Multirate Digital Signal Processing
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Lecture – 06
DT Processing of CT Signal Example

Good morning, we begin lecture 6 as always we begin with a quick summary of lecture 5, we will look at some examples and build on what we have already developed in the last lecture. The key components from today's lecture will be the 2 examples we were in the process of discussing one of the examples in the last lecture, we will complete that look at one more example which shows us that how using DSP and in particular eventually, multi rate signal processing can help us mimic some of the analogue processing that we have.

This then leads us to the 2 most important elements in multi rate signal processing that where we actually the blocks that change the sampling rate, so the up sampler as you will be familiar with is denoted by a block which is indicated with an up arrow.

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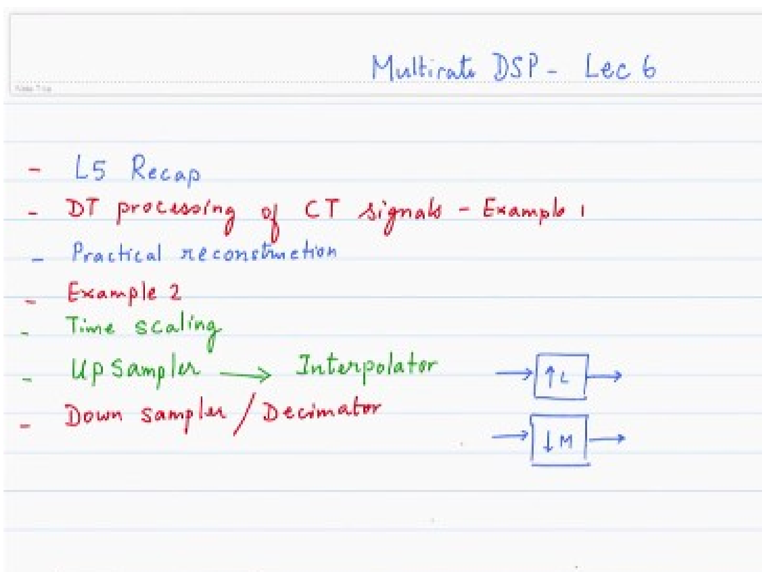

So, the input signal compared to the output signal, output signal has got a sampling rate which is L times higher, an integer factor of L higher than the input rate and the down sampler is the operation that goes in the other direction, it reduces the sampling rate increasing the sampling period and it increases by a factor of M, so the up sampler and down sampler are our basic building blocks and multi rate signal processing and we will be looking at that in a fair amount of detail.

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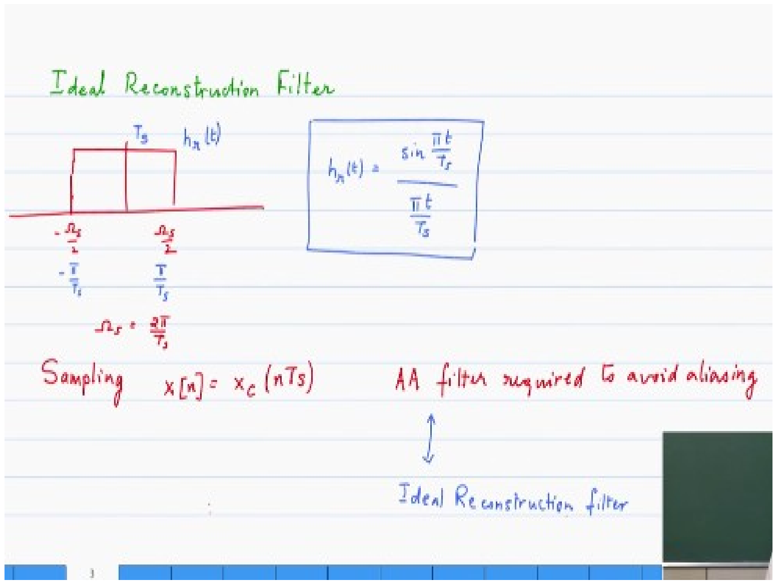

So, let us quickly summarize the key contents of lecture 5 and then build on that so the; if I have a signal x of n, all possible sequence operations on this discrete time sequence can be captured in the following, sequence operations can be represented as y of n equal to x of + - M by N times n + - N naught, okay so I can have positive shifts, negative shifts, I can have increase in sampling rate, decrease in sampling rate, I can have time reversal as well.

So the all possible combinations basically boiled down to 3 operations; shifting, time scaling and time reversal and as we mentioned in the last class, the preferred sequence of operations one would be; the first one would be the shift operation, second one would be the time scaling and then that and the third one would be the time reversal okay, so this is something that we looked at a few examples again, we would not spend a dwell much on that assuming that is a concept that you are familiar with.

Then in the last class, we also looked at one application of multi rate DSP, we looked at a case where we needed to have a shift by .44 times Ts, okay in the context of a mimicking the acoustics of an auditorium so basically, just this is an illustration this one we cannot achieve in our existing DSP framework because we can only get multiples of the sampling period and this multi rate signal processing says that yes, I can get the required fractional delays in very simple, very elegant methods and that is one advantage that we have, okay.

Then, we also established a relationship between discrete time and continuous time and let me just rewrite that in the following way, so we are constantly going to be going back and forth or at least working in the discrete time domain but keeping in mind that there is an underlying continuous time framework which you want to keep so basically, there is a linkage that we always want to keep.

