Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology- Madras

Lecture – 05 (Part-2)
Discrete Time Processing of Continuous Time Signal – Part2

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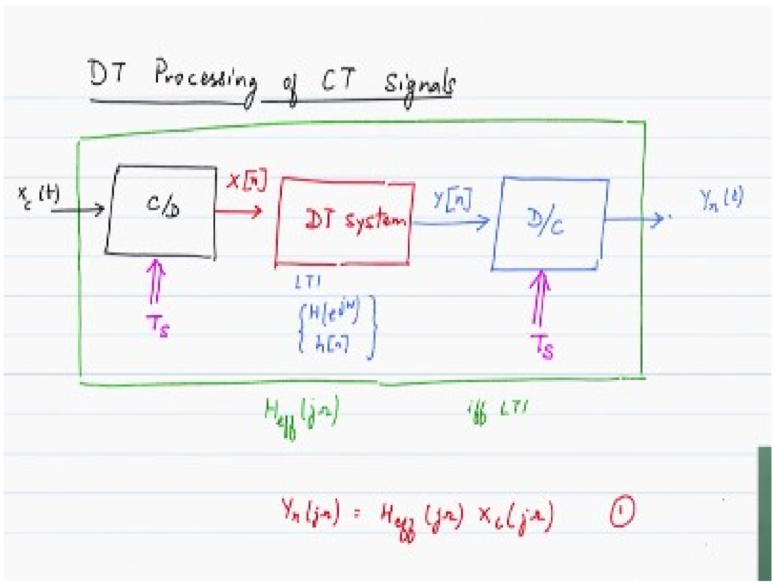

Okay, so this more or less leads us to the key part of today's lecturer which is the discrete time processing of continuous time signals, so this is the first of the 2 components, the other one will be the continuous time processing of discrete time signals, discrete time processing of continuous time signals, so I have xc of t, which I want to take into the discrete domain through a C to D converter sampling period specified Ts to avoid aliasing produce a discrete time signal x of n pass it through a discrete time system.

We would always wanted to be an LTI system where we can be represented in terms of the impulse response or the frequency response H e of j omega, so either h of n or H e of j omega that will produce for me y of n which are then take it through a D to C converter and then this will become my reconstructed signal, notice I am not calling it x, I am calling it y because it has been processed I mean, I filtered it.

I may have done something to it, so it is a different signal an x of n output y of n that is the input output of the discrete time system, so now if you were to encapsulates the whole thing into an equivalent processing block, if you did not know that you had gone into the discrete time domain and had just said okay, I fed in to this green box xc of t, a continuous time signal, then I obtain a output yr of t, okay.

Now, is there an input output relationship yes, if this is LTI, I should not do anything inside the block that violates linearity or time invariance, so if I preserve the LTI property then, I can say that this overall green block actually behaves like a continuous time filter which is H effective; Heff, H effective of j omega, so I cannot tell the difference between an analogue signal being processed by analogue continuous time LTI system, a H effective of j omega producing yr of t.

And this equivalent block diagram that is happening inside, okay and a very, very interesting sequence of developments that we can study in this context and let me just sort of give you a flavour for it definitely will encourage you to read Oppenheim and Schafer chapter 4 some very interesting applications are present.

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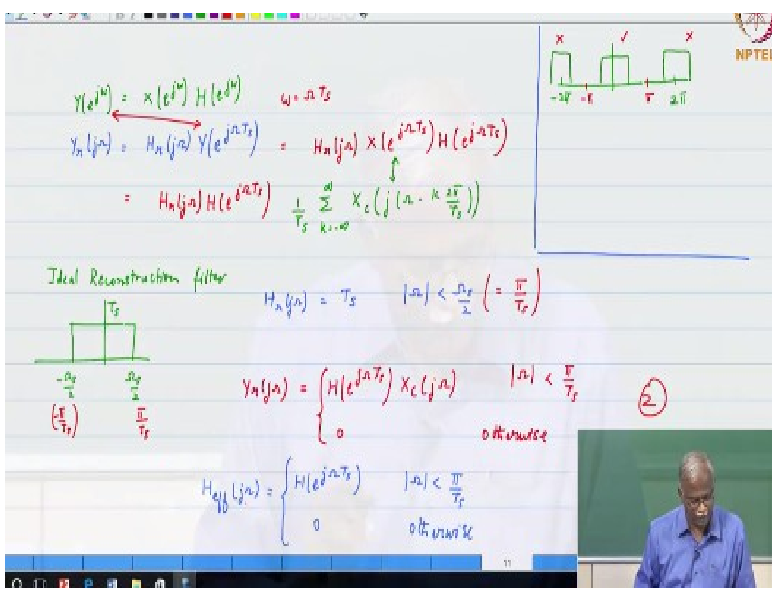

