Multirate Digital Signal Processing

Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology - Madras

Lecture – 02 (Part-2)
Sampling and Nyquist Criterion - Part 2

Okay, so let us take couple of examples which are helpful for us in the context of the problems that we are looking at. Okay. So an example.

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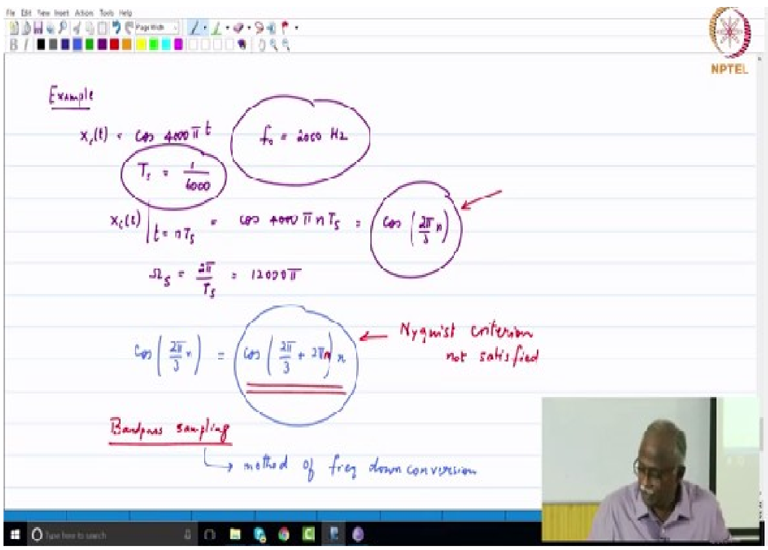

Now if I had a continuous time signal, at cosine 4000 pi t, that is my signal, okay. This means the frequency is 2000 Hz. If I assume a sampling period of 6000 Hz, or sampling frequency of 6000 Hz or sampling period of 1/6000, what is the discrete time signal that we receive? So Xc of t evaluated at t=nTs, that becomes cosine of 4000 pi nTs= cosine 2pi/3n, that is the, okay. So the sampling frequency that in this case was used was sigma s=2pi/Ts=12,000pi, okay.

Now would any other continuous time frequency, not 2000 Hz, produce for me the same discrete time signal when sampled at the same rate? Is it possible for another discrete time signal to produce the same thing? So are the conditions clear? I want the same discrete time signal and I also want the same condition to be satisfied, okay. So is it possible? Let me suggest something. So I have cosine 2pi/3n, this is the same as cosine of 2pi/3+2pi*n, okay. In fact, it can be any multiple of 2pi as well, does not change anything, okay.

So now what is the relationship between the sampling frequency and the, so basically if I have a signal, a discrete time, basically this corresponds to if I were to redo the calculation, this will correspond to a much higher frequency but will still map to the cosine 2pi/3n. Do you see that. You can find, in fact, you can find infinite number of frequencies which will map into the; so it is very important for us to keep in mind that the process of sampling or the process of producing a discrete time signal sometimes can produce ambiguities.

But these are ambiguities that we would like to leverage and therefore take advantage of that, okay. So basically what we are saying is that the process of sampling produces for us a discrete time signal. This discrete time signal can be interpreted in terms of a continuous time signal only under the assumption that you have satisfied Nyquist. If you have Nyquist, then you have a unique representation.

If you have, obviously these are cases where Nyquist will not be satisfied, Nyquist criterion not satisfied. So when Nyquist criterion is not satisfied then you do not have the guarantee that the discrete time signal is unique. However, this is not a bad thing. It is not a negative thing. This actually turns out that this is one of the multirate techniques that we will take advantage of. So this, by the way you probably know it with the reference name as bandpass sampling.

So bandpass sampling is another flavour of a multirate technique. Basically it can be viewed as a multirate technique which does 2 things; you create samples of a signal without aliasing, without overlap of images. Samples will be produced all over the spectrum. You take that portion of the spectrum which is closest to what you consider as baseband. So actually bandpass sampling or in our context, just the sampling process can also be leveraged as a method of down conversion, method of frequency down conversion, okay.

