Multirate Digital Signal Processing
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Lecture – 01 (Part-1)
Introduction to Multirate DSP - Part 1

Good morning and welcome to the course on multirate digital signal processing. This is lecture 1 and a very warm welcome to all the students. In today's lecture, we will be giving a brief outline of the course and also introduction to what will be the elements of the course. This is a course that I believe has a lot of very practical and intuitive applications and will be very helpful in understanding why multirate signal processing is beneficial to us. And even in this lecture, we will be looking at a few examples.

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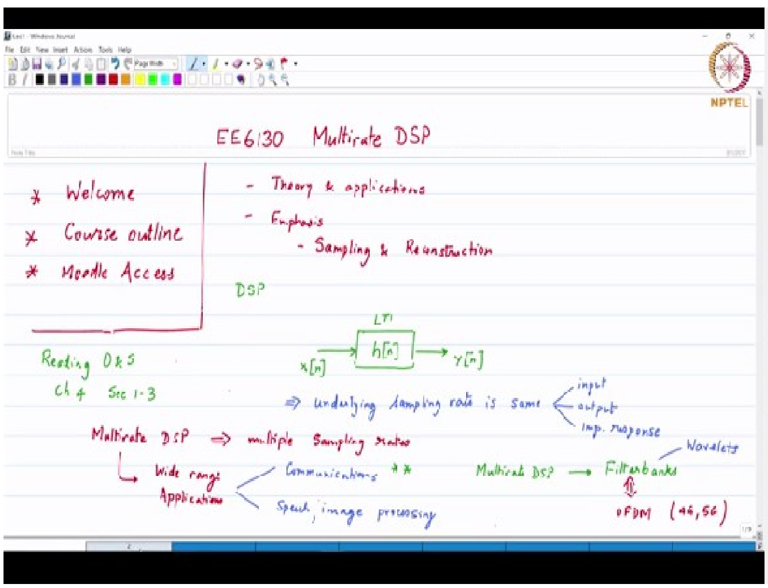

So let me start by saying that this is a course that has a good blend of theory and applications. And that is what makes this course an interesting course. And there are several books that you can use. The course outline that has been uploaded in module, recommends too that you follow, one is the book that you have used for the basic course on DSP Oppenheim and Schafer and because the chapter on sampling in Oppenheim and Schafer is a very good foundation for us to start and to build on the course.

And we are in essence building on the foundation that you have had in DSP. So the theory and applications of multirate signal processing, we do have a point of emphasis that was probably not as much present in the DSP course. In DSP course, sometimes we do spend too much time on the aspects of sampling and reconstruction. We do study it, we do study what are the implications, we look at the time domain aspects, the frequency domain aspects but this is something that I believe will be a very useful part for us in the study of multirate signal processing.

So in this context, maybe I could even suggest that there is a very useful reading assignment that you could begin in terms of preparing for what we are going to be looking at in the early lectures by reading Oppenheim and Schafer, O&S stands for Oppenheim and Schafer. Chapter 4, that is the chapter on sampling and in particular sections 1 to 3 and that will be a good compliment to what we will be covering in the class.

So let me start by giving a flavour of what exactly that we are referring to. So when I talk about DSP, probably the picture that comes in the minds of most students is that we are talking about LTI systems. Am I right? LTI systems, this is an LTI system. LTI systems can always characterize by means of an impulse response. You have a discrete time input, x of n which gets processed by the LTI system and you get discrete time output y of n.

Notice that the time index is the same input and output which means that the underlying sampling rate. So in essence there has been a constant sampling rate that is assumed for the representation of the input signal, for the representation of impulse response and for the output. So underlying sampling rate, though we do not really, I associate a sampling rate, but there is a sampling rate behind these discrete time signals and the underlying sampling rate is the same.

And I think that is an important element to just make note of because that is not something that you would have emphasised when studying the DSP aspect. So underlying sampling rate is the same. Is the same for the input, for the output, for the impulse response. All of them seem to have the same notion of time and the spacing between the samples, impulse response, okay. Now by the very nature of the title, multirate DSP will take us into a domain where the sampling rates will change.

So multirate DSP means that within a system, as in this case input and output, there could be more than one sampling rate. Multirate DSP implies we have multiple sampling rates present. And why would you introduce multirate sampling rates? What are the advantages? When would you use it? Do you increase the sampling rate or decrease the sampling rate? All of that is part of the study and our understanding of when is it advantageous to change.

So multiple sampling rates are very important for us and this is what enables multirate signal processing to have very interesting applications. So multirate DSP, the reason for its study, I would say is primarily the wide range of applications. And the range of applications include primarily in the area of communications and that is probably one of the areas where we at IIT have been doing a lot of the study.

