**Multirate Digital Signal Processing**
**Prof. David Koilpillai**
**Department of Electrical Engineering**
**Indian Institute of Technology - Madras**

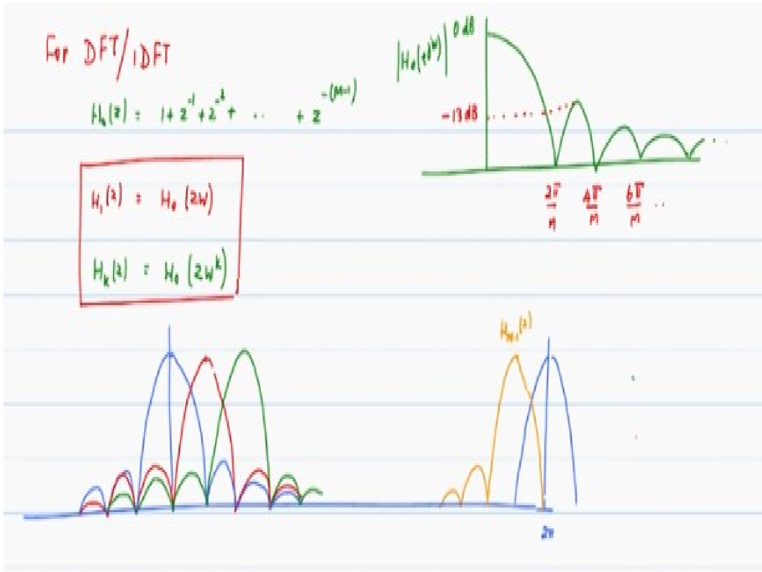
**Lecture – 20 (Part 1)**
**Maximally Decimated Filter Banks 2 – Part1**

**(Refer Slide Time: 00:22)**


Good morning, we in today's lecture we will be covering a lot of ground on the Maximally Decimated Filter banks. We will look at the special case where M=2 channels and it actually requires a fair amount of work to do the, to understand the mathematical analysis how the filters behave how do we design them and how the whole system works together. So but before that there were a few questions after the lecture yesterday.

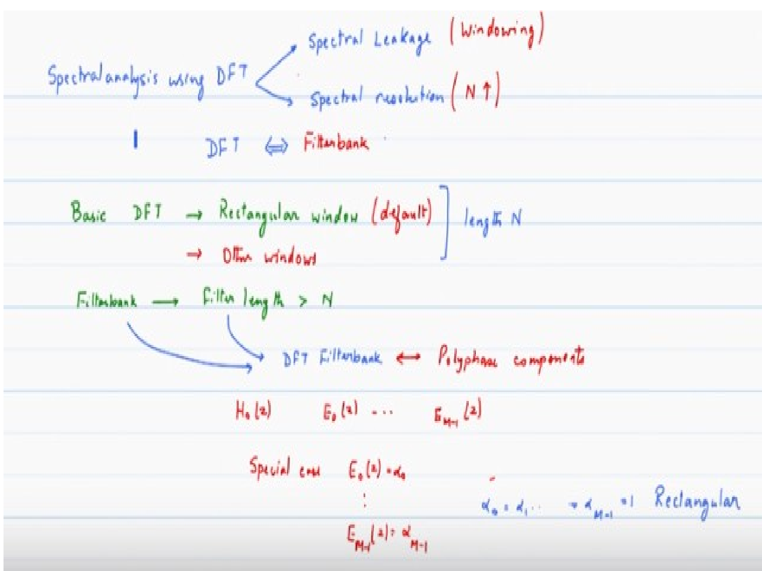
So I just thought I would clarify those doubts so that incase those are a broader doubt among the other students as well.

**(Refer Slide Time: 00:54)**




So just as a quick summary whenever we do the DFT or IDFT basically we view it as a transformation but from a that is from a mathematical view point or from a electrical engineer view point but from signal processing view point, we think of it as some operation with filter banks. So basically you have a bank of filters which are shifted in terms of their center frequency identical shifted in terms of their center frequency. And this is a important element to keep in mind.

**(Refer Slide Time: 01:26)**




And along with that maybe an important aspect is that whenever we do spectral analysis using DFT spectral analysis using DFT. Here are 2 points that always are worth remembering. The first one is that there is inherent spectral leakage. So it can get some misleading interpretations. And goes back to your filters not having just a pass band but they have got a kind of ripples so therefore you can get some erroneous conclusions.

