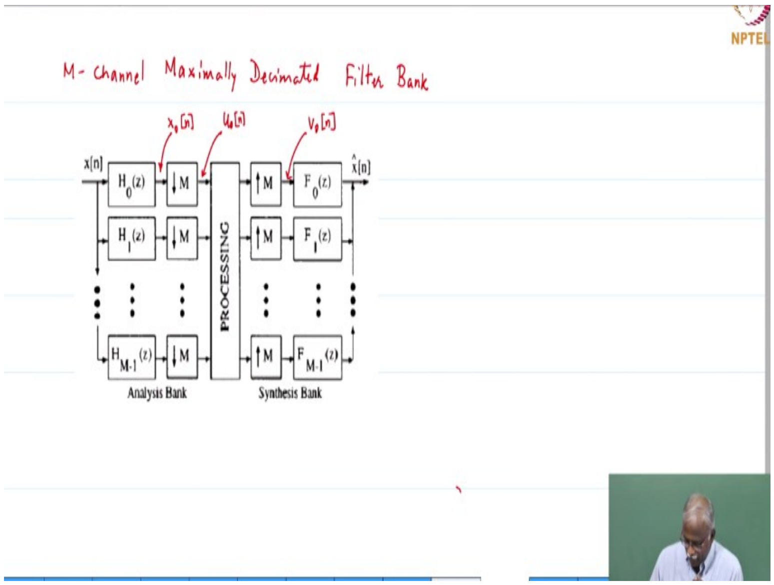
Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology – Madras

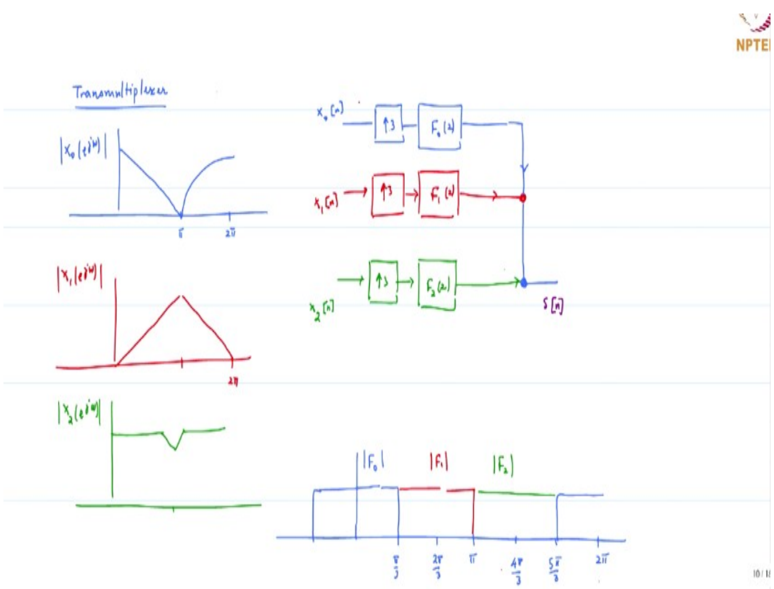
Lecture – 19 (Part-2)
Transmultiplexer and Maximally Decimated Filterbanks – Part 2

(Refer Slide Time: 00:16)



Okay. Now I would like to introduce to you a very interesting application called the Transmultiplexer.

(Refer Slide Time: 00:29)



So the transmultiplexer application. First let me give you a figure for you to analyze and do it very quickly and then we will; so I have three signals; let me call them as X0 e of j Omega; the spectrum of this signal between 0, pi and 2pi is as follows. Straight line here, okay so it is a complex signal.

X1, choose a different color X1 e of j Omega magnitude 0 pi 2pi the spectrum it is a real signal but it is a sort of a, it does not have low frequencies it is got high frequency content, okay. Third signal, X2 e of j Omega magnitude and again this has got a very different type of behavior it is flat a small dip around pi and then goes back up, okay. These are just randomly chosen spectra nothing special about that the spectra themselves. Okay.

