Multirate Digital Signal Processing

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Lecture – 11 (Part-2)
Fractional Sampling Rate Change – Part 2

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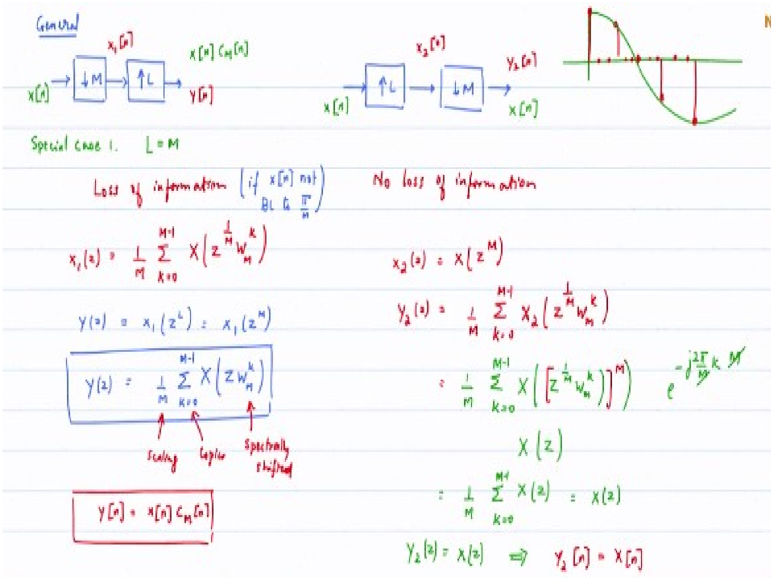

Okay, now we will go back to our earlier problem.

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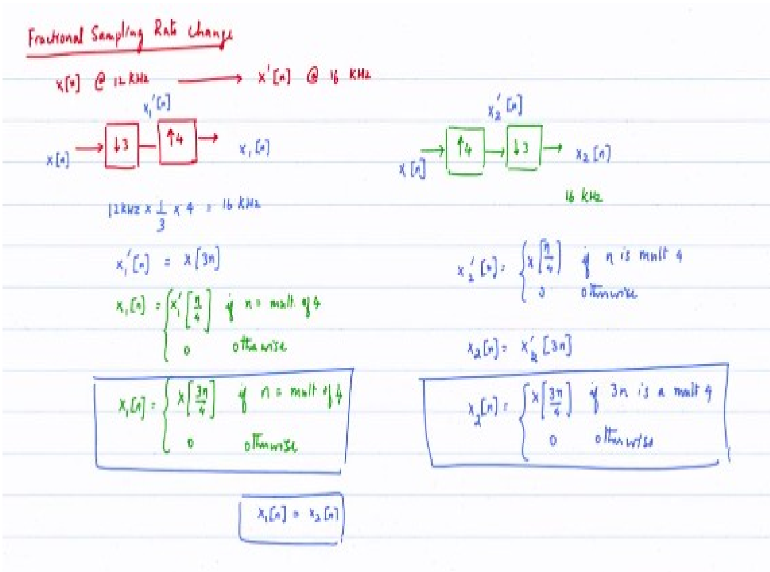

So we will look at this one and say well time domain sequence when they are different and this particular case happen to be the same. So let us see how do actually we go about the process of sampling rate change. So the question to make sure if I understand you correctly, now if I take input sequence and I do a sampling rate change I produce a new signal in this particular case x1 of n strictly does not satisfy the sampling rate change.

Because it is something which has got the correct sampling rate, but spectrally is not correct. So how do you get the spectral correctness or how do you get the same input signal continuous-time signal sampled at one frequency translated in to the same continuous-time signal at a different frequency, that is possible if you associate the filtering with each of those steps.

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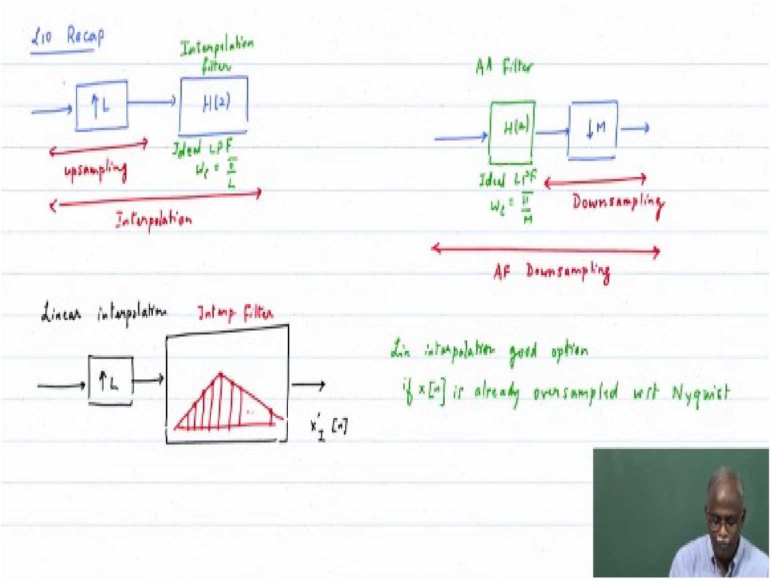