In the discrete time domain, it is a sequence x of n which can be represented in terms of its discrete time Fourier transform X e of j omega, okay, now we said that this is the we can link it to the corresponding continuous time by saying this is equal to 1 over Ts summation k equal to - infinity to infinity Xc of j omega by Ts - 2pi by Ts into k, this was we established this linkage in the last class. Because this is nothing but the sampled signal Xs of j omega where I have substituted omega equal to omega by Ts.

So again we know how to get x, the sample signal, you multiplied by the Dirac deltas and then that produces the sequence, so you can relate it to the discrete time Fourier transform and the continuous time transform through this link because this is the; this is related to the sample signal xs of t. So, ultimately x of n and xs of t are linked, see that is our ultimate element, then the last part of the lecture yesterday was the discrete time processing of continuous time signals; discrete time processing of CT signals.

We said that if LTI is preserved that you if the overall system is LTI, then we can write down the following result that the input output of the analog system, in input analogue continuous time signal and output continuous time signal can be thought of having an effective overall transfer function, H effective of j omega which is given by; which is related to the discrete time LTI system that is sitting inside the processing block.

So that would be equal to H e of j omega Ts mod Omega less than pi over Ts equal to 0 otherwise, so we showed that under certain conditions, we can treat the overall system as LTI and also be able to link the overall transfer function, the H effective j omega to the discrete time transfer function; transfer function of the discrete time system okay, so this is a summary of the key points, let us just build on the element.

But maybe one more observation which I just wanted to highlight before we move on, the ideal reconstruction filter, we have talked about that on a couple of occasions is something that goes from - omega s by 2 to omega s by 2 and has got a scale factor of Ts, okay. Now, this is what removes the unwanted images outside of omega s by 2 and gives us the reconstruction and we know the reconstruction filter.

The anti-aliasing filter; if I am going to sample at omega s, what does that; what does the requirements of that? It is also a brick wall filter going from - omega s by 2 to omega s by 2, the only difference is it does not have a scale factor of Ts but in terms of the spectral properties they both are exactly the same, so the anti-aliasing in the ideal case, so the ideal reconstruction filter, so the anti-aliasing filter is; we can say that is linked to the ideal reconstruction filter.

So, the both the process of sampling and reconstruction again, show similar things that requirements and ideal reconstruction filter okay, so keep in mind that is for a particular sampling frequency, anti-aliasing and reconstruction require filters of similar properties, ideal reconstruction filter, okay I thought you may have already noticed it but I thought I should mention because they are actually identical for a given omega s.

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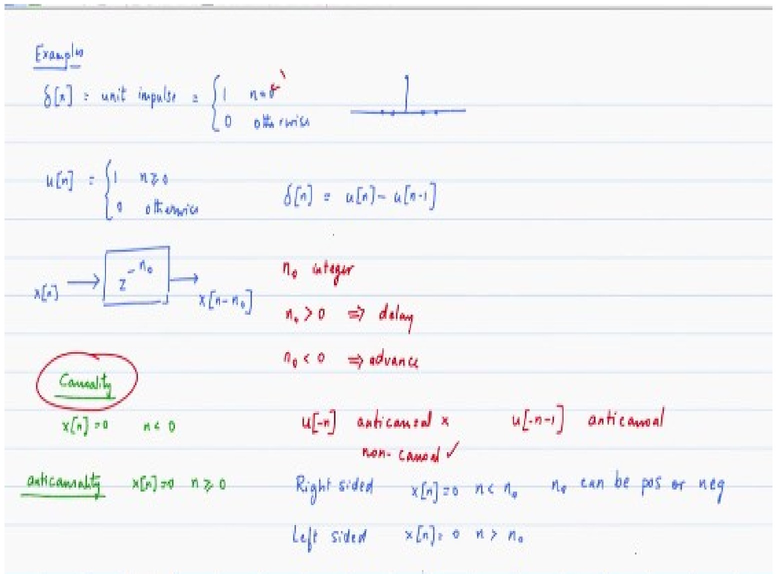

So, now a few examples so that sort of just to apply the things that we have, run through of some examples which I am sure you will be familiar with but I thought it is good for us to okay, we do talk about a discrete time impulse, unit impulse, so this is unit impulse very different from the Dirac delta that we have in the continuous time and this is defined as a signal that is equal to 1 for n equal to 0 and 0 otherwise.

So, again it is a very useful signal basically, it has got and it is 0 everywhere else okay, then we also have a unit step function which is equal to 1 for n greater than equal to 0, 0 otherwise, again another very, very useful function when you want to ensure causality, right sidedness all of this, this comes into play now, there is a form of writing the unit impulse in the; using the unit step which comes out to be actually a handy method; handy tool for some of the applications.

So, delta of n can be viewed as u of n - u of n – 1, unit step - the unit step shifted by 1 unit of time, okay again several such simple sequence manipulations come in handy for specific applications and then we will highlight that but something for you to keep in mind similarly, there is a unit ramp function which can also be related and several interesting results can be obtained.

I am sure you can look up the references, the aspects of delaying a sequence; very, very important for us, so if I have a sequence x of n and I pass it through a block z power – n naught then what I get at the output is x of n – n naught, okay that is the sequence that we get and n naught has to be an integer, so we only are dealing with those cases of course, with multi rate techniques we can produce a fractional delays.

But when you have an input output given by a relationship shown by this block n naught has to be an integer, so n naught has to be an integer typically, n naught greater than 0, this corresponds to delay, a shift to the right in terms of the indexing, this is a delay operation. Now, unlike continuous time we can actually shift the sequence to the left as well because it is a sequence, it can be shifted in either direction.

