Multirate Digital Signal Processing

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Lecture - 25
Study of All-Pass Lattice

Good morning. Let us begin lecture 25. Today is the day we focus on allpass lattice with the hope of addressing the issue for the 2-channel maximally decimated filter bank, how do we eliminate amplitude distortion.

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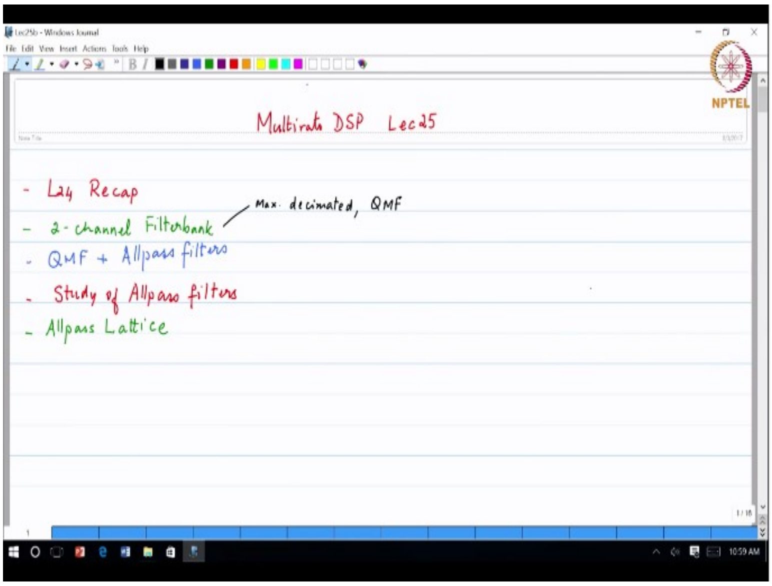

Again, as I mentioned this is not the end of the or this is not the end goal. The end goal is something beyond; we are basically building up a set of tools that will help us go beyond the magnitude distortion itself but for now the focus is on the magnitude distortion.

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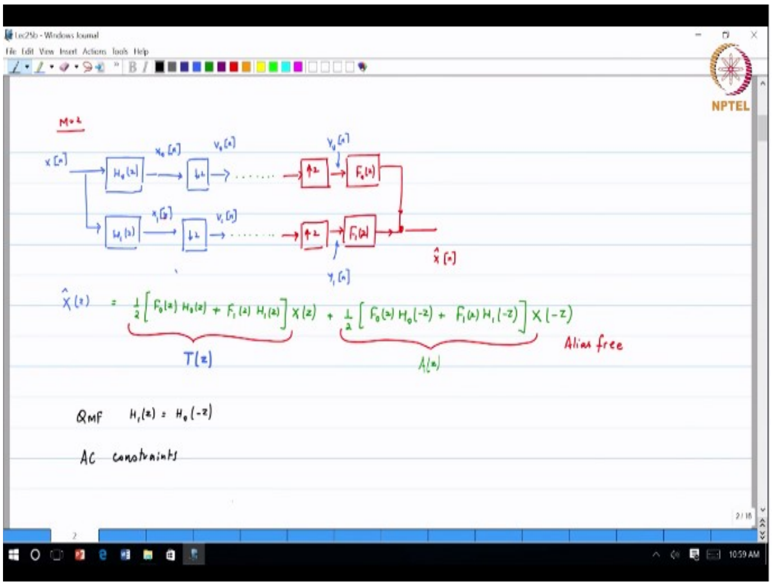

So quick run-through of the key results, we are talking about the 2-channel filter bank aliasing eliminated through the choice of the filters.

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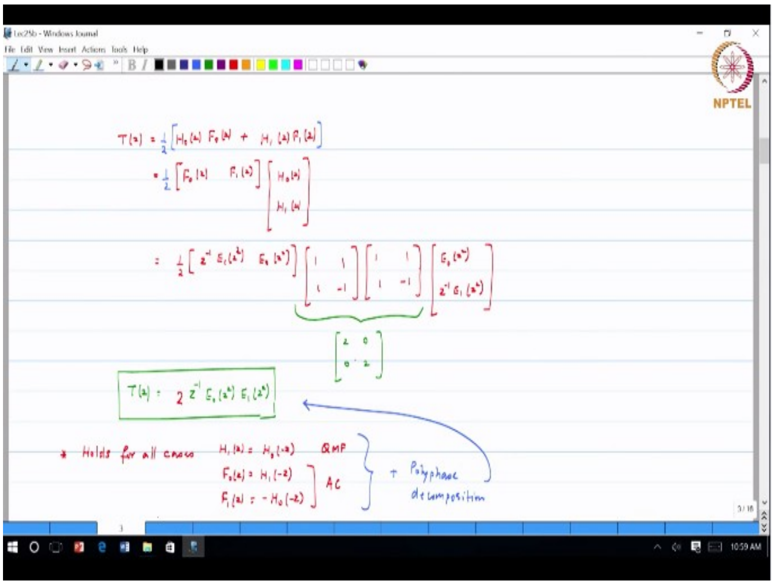

What we are left with is if you do the polyphase decomposition, the transfer function is two times z inverse E0 of z squared E1 of z squared. We are not going through the traditional approach.

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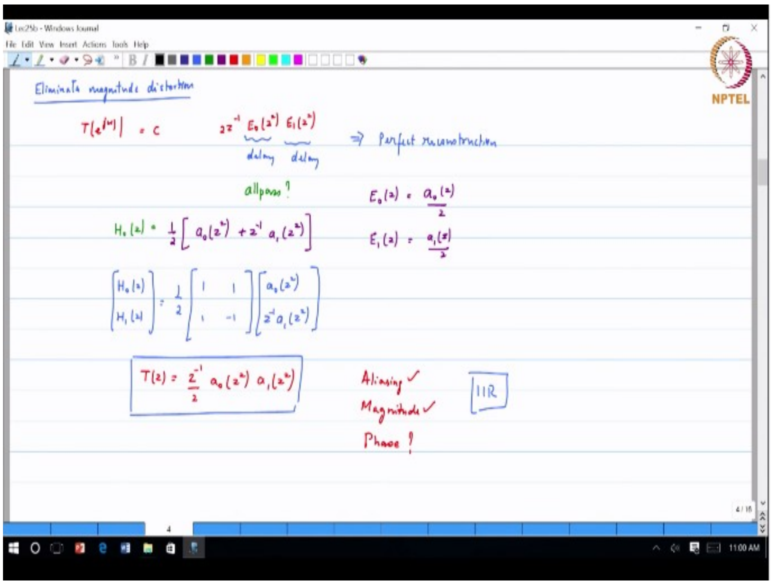

We are going to look at the possibility of representing the filters in terms of a sum and difference of all pass functions a0 and a1. In that case, the transfer function will eliminate aliasing. Now we are working backwards, if we have all-pass filters, how do we get to the good low-pass filter? So I think a kind of meeting it from both sides. So this is what we know already.