Because once you sample it, so for example if this was your original signal, you sampled it at 6000 Hz and then reconstructed it, you would basically get a sinusoid at 2000 Hz. It may have been a sinusoid at much higher frequency. Again, this is an example we just a tone, but what we would like to demonstrate and build upon are the ability to work with.

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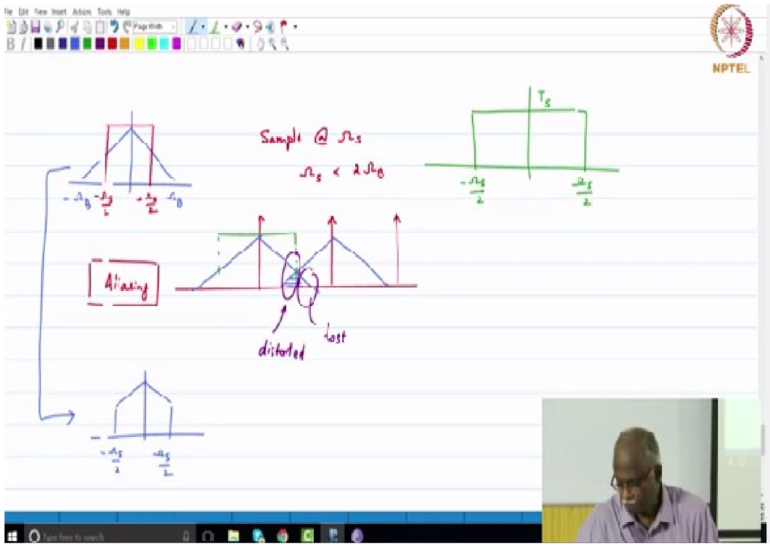

So let me leave you with that. I am sure there are several examples we can look at, may be in the next class. I would revisit it with 1 more example where there is a little bit more explicit problem where, we will take a case where we are deliberately under sampling and look at where the penalties are, what is the cost that we are going to pay. Now let me ask this question, supposing I have a signal, band limited, omega B to -omega B.

I want to sample at omega S, okay. But omega S is less than 2 times omega B, okay. So in other words my copies of the or the impulse train are at multiples of omega S. The signal bandwidth is going to, okay. So obviously there is a problem with this portion because the copies are overlapping. Now when I reconstruct, reconstruction filter if you remember we said was a ideal low pass filter all the way to omega S/2, okay.

May be I did not specify that. The reconstruction filter is an ideal low pass filter, -omega S/2 to omega S/2 and because the copies of the signal have got a scaling of 1/Ts, this one actually has a scaling of Ts to take out the effect of the 1/Ts sampling. So notice that the reconstructed filter has 2 things; one it does not have the original bandwidth because I cannot reconstruct anything beyond omega S/2, right.

That is my limit, and my input portion of my input spectrum has been lost. This portion of the input spectrum is gone. This is lost. And this portion of the input spectrum is actually distorted. Some part of my desired signal is gone. The part that remains is distorted and therefore, this is a scenario that is highly undesirable and this is the problem of aliasing. Aliasing is when you create overlap between adjacent copies of the input spectrum because of insufficient sampling rate.

Now same question. I have a band limited signal from omega B to -omega B. I want to sample it at omega S where omega S is less than 2*omega, B but I do not want aliasing. What do I do? Anti-aliasing filter. So the only option for me is to apply a filter which limits it to -omega S/2 to +omega S/2.

So what I will be left with is a spectrum that is in the range -omega S/2 to omega S/2 and now if I sample it at omega B, then I basically will get aliasing free, right. Basically I avoid aliasing. Reconstruction is still tricky because you are going all the way to the band edge but at least, I have avoided aliasing.