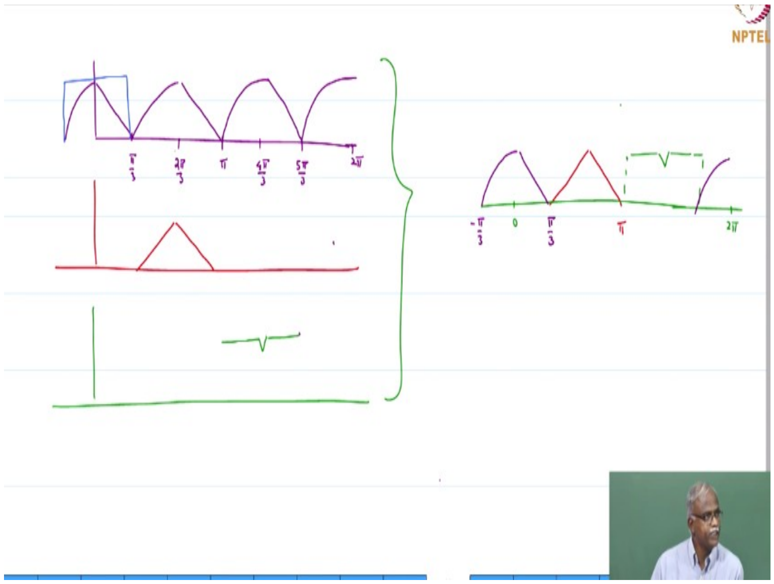
Just some of them are complex some of them a real low-pass one this one looks like a signal that has got all frequency components. So the important thing is the; what you are going to do with the processing that we are going to do with these three signals. I am going to feed x0 of n to a multirate structure which is first going to sample up sample by a factor of 3 or in insert 0s followed by F0 of z, where F0 of z.

Let me just draw the filter here. This is pi/3, 2pi/3, pi, 4pi/3, 5pi/3, 2pi, okay. So the first of the filters F0 of z goes from –pi/3 to pi/3. So the image would be located here. The second filter is H of that one goes from pi/3 to pi. And then the 3rd one goes from pi to 5pi/3, okay. So this is F0 magnitude F0; this is magnitude F1; this is magnitude F2, okay. And the structure that that we have is this output gets added along with take x1 of n up sample by a factor of 3.

Pass it through a filter F1 of Z and then add it to this point and the third and the arrows are going in this direction. Okay. The third one is the green; it is x2 of n up sampled by a factor of 3 passed through a filter F2 of z which is then added on to this original signal. Okay. So here is the signal flow please tell me what the output signal let me call this as s of n, s of n is a combination of these three signals. I would like to understand what the spectrum of s of n looks like. Okay.

And so the first thing you would do is each of these gets up sampled by a factor of 3. Let us do it for X0; I will assume that this you will be able to do it for the others.

(Refer Slide Time: 05:10)



So the first step would be s; X0. Basically, this is pi/3 pi and 2pi just equal equally spaced easier for me to sketch it. So pi/3, 2pi/3, pi, 4pi/3, 5pi/3 and 2pi, okay. So now if I up sample by a factor of 3, I am going to get 3 repetitions of the image. That is one first repetition, second repetition; third repetition. Okay. And I am going to pick off the; so before that is this one. So the first branch is going to pick off the portion from –pi/3 to pi/3.

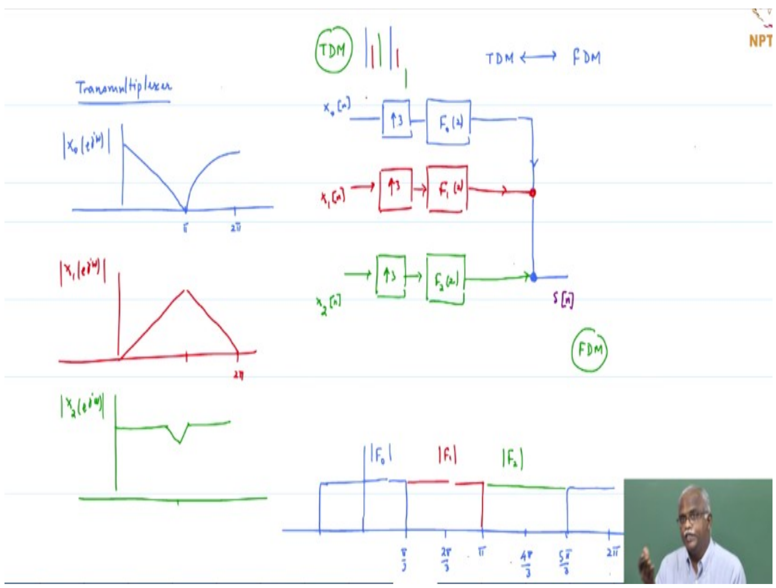
Likewise, if you look at the other two branches as well then what we get is in the second case you will get the this portion of the spectrum second branch. The third branch the portion of the spectrum that gets retained is from here. And so if I add that three together the combination of these three then the signal that I; that we get is a signal that looks like this from –pi/3 to pi/3 then the red portion.