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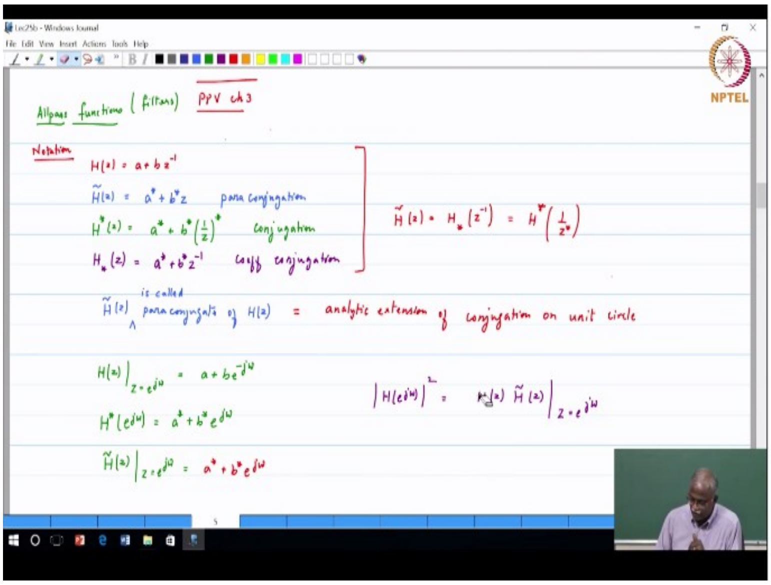

So what we are developing now is the theory of all-pass functions which then says how do I construct good low-pass and filters for the filter bank. So the study of all-pass filters, all-pass functions introduces a few notations which is very helpful for us. The first one is para conjugation and para conjugation represents conjugation of the coefficients and z replaced with z inverse.

And we said that this basically gives us 2 alternate definitions of para conjugation where you do conjugation of the coefficients and replace z with z inverse, that is a direct representation, the other one is you conjugate the entire function which means that z being complex will also get conjugated and then you would have to replace z with 1/z conjugate. So that is the other representation that we have represented.

So hopefully conjugation or the para conjugation is something that is you are comfortable with and we will use this in the development of the theory going forward. An important result is that the para conjugation is the analytic extension of conjugation on the unit circle which basically says that if I have H e of j omega magnitude squared, then this can actually be written as H of z H tilde of z, z=e of j omega that is on the unit circle because that represents conjugation.

But once you go outside of the unit circle, then it still has the same flavor of taking the magnitude squared but it is now H of z, H tilde of z where H tilde represents the para conjugation.

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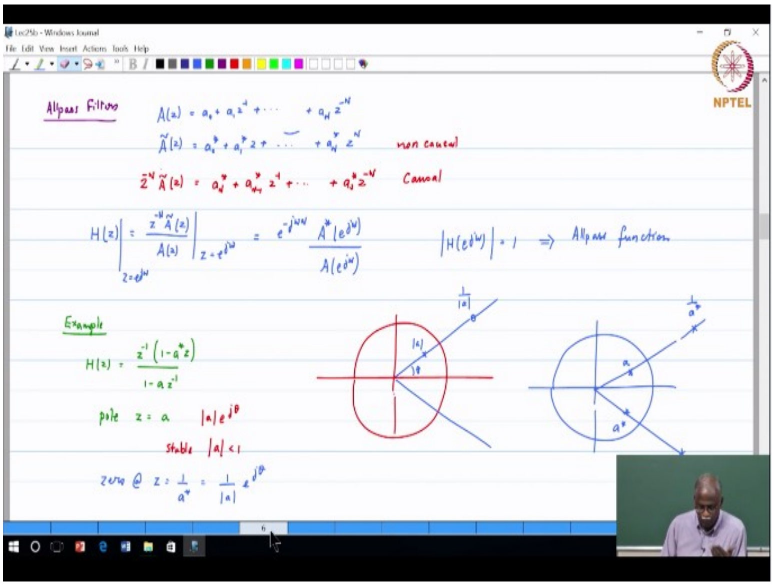

There were 3 properties of all-pass functions which we were trying to prove.

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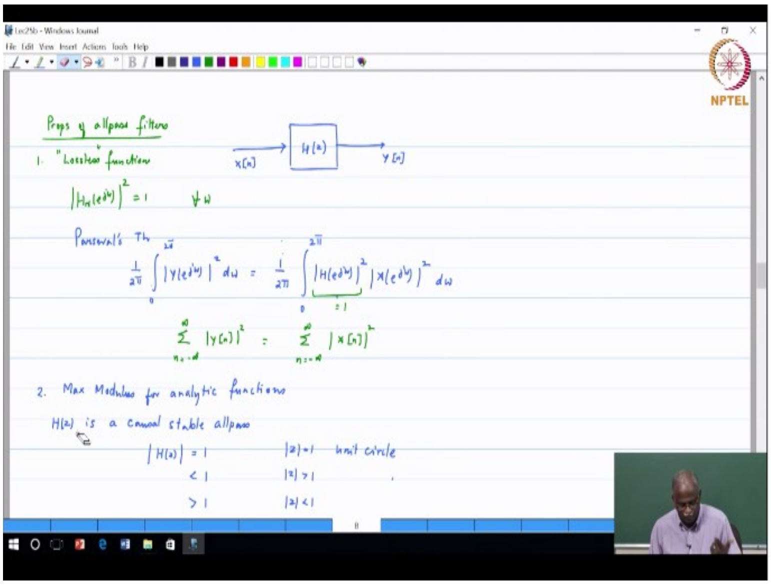

The first one is the fact that let me just jump ahead that all-pass functions are lossless in terms of the energy of the input signal being preserved at the output. Second one is the maximum modulus property where H e of j omega magnitude is=1 on the unit circle, it is<1 outside the unit circle and >1 inside the unit circle. Again, this is the property of analytic functions and the analytic extensions that we have done but please take this as a result in case you have not studied the maximum modulus property in the study of complex variables.

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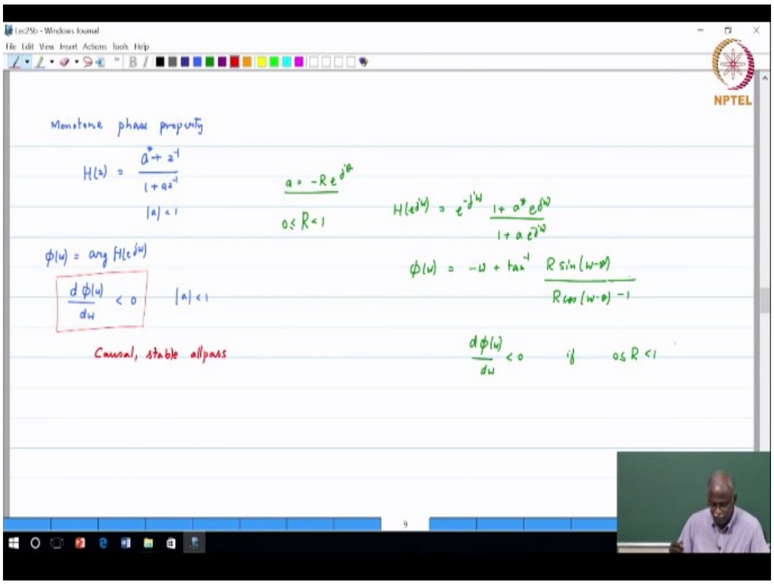

The third property of course is H e of j omega, if you look at the phase of that transfer function then we said that the derivative with respect to omega is less than 0, that means it is monotone decreasing function and this is something that is in property of causal stable all-pass functions okay. So this is true for causal stable all-pass functions okay. So I am assuming that these are properties that you can verify if needed and be able to use it causal stable allpass. So this more or less gives us a good framework for the all-pass functions.