So to improve upon this we use windowing. So without windowing its not a good idea to attempt the DFT.The second element that to keep in mind is that the number of the length of FFT determines your spectral resolution. So that is depends on the size of the DFT and if you want to increase the spectral resolution you should increase the size of your DFT okay. So maybe the way to look at it is that we do a DFT operation.

But there is an underlying filter bank function that is actually being carried out and it is good to always link the fact that why spectral leakage occurs how do I get more spectral resolution those are elements that you can obtain from the filter bank concept. Now if I have to increase the size of N what does it do to the filter bank the size of the DFT increases the number of filters makes them narrower. That is why the spectral resolution goes up.

So okay so that is where now an another important point to do or to keep in mind is that whenever we do the basic DFT that is you take a block of data and then you process it there is implicit in your operation a rectangular window. So, it whether it was a finite length data, or it was an infinite length data the fact that you took a segment of length N effectively means that, so this is a default.

Never forget that underlying any DFT operation if you just take that DFT yes, its an orthogonal transform but at the from a filtering point of view you have windowed it and you have done that. Okay. Now what happens if you use other windows? You get slightly better in terms of spectral leakage but in all these cases your underlying spectral resolution depends on the length of the window or the length of the DFT as well.

So basically, we are talking about a length N. The underlying filter has only length N it can be a rectangular repulse response or a shaped impulse response, but the underlying length is N. Now where does the filter bank come in and a filter bank on the other hand is a special case of these windows. If you constraint the length of the filter to be equal to the size of the DFT then it becomes N.

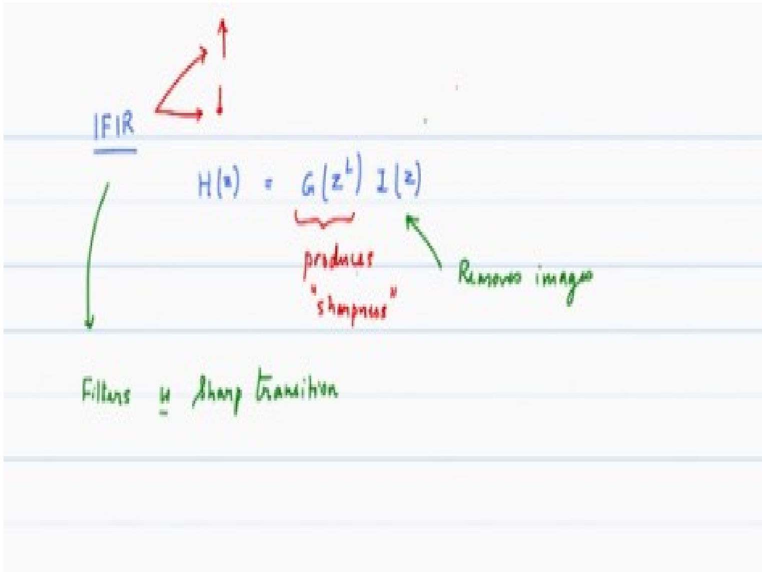
So, filter bank actually allows you to have filter length arbitrary it can be greater than N. And that is why a filter bank can reduce leakage and gives better resolution and give you the additional benefits. And this is any filter bank, but a special case of the filter bank is when you have a DFT filter bank. You have designed each of the filters as shifted versions. So in that case you get a DFT filter bank.

And whenever we want to compare it with the DFT then the filter bank cannot be an arbitrary filter bank. It has to be a uniform filter bank it has to be a shifted versions; it would have to be a very similar to what the basic DFT itself is doing. And if you have a DFT filter bank then the added benefit is that if you now break it up into its Polyphase components then you get very efficient implementation, polyphase components okay.

So basically the Polyphase components of your filter H0 of z If you have to label them as E0 of z to EM-1 of z these are polynomials. The special case is when each of these is equal to a constant. This is equal to alpha 0 E M-1 of z is equal to alpha M-1 that is an arbitrary window and if you take alpha0 =alpha 1= alpha M-1 then you get the rectangular window. So, you see that the filter bank is the most general you restrict length to N then it becomes a windowed filter bank windowed DFT.

If you set all the window coefficients to be equal to 1 then you get your normal DFT. So, when you view it as a sequence of operations then you get the insights and there is advantages to that. So, I would not repeat this hopefully this is the good enough reinforcement of what we have already covered.