So, we do make a note that we can produce sequences with n naught less than 0 which would be an advance operator okay, so this is something that we can do very easily in DSP which we cannot do typically in a continuous time system, so this is feasible for all sequences, we can shift them forward to the right or to the left you can either delay them or we can advance them. Now, as part of the examples, I just want to you to just, we will include some definitions, basic definitions.

It is useful for consistency when we talk about these things later on in the course now, definition of causality; we have to be very careful, very strict in terms of our definitions because these are non; these are tight definitions so, causality basically says a sequence is causal if x of n is equal to 0 for n less than 0, okay that is a condition for causality and likewise there is a condition for anti-causal or anti-causality, okay.

And that says that x of n must be equal to 0, for n greater than or equal to 0, so there are sequences that will be that can be neither causal or anti causal which means that they are non-causal sequences and those are very common that we work with okay, so let me ask you to answer a few simple questions. So, u of – n, is it an anti-causal sequence? No but it is non-causal okay, so this is not anti-causal because it has got a value at n equal to 0, so it is not anti-causal.

But it does satisfy the property that and it is not causal either so, it is actually non-causal okay and what about u of - n -1 that would be anti-causal because you shift u of n by 1 unit of time and then time reverse it then you will get a anti causal sequence okay, so strictly this is the definition and it is good that we are comfortable with this result okay and again how to get u of -n – 1, I am sure you are familiar with.

Now, relating to causality is a another property which is called right sidedness, okay, right sided right sided is in a way it is like a generalization and this one says that x of n is equal to 0 for n less than n naught, okay and n naught can be positive or negative, okay so basically it says that at some index it starts and then it exists only for these index values greater than some value and this if this n naught happens to be 0 you get causality.

If it is something that is negative then you will get non causal sequences but it still it can be non-causal and right-sided, okay and similarly, non-causal and left-sided so, a left-sided sequence would be something that is 0 for n greater than n naught, again n naught can be positive or negative and therefore we can get, so one example.

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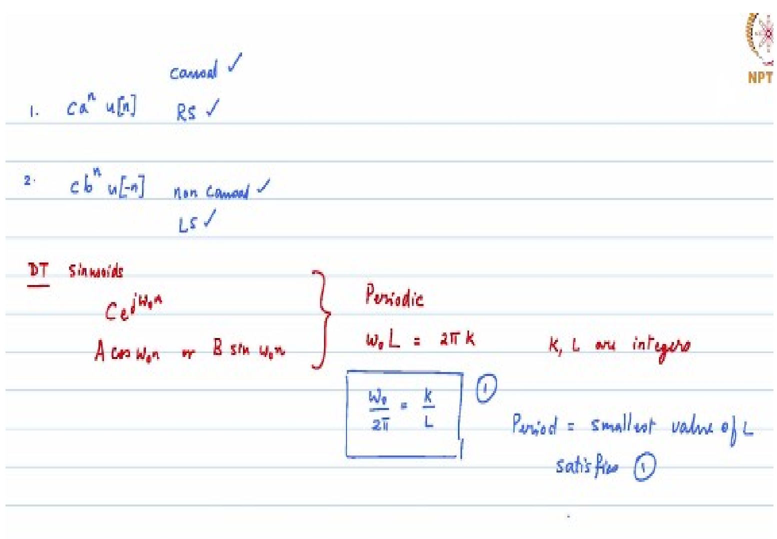

If I have a complex exponential, c a power n u of n okay, so this would be causal yes and right sided of course, causal means it would be right sided, the other one would be a second example would be c times b power n u of – n, this is left sided clearly and it is not anti-causal, it is non-causal, okay because it has got, it is defined at n equal to 0 again, the same property that we discussed earlier.

One of the things that we will not review spend time reviewing but I am I will assume that you are familiar with is discrete time sinusoids, we do extensively leverage the properties of sinusoids particularly the property that they are Eigen functions of LTI systems, so any sinusoid of the form c e power j omega naught n discrete time sinusoids of a frequency omega naught, complex sinusoid or you can have a times cosine omega naught n or b times sin omega naught n.

Again, all of these forms are have the same property now, unlike continuous time sinusoids these are not guaranteed to be periodic, only under certain conditions on omega they are periodic and again this one to make sure that we are familiar with the basic definitions of periodicity for the discrete time sinusoids, so periodic properties is valid or holds for a discrete time sinusoids, if we can find an integer L such that Omega naught L is equal to some multiple of 2 pi; integer multiple of 2 pi.

So, basically K, L are integers, okay and very often this is stated as a condition which says discrete time sinusoids are periodic, if Omega naught by 2pi is equal to a ratio of integers which is K by L and the period itself; period is the smallest value of L which satisfies this equation 1, okay, this period is equal to the smallest value of L, value of L that satisfies 1; equation 1 okay, so again this I am assuming is something that you are that you will be familiar with something to keep in mind that we are very much interested in sinusoids.

We are very much interested in periodic sinusoids and multi rate signal processing leverages that in a very rich manner okay.