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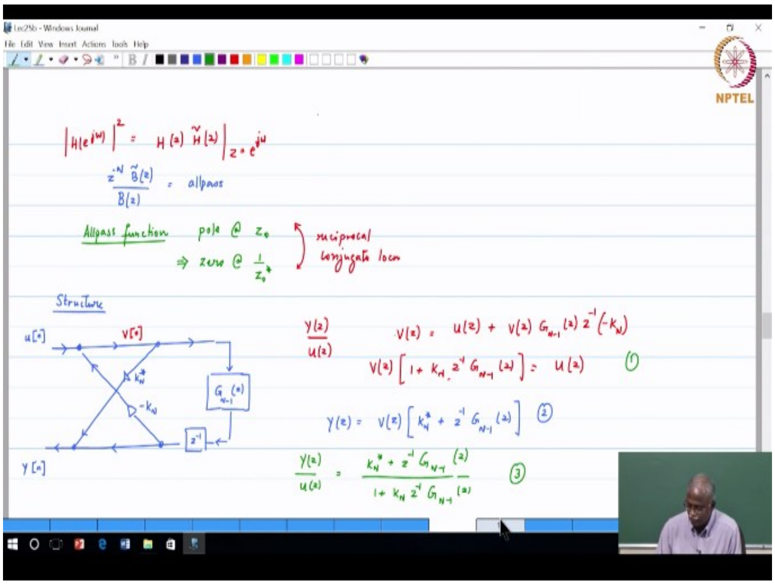

Let us now dive into what is it that we are going to need going forward from here. So we said that magnitude H e of j omega magnitude squared can be written or represented as H of z H tilde of z on the unit circle, so para conjugation on the unit circle. Now this also tells us that whenever I take some function of the form B of z divided by z power -N B tilde of z, this is going to be allpass because on the unit circle they are conjugates of each other.

And therefore the magnitude will be equal to 1, therefore this is the form of the transfer functions that we are interested in. The other element which is also sort of emerges from here that if you have all-pass functions and they have the structure shown above, this also means that if I have a pole at z0 okay pole at z0 then this necessarily requires because of the structure there will be a 0 corresponding to because they are both of the same base same polynomial.

So there has to be a 0 at 1/z0 conjugate, so there is a reciprocal conjugate relationship between the poles and zeroes. So there is so please make a note of that reciprocal conjugate locations yeah and this is universally true whether it is a stable allpass or not you must have this property in order for it to satisfy and then we also justified that how this pole and zero are on the same radial axis and therefore on the same radial line.

And therefore it enables us to work with, this is of course we have the 3 properties okay. Now we would like you to study the following structure because it is going to take us into the domain of all-pass structures and it is very insightful, it is a simple exercise but also very useful exercise. So if I have a structure which is of this form, let me represent this transfer function as G subscript n-1 of z.

Again, the subscripts will become clearer once we have gone through. There is a delay element z inverse and then there is going to be a crisscross which is a slightly different crisscross from the FFT butterfly that you are familiar with. So please draw the arrows in the right direction. So this one is a path going from the input to the output and this one is a path going from the output to the input okay.

So need a minute to get my lines straight okay, so 2 multipliers, so notice that this is a feedback type of structure. Let me draw the input line, let me call that as u of n, the output line y of n, make sure all the branches are labeled with arrows so that we have no confusion. This is -K subscript N and this is K subscript N conjugate okay. So basically my input is u of n, output is y of n.

I would be interested in computing the transfer function Y of z divided by U of z that is my ultimate goal, so let us work towards that and it is helpful for us to define an intermediate signal v of n because that helps us get the transfer functions. So let me just write down the steps I am sure analyzing a lattice structure is something that you are familiar with but just as practice let us do that together.

Please help me write the equation for V of z, so basically v of n okay. v of n is u of n+v of n passing through G n-1 of z z inverse and –KN both of those get added together. Basically, these solid dots means that they are either signals are adding or if it is a branch from which they are branching out, it is the same signal is going on both those branches. So V of z is equal to U of z+V of z itself passing through G N-1 of z then z inverse and then -K subscript N okay.

So if you were to group the terms, this would be V of z times 1+KN z inverse G N-1 of z is equal to U of z okay that is the equation 1 okay. So please do the same similar exercise and verify equation 2 which says Y of z can also be written in terms of V of z can be written as V of z * K N star+z inverse G N-1 of z okay. So that is equation 2, so between equation 1 and 2, we have the essentially the representation that we are looking for.

So Y of z by U of z can be written as KN star+z inverse G N-1 of z divided by 1+KN z inverse G N-1 of z okay. So again basically it is just an analysis of a lattice that has got some feedback paths. So therefore the transfer function will be an IIR transfer function. Basically, you will have numerator and denominator and this is what we have obtained okay. This is let us call this as equation 3.

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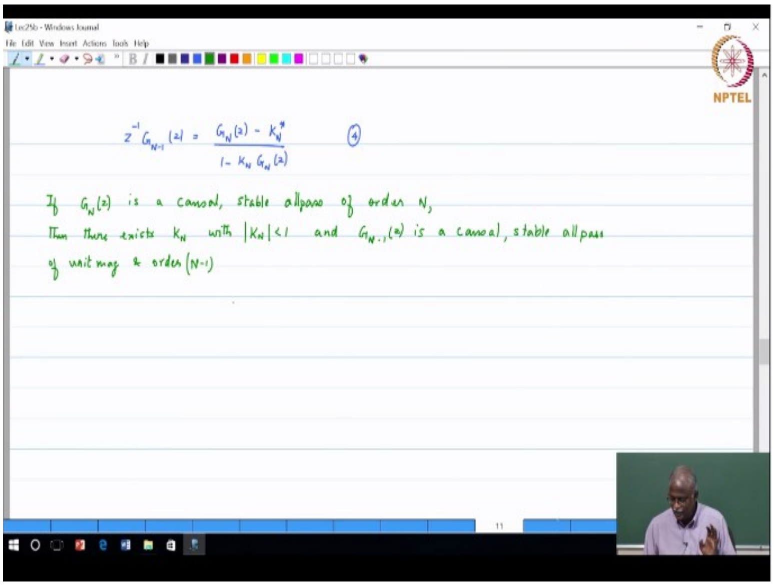

Now from equation 3, it is just a simple rewriting of the equations to verify that z inverse G N-1 of z can be written as G N of z -KN star divided by 1-KN G N of z okay. That is basically rewriting the equation. So maybe we call this as equation 4. So 1, 2, 3, 4 from the lattice structure that has been given to us okay. Now here comes the interesting part. Now supposing GN of z is a causal stable all-pass function of order N GN of z.