I am sure you have already probably finished before me. This is the red portion going all the way to pi and then the green portion and then finally you will get one more the other part, okay. So this is what the spectrum looks like, okay. So this is what the spectrum looks like. Okay. I hope you basically got the structure this is 0 and this is 2pi. Okay. And just to show that the frequencies below -2pi I have just shown below 0 just for a short portion so you know how the signal pattern repeats itself. Okay

So the; what the structure actually did was to do something like equivalent of modulation; what it does; because it took; these are all baseband signals. It took; one of them it retained it kept it as a baseband signal the other two it actually moved it up in terms of the center frequency. So what you find here is a translation of the basic spectra. And so in some sense multirate signal processing can be used for modulation because it is very easy for us to shift the center frequency of any signal spectrum. So this is one principle of that.

The other element of what this structure is doing is that it took 3 signals, right. It took X0, X1, X2. So you can you can think of these as 3 signals in the time domain there you can think of them as time division multiplexed signals.

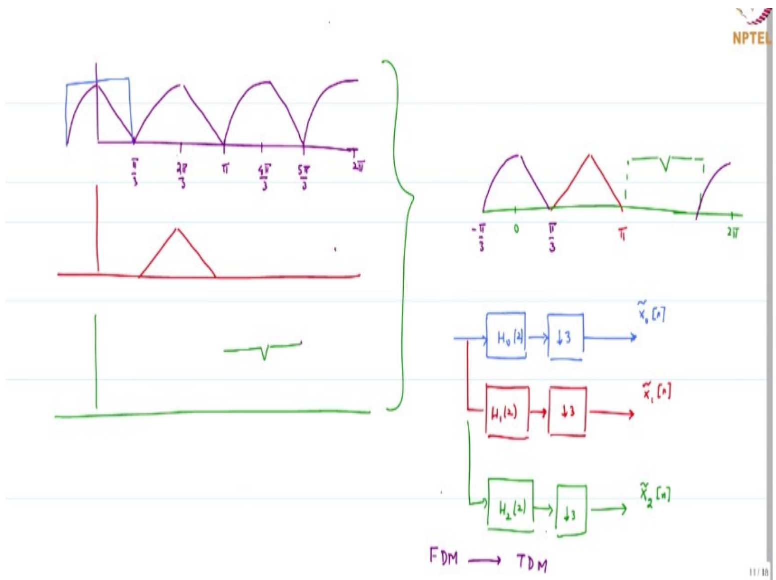
(Refer Slide Time: 09:04)



So basically you could have a part where there is a green sample there is a red sample and then there is a green sample then again the blue repeats, red and the green. So basically you can think of them as interleaved, you deinterleave them pass it through each of those branches and then what you get here at this point so from time division multiplexing you go to frequency division multiplexing. You get a single signal where the spectra of these signals are suitably shifted so that is called a frequency division multiplexing.

So this is why the name transmultiplexer is given because you went from time division multiplexing or time domain multiplexing to frequency domain multiplexing, frequency division multiplexing. And this is a very useful structure in multirate signal processing. It also turns out that it is very useful in communications as well. We will establish the link in a minute. So anytime you do the transformation from time division multiplexing to frequency division multiplexing we must be able to do the reverse operation.

(Refer Slide Time: 10:19)



The reverse operation basically says take each of these signals or take the signal s of n; pass it through a filter, it could be F0 itself just for; I will just call it as H0 of z. Where H0 of z shares the same spectrum as F0. And H1 shares the same spectrum as F1 and H2 shares the same spectrum as F2. So basically the structure is going to be H0 of Z which is a low-pass filter followed by a down sampling by a factor of 3, okay. Please confirm or verify that you will get back the original X0 of n assume the filters are perfect.