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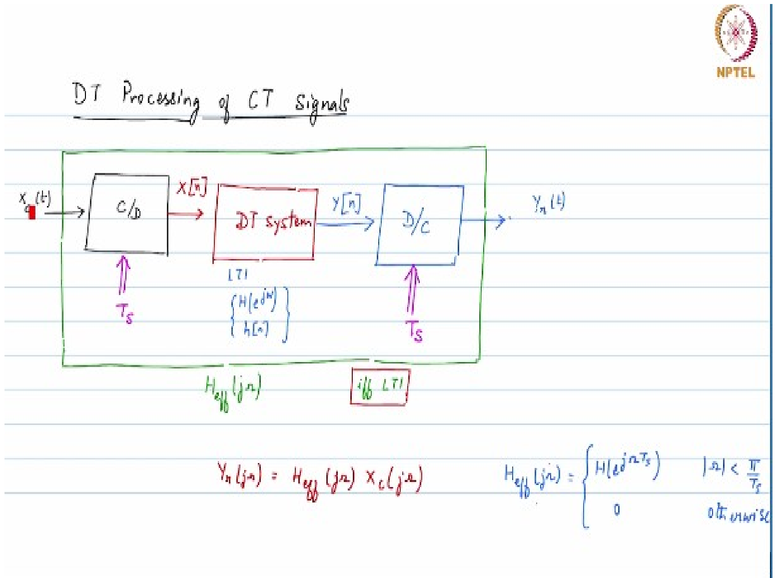

So the we pick up the example that we were talking about in the last lecture discrete time processing of continuous time signals, I have a continuous time signal that I am feeding into my block, first block converts it into a discrete time sequence, process it using a discrete time system LTI system. Then, we have a D to C block; D to C converter that converts it back now, under the conditions of LTI, we showed the effective transfer function from the input to the output given by the related to the discrete time; transfer function of the discrete time system okay.

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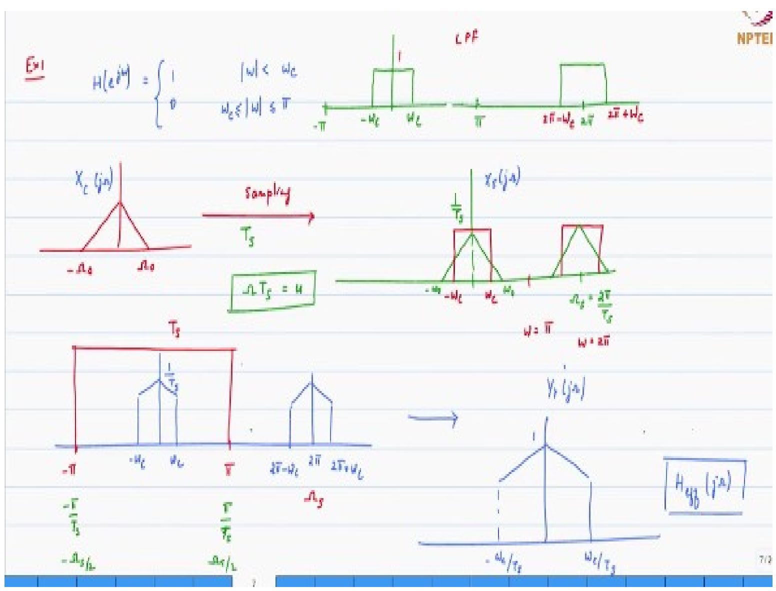

Now, yesterday's lecture, the one that we were trying to develop and you know we could not reach almost the last step but we will let us just pick it up from there and conclude our discussion. The discrete time system has got the following transfer function which we sketched and observed that this is a low-pass filter and that it will have periodic repetitions of the spectra, so from – omega c to Omega c, we have a transfer function that has got a amplitude of 1.

And then basically, it is 0, the rest of the region between - pi to pi, so if I have a continuous time signal xs of t, x; this would be Xc of j omega and this is a band limited signal, I am assuming, I am choosing a suitable sampling period that will avoid aliasing it produces for me the spectrum which is Xs of j Omega scale factor of 1 over Ts, we have got multiple replicas, those replicas are at multiples of omega s which is equal to 2pi over Ts, okay.

Now, we said that we could map the continuous time frequency to the discrete time frequency, so it basically just multiplying the continuous time frequency by Ts, so if you multiply by Ts, we find that this corresponds to Omega s corresponds to 2 pi and then the midway point is omega equal to pi. Now, with this if you now superimpose the once you have translated it into the discrete time frequency we can take the filter and superimpose it

And we said that if this was what it worked out to be, so if this was - Omega c to Omega c, so that would be a periodic spectrum but we will focus between from - pi to pi, what comes out between -pi to pi is a filter of this form, so basically what gets stop; in any portion of the signal that is outside of Omega c gets removed by the digital low pass filter, okay. So, what you get is periodic spectra which are looks like this blue figure that we have.

And then when you come to the last stage of reconstruction, you multiply it by a low pass filter which again corresponds to a pi over Ts, continuous time frequency will be pi over Ts, - pi over Ts to pi over Ts, pi over Ts or - omega s by 2 to omega s by 2, all these are equivalent representations of those frequency markers and then all other images would be removed and so what we will be left with is the scale factor is now gone.

And what we are left with is a portion or a replica of the original spectrum but with portions outside of Omega c by Ts have been removed and that is because of the low pass filter that was there in the continuous in the discrete time, this is where we stopped in the last lecture, so is the process of or the basic framework of this example where we applied discrete time low pass filter to analog signal and the reconstructed it to get an analogue signal where portions of the spectrum have been removed.

Basically, if this would be the same as if you had a continuous time filter which was operating on your continuous time signal, okay is this basic framework is it okay, first thing that would like to reinforce is what is the H effective of j Omega?