**(Refer Slide Time: 07:21)**

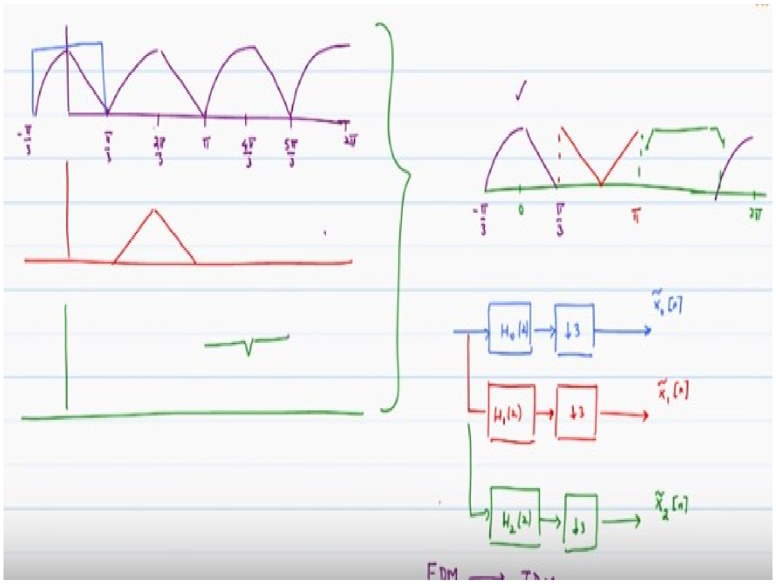



The other concept that we talked about is IFIR filter design Interpolated FIR filter design. So, the general structure is H of z is equal to G of z power l I of z. Now this can be used both for up sampling and it can be used for down sampling. Both of them require filters and IFIR technique is particularly advantageous if you have to have one of these. The key point to note is that this is where you get the sharp filters.

This is what produces sharpness. But along with the sharpness it produces some unwanted images, and this is what removes the images. Okay so we take advantage of the fact that the insertion of zeroes compresses the spectrum and then basically you need to take care of it by removing. This is the IFIR technique is primarily used when you need filters with sharp transition. And it is definitely advantageous if you have sampling rate conversion as well.

Even without sampling rate conversion this maybe an attractive method particularly if you have sampling rate conversion this would be the way to do it. And again I do hope that if there is a need for you to design a sharp filter you will keep this concept in mind.

**(Refer Slide Time: 09:04)**



Ok the next one which we did spend some time yesterday was the Trans multiplexer where we took 3 signals up sampled them by a factor of 3 filtered them and then produced the FDM signal. After the class one of the students few students pointed out that there was a mistake in the figure that I drew so let me just correct that figure so that you will not have a confusion. So this is the spectrum that we have drawn from 0 to 2 pi.

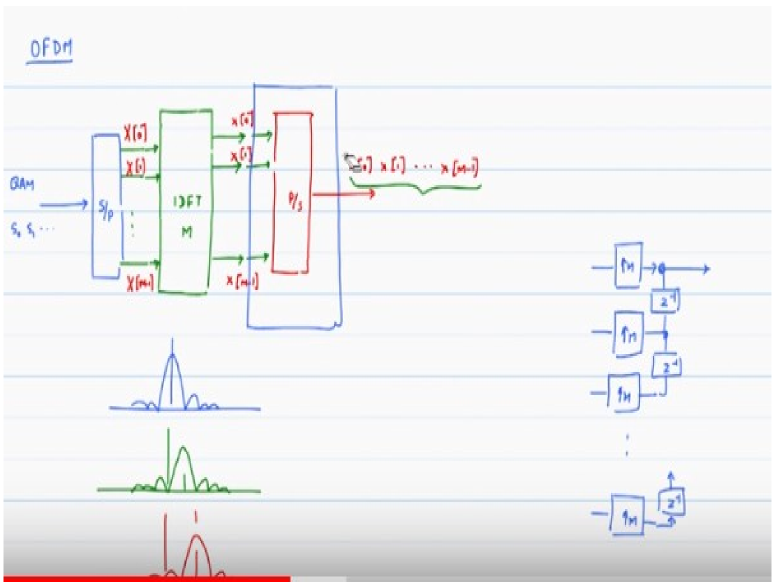
And generally when we do the deformation or interpolations it is good to keep in mind what it looks like –pi to pi. And I think that is where the mistake occurred. If you look at it from point of view of the first branch the first branch produces three copies of the signal and then I filter out the portion from –pi/3 to pi/3. So this portion is correct so from –pi/3 to pi/3 you get a curved portion and then you get a straight portion.