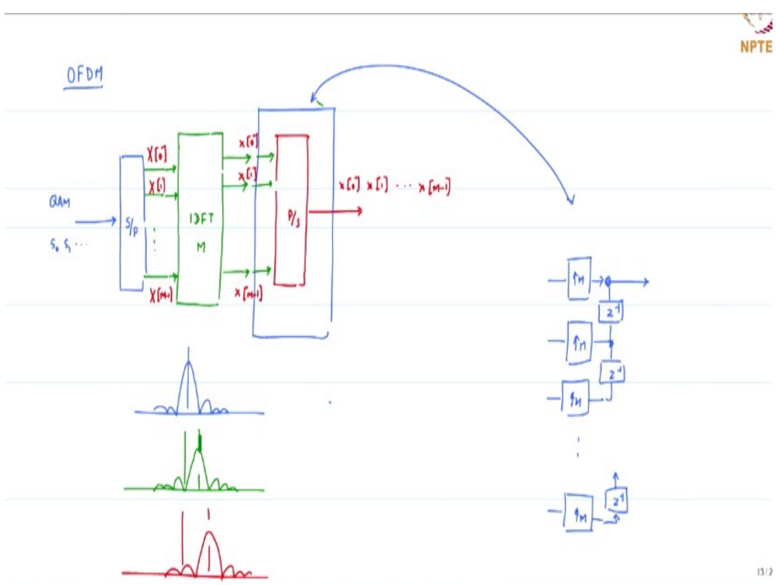
So the same input signal I pass it through a band pass filter H1 of z; it has a total bandwidth of 2pi/3 so I can down sample without aliasing down sampled by a factor of 3 and then get back the original signal. Hopefully this is X1 of n with a tilde that means it should be as close to the original signal as possible. This is X0 tilde of n and the third branch same input signal pass it through a bandpass filter H2 of z.

This can also be down sampled by a factor of 3 and this should give me X2 tilde of n and these are hopefully very close to the original signals themselves. Notice that once again you have brought everything back to baseband because these are not at some higher center frequency they are all centered around 0. So this operation basically does the reverse of a conversion it does the frequency division multiplexing back to time division multiplexing.

You basically have separated out the signals in; so that they can be treated separately or independently in the time domain, okay. So what have we done what is the; what have we discussed so far? We have taken a set of parallel signals up sample them pass them through uniform Filter Bank, right. Appropriately shifted filters, we showed that this process or this computational step translates each of those frequencies into non-overlapping signals with suitably shifted center frequencies.

At the other end if you want to separate them out and bring them back to baseband you have the reverse operation. The filters look identical, basically they have the same responses and then the down sampling brings them all of them back to baseband. Okay. Now here comes the here comes a very important element in our discussion. Why did we do this discussion so far?

(Refer Slide Time: 13:32)



Now if you have already; I am sure all of you would have studied OFDM in digital communications, right many; everyone would have studied. You can open any transmitter OFDM transmitter, so this is what you will find. You will get basically you will let me write it as some QAM symbols, okay. Let us call them as S0, S1… these are QAM symbols. Then the first step of this process will be a serial to parallel converter where you will split this into parallel streams.

Let us say that there are M parallel streams. So this is number 1, number 2, number M. And what we do next in the OFDM transmitters is that you take a M point IDFT. This is a IDFT of size M, okay. Now these signals; so let me just label them I think maybe the convention that you would have used let me just use the same convention. You probably would have labeled these signals as uppercase X of 0; this one as uppercase X of 1; this is uppercase of M-1, okay.

Even if you did use something slightly different we will assume that this is the labeling scheme that you have followed. This will produce for you after doing the IDFT; it will produce for you a set of M samples those are typically referred to in terms of lowercase x of 0; x of 1; x of M-1 okay. And after this you would have had a parallel to serial converter; parallel to serial converter. So what would be the output coming out of this block would be the lowercase x0 of 0; x0 of 1; x0 of M-1.