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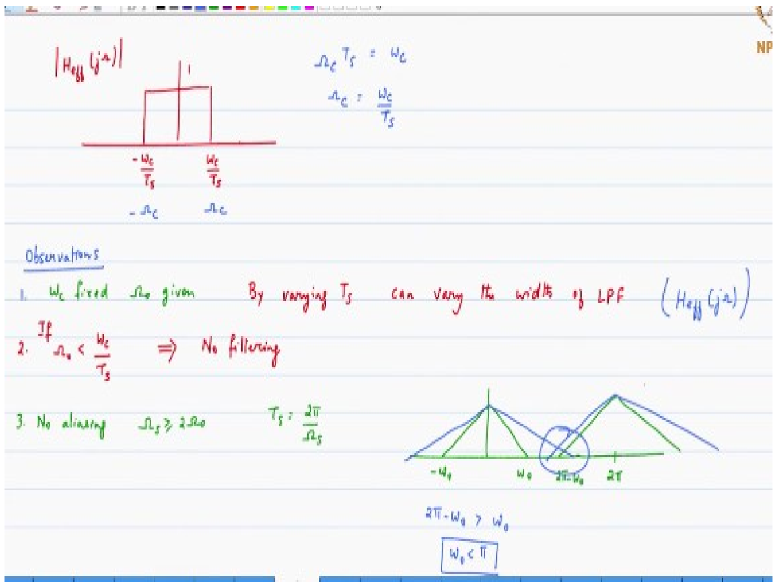

So, if I were to ask you please sketch for me H effective of j Omega, let us say I am interested in the magnitude, what would be the overall response, so the overall response H effective of j Omega would be a low pass filter with the magnitude of 1, it goes from - Omega c divided by Ts because that is what I converted into the continuous time frequency, this would be from Omega c by Ts that is exactly what H effective of j Omega is

And this is; this we can call if you want as the supposing, I had Omega c times Ts is equal to Omega c, so basically the cut off frequency of the discrete time filter, if I want to map it then I can define a cut off frequency in the continuous time which is equal to Omega c by Ts that is how you would specify a low pass filter in continuous time with a cut off frequency, so this would be - Omega c, this would be Omega c, okay.

Notationally, I am repeating it so, we are comfortable going, drawing a particular spectrum and labelling them either in continuous time or in this equivalent discrete time notation, so that we are okay now, the key element comes in the discussion that is now to follow, so please note the following we will be making a set of four observations, okay and I will take time to make sure that each of those observations are clear okay.

The first observation is that Omega c, we can treat as something that is fixed because that is the low pass filter that has already been designed and it is given to us, Omega c, I want to treat it as fixed, second one is that Omega naught is given, right I am given a signal with a certain band limited property, okay. Now, a very key observation based on the figure that is on this picture is that what happens when I vary Ts.

Ts is something that I get to choose, I know what I should choose to make sure that there is no aliasing, so by varying Ts what happens; by varying Ts, I cannot change Omega c, Omega naught is given to me but notice the effective filter H effective of j Omega is going to get scaled made narrower or wider by the choice of Ts, by varying Ts we can vary the bandwidth; effective bandwidth, can vary the width of the low pass filter.

Because the width of the low pass filter depends on Ts, I get to choose Ts, okay, so the width of the low pass filter, now this is one of or effectively the bandwidth of H effective of j Omega, now this is actually one of the advantages why you would want to go to the discrete for discrete time processing. Let us say that you are not sure what is the bandwidth that you; how you need to cut off the; where you need to cut off the continuous time signal.

And you want to keep that flexibility but you do not want to keep redesigning your analogue filters then what you would do is; go into the discrete time domain and the only thing that you would need to choose would be Ts, you want it very narrow then, what you would you do; you would choose a high value of Ts, large value of Ts, large value of Ts basically says lower sampling rate, large value of the time period, lower sampling rate.

So, then the filter if you go the other direction where you choose the small value of Ts, then you allow more of the signal to pass through okay, so if you are comfortable with that observation let us then write down the second observation, so the second observation is the following; now, if Omega naught if you have by; let us say if you did end up choosing your Ts such that this relationship holds Omega c by Ts, okay what would happen?

If what happens; this happens what; there is this would mean that there is no filtering why because the Omega by Ts is wider than Omega naught, your input signal so basically, your input signal will go through and come out at the other end with nothing happening to it okay, again interesting observation because you can choose to not filter in the process so, even though it is going into a filter you can make sure that nothing happens to the input signal.

Again, it is an interesting observation basically; this means that there is no filtering, so you actually can see that you can vary the filtering all the way from something where there is very tight filtering to almost no filtering, okay. Now, here comes a couple of very interesting and important observations in the context of our discussions so far now, supposing I want to have no aliasing, okay I want to have no aliasing.

So, now there is a discrete time filter, there is a I would have to choose a sampling period and I know what my sampling frequency is, how I am going to choose my sampling frequency, so if I want to say that there is no aliasing, Omega s must be greater than or equal to 2 times Omega naught from that I will get my sampling frequency, so Ts is equal to 2 pi by Omega s, so this is very clear but I want you to look or at least think about what this means in the discrete time.

So, my discrete time filter is from - Omega c to Omega c, right that is my discrete time filter, now if you were to think of the input spectrum signal spectrum as something that went from – Omega naught to Omega naught when I when I represent it in after the sampling process and I translated my signal into; now, what is the condition in the discrete time domain that I do not get aliasing, you understood the question.