So you must go back and look at we mentioned that at the beginning of the class that there is a combination of the filtering that is associated with the up sampling, there is a filtering that is associated with the down sampling, if we do both and in fact since you asked the question it is a good place to answer that particular one.

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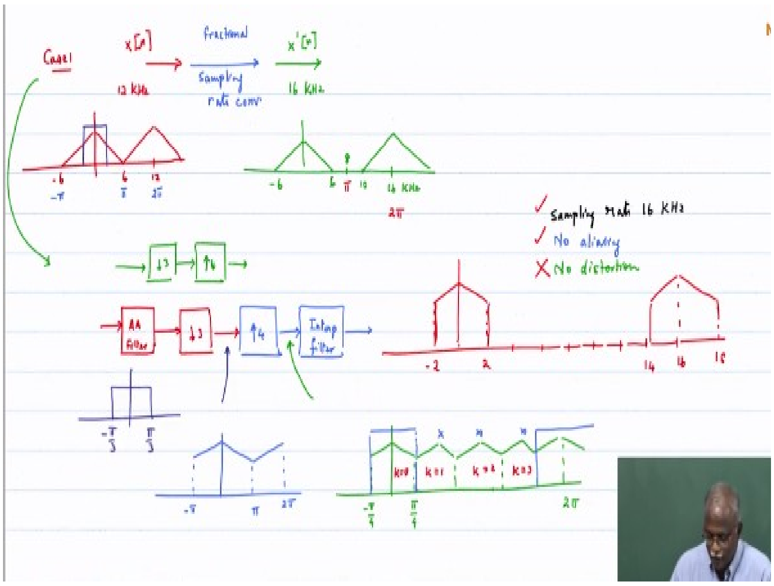

So let us take case one okay, so I have let us say signal at x of n which is at 12 kilohertz let us just, the spectrum looks like this -6 kilohertz to 6 kilohertz it is sampled at 12 kilohertz so basically Nyquist sampling. So this is 12 kilohertz. So if I expressed it in in terms of the discrete-time frequencies this would be 2pi, this would be pi, –pi. So your signal is contained between – pi to pi and you have sampled it at the Nyquist rate.

So therefore the spectrum is fully occupying the space between – pi to pi. Now this is what I want to translate into another signal through a process of fractional sampling rate change. So this is a fractional sampling rate change, sampling rate conversion okay and I want to get another signal which is x prime of n which is at 16 kilohertz. So to answer your question what we are this is exactly the problem that we are trying to address.

So I want the original spectrum between – 6 to 6, instead of being Nyquist sampled I now have a gap because I have chosen to sample it at 16 kilohertz okay. So this will be at 10 kilohertz, if this is 2 pi, this is pi and this corresponds to 8 kilohertz okay. So this corresponds to 8 kilohertz. So there is a little bit of spacing between these spectral copies. So as we have mentioned we have to do downsampling by a factor of 3, upsampling by a factor of 4.

So case one I would like to take up the downsampling by a factor of 3 that is downsampling first followed by the upsampling. We said that we must introduce the associated filtering. So which means the actual implementation of this will be the anti-alias filter. Anti-alias filter followed by downsampling by a factor of 3 followed by upsampling by a factor of 4, followed by the interpolation filter, interpolation filter which will remove the unwanted images, interpolation filter okay.

So this is the sequence of operations I just like to see what happens, what happens to the input spectrum so that, so first of all if I want to downsample by a factor of 3 then the anti-aliasing filter is already specified, anti-aliasing filter will go from –pi by 3 to pi by 3, -pi by 3 to pi by 3, now the problem is it is already sampled at Nyquist rates so input signal is sampled at Nyquist rate, but if you tell me down sample by a factor of 3 I am going to apply the anti-aliasing filter.

So the scenario is that the down sampling by a factor of 3 if pi corresponds to 6 basically this filter will keep from -2 kilohertz to 2 kilohertz correct and then it down samples by a factor of 3. So now the spectrum at this point, spectrum at that point is going to be as follows, … after the downsampling this is –pi, this is pi and this is 2pi. So basically I have chopped off a portion of the spectrum because of the anti-aliasing filter.

