response. We are going to say H1 of z=-z power -N H0 of -z tilde, para conjugate. With this combination, you can guarantee that T of z is going to be z power –N/2 if H0 of z=G0 of z where G0 is a half band filter okay.

And maybe one last time we will look at the design of the half band filter and the lifting and all of those things but this is what assures you that X hat of n is=1/2 of X of n-N just a delayed version with the scale factor which we can take care of, perfect reconstruction okay. So the two-channel case is now over. Well, let us think of now extending it to 3, 4 and arbitrary, it is non-trivial.

But the two-channel case at least there is some intuition that we could follow the way through. So beyond two-channel we more or less have to rely on the mathematics. There is no intuition that can, because what cancellation happens all of that has to be based on. So we will now go back and when we look at M-channel case, we will now go back and just do the mathematical analysis of the two-channel case extended to the M-channel and say okay only under this condition we will be able to satisfy perfect reconstruction okay.

So the M-channel case comes as an extension, but not necessarily in terms of the insights that we got from the two-channel case but from the mathematical framework of the two-channel case. We will pick it up from there in the next class. I am also thinking of making a switch rather than doing the M-channel case at this point, let us leave filter banks for some time, let us to OFDM something a little different, little that gives you a different perspective.

Once we finish OFDM, we will come back and finish the M-channel case because the M-channel case is somewhat mathematical. Let us focus on OFDM which is a lot more intuitive. Thank you. We will see you tomorrow.