The process of sampling takes me from continuous time to the sample domain, the frequencies map based on the relationship Omega times Ts is equal to Omega, this is the one that maps the relationship, so which means that the input spectrum which is from – Omega naught to Omega naught can be mapped to a corresponding discrete time frequency – Omega naught to Omega naught and just like you have a condition for no aliasing in the discrete in the continuous time domain, what is the corresponding condition for no aliasing in the discrete time domain, okay.

So, if I had a drawing of this, this is from – Omega naught to Omega naught, the repetition of this is going to come at 2pi, this frequency is going to be 2pi – Omega naught and the condition for no aliasing will be 2pi – Omega naught just like we wrote down the condition for aliasing in the continuous time domain now, you can write it down as 2pi – Omega naught must be greater than Omega naught or in other words Omega naught, the highest frequency component must map to less than pi that is a very important observation.

Because you will find that if you did not satisfy this condition, what will happen is you could have gotten a situation where this was wider, this would have been wider in this case and then in this portion, you would have ended up with aliasing and again, so the aliasing notion is not only in the continuous time, it is actually when you are completely working in the discrete time domain also you can still have underlying understanding of aliasing, okay.

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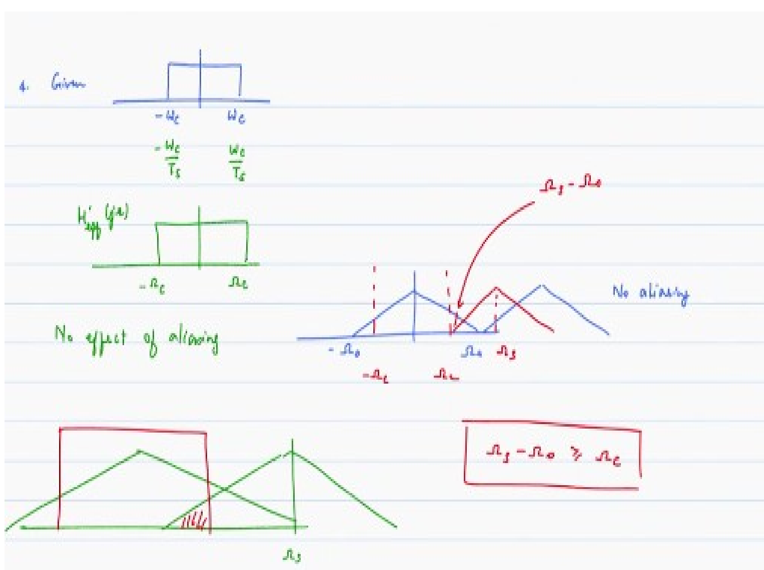

The most important part of this is that I know that I am going to; this the fourth observation, we are given that we are going to be doing the following filtering okay, from - Omega c to Omega c okay, so or in other words the effective filtering is going to be from - Omega c by Ts to Omega c by Ts, okay or in other words if I can relate it in to the continuous time, I am going to be filtering with a filter which basically, goes from - Omega c to Omega c, this is H effective; H effective of j Omega okay.

Now, I want to have the condition that there is no effect of aliasing; no effect of aliasing, it is not no aliasing, please note that I am only saying that I know that I am going to do some filtering and it is going to kill all frequencies outside of Omega c okay, mod Omega c, now what is the condition for no effect of aliasing? So, in other words I am going to allow aliasing to come in as part of my sampling process.

But I am going to make sure that so basically, this would mean that if I had a input spectrum which is of this form, Omega naught; - Omega naught to Omega naught, let us say somewhere here is the boundary for Omega c that is - Omega c to Omega c, no aliasing requires you to create your copies beyond this point, this is no aliasing okay, so this is the condition for no aliasing but the condition for no effect of aliasing basically, says your image can be here, right.

Aliasing will occur but my low pass filter is going to take it out so therefore, I am still okay, so if this is omega s, this edge is Omega s – Omega naught, so Omega s – Omega naught is going to be my band edge that has to be greater than or equal to Omega c, okay because if that is true then I know that though aliasing will occur through the process of sampling because my filtering process is going to anyway get rid of some portions of my spectrum, I am okay to satisfy this.

Now, you may wonder why you take so much of effort to talk about the case where you have aliasing but no effect of aliasing, okay this is because if you want to keep your sampling rate to the minimum, okay you do not want to unnecessarily increase your sampling rate and you know that the certain processing you are going to do, then you can take advantage of it, this is exactly similar to what we did in the CD player, when we looked at the creation of CD content and this is what we were looking at okay.

So, this is important for us as we build and then as we move forward to show that this is the condition that we would need to satisfy to have no effect of aliasing well, I am assuming that it is band limited to begin with, so I would assume that if as long as I had satisfied the anti-aliasing filter is a requirement only if you were; if you had something that was not band limited and you were going to do a sampling process and you did not want to have aliasing.

“Professor – student conversation starts” In this condition, yes because you can think of it as a situation where the anti-aliasing filter would basically filter off up to Omega naught but you are choosing a sampling period which is going to cause aliasing, if it was; see, if you are not band-limited to Omega naught, then there is a problem because what would happen if you are not band limited to Omega naught, can you please sketch for me what that would look like, okay.

If it was not been limited to Omega naught, then my input spectrum would be something much wider okay and I would pick Omega s as before, then what would happen; the copy would come this far and my low pass filter is; so, my signal of interest actually has been corrupted in this portion, so in order for this particular example to work the way we have presented it, your input signal must be band limited to Omega naught.

So, the anti-aliasing filter has got a role to play there but you have deliberately violated Nyquist because you know that you are interested only in a narrower portion not the entire input spectrum, right that is a good question. **“Professor – student conversation ends”** though yeah, so as long as the input signal was band limited to Omega naught, this framework would work; this would work.