Then the down sampling process basically stretched the signal and so now it is sitting between –pi to pi okay. Now I do the up sampling by a factor of 4. So at this point the spectrum is going to look like this, input spectrum will get limited to pi by 4 to –pi by 4, -pi by 4 to pi by 4 that is the, because you will get Z power 4, then I will get 3 copies that is K = 1, K = 2, K =3 and then you get the 2 pi.

So just draw these lines. So this corresponds to K = 0, K = 1, K =2, K = 3 and the interpolation filter is going to go between –pi by 4 to pi by 4 and remove everything else outside. So all the way up to. So this will get removed, this will get removed, this will get removed. So the weak constructed signal at this point has the following spectrum. It has got from this to this.

Corresponding frequency is -2 to 2, 4, 6, 8, 10, 12, 14, 16 and at this point it has got the copy. This is at 16, this is at 14, this is at 18. Let us evaluate what we have done I hope you are comfortable with what has been drawn. There was one image between 2 and 6, a second image between 6 and 10, a third image between 10 and 14, those 3 got removed through the filtering process.

Now is your sampling rate 16 kilohertz, final sampling rate? Sampling rate 16 kilohertz? The answer is yes, right. You can give a tick mark for that. Did you avoid aliasing? So condition of no aliasing? tick mark, yes, because we did the filtering the correct way with the anti-aliasing filter. No distortion of the signal or basically did we get what we wanted to get? The answer is no.

Distortion, the answer is a big no and the reason is anytime you do downsampling first there is a risk of loss of information which is very nicely illustrated that yes you did lose information because you did the anti-aliasing filtering you did not run into aliasing problems but the price that we paid was that there is a loss of information. Now on the other hand, had you done the up sampling first?

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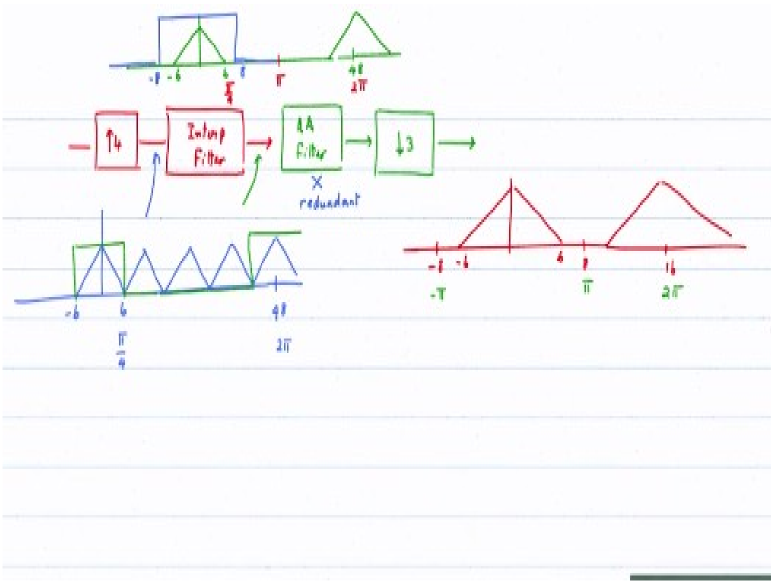

Very interesting to see, you know, and we will just draw it very quickly. So if you had done the upsampling by a factor of 4 followed by the interpolation filter, interpolation filter, then you have to follow it by the downsampling. So the down sampling says pass it through the anti-aliasing filter first, AA filter first, followed by the down sampling process. Make sure that input signal is band limited to pi by 3 to pi by 3 and then do the down sampling and then you should get the signal that is of interest to us okay.

So here is a quick run-through of the just we will just do it in the spectral domain that helps us. So the first step after the upsampling process. I get 4 copies of the signal, so the input signal was at 12 kilohertz. So when I upsample by a factor of 4, it became 48 kilohertz. So this is still 6 kilohertz -6, this has now become 2 pi for me okay. If this is 2 pi then this point is pi by 4, am I right.

So the first stage of the process is here. Now the second stage after the interpolation filter. Interpolation filter says it is a brick wall filter from –pi by 4 to pi by 4 which will then remove all of these images and then retain for you this one. So at this point the spectrum looks like this, it is -6 to 6 with the sampling rate of 48 kilohertz or you can write it in discrete-time. So this corresponds to 2 pi.

This corresponds to pi by 4 and pi by 2 is pi is somewhere in the middle, okay it is not strictly to scale. Now what is the role of the anti-aliasing filter? Anti-aliasing filter says whatever is the input signal that is coming in filter off from –pi by 3 to pi by 3 okay, -pi by 3 to pi by 3 is 48 is 2 pi, pi is 16, pi is 24, so it is actually between -8 to 8 kilohertz, that is the anti-aliasing filter. Now is anti-aliasing filter doing anything at all? There is nothing for it to filter.