Now this figure is wrong okay what is the figure that actually should have been if you look at it from –pi to pi in the previous graph. Let me draw it with a different color. –pi to pi actually looks like this correct. So when I shift it what I should have gotten was a spectrum like this -2pi/3 to pi and one more copy so this red line is not correct the purple line is the one that is correct and when you actually apply the filter then what you pick off is this portion of the spectrum.

So, which means that the line looks like a V not like an arrow head. So, this is the correct spectrum that you should have. And in the same way if you go back and draw the third signal then what you will find is that the third signal actually looks like this from –pi to pi. So, when copies are generated then again it is of this type this green line is wrong what you should get here is again and therefore the correct spectrum is here.

So, this is again make sure you do note why this is the correct version make sure we are able to get that. Basically the multiplexing operation does keep the copies, but the shape of the copies is equally important. So, make sure you preserve it correctly. We said that the inverse operation of the translating from frequency division multiplexing to time division multiplexing can be shown to be the counterpart.

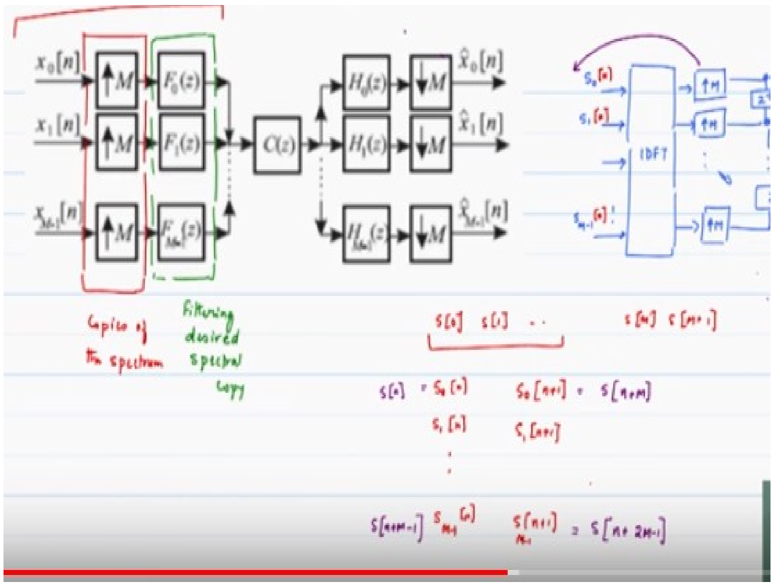
**(Refer Slide Time: 12:03)**




Now there were couple of questions with respect to how the OFDM and the filter bank are related. And whether there was some gap in what was presented in the class I just want to make sure that we clarify that. So, first let us just make sure that we are comfortable with this portion of it. OFDM the transmitter again if there is any questions feel free to raise it. There is a serial to parallel conversion followed by the IDFD.

The dimensions are the same as the number of parallel lines. After which there is a parallel to serial conversion. Now the parallel to serial conversion is something that can be represented in terms of up sampling by a factor of M followed by a delay chain this will basically reorder if the vector came in as X0 X1 to XM-1. This will basically make it into a serial vector and presented to us. So this portion of the figure is clear.

**(Refer Slide Time: 13:08)**




Okay now what I would like to do is built on that so that we can link the two concepts that we have. Now ignore for a moment the second half the first portion of is what we are interested. This is the portion of interest okay this is the Trans multiplexing operation. Now this is the segment if I were to draw a line around this block this is where the copies of the spectrum are getting generated.

Am I right so this is where the copies of the spectrum as we saw in the 3 channel case. And this is the second part is where the filtering occurs to keep the correct copy of the spectrum depends on how you have designed the filters you get a bank of filters and this is the portion where we do the filtering and filtering the desired spectral copy you keep the desired spectral copy and throw away the others okay.

So I am sure that this part of it is comfortable with this being the set up that we have. Now what does the OFDM transmitter looks like I will spend a little bit of time just to make sure that this is clear. So, there is an IDFD followed by up sampling followed by a delete chain. So if you do not mind let us just draw that because that is very helpful for us. Keeping this picture in mind the portion of interest is starting from the IDFT.