And that is one of the reasons why you would always have an anti-aliasing filter preceding your sampling process because you always want to make sure that you are in control of how much or when aliasing will occur and how much aliasing will occur because that is something that you want to be always in keeping in that okay, so what we would now like to do is; move on to one aspect that I am sure that you are reasonably familiar with.

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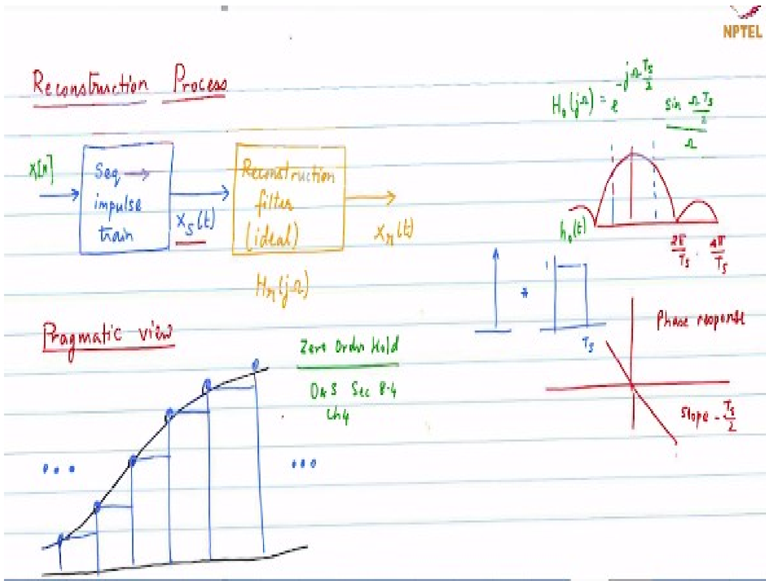

And I would like to pick up on that a part that is the aspect of time scaling because time scaling; did I want to do this first, okay yeah I actually wanted to finish this discussion before we went to that okay, so we said that the process of reconstruction conversion from D to C requires us to convert the samples x of n; x of n are just a sequence of numbers, you must convert them into an impulse train Dirac delta 's where you have the height of the impulse scaled by the x of n that would give you xs of t.

And then you pass it through an ideal reconstruction filter which we talked about and then now in the practical case, how will we do that because creating this impulse sequence train is going to be difficult, so in the practical scenario we do what is called sample and hold, so we take this value of x of n and then create a staircase function which is not the Dirac delta but basically a staircase approximation to the signal that we want to create, okay.

This is called a zeroth order hold and it is a very important one and it is something that most circuit designers are very, very careful in their design and some of the elements of this you can see in Oppenheim and Schaeffer section 8.4, okay that is of chapter 4, okay please do take a look at but what I want to do is just sort of indicate to you that yes we can take care of these sort of impairments quite easily in our multi rate context.

So, what is the staircase function? The staircase function can be viewed as follows as a Dirac delta convolved with a signal which has got the following impulse response from 0 to Ts, it has got a height of 1 and this is a Dirac delta now, if I convolve these 2, then we get the staircase approximation, so wherever the delta occurs I convolve and I get this sort of staircase approximation.

Now, if I call this as H naught of t that h naught to indicate that it is a zeroth order hold, can you; you can I am sure you will be able to do the Fourier transform as H naught of j Omega okay, what you would do is you would advance it by Ts by 2, then you would get a symmetric waveform which you can easily compute, so e power - j Omega Ts by 2 that is the shift, then you get a rectangular function which will approximate the following, you get a sine Omega Ts by 2 by Omega, okay.

So that is the expression of the Fourier transform now, why are we interested in this because we are interested to see what is this spectral characteristics, the spectral characteristics is of course a sinc function which is 2pi by Ts, 4pi by Ts, those are the zero crossings but the important question here is what portion of the spectrum am I eventually interested in the process of reconstruction through the process of reconstruction from – pi by Ts to pi by Ts.

So, a lot of the spectral effects of this sinc function are outside my range of interest okay, so this is – pi by Ts to pi by Ts, now where will that come from; that will come from the reconstruction filter okay, so the key point to note is this particular rectangular waveform has got a sinc response maybe another factor to note is if you look at the phase response basically, -j Omega Ts by 2.

So that means it has got linear phase with a slope of – Ts by 2 that is the; so this is the phase; this is the phase response, another one is the magnitude response okay, now the key point to note is that we are interested in a cascading so, basically what will happen; this impulse train now got replaced by the staircase function, after this will come a reconstruction filter, so the we now cannot use the old original reconstruction filter.

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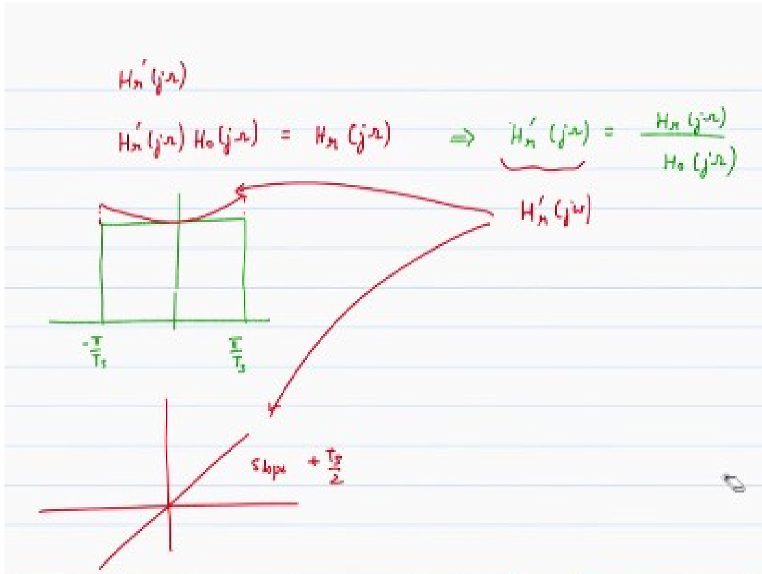