So 1 block of data you converted from input stream you paralyzed it; took the IDFT and then did a parallel to serial conversion, okay. Everyone agree that this is the standard transmitter of an OFDM block, okay. Now here is where the interesting part comes. Can you please tell me how you would having studied the multirate part? How you would characterize this portion? Parallel to serial converter we have already seen is nothing but a parallel you up sample by a factor of M.

Each of these streams you up sample by a factor of M and then add them with suitable delays. Am I right? So the next stream will be up sample by M it will pass through two delays and so it gets two units to the right, so… the last one is up sampling by M passing through M-1 delays and eventually getting to the output, okay. So this portion is this. So this parallel to serial converter in our structure is nothing but a bank of up samplers followed by delays and then you add them to the other side. Okay.

Now, IDFT followed by a Filter Bank. IDFT followed by a Filter Bank, okay. So basically if you go back and look at the structure that we derived followed by delay chain followed by the IDFT basically tells us that the filters are shifted versions of it. So in other words if you now combine this multirate structure that we have with the IDFT this basically says that the first branch basically is a Filter Bank that has this type of a response that is the first branch.

The second filter is going to be shifted, is a shifted version, okay. The third one is a basically it is one more shift, okay. So you can see that these are the shift and if you now want to think about it as moving the moving the interpolator to the front, okay. What is it doing? It is a transmultiplexer, because a transmultiplexer structure what does it have? It has got basically the one that combines them has got the up samplers followed by a Filter Bank. So what is the OFDM transmitter what the OFDM transmitter format?

Basically is nothing but taking these symbols X0 X1; X of M-1 and applying it to these carriers and then doing a frequency division multiplexing that is OFDM. Okay. So in other words the tools that we have developed the insight that we have developed in our multirate framework where we say that okay if I have the IDFT it is actually not just a transformation it is actually a Filter Bank. The underlying filters are a DFT Filter Bank.

They basically span from 0 to 2pi. So once you have that understanding and you can say that ok parallel to serial conversion is nothing but is nothing but a delayed up sampling block followed by a delayed chain. The IDFT combined with it is a bank of filters. And then actually what we have done is; so in other words what have you done in OFDM.

(Refer Slide Time: 20:51)

So OFDM the basic principle is as follows. Whatever is the bandwidth 0 to 2pi you think of it as divided into number of narrow carriers, okay. And the number of carriers, this is; so basically each of these lines is the center frequency so this would be carrier number 0, carrier number 1, 2 all the way to M-1 because this would again limit to this is 2pi, okay. So that would be the carrier frequency. So now if you were to think of it in terms of the bandwidths it you would have to draw it in this fashion.

So the bandwidth is actually; this is channel 0; this is channel 1 and the center frequency channel number 2 and channel M-1 is situated here, okay. So what is the OFDM transmitter actually doing? The OFDM transmitter says, okay channel number 0 you please transmit for me symbol S0 some QAM symbol, okay and it says okay channel 1 you transmit S1, you transmit S2… and at the other end what would you do; this is an FDM signal.

What would you do on the other end? You would do the inverse operation. You would basically convert it from a center frequency back to baseband and then pass it out. And what will that what will that structure be structure will be the reverse operation that we have done where you have filters followed by down sampling, down sampling is a demultiplexing operation which is very straight forward we know that, that is a series of delays followed by a down sampling chain.

Now when you; to cancel out the IDFT you will have the DFT sitting there and once you once you cascade this together you find that you now have a very, very different interpretation of OFDM. It is not just the IDFT or DFT. So it actually is taking its leveraging the fact that you have a filter bank sitting there; the operation is that you have taken your spectrum divided it into narrow channels and you are transmitting symbols on each of those carriers, okay.

Now the advantages of OFDM become very, very simple very, very powerful. If one particular channel is not good let us say this channel was not good it had for some reason it had either interference or a lot of power, you can choose not to transmit anything on that. So in other words you can transmit S2 on the next carrier, so that is point number 1. You can choose to omit certain carriers which are undesirable okay that is a huge advantage.

And where does the inside come from when you do when you treat is as IDFT; I do not know maybe one channel got 0 sample. No, no that is not the way to interpret it; you interpret it as one of these channels has got high lot of interference or the signal-to-noise ratio is not good so you have chosen not to transmit anything on that particular carrier, okay. The other element is that you can transmit all of these signals with the same power level that is normal DFT, okay.