Again we want to show that there is equivalence and that is the idea of this discussion. And then the parallel to serial conversion which then says there is an up sampling by a factor of M…the last branch also undergoes up sampling by a factor of M. And then they are gathered together by means of a delayed chain. Okay there is a delayed chain which gathers them together okay and we get a sequence of data has been presented to the inverse DFT yesterday.

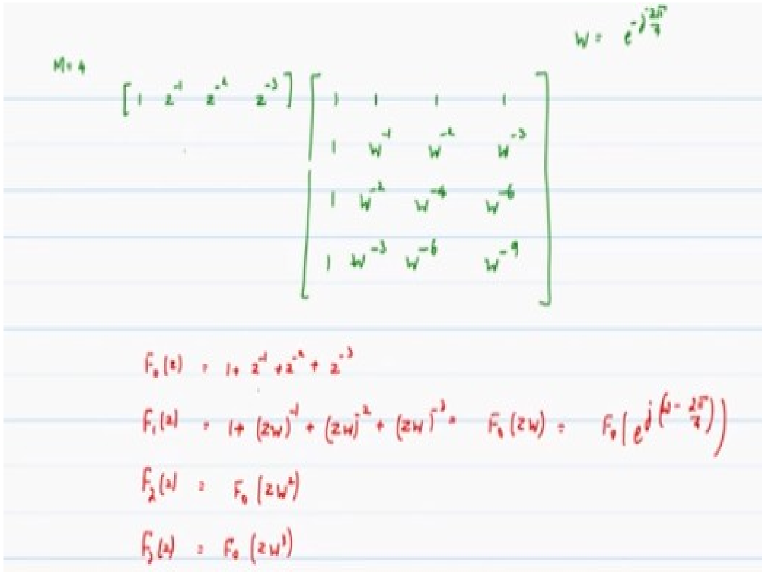
We labelled them as a S0 okay a symbol S0 symbol S1 a symbol SM-1. Now just so that we are able to get the timing also correctly. Please label them as n so at the time index n=0 you get a vector. Where did this vector come from if you think of the S as a sequence of symbols starting from S0 S1…SM SM+1 your data, you have splitted into segments and then you have mapped them into the QAM mapper then you take portions of them M vectors then you block it.

So, in the general case you get S0 of n S1 of n SM-1 of n and basically this would correspond to if this corresponded to S of n some index this would have corresponded to S of n+M. so that would be the 1 st vector and how it is formed. The second vector basically we would label it as S0 of n+1 S1 of n+1 basically the next time index SM-1 of n+1 so this would correspond to again it is a continuous stream of data.

This would correspond to S and this would be M-1 I am sorry. If you spot a mistake, please flag it so it will not confuse everybody. So, this would be S of n+M all the way to S of n+2M-1 okay. Basically the blocking occurs I just wanted to make it sure that you are comfortable with the notation. Yes, this is a continuous stream of symbols, but they are getting blocked into non over lapping vectors and then we obtain the result okay.

Now the first step that helps us in our analysis is that am I allowed to move the up sampler to the left of the IDFT block. The answer is yes why because IDFT is an inter connection of adders and multipliers and the up sampler can be moved to the left.

**(Refer Slide Time: 19:03)**



Okay so please redraw this just so that we are comfortable with the sequence of steps one more step. So the first step; step1 of proving the equivalence is to move the up samplers. Up sampling by a factor of M up sampling by a factor of M . . . up sampling by a factor of M and no confusion as far as the data is concerned this is the sequence S0 of n this is the sequence S1 of n this is the sequence S1 of M-1 of n.

And after this comes the IDFT followed by the delay chain and the adder. We need 3 steps this is the 1st step the other 2 follows very easily from here okay. That is the output, so the key element is now focus on this portion okay it looks a lot like these figure that we have drawn here. Basically there is a sequence of up samplers followed by a bank of filters. Now the question that remains to be answered is what is within the green box a bank of filters.

Which is doing the same thing that what those filters are doing there for us. So the key element is that what are the signals at this point it would be S0 Z power M right up sampling. So, the signal at this point would be SM-1 Z power M. So what we want is that this signal if I call this as s of n, we want S of Z to be a filtered version of this up sampled signals. So, this should be F0 of Z F1 of Z.

And FM-1 of Z multiplied by the column vector where you get S0 of Z power M S1 Z power M . . . SM-1 Z power M okay this is the Trans multiplexing operation. Whatever signal comes in you up sample it that gives you the column vector and then you combine it using appropriate filters okay. Everyone is comfortable with this basically what you will get is a scalar after this operation and that is what is represented as the frequency division multiplexed signal.