So, for the input output of the discrete time block, so we will have Y e of j Omega equals X e of j omega times H e of j omega that is the discrete time LTI system, okay and then the y reconstructed signal, Yr of j omega, the Fourier transform of the reconstructed signal will be the reconstruction filter Hr of j omega times the analogue representation or continuous time representation of y of n which we have just told a, start of this class can be expressed as Y e of j omega Ts, it is Y e of j omega from here to here, we have gone using the transformation omega is equal to omega times Ts, okay.

So, basically that is spectrum, it has multiple copies, the role of the reconstruction filter is to retain one and remove the rest that is what we have here, now Y e of j omega can also be written in terms of the equation number 1, so this can be written as Hr of j omega X e of j omega Ts, H e of j omega Ts and next step further simplification or you know getting it little bit more insight into the signal that we are interested in.

So, this can be written as Hr or j omega, H e of j omega Ts that is the discrete time LTI system and the signal X e of j omega can be written in terms of the corresponding continuous counterpart which is 1 over Ts summation k equal to - infinity to infinity Xc, basically x of n is related to xc of t and I am writing it in terms of the spectrum of xc of t, this would be j times omega – k times 2pi by Ts, okay.

Basically, scaling and the replication of spectrum which is the relationship for this and this, okay so here comes the insights that all the things that we have put together, so let us assume that we have an ideal reconstruction filter, so I have the ideal reconstruction filter again do not want to complicate things by looking at the practical aspects just for this is more of a conceptual, so ideal reconstruction filter immediately we recall that this has to be a brick wall filter from –omega s by 2 to omega s by 2, this will have an height of Ts.

So, this can be written in equation form as Hr of j omega is equal to Ts in the range omega less than omega s by 2 but that is the same as saying this is equal to pi over Ts, okay, so if I can equally well replaced this with –pi over Ts and this as pi over Ts, okay so this is a filter that will cut off everything that is outside, notice that even the first image of the copy of Xc of; will be outside of this range, so if I implement this the property here, then Yr of j omega.

Using the property of reconstruction filter, we can then say is nothing but H e of j omega Ts and only the first copy of the spectrum that of the expression that is in green, remember that the Ts factor has been cancelled through the reconstruction process and what we are left with this Xc of j omega and this is in the range mod omega less than pi over Ts and is actually equal to 0 outside again because otherwise or outside this range because of the reconstruction filter property.

So, now comes the most crucial step since we are saying that we are preserving linearity and time invariance, we also know the input output relationship for this block can be written as Yr of j omega is equal to H effective of j omega times Xc of j omega, okay that is the input output relationship for a continuous time LTI system, comparing this with the expression that we have now obtained, 2 expressions for Yr of; call this as equation number 1, this is equation number 2.

Then we can write down a very important and useful result H effective of j omega, the continuous time system equivalent or effective continuous time system actually is related to the discrete time system H e of j omega Ts in the range mod omega less than pi over Ts and equal to 0 otherwise, okay, just as a quick reinforcement of what we have said let me just take this upper portion okay, if I were to have discrete time filter which is a low pass filter, okay this is the discrete time low pass filter.

Now, this spectrum will repeat with period of 2pi, so I will have a 2pi, I will have a -2 pi, this will repeat now, if I map this to the continuous time frequency that part that signal also will have multiple, all these copies, what the H effective filter says only within -pi over Ts to pi over Ts in the discrete time domain, we showed yesterday that corresponds to –pi to pi, so that means only what is there within this portion is retained, everything else is removed.