If GN of z is a causal stable allpass okay, causal stable allpass of order N okay, so here is the key result. Then, there exists a coefficient, there exists K subscript N a coefficient such that with mod KN<1 strictly less than 1 and so basically there exists a KN such that mod KN will be less than 1 and G N-1 of z is also a causal stable allpass of order N-1, allpass of unit magnitude that means there is no gain okay.

That maybe is not as important but more important it has got order 1 less, order N-1. Now we need to verify this okay. Is the statement clear? Lattice structure obtained the GN of Z in terms of G N-1 of z. Now what is GN of Z?

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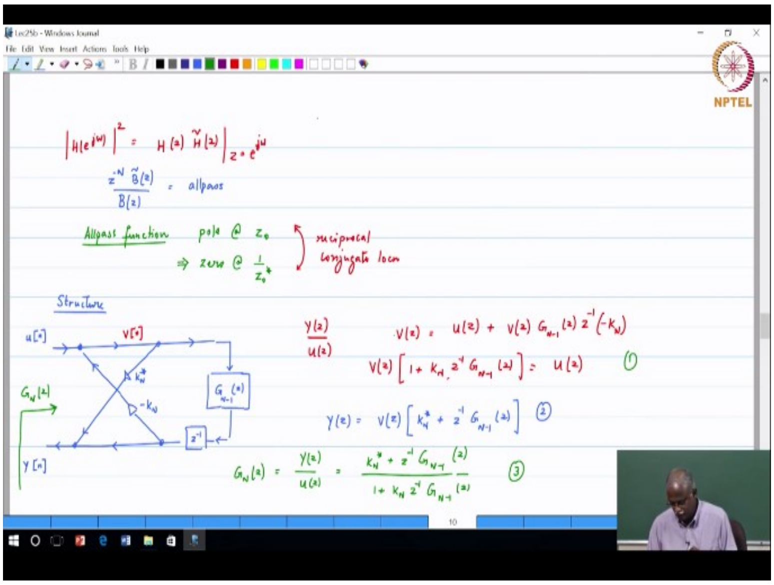

In the previous graph, let us just go back and label this, so this is what we are labeling or we are calling as GN of z. So basically the transfer function in this input-output relationship or if you are used to viewing it as this is the transfer function looking in okay, input signal going in, output signal coming out and again when the two-port networks you would have studied transfer functions of this type. This is the representation okay.

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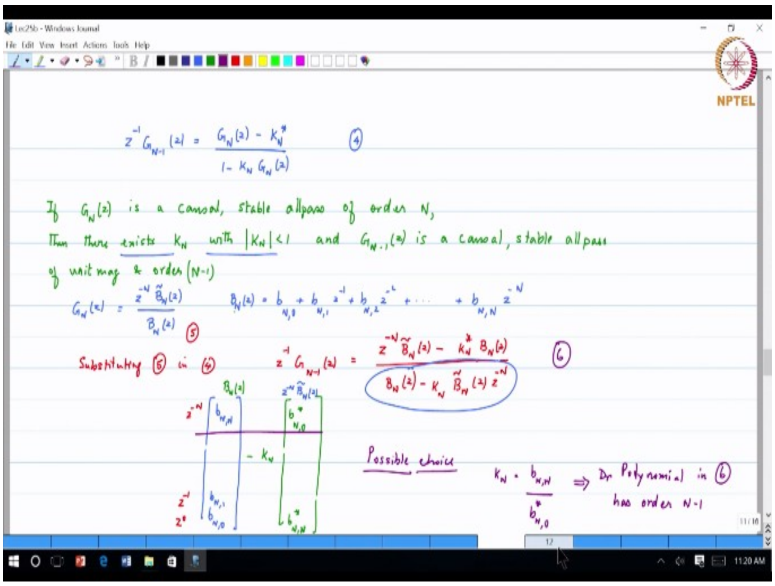

So our task now is to quickly verify this. Of course, if you have worked with lattices, this would be quite an easy task but in even if you have not worked with it, it is a very interesting exercise. So let us look through that okay. So GN of z is a causal stable all-pass function. So we would like to write down GN of z in the form of GN of z is of the form B subscript N of z just to indicate the order of the polynomial.

The numerator would be z power -N BN tilde of z, that would be the and if you wanted to write down the polynomial BN of z, we would write it in the following manner, bN, 0 that means it is the 0th coefficient of the polynomial+bN, 1 z inverse+bN, 2 z-2 dot dot dot+ bN, N the Nth coefficient of the polynomial z power -N okay. Now if I substitute this form in the expression that is given to us in 4.

Substituting 5 in 4, what we get is z inverse G N-1 of z is of the form, the denominator is BN of z, did I get that correctly 1-KN yes correct. So BN of z -KN BN tilde of z that will be the denominator, numerator will be z power -N BN tilde of z -KN star times BN of z. Basically, I have just substituted the expression for GN of z and just rewrote the equations. **"Professor - student conversation starts."** I am sorry, oh absolutely, absolutely, thank you.

Yes, okay. **"Professor - student conversation ends."** So what I would like to do is focus in on this portion of the expression okay. So I am going to write the polynomial, I am going to write it in the following way. I am going to write the coefficients as a vector. Again, you will see in a minute why that is helpful for us. So the coefficients are bN, 0; bN, 1 all the way to bN, N okay.

So in terms of the coefficients, this is the coefficient of z power 0, this is the coefficient of z-1, this is the coefficient of z -N okay. That is basically that is BN of z. So this is a representation of BN of z. Let me use a different color. This is a representation of BN of z okay. Now the second term is BN tilde, so I will do a -KN and I now have to represent BN tilde and z power-N.

So basically the coefficients will go in reverse order and they will get conjugated. So the topmost coefficient, the coefficient of z power-N now will be bN, 0 conjugate. Then, all the way down and the last coefficient will be bN, N conjugate okay. That is a representation of z power -N times BN tilde. This is a representation of z power; I am using a color that is already there.

Let me use blue, so this represents z power -N BN tilde okay. So put them both side by side, it is very easy to sort of see what the coefficients come out to be and you can do something similar for the numerator also but the important thing is the following. What did the statement say that there exists a K of N with mod KN<1 such that the G N-1 has got is first of all lower order and so here is the possible choice.

So possible choice that means you have to show the existence, possible choice for KN, so if I choose KN to be equal to bN N, bN N/bN, 0 conjugate okay. KN to be bN N - so notice that that means the first term the topmost z power -N power will get knocked off. So this more or less guarantees that the denominator polynomial in 4 or in you can also label this as 6, denominator polynomial in 6 has order N-1 okay.

This choice will ensure that the order of the polynomial gets decreased okay. Now spend a minute to tell me what the coefficient on the numerator will be? What the coefficient on numerator will be? So basically it will be z power -N BN of z that is sitting there-KN conjugate times the coefficients of BN of z okay. If you write that down and substitute what you have here, you can verify that in the numerator term.