So in the area of communications, in the area of speech and image processing also very useful for us to speech, image processing. These are areas. Also you will find that multirate signal processing has got many applications in the modem side, on the receiver side, on the transmitter side. How do you generate a transmitted signal? So multirate signal processing you see comes into many interesting flavours.

But probably one of the things that makes it more interesting and contextual, in the context of OFDM, in the context of communications, is that multirate signal processing, multirate DSP has got one of its core components, a topic called filter banks where you have multiple filters which are operating simultaneously on an input signal. It turns out that filter banks are a framework for us to understand OFDM, orthogonal frequency division multiplexing.

This is a waveform that you see a lot in the 4G and the 5G cellular standards. And so of course, there is a communications viewpoint that these are, OFDM is nothing but multitone transmission. However, there is a very rich, multirate DSP framework which tells you how to understand, how OFDM works, what are some of the advantages, disadvantages and how do we benefit from that?

So let us spend a few more minutes on by way of introduction. By the way, just like filter banks help us to understand OFDM, it also helps us understand wavelets. So filter banks is an umbrella of framework which helps us understand OFDM, understands wavelets and it has a very extensive use in a number of communication applications.

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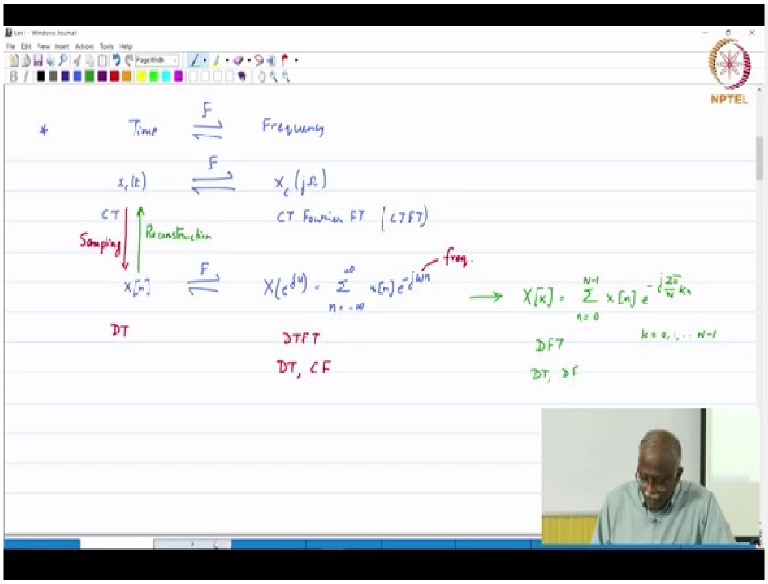

So if you were to look at it in the context of at top level, so one of the key things that we would like to be very comfortable in is to move between the time and the frequency domain, just as in the DSP case indicated by the, be able to go back and forth. All of the concepts, we would like to be able to relate in both the dimensions. Now the only differences in the case of DSP, there was only a single sampling rate.

So there were certain relationships that form the foundation. For example, so if you in the context of a continuous time signal. So this would be CT, a continuous in time, if I took the Fourier transform of this signal, I would get the subscript c stands for continuous because if you drop the subscript, we will assume that we are in the discrete time domain. So this has a continuous time Fourier transform.

So this is the continuous time Fourier transform. This is something that you would have studied in the course on communications referred to as CTFT. Now when we come to the discrete time signals, we have x of n, the corresponding Fourier transform, two variations of that. You can have a form that has Xe of j omega which is given by summation n equals -infinity to infinity x of n e power -j omega n.

And this transform as you will recall is called the discrete-time Fourier transform. Why is that? Because the signal is discrete time as opposed to a continuous time signal and I have taken the Fourier transform, okay. This can also be characterized in the following way. It is discrete in time. It is continuous in frequency, CF, DTCF. Because omega is your frequency variable and that is a continuous variable.

Now the DTFT also leads us to another representation where we have the following, X of k is equal to summation n equals 0 through n-1 x of n e power -j 2 pi over N k n and this is, what is this transform? DFT, the discrete Fourier transform. This is discrete in time. It is discrete in frequency, okay. So that is our, and of course, k goes from 0 through n-1, okay. Now very often in our study of DSP, we kind of have these 2 parallel forms.

If it is in a continuous time, we have the upper branch. If it is discrete time, we have the lower branch. And may be occasionally, we talk about the link between the two. But in the context of multirate signal processing, what is extremely important is that we always have a link between these two and going from continuous time to the discrete time, would be the process of sampling and in the reverse direction, it is the reconstruction.