Now we have S0 S1 SM-1 I just need to see whether these 2 are linked and it is easy for us to do because the summation happens through a delay chain so that can be captured in the following form 1, Z inverse, Z -2, Z -M-1 okay and followed by the IDFT matrix which I will denote as W* okay W dagger. So, what is there in the figure not done anything else. Now that is step 2 step 2 is when we have shown this.

Now I found it easier to try a specific example let me just highlight that M=4. In that case I get 1, Z inverse, Z-2, Z-3 please look at this matrix 1 1 1 1, 1 W inverse W-2 W-3 in the case of W is actually e power – j 2 pi/4. It does not really matter just completeness 1 W-2 W-4 W-6, 1 W-3 W-6 W-9 that is the W dagger matrix. Okay so from the previous comparison F0 of Z is 1+Z inverse+Z-2+Z-3 F1 of Z that the row vector times the second column.

What you get is 1+ZW inverse+ZW-2+ZW-3 which you can verify the same as F0 of ZW and this is the same as F0 e power j omega -2 pi/4 basically shifted by the 2 Pi/4 center frequency. Similarly, you can you can do the rest F2 of z is F0 of ZW squared and F3 of z is F0 of ZW cube. Okay so what did we say F0 is 1+z+z inverse that is our familiar filter bank. Where is my filter bank? Yes, 1+z inverse+z-2 this one.

The next filter is shifted by ZW you will get that so in other words what is the OFDM transmitter doing this block where you have the IDFT followed by the parallel to serial conversion can be represented as the DFT followed by the parallel to serial conversion The up samplers can be moved in the front if you move it to the front then what you are left with the IDFT followed by the delayed chain combination.

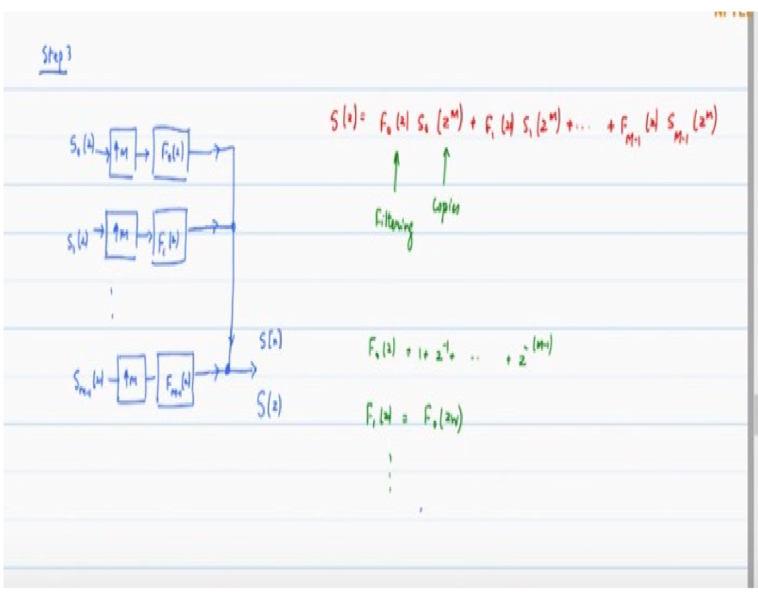
What is the transfer function from each of these input points to the output? that is F0 and I will label that as F0 of z that is how the trans multiplexer diagram is defined right output after the up sampler the transfer function is F0 of z. What is my F0 of z we have shown that it is a filter bank starting with a rectangular window and then shifted versions of it. So, basically what you are doing is if you look at this operation if you apply a vector to it input to the IDFT block.

So, that means the vector gets operated on by the IDFT and then gets combined by means of what will come out is a column vector and that gets combined into a scalar using the delay chain. So, how do you get a scaler so basically if you are taking one more step so apply the input to this IDFT matrix so let us call this as S0 S1 S2 S3 okay when you do the DFT you will get a vector a column vector.

If you multiplied by a row vector which is 1 Z inverse Z-2 Z-3 is basically adding them with appropriate delays. So, what we have shown here is an equivalent representation of what a block diagram is implementing. The S0 S1 S2 S3 is that is an input to IDFT computation is done and then the summation is done through a delay chain. So, now we are saying that so what is the transfer function that is represented by this.