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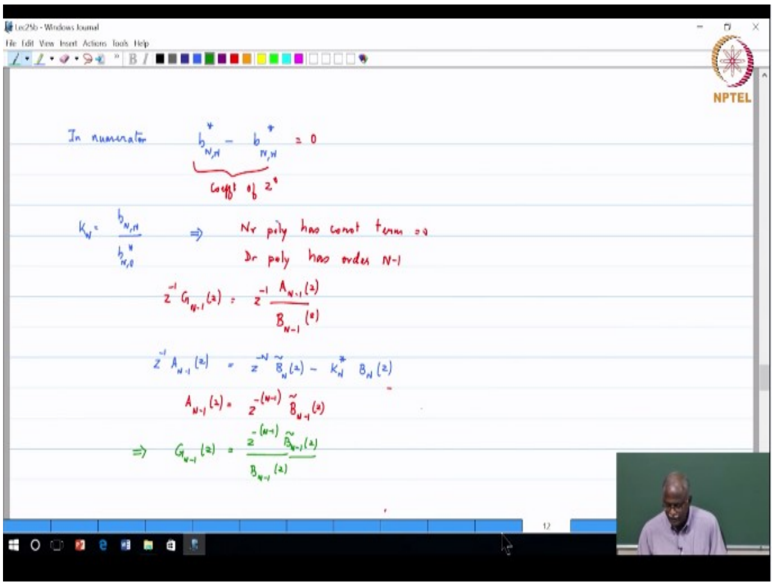

Yes, the constant term gets knocked off. In the numerator what you will get is bN, N; bN, B conjugate-bN, N conjugate, this is the coefficient of z power 0, this is the coefficient of z power 0 or the constant term and this is equal to 0 okay. So the choice that we have made choice of KM KN sorry KN=- sorry=bN, N/bN, 0 conjugate within the structure and the framework that we have been working with.

This guarantees the following, the numerator has got the constant term equal to 0, numerator polynomial has constant term equal to 0, denominator polynomial has got order N minus 1 has order N-1, so therefore we can write the transfer function, z inverse G M-1 of z, that is the transfer function that was there and the numerator has got order 1 less so therefore I can write this as some z inverse/it is not M it is N okay.

Some denominator polynomial which is B N-1 because it is a lower order polynomial and the numerator is some polynomial of order N-1 also because you have factored out z inverse right because it will have starting from z-1 all the way to z power –N if you factor out z inverse that will also become a polynomial of order N-1 okay. So now please go back and verify that z inverse A N-1 of z from the expression is actually equal to z power -N BN tilde of z-KN star BN of z.

This is the expression for the numerator of the and you can verify. Again, it is absolutely straightforward but very, very important that you verify that this gives you the following result that A N-1 z is actually equal to z power -N-1 B N-1 of z tilde okay. Very, very important that you are able to verify. It is straightforward but please do verify that this is the case. If this is the case, then we have the following result.

This basically means that G N-1 of z is of the form B N-1 of z z power -N-1 B N-1 of z tilde okay. Let me give you a minute to sort of assimilate that if you want to make any additional notes.

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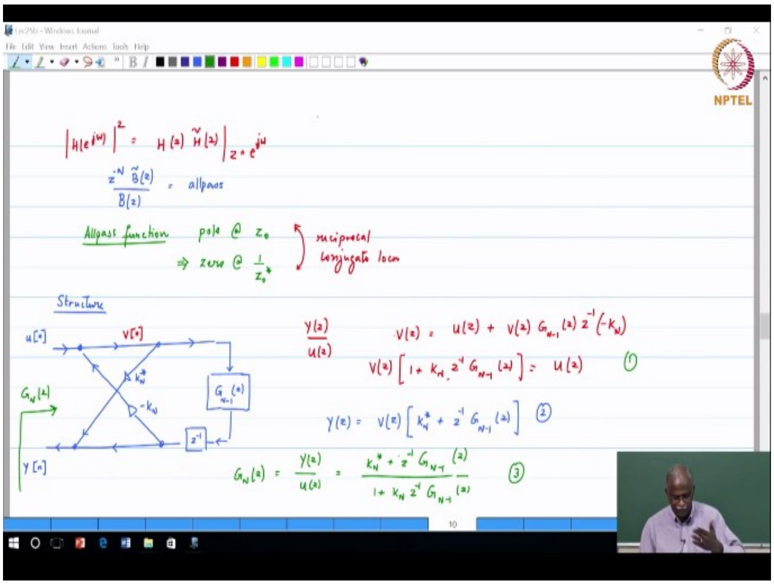

So let me go back to the previous structure. The structure is defined as follows. We have assumed that GN of z is a causal stable all-pass function, so which means you can write it in the using the para conjugation form.

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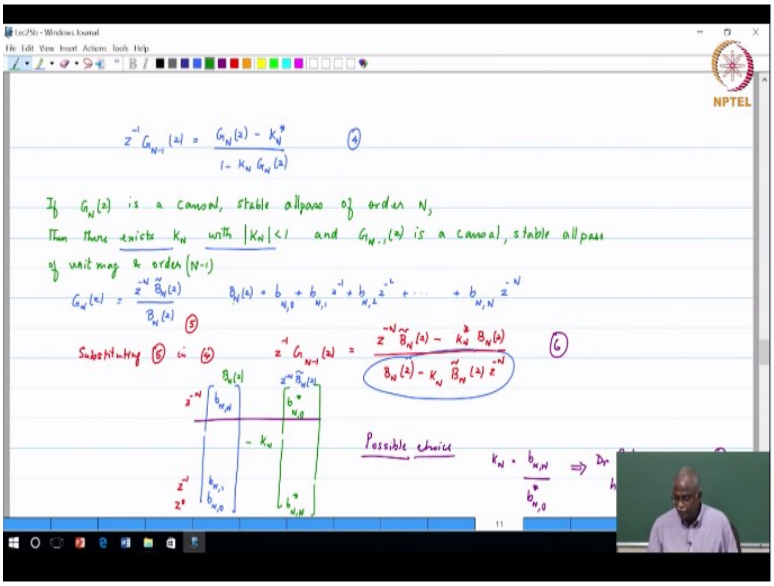

If you do that and choose your KN appropriately okay, so we have not yet shown that mod KN is<1, we have not yet shown that G N-1 is stable. So those are those are yet to be shown but at least we have come to the part of saying that okay. So now at least we have shown that G N-1 is also allpass. GN allpass, causal stable allpass, we have been able to show that G N-1 with the appropriate choice of KN will come out to be an all-pass function.

"Professor - student conversation starts." Yeah, the second-last equation, yes, is that how you are defining it or, which one? No, you actually have to verify. So basically we have shown what B N-1 of z is right. We have shown what the expression for B N-1 of Z is, using that substitution for that value similarly you can get an expression for the numerator and you can show that the numerator.