Or if you can look at your constellation S0 S1 or you apply them all these symbols are coming at the same power level. Now if you found out that for example S1 had got very good signal to noise ratio, okay then how would you take advantage of it in to maximize capacity. So what you would do is you would boost the power of this particular signal, right. You transmit more power on S1, so in other words the spectrum would now look like you had this is S0.

And then S1 alone you are transmitting with a lot of power. And then all of the others at normal power let us say, okay. Now how do you achieve this? You could scale S1 apply more power to S1 and then pass it through a normal DFT where but that would basically mean that you are transmitting more power on this channel. Now why would you transmit more power if the signal the channel is already good, in that case if this was a for QAM you may be transmitting a 64-qam on this particular carrier, okay that is how you achieve capacity.

This is this is related to the concept called water filling, okay. Water filling basically means that when you have certain channels which have got good signal-to-noise ratio you allocate more power to them compared to the other channels. And why would you allocate more power because I can send more bits per symbol on those channels. So you can see how the OFDM and all of its advantages actually come into play in a very nice way in terms of our understanding of the OFDM.

And its use in a wideband system primarily from the transmultiplexing operation, okay. **"Professor - student conversation starts"** Sir, what are these QAM signals you are talking about? **"Professor - student conversation ends"**.

Okay. So basically if you have a sequence of bits that you want to transmit so the modulation method basically says combine them into sets of; if you want to combine as 3 bits then you can you will have 8 values and you may transmit these; let us take 4 bits it is easier to do 4 then; so if I take 4 bits then I will have 16 possible values and the possible combination symbols that I will transmit are one of these 16 QAM symbols, quadrature amplitude modulated symbols.

So rather than transmitting it as 4 bits I transmit one symbol let us say 0111 let us say it was this particular 1 this amplitude and phase the information is carried in the amplitude and phase. So I transmit one symbol with a particular amplitude and phase which the receiver will then detect and then we will map it back to 0111. So I do not transmit four bits I transmit one symbol which represents multiple symbol.

So each of these are actually these QAM symbols, you have taken the bits and you have grouped them into the appropriate constellation and then you are transmitting this information not on a single carrier this is why it is called a multi-carrier transmission OFDM is called a multi-carrier transmission. Because what you have is actually a bank of carriers and each carrier you have placed one symbol appropriately and then you combine them. How do you combine them?

The transmultiplexing operation. Basically, you shifted them in frequency and combined them. But that becomes the kind of comes for free when you use the IDFT with this particular structure. IDFT and this structure actually implement a filter bank for you. So when you apply them you automatically get the translation from time division multiplexing to frequency division multiplexing.

And therefore and of course this also gives you a lot more insight into when do you not transmit on a particular channel or one a particular carrier when you boost the power of a particular carrier again all of these are flexibilities that come into play when you have an OFDM system. Now the other element that which we have not touched upon today is when you do an OFDM transmission I have; there is one more element that is in this one.

What is that? You insert something called a Cyclic Prefix. You take this data output; you take some portion of it and then move it back and call it the Cyclic Prefix, okay something called a Cyclic Prefix, CP. Now CP also has got a beautiful explanation on the context of multirate signal processing. Okay. So when we actually get into with it this is not the entry point into OFDM this was more of okay we have finished filter banks.

We are going to actually look at the maximally decimated filter banks for some time. So it is not it is not the application area. OFDM is one of the application. I just wanted to already establish for you the link to OFDM. Now once we have this in place we will come back and analyze it the next chapter that we will be studying is actually OFDM. We will once again revisit this issue and understand you know how OFDM works.

Why is the multirate signal processing? Where are the insights that we get? Why Cyclic Prefix? How long should the Cyclic Prefix be? All of those questions get answered once you have a DSP perspective on what is happening in the channel, okay. So what is the actual topic that we are going to do next is an M Channel maximally decimated Filter Bank. So basically you have the analysis filters each channel is each of them has bandwidth 2pi/M and each of them can be down sampled by a factor of M and then reconstructed.

So basically this is the broad framework that we would analyze and we will spend our time analyzing M=2 that gives us a lot of insight and basically we will leave the general case as a basically a theoretical framework not spent too much time on that. But good insights that we will get from here, we will pick it up from here and tomorrow's class. Thank you.