So by the way this is Z do not get confused you have to write this as Z I am trying to get the transfer function okay so basically that says that F0 F1 F2 F3 are the so in other words this is step 2. First step was moving the up samplers then analyzing the DFT the second step was to show that each of these combinations actually are.

**(Refer Slide Time: 28:12)**



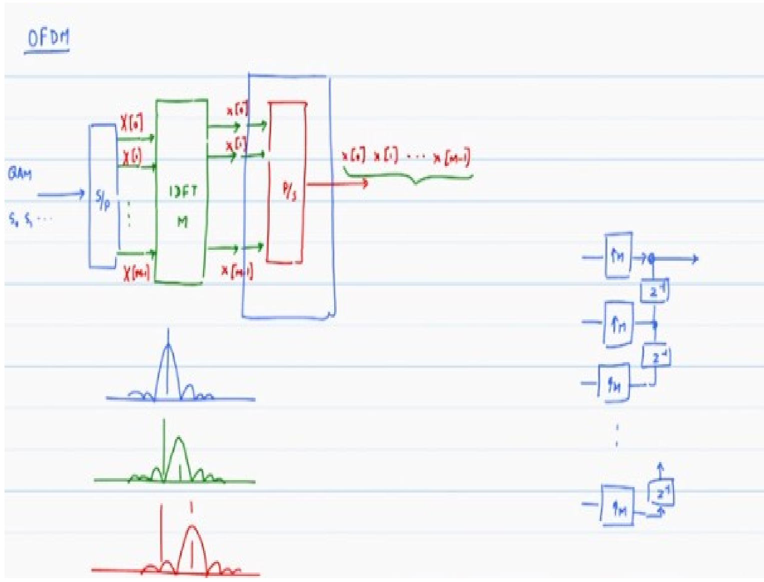
So, the final step 3 is to confirm the equivalence so in the general case we get up sampling by a factor of M followed by F0 of z and up sampling by a factor of M followed by F1 of z this is S0 of z, S1 of z, SM–1 of z, up sample by a factor of M. This is FM-1 of z in most trans multiplexers the summation is not at the top it is shown at the bottom and do not get confused it is exactly the same thing it does not matter at all.

But there is a reason for this anytime a synthesis is happening you kind of show that all the signals come, and it gets sort of collected at the bottom it is just a convention that is done so instead of keeping the summation at the top and basically this is what is going to be fed to your channel. Whatever your channel is in this case it will be a mobile fading channel to which you will you will transmit the signal.

So, this effectively is your S of n the combined signal or if you want to represent it in terms S of z transform. So S of z is F0 of z times S0 of z raised to the power M F1 of z times S1 z raised to the power M... +FM-1 of z S M-1 of z raised to the power M. Now this is the general transmultiplexing operation. Where you convert into a frequency a division multiplex signal as you can see this is where the copies are getting generated.

This is where the appropriate filtering is taking place what we have shown that is that if you replace it with the OFDM type transmitter IDFT followed by a parallel to serial conversion that is the same as up sampling followed by the IDFT block where the filters can now be represented in terms of the transfer functions that we have just now shown. F0 of z is 1+z inverse +z or - M–1 F1 of z = F0 of z w .... I would not repeat the rest of it. So, I hope this part of it is comfortable.

**(Refer Slide Time: 31:19)**



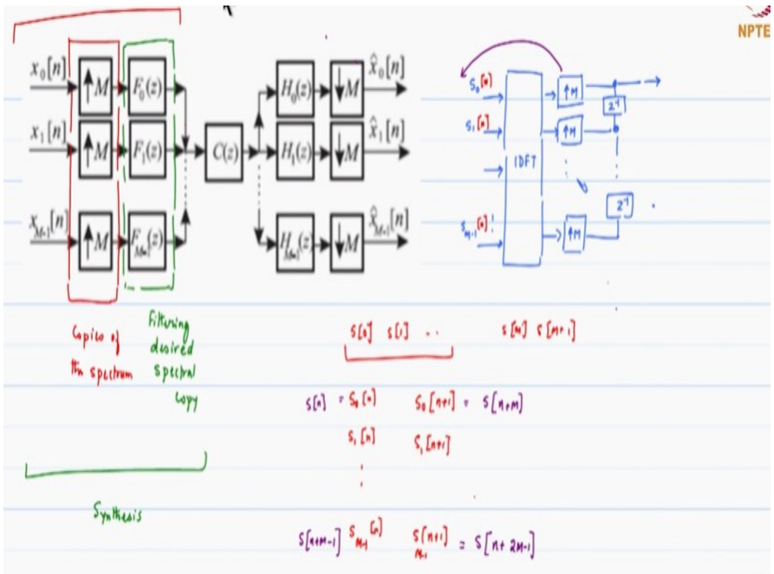
And that is where we said that the OFDM is actually the transmitter portion of the OFDM is actually doing is generating a frequency division multiplex signal and it is beneficial for us to think of what those filters actually look like and those filters these filters these shifted versions of these filters. And what is it that is being applied to these carriers those particular symbols QAM symbols.