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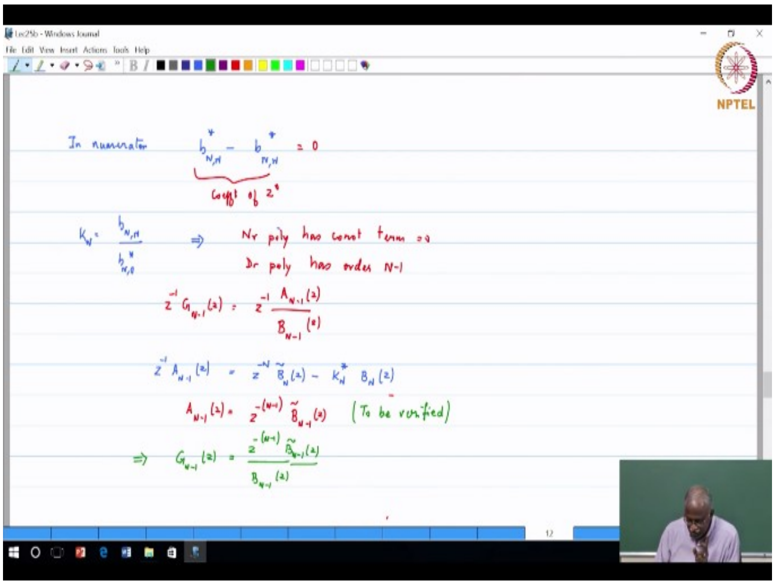
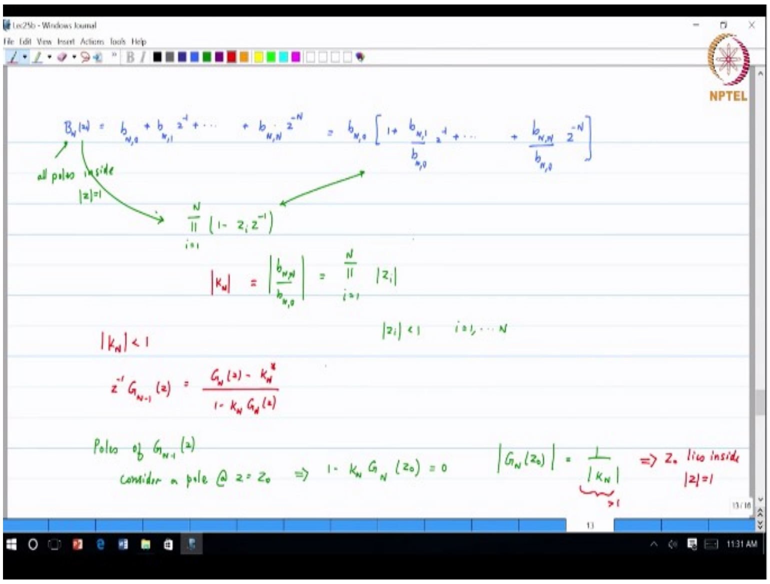


So yeah this is not an assumption, this is to be shown. This is to be verified. You have to verify that this is true. Yeah, if you assume that then of course it will become allpass so but it is actually something that it is not difficult to verify okay. **"Professor - student conversation ends."** Now a very important observation that comes now again this is where the beauty of the lattice structures lies and all of the theory that you have studied in signal processing networks and systems all of it come together. **(Refer Slide Time: 27:50)**



Let us take a closer look at the polynomial BN of z. BN of z is a denominator of a causal stable all-pass function, so it has to have all poles strictly inside the unit circle. So all poles strictly inside the unit circle, inside mod z equal to 1 okay. So now BN of z, we have written it down as bN, 0+bN, 1 times z inverse dot dot dot+bN, N z power –N. If I factor out bN, 0, again we seem like a trivial step but it is very important in the overall.

This becomes bN, 1 divided by bN, 0 z inverse all the way up to the last term is bN, N divided by bN, 0 times z power -N okay. Now you may wonder, you know just factoring one element out does not make any difference at all but if you were to factorize the polynomial in terms of its poles in terms of its zeros B of z has its zeroes then if you had to write it down in the following way, product there are N, it is an Nth order polynomial so there are N zeroes, i=1 through N 1-zi z inverse.

Basically, I am writing it in alternate form and now compare it to this expression okay. You can verify that modulus of bN, N/ divided by bN, 0 is actually equal to the product of the poles, the radius of the poles okay. So this is equal to the product of i=1 through N mod zi okay. Now we are told that the allpass is causal stable, so all mod zi <1 for all values of i, i=1 through N and this if you remember is also the definition of KN.

So this is equal to mod KN, so the second assumption we have also validated. First assumption was that the lower order allpass was obtained. So mod KN<1 is also satisfied because G N-1 is a causal stable all-pass function okay. Now just hold on for the next step because this is also very, very important. Now the expression for G N-1, so basically if you go back and look at I do not remember the equation number.

But I just request you to look at it up z inverse G N-1 of z we wrote down as GN of z-KN star divided by 1-KN GN of z okay. We wrote this down. So I want to now look at the poles of G N-1 of z, poles of G N-1. I am hoping to see if I can show that the poles of G N-1 are inside the unit circle that means stable, gave me stable of order<1 in the lattice structure. So G N-1 poles of 0 supposing is a pole at z0 then supposing consider a pole at z0, consider a pole of G N-1 at z=z0.

This would imply 1-KN GN of z0 is=0 that would have to be 0. From this, we can also verify that mod GN of z0 is=1/mod 1/KN correct. I have just rewritten that equation. Mod KN is<1, so 1/mod KN will be>1, so this will be greater than 1. Now what can you tell me from maximum modulus theorem? It has to be inside the unit circle, z0 has to be inside the unit circle okay. So this implies z0 lies inside the unit circle okay.

So G N-1 is stable okay, so this is a very useful structure. So what it says is if I have GN as a causal stable allpass, I can find a KN such that G N-1 will be one lower order, it will also be causal stable allpass okay. Now you tell me by induction what can you take it to the next level. G N-1 stable, I can find K N-1 such that it becomes K N-2 and of course you can go all the way down and the last term will be the most trivial all-pass function will be a constant. So if I were to generalize this structure I get the following.

"Professor - student conversation starts." Yes, okay. Let me just make sure okay this expression G N-1, I am sorry. Right, even you have a rational transfer function, No, the numerator is a polynomial, denominator is a polynomial. No, no when you rewrite it, you can rewrite it as polynomials. Basically, it is a polynomial on the numerator. Yeah, but you can rewrite it as a polynomial right.

Look at it because there is a relationship between the two okay. **"Professor - student conversation ends."** So here is the way we would look at it.