So, I have a discrete time filter which is a low pass filter which has got periodic repetitions of its spectrum but through the process of reconstruction, I have removed the other copies and what is left for me is only the central or the main component, so it behaves or now behaves not like a discrete and filter but now as a continuous time filter whose overall transfer function is H effective of j omega.

So, the conditions for LTI property to hold the discrete time system He of j omega that has to be a LTI, basically I am doing a transformation from x of n to y of n, the discrete time system must be LTI, the input signal must be band limited and I must have satisfied Nyquist because in the process of going from the continuous time to the discrete time, I should not have done anything which will prevent me from going back to the continuous time, right.

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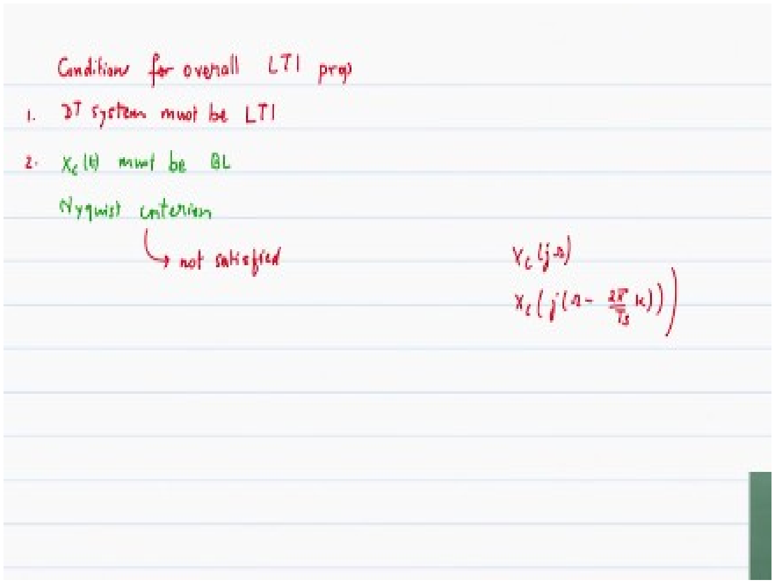

So, those are the property, so maybe we just write it down here, so the conditions for overall LTI, again this is very, very important because you are now saying what I needed to do in the continuous time, I am actually can do it in the discrete time provided I can satisfy these conditions, so these conditions are very important and useful, the first one is that discrete time system must be LTI okay that is obvious or otherwise we cannot represented by an impulse response.

And of course, it will not be helpful for us either, now on the continuous time site, we have to ensure the following one is xc of t must be band limited, then the second part of it will be that once it is band limited that you satisfy the Nyquist criterion may be having studied the practical aspects, we would say also over satisfy the Nyquist criterion by a little bit it is easy for us to build because if Nyquist criterion is not satisfied, okay if this is not satisfied then what we will get is copies of the input signal that are aliasing.

And therefore, linearity will be affected, linearity basically says what I applied at the input there is a corresponding something that appears at the output, I should not have anything which does not directly depend on the input itself, so if I have xc of omega j at the output that is perfectly okay but if I have xc of j omega -2 pi by Ts or some dependence on any of these things, then I have a problem because that is not directly the input, it is the shifted version of the input.

So, therefore if any of these components are present, then I will lose linearity and therefore that is why Nyquist criterion is very important for us to be satisfied, okay, so given these you can guarantee that the Nyquist property will be satisfied and that this is a good way for us to move forward. Now, let us build on this very quickly because I want to make sure that you are comfortable with what we have said so far and actually see the benefits or the power of such a mechanism.