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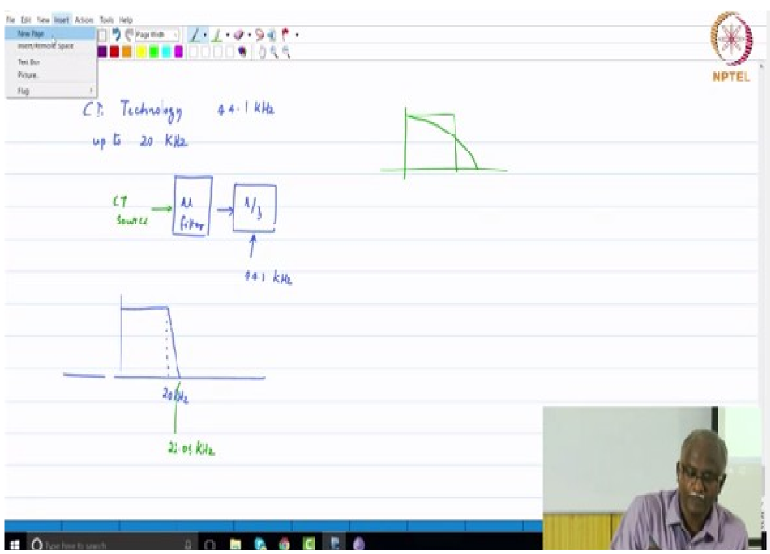

Okay, but all of what we study is useful only if it helps us understands some of the practical challenges that we encounter. So let me take you back to the technology that all of you enjoy. Music technology using CDs, compact discs, okay. Compact disc, we said had a sampling rate of 44.1 KHz and we are trying to preserve all frequencies up to 20 KHz, okay. So the CD technology when you are creating CD content, this will be A/D, analog to digital converter and it will be sampling at 44.1 KHz.

Am I right? It is an A/D because you will store it at 16 bits per sample or 20 bits per sample whatever is your rate. Now if I am going to sample at 44.1, you know that you have to have an anti-aliasing filter, AA filter. I want you to help me design the anti-aliasing filter. Anti-aliasing filter should not cause any distortion in the portion of the signal of interest. So all the way up to 20 KHz if that is what I am trying to preserve, it should be flat in response.

Where should it cut-off? What should be the stop band edge? 22.05 because one-half of the sampling frequency. Because anything beyond that will create aliasing for me. So the cut-off frequency for this is 22.05 KHz, okay. Now those who have studied analog filter design, what are the things that make analog filters complex to design? What are some of the aspects, if I impose these conditions, the analog filter becomes complex?

Give me a few criteria that makes the analog filter difficult to design, difficult to implement. Okay, one is the sharpness with which you transition from the pass band to the stop band. In different context, it is called by different names. Basically if you have a very sharp transition filters, also called a high Q filter. Anytime you have sharp transition, that is a problem. Anytime you impose a very large flat constraint, that also is a problem.

Because that also makes it difficult to achieve. Because most analog filters tend to do something like this. So which means that instead what you are asking is something like this. So again, that is where it becomes a complex design. Now obviously when you have a CD, you are trying to create CD content, you have an A/D beautifully designed at 44.1 KHz. You have to precede it with an anti-aliasing filter.

Now is it anti-aliasing filter or analog filter or digital filter? It is an analog filter, because you have not yet gone to the A/D. So basically you are before. So here you are at the continuous time source. Whatever it is, you cannot say that well you know only voice will be there, so it is okay, my analog filter can be a little sloppy. You have to design a filter of this kind. So this becomes a very difficult and challenging problem, makes it the anti-aliasing requirements and subsequent A/D becomes a complex issue. Now how does multirate help us in this case?