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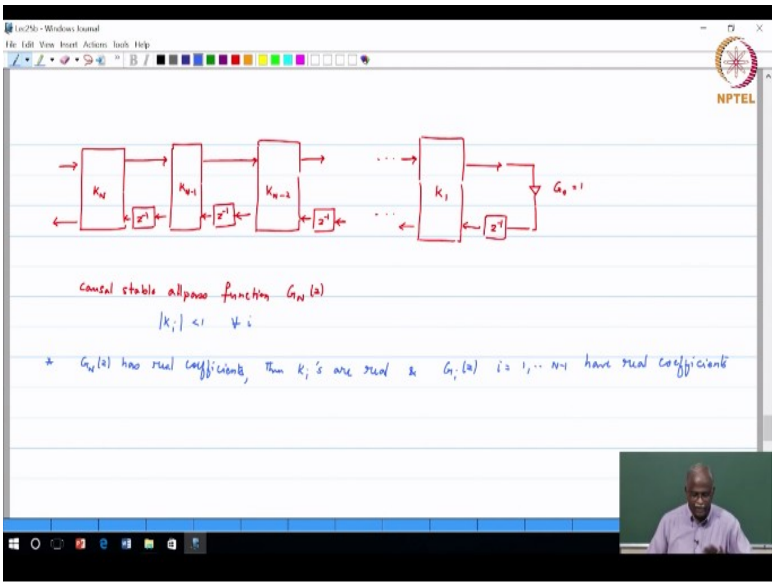

I will have an input, there is a that crisscross form, I am just going to label this as K subscript N and the delay is outside okay. So what is inside is only the crisscross portion, the transfer function of this would be G N-1 but now I am going to replace it with another lattice structure or crisscross which is K N-1. This would be z inverse, this would lead to K N-2 and dot dot dot on the numerator and on the denominator side and the last stage I will call this as K1.

This is also a crisscross and this goes to a transfer function which will be G0, G0 let us take it to be equal to 1 that is a trivial all-pass function, is just a constant and what we have here is z inverse okay. So what it says is that if I have a causal stable all-pass function GN of z, causal stable all-pass function GN of z, then actually we can obtain a lattice structure where we can show that all the KN's are modulus<1, modulus K lower case or let me call it as K subscript i<1 for all values of i okay.

We can also show the following, again it is a trivial extension of it, I am sure you can do that. If GN of z has real coefficients, real valued coefficients then KN which is a ratio of two of the coefficients of the polynomial, KN's will also become real, then Ki's are real and the lower order polynomials Gi of z, i=1 through N-1 also have real coefficients also, have real coefficients.

So actually you can come up with a lattice that has got basically real coefficients everywhere okay. Now here comes a very, very important observation. This also says that if you take this structure and ensure that mod K's are<1, mod K's are<1 then you are guaranteed that your transfer function will be allpass and it will be stable okay.

Now if you do anything with adaptive filters where all pass functions are involved, this is the structure you should be using because all you have to keep in mind is that the KN's do not have magnitude>1 and of course you can use, so the very, very useful structure. Now have you seen the structure anywhere other than signal processing? You have. You have seen it somewhere, something called Routh–Hurwitz criterion for stability of transfer function.

What do you do? You construct a numerator, you take a polynomial, construct a numerator polynomial which is time reversed and then you compute the K coefficients. What are they? Those are just the reflection coefficients. That is actually what you do for the allpass structure. It is actually the test for stability of a polynomial. How do you test for stability of a polynomial?

If you take any polynomial and just basically construct an allpass, take your polynomial to be B of z and then compute all the K coefficients, all of them come out to be less than 1, you are guaranteed that it is a stable transfer function, all the zeroes are inside the unit circle right and this is actually this is the basis for that test to actually basically there is a lot of theory that goes behind how do you prove the necessary and sufficient conditions and all that.

We have kind of skipped that but again to show you that this is a very useful structure that we that we obtained okay. So all-pass functions, structurally all-pass functions and what we can do with them are shown in this particular section okay.

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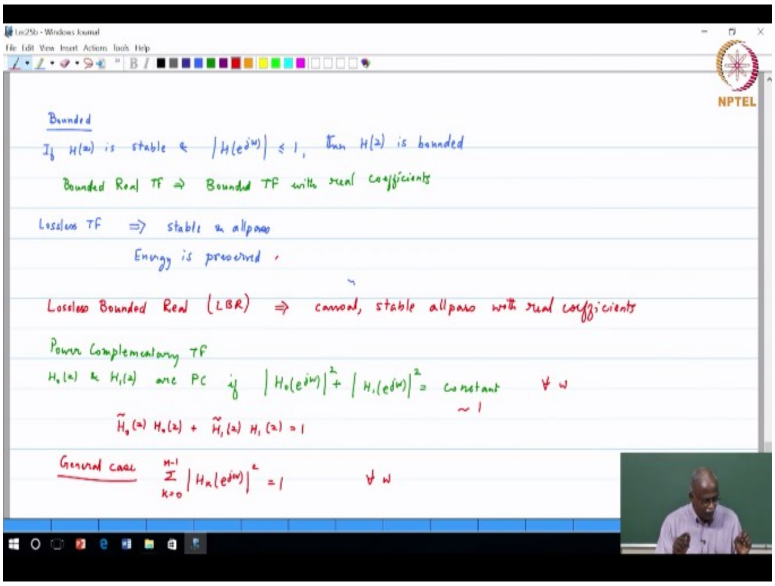

Now very quickly I would like to introduce some additional terminology which is also important for us. I am going to talk about bounded transfer functions okay. Again, these are important for us in the study of filter bank, so that is why we are introducing them. So a function is said to be bounded if H of z is a stable transfer function, is stable and it satisfies mod H e of j omega<or=1 or some constant. Again, for us 1 is as good.

Then, we say that H of z is bounded, it is bounded in terms of the magnitude response viewed on the unit circle. H of z is a bounded transfer function because you are always looking at it in terms of its behavior on the unit circle. Now this clearly tells us that this is a well-defined transfer function on the unit circle. It does not take very large values, it is a well-behaved function and then by analytic continuation you can get all kinds of extensions of H of z but I will leave that to you.

There is another subset of this which is called bounded real okay, bounded real transfer function that is the class of functions where the coefficients of H of z are real okay. So again nothing very different, bounded real transfer function implies a bounded transfer function with real coefficients okay. So bounded actually has got a very strict definition in the context of the signal processing framework that we are dealing with.

Bounded transfer function with real coefficients okay, so we will be focusing on the class of bounded real transfer functions, very useful for us. Of course, the next one that I am going to write down is already known to you, what is a lossless transfer function? A lossless transfer function basically preserves energy. So basically it means that it is a stable transfer function and is allpass okay.

Energy is preserved, energy is preserved okay. Now I want to ask you to think about the following. Does this statement make sense? A class of functions that are lossless bounded real transfer functions, LBR, does that make sense? It is a transfer function that preserves energy and is well-behaved on the unit circle okay, so well if it is a lossless and allpass, it has to be<or it has to be=1 on the unit circle but it has got real coefficients as well okay.

So basically this would point to a causal stable allpass with real coefficients okay. So just lossless could mean an all-pass function with complex coefficients and another reinforcement of the property is that we also want things that are bounded okay. Now here comes a very interesting extension of what we have been studying so far. We call a set of transfer functions as power complementary, power complementary transfer functions if they satisfy the following.

H0 of z and H1 of z are 2 transfer functions, they are power complementary PC if they satisfy the following property, magnitude H0 e of j omega magnitude squared+H1 e of j omega magnitude squared is equal to a constant. I would like to take it to be equal to 1 for all values of omega. We would like this property to be satisfied and now we have seen this condition in the 2-channel case when I had linear phase.