Because already the interpolation filter took care of it right. So this is actually redundant. It really did not do anything in terms of the, so then I do the downsampling. Downsampling basically says that this frequency which corresponds to 8 kilohertz which is actually the pi by 3, pi by 3 will become the same as pi okay. So effectively what happens at the output of the downsampling process input signal goes from -6 to 6, -8 to 8, this actually becomes the new pi because of the frequency stretching.

This is –pi, okay, and then another copy of the signal will be there at 2 pi, another copy of the signal at 2 pi, if this is 8, 2 pi will be 16, 16 kilohertz okay. Let me write this one in green and in red let us write the 16 kilohertz okay which is exactly what we wanted. So the 2 things to remember, always keep the interpolation or upsampling with the corresponding filter because those 2 go hand in hand in the sampling rate conversion process.

Even if it is just an integer sampling rate conversion we prefer to look at it not just as inserting of 0 valued samples. We would like to look at it as insertion of 0 valued samples followed by filtering so that you get the interpolated signal. Likewise, downsampling is more straightforward because you just retain some samples, but we always associate an anti-aliasing filter if you want to make sure that you do not want to run into the distortions caused by the aliasing. So this is a very useful hopefully interesting example and exercise.

Now the next step that I want you to look at maybe you can start to take a look at this by yourself, case 2 of our earlier discussion. Case 1, special case 1 was L = M. So now what I would like you to do is go back to that particular example and look at case 2.

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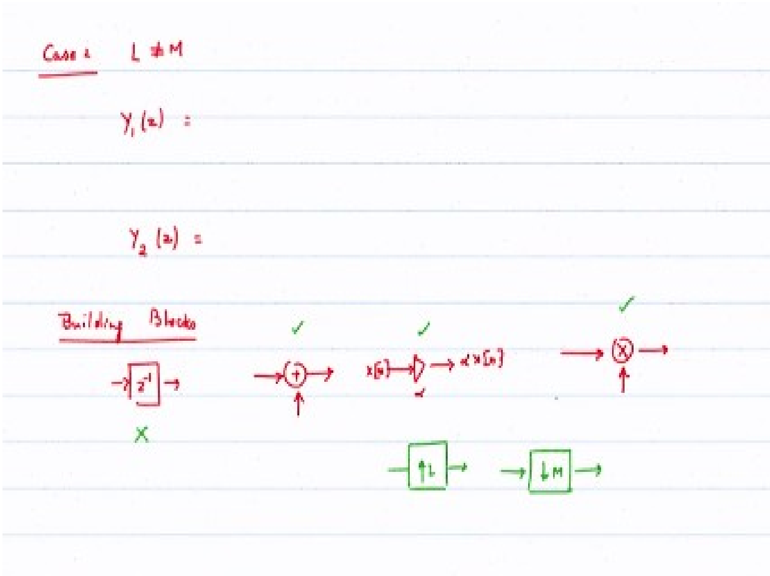

Where L not equal to M, L not equal to M. Now just like we did last time please get the expression for Y1 of Z, get the expression for Y2 of Z and obviously we should be able to, they will not be the same in general, but there will be special cases where they will be the same and that special case must be satisfied by this M = 3 and L = 4. That is one example where we saw that the output signals are the same.

So please do take a look at this and we will build on this in the lectures going forward. So let me just conclude with important point that we keep in mind as we go. So what are the building blocks that you have encountered in DSP? Building blocks in DSP? You have encountered delays right, you encountered delays. You encountered addition blocks, you encountered scaling if you take alpha if this is x of n, this is alpha times x of n a scale factor.

So this is input and output and of course you would have also encountered multiplication. This block we have to leave out because we are dealing with time varying blocks. So delays is actually going to cause a problem. Now this block does not cause any problems in a multirate context. This one does not cause any problem, this one does not cause any problem. So basically any interconnection of these 3 elements along with the multirate blocks.

The multirate blocks are upsampling by a factor of L, down sampling by a factor of M, the most general forms, this combination is going to be the powerful one that we are going to work with. Now when delays come into play then we have to be a little bit careful. So for us the interconnection of these building blocks with the multirate blocks we will just sort of show that all of the properties that you have with these 3 blocks carry forward into the multirate domain.

Basically addition means linearity, scaling again refers to linearity and multiplication is a memory less operation which you can do in the presence of multirate blocks as well. So we will expand on this, expand on the fact that when can you interchange the downsampling and upsampling and then how do you extend it now beyond the just the building blocks into domains where you have filtering also into play. Basically these delay elements also coming into play. So those will be the next step that we will be looking at. Thank you.