But we need now need to use Hr prime of j Omega and the condition on Hr prime of j Omega is that Hr prime of j Omega since it is a cascading of filters multiplied by H naught of j Omega must be equal to the ideal reconstruction or I would like it to be as close to the ideal reconstruction filter as possible okay, so this basically tells me that what is the response of this Hr prime because now we have to design Hr prime to be equal to Hr of j Omega divided by H naught of j Omega.

That means I must compensate for whatever distortion that the staircase approximation is going to do so, basically the ideal reconstruction filter would be of the following form - pi over Ts to pi over Ts, now the staircase approximation caused a droop in my signal, so what I need to do is the basically this compensated filter will now have a response that compensates for the droop okay and of course, that is it; that is the magnitude compensation.

And the phase response of the ideal reconstruction filter is a zero phase filter, so what it will do is to compensate for the phase, it will introduce a phase that has got a slope of +Ts/2 okay, so this is now my reconstruction filter, so the Hr dash of j Omega has got a magnitude response given by the red line has got a phase response given by this line, if I now cascade Hr of j Omega with H naught of j Omega what I will get is effectively H of the ideal reconstruction filter okay.

Now, designing an analogue filter with these sort of compensations may be difficult, so that is where once again DSP and multi rate DSP can come because you can do the pre compensation basically, you can compensate for the input signal x of n before it goes into this staircase approximation, so that you can compensate for this, so basically the compensation is actually done before the distortion occurs and you take care of it in the discrete time domain itself.

So, just sort of to leave you with the thought that there is a lot of power or you know the ability to do things in the discrete domain really manifest itself when we actually have to go back and forth between the 2 segments between the 2 domains and so this is a place where we will leverage and build a lot of the things that we are interested in okay.

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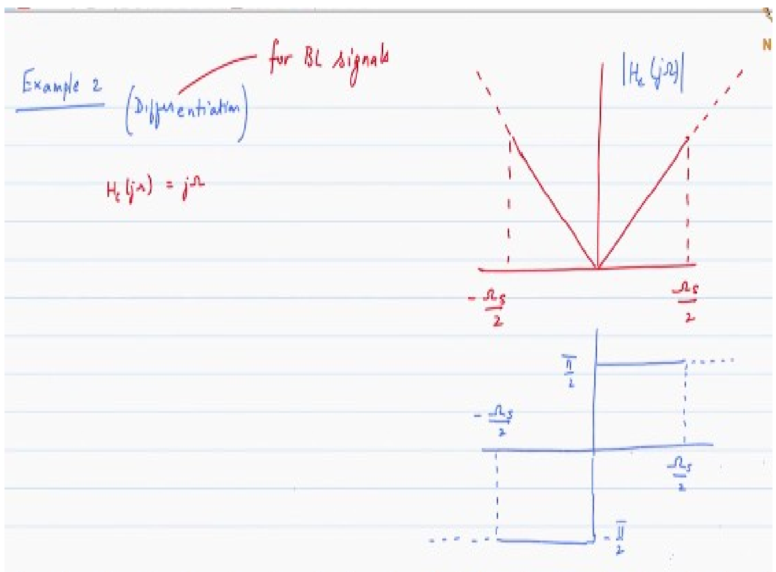

So, there is one more example that I would like to discuss with you this is a worked out example in Oppenheim and Schaefer, so let me just give you the problem statement, the problem statement basically says that we are interested in designing a differentiator, okay now if my input signal is band limited only then I can do sampling, I can only sample that now, if my input signal is band limited the ideal reconstruction; ideal differentiator if you the an ideal differentiator has got the following response.

It is Hc of j Omega equal to j times Omega so basically, it will be a line that goes at 45 degrees that is a magnitude response and beyond the dotted and the dashed line and the phase response will be +pi by 2 for positive frequencies, -pi by 2 for negative frequencies now, if it happens to be that your input signal is band limited to some value Omega naught, you do not need to satisfy these properties beyond that.

So which then says that if it is band limited, I can first of all sample my signal, so I can go to the discrete time domain and if I can go into the discrete time domain, then I can design a discrete time differentiator and then come back to the; so basically my ability to do at the discrete time differentiator is I have a lot of tools to design those, so how will our discrete time differentiator look like, it will have up to pi it will have this line going up.

And similarly, you will have this phase response up to from -pi to pi that is it and then and so using that you can then say that I will sample my signal, pass it through a discrete time differentiator and then convert it back and I would not know whether I did a continuous time differentiator or a discrete time differentiator and this would be a much easier and much more flexible method for me to design a differentiator in practical application.

So, again please look at this we will pick it up from here in the next class, what we are going to be looking at or going, entering into is the area of the time scale when how can I change the sampling period to a lower sampling period or to a higher sampling period, what are the implications in terms of the spectral effects and how do I deal with it and how do I leverage it to our advantage. Thank you.