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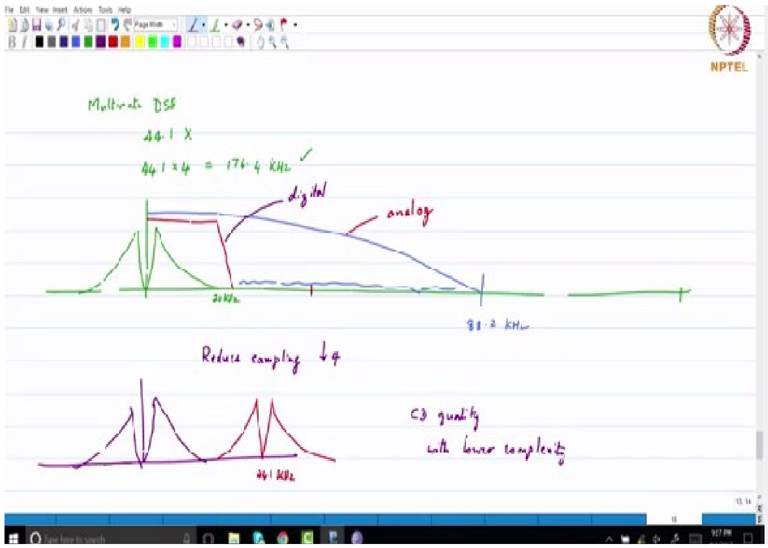

Let me just give you an insight and then we will pick it up from here. So multirate says, multirate technique, in fact all your CD players actually use this; multirate DSP says a 44.1 do not want, do not do that. Instead do 44.1*4, that is 176.4 KHz sampling, okay. Do this, okay. So now what will be the scenario? If this is my audio spectrum that I am trying to. So this is somewhere around 20 KHz.

Previously I was trying to design an analog filter which was like this, falling off at 44.1 and the reason for that was because my sampling rate was 44.1. But now 44.1 is here and 176.4 is somewhere way out there, okay. So basically what that says is the analog filter can actually be something of this kind. You can start to be, let me use a different colour, blue. So it can be here and then eventually gradually drop off, okay.

So somewhere may be one-half of 176/2, that would be 88.2 KHz, okay. So I should change this a little bit, okay. Actually it turns out to be that it can go beyond 88.2 as well but for now this is the. So now what have we done? We have created a digital sample scheme at 176.4 KHz, okay. Now from here, how do you go to, basically you want to reduce your sampling rate. So what does this analog filter do?

It allows some other signals also to be present here because that is unwanted. So if I want to bring it down to the correct sampling rate, now what should I do is, I must follow it up with a digital filter that looks something like this. A digital filter which will then cut-off the unwanted portions that the analog filter gave me. And after that, reduce the sampling rate by a factor of 4, which turns out to be a very simple one, retain 1/4 samples, okay.

And what will you get as your resultant signal? You will get the spectrum and you will get the sampling at 44.1. So the first image will be just slightly away from here. This will be centered at 44.1 KHz. Now instead of 1 analog filter, you had an analog filter + the digital filter. So this is the analog filter, easy to design, not complex at all in terms of the requirements. It is a very low Q filter, digital filter, not a problem, easy to design.

We can implement it. We can implement it with linear phase. So does not distort the signal. So therefore, what we have actually ended up achieving is obtaining the CD quality with lower complexity, okay. So one of the things that you will find is that analog processing of any kind is actually difficult. In the study of DSP, very often you do not even worry about the analog. You just talk about a continuous time signal.

But in multirate when you are talking about changing the sampling rate, we always have to refer back to the continuous time signal. So therefore, it is very important for us to be able to always talk about how do I go from the analog domain to the continuous time domain and then back. So in this context, always very important for us, that we have reduced the complexity of the overall process of conversion from the continuous time to the discrete time and in the reconstruction process.

So anytime there is analog filtering, there is a possibility that you could reduce its complexity by moving it into the discrete time domain. And that is a branch of study by itself called discrete time processing of continuous time applications. So but again, what I wanted to show you today was that when once we have understood aliasing, we know anti-aliasing can be done. Anti-aliasing has to be done before you can create any digital content.

That is what a CD has to do. And the requirements of anti-aliasing are quite stringent except if you take advantage of a multirate technique because anyway your signal is going to be in the digital domain which means that you can do digital filtering without much difficulty and therefore, get an advantage of that, okay. Thank you. We will pick it up form here in the next class.