If I could construct those pair of polynomials to be power complementary then I would not have to be doing this magnitude constraint, it would come out to be in fact I would get perfect reconstruction because I have already eliminated aliasing, eliminated phase and now with if they had been power complementary but you kind of see where we are heading. Now by analytic continuation, this basically means H0 tilde of z, H0 of z+H1 tilde of z H1 of z is equal to 1 okay, that is the power complementary condition okay.

Now of course if you can generalize the power complementary to more than two transfer functions, in the general case of power complementary transfer functions, we get the following. Summation K=0 to M-1 magnitude HK e of j omega magnitude squared equal to 1. So basically they can be M transfer functions which are power complementary, you just have to make sure that their magnitude squares when added up over all the functions is a constant that we have okay.

Now very quickly we need to put several pieces that we have already studied together okay. Here is a result. I just want you to quickly think about. We will put the pieces together in tomorrow's class.

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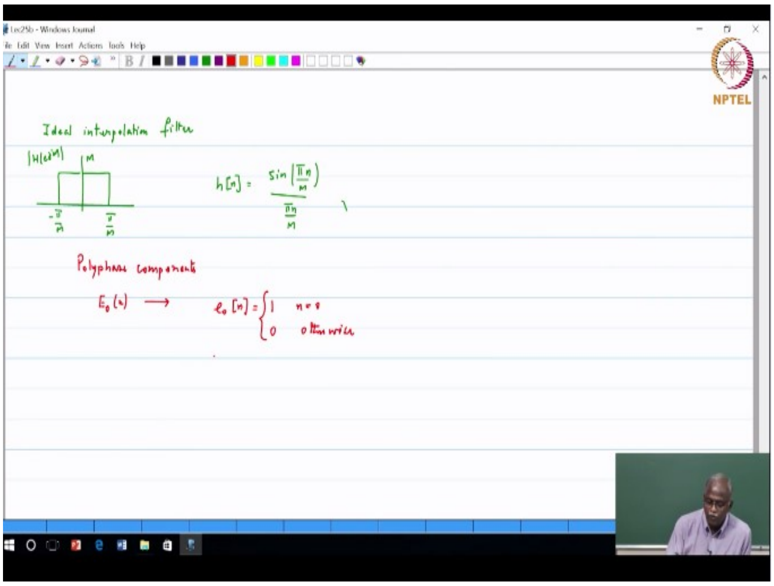

What is the ideal interpolation filter, Nyquist filter? Okay, we said that this will be if I want to interpolate by a factor of M, this will have a height of M, this is a frequency response, magnitude H e of j omega, it will go from –pi/M to pi/M. If i take it to be a zero phase rest filter then I get an infinitely long sinc function, sin pi (n/M)/ pi n/M. Basically, that is a sinc function form.

Now did we say something about the polyphase components of these ideal filters in any observation that we had made? I believe we had made an observation about the polyphase components. If you look at this structure, what you will find is that there is a polyphase component E0 of z such that E0 of n the corresponding is equal to 1 for n=0 and equal to 0 otherwise okay. That is the central point of the impulse response.

That is the only one that is good and remember there will be zero crossings at regular intervals, so all other coefficients will be 0. So this particular polyphase component has got only constant coefficient okay. Now this has got a very significant importance in our study and basically maybe that will come in the next lecture. The second element that we would want to I want to build upon is the underlying interest that we have in the following.

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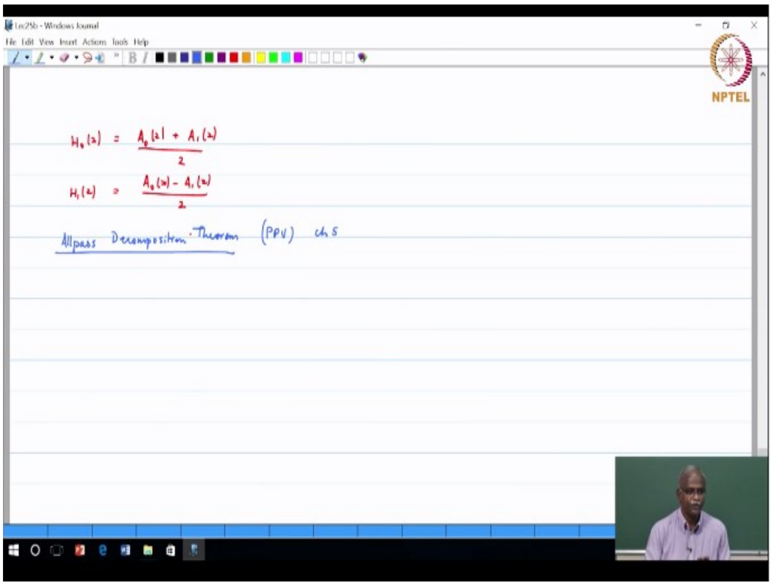

Our goal is to come up with a set of transfer functions H0 of z to be equal to sum of 2 all-pass functions. A0 of z+A1 of z/2 H1 of z again this is why we even studied about the all-pass functions. H1 of z to be equal to A0 of z-A1 of z/2. Again, this is because of the relationships that exist between them H0 and H1. So basically if we can write it in this form then we are able to get a lot of the results that come from there.

So what we will refer to as the allpass decomposition theorem? When can we achieve these conditions, allpass decomposition theorem? When can we write 2 transfer functions that are power complementary and will satisfy this allpass decomposition theorem? Again, this is a result that has been derived in the book by P.P Vaidyanathan chapter 5. I will use the result without proof.

But again there is enough of the justifications for what we are going to be actually implementing and so what are we going to do now? We have studied all-pass functions, we will now look at the solution that gives us a good set of filters which are H0 and H1 which can be used in an IIR 2-channel filter bank and hopefully this maybe gives us some clues to go beyond.

Because this one will not eliminate phase distortion, this will have phase distortion. Now we will sort of try to see whether we can look at the linear phase part and then the allpass part and then say can I combine these two in some ways and get a perfect reconstruction filter bank because I want to eliminate phase distortion and magnitude distortion in the solution that we are trying to work with.

So one more step and we will get a work towards the perfect reconstruction okay. After we do 2-channel perfect reconstruction, we will only briefly mention what it takes to do arbitrary channel perfect reconstruction. Basically, will say how difficult a problem it is and then once we finish that then the last two chapters of the course, one will be on wavelets, the other one will be on OFDM okay.

So those are the two chapters remaining and again what I would like to request is your cooperation to schedule the makeup classes. It has been very, very difficult for the TAs and me because you know the studio is not available or some student has got something or the other. So the only options that seem at this point are October 28th is a Saturday and October 31st is a Tuesday.

So between those two, we would have to make sure we make up 3 hours of missed classes because only then we can finish whatever we have planned as part of the course. Others will be left you know and unable to finish what we would like to do. So again we pick it up from here in tomorrow's class. Hopefully, we will reach perfect reconstruction tomorrow. Thank you.