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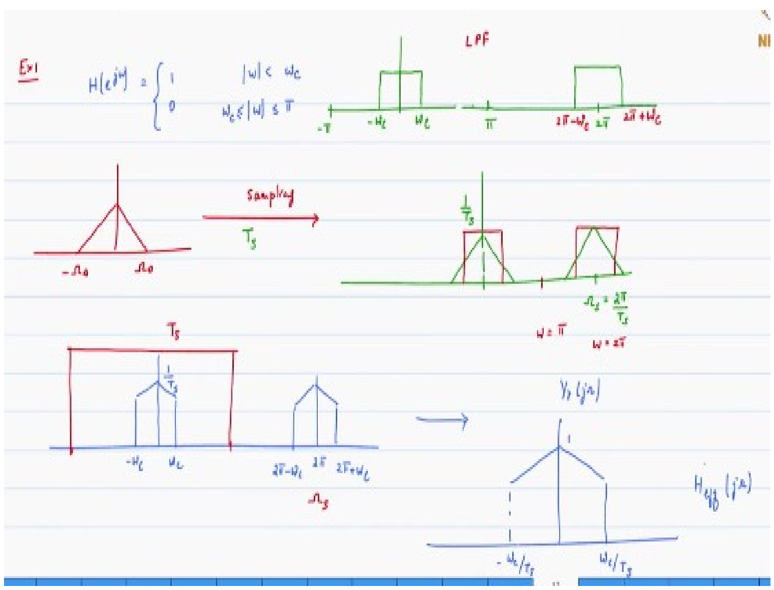

So, here is the first example, first of many example we will discuss this example request you to read the other examples that are there in Oppenheim and Schafer, okay so if I have a discrete time filter H e of j omega which is equal to 1 in a range mod omega less than omega c equal to 0 outside; outside, basically means you have to specify it, its greater than omega c less than or equal to pi, after that it will repeat, so we cannot just say otherwise, we have to say you know, so basically we have to specify a digital filter from –pi to pi, otherwise it is the incorrect okay.

So, this one would have a frequency response that looks like the figure that we drew –omega c to omega c and then from –pi to pi, it is 0, so this would be clearly low pass filter, you will be passing the low frequencies, okay so again it has components of the pass band, sorry not at pi, it should be at 2pi, so if you extend this line pi, 2pi, we should get a copy of the signal, again keep in mind that we have plotting from –pi to pi.

If you were to plot from up to 2 pi, you would have to show this particular, this version also and the band edges are 2pi – omega c, this would be 2pi + omega c, okay keep that picture in my mind, okay now I have a continuous time signal which is band limited – omega naught to omega naught which through the process of sampling; sampling at a sampling rate of Ts, the sampling period of Ts which satisfies the Nyquist criterion gives us the following.

I get replicas of the spectrum scaled by the sampling frequency and so, basically there is a scale factor 1 over Ts and they are separated by the sampling frequency, so this is omega s that is equal to 2 pi over Ts okay and yesterday we said that once you get it into this domain you can also relabel the frequency access in terms of the discrete time frequencies, so this would correspond to omega equal to 2pi, the midpoint at omega equal to pi.

And correspondingly, you can look at the band edges as some pi by L, some value that is easy for us to represent, okay, so here is the discrete time signal where you have; so now we can forget about the continuous time apart and you can say that okay here is a discrete time signal whose spectrum is given in this by this drawing and this is to pass through a low pass filter, okay so the low pass filter let us say has this property, okay.

And because of the discrete time property, it is going to cut it off for all of its copies as well, so what we get as after the discrete time filtering has happen is the following signal, so I get the following, I have the main copy which has this shape and this has been cut off at – omega c to omega c; - omega c to omega c , then at 2pi, there is a copy which is again at the same band edges this is 2pi – omega c, 2pi + omega c.

And the reconstruction process so now, for that I need to remap my discrete time frequency back to the continuous time frequency, so this becomes omega s once more, this and I will do my reconstruction filter which would be from – omega s by 2 to omega s by 2, it will have a sample height of Ts, notice that the original spectrum has got 1 over Ts representation when I pass it through this.

Then, I now get a continuous time output, so Yr of j omega is the following spectrum, it has the shape like this and if you were to apply reconstruction filter that is the indicator I have shown here, reconstruction filter that goes from –omega s by 2 to omega s by 2, then what we and it will remove all of the other copies and what we will get is the signal at this point which we can label as follows, this has a height of 1.

And this is at –omega c by Ts to omega c by Ts and this is the overall the output response and we will now relate this to the effective filter; H effective of j omega and this we will take up in the next lecture, thank you.