So, each of these and we said that you can do power allocation you can choose not to use a particular channel if there is interference. So, lots of interesting things that you can do with OFDM again those are things that you would have already seen in the communications course what we are looking at is to give a signal processing. And in particular a multi rate DSP interpretation for the OFDM.

For the frequent of the generation of the frequency division multiplex signal in our terminology it is a trans multiplexer and or FDM actually is a trans multiplexer a in the true definition of the afterword. Okay any questions? because there were there are several things which were the clarity of where questions were asked, and I was thinking we should probably make sure that this concept is clear.

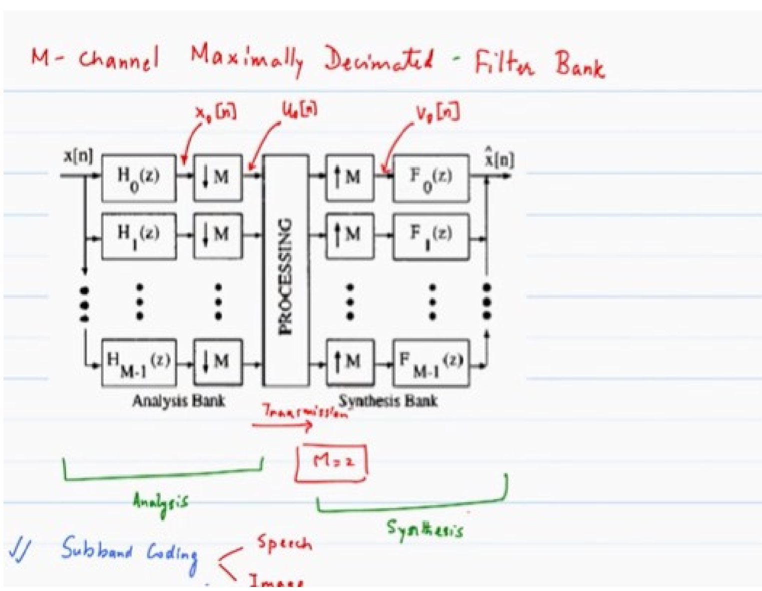
We will come back to study OFDM in its completeness like now just to indicate that whatever we have developed is actually a very useful tool in even in analyzing a application such as OFDM Any questions? Okay so now we move onto the filter bank okay.

**(Refer Slide Time: 33:21)**




We leave the notice that there is a transposition is happening in a transmultiplexer what you have at the beginning the first portion this is the synthesis portion. Synthesis portion and the second half is the analysis portion analysis is where the signal gets split synthesis where the signal gets combined.

**(Refer Slide Time: 33:31)**



Okay now the conventional multirate M channel filter bank is shown here the analysis comes first whatever processing you do and then followed by the synthesis. Analysis synthesis so again we see that the transmultiplex operation is a transposition of the analysis and synthesis blocks. Most of the times when we are doing multi rate filter banks we are trying to split a signal, so this is also linked.

Those of you who are familiar with the speech you have a technique called sub band coding where you say that you split the signal into different frequency bins. And whichever frequency been has got more information you use more bits to encode that information, right? So for example you split it into high frequency and low frequency. You find that the low frequency has got more information, so you use more bins to indicate the low frequencies and high frequencies.

So, therefore you get a form of compression so the whole filter bank theory came from its applications. In a sub bank coding has been used extensively in speech and image and continues be used in these applications. So, this is this is a very foundational block that we are looking at. And in this context what you have is a composite signal basically the speech signal you split it into different frequency bands.

And then do the processing and then at the other end. So, this may not happen at the same and you may have a transmission happening in the middle for example this could be a transmission that is happening and at the other end you will do the reconstruction. Okay so this is a very gender framework a very important framework and a useful framework as well.