So this is where some of the key elements of the multirate signal processing come into play and make it a very interesting and a very rich area for us to work, okay.

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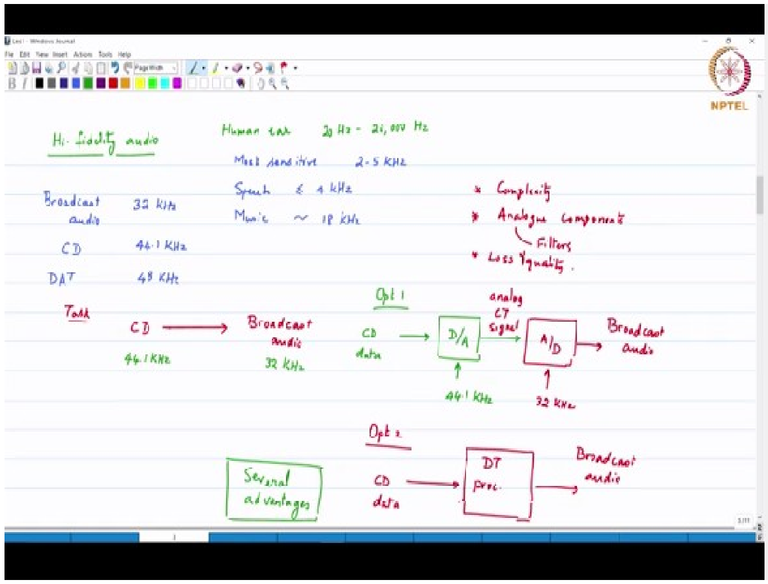

May be, example is helpful for us to even start talking about multirate signal processing. So to introduce multirate signal processing, let me just look at the application of multirate signal processing in high fidelity audio. Many of you are music fans. So I am sure you appreciate what it takes to produce a high quality music and you will find that multirate signal processing plays a very very important role in producing music of that quality.

So the auditory range of the human ear is? What is the auditory range? What is the range that your, human ears sensitive to? 20 Hz to 20 kHz, 20 Hz to 20,000 Hz, okay. For most people, it does not go that high but that is more of less the range that we have. The range that we are most sensitive too. Is in a much lower frequency range. It is 2 to 5 kHz. And most of our speech communications has frequency content less than 4 kHz.

And typically it is music that has got range all the way to maybe 18 kHz, okay. So musical instruments go much higher than speech. And so typically if you want to have very good representation of music, that is where the aspects of high fidelity applications come in, okay. So we have 3 standards for representing high fidelity signals and we have 1 which is broadcast audio which does the representation stores the information at a sampling rate of 32 kHz.

CDs, your compact disc stores information at a rate of 44.1 kHz and a format that is probably not as common today, called digital audio tape, but still used in studios, has recordings at 48 kHz, okay. So one of the cases where you may have to, where you will come across the introduction of multirate signal processing would be that if you had, so the task is that you have a CD, so some music that you have on a CD, you want to convert it into a format that can be played on broadcast audio.

So this basically means that you have to convert the sampling rate from 44.1 kHz to 32 kHz. Two options for us. One would be the brute force option. Let me call it as option 1. You can take the CD data. You play it back, you basically convert it into an analog signal that would mean that you would pass your data through a D/A converter where your D/A converter is running at 44.1 kHz so that it can produce the corresponding analog signal, okay.

Once you do that, then at this point what you have is an analog signal or continuous time signal, CT signal which you can then pickup like any other analog signal and sample it at 32 kHz and then get your broadcast audio, okay. So this is a very simple practical application where you would have your force to convert. Now notice that any time there is a conversion from digital to analog and from analog to digital, there is some loss of information and some loss of fidelity.

So first and foremost some of the aspects that we would note about this is that there is some complexity involved because D/A and A/Ds are running. There is complexity. D/As and A/Ds actually have in them analog filters. So there is some complexity and power consumption because you have analog components in the blocks that we are dealing with. And what we refer to are primarily analog filters because an analog filter is needed for reconstruction.

Before you do the sampling, you need another analog filter. So basically there is some analog filtering that, and of course, there could be a potential loss of quality, okay. Because we are looking at very high fidelity representation of signals, you do not want to lose any information that is there in your original signal. Potential loss of quality, okay. So these are things that you; now on the other hand, if you had a way of doing the following.

Option 2, you take your CD data and you do some processing which would be completely in the discrete time domain and you come out with the broadcast audio. So which essentially means that you did not lose any or you did not attempt to convert it into analog. You stayed completely in the discrete time domain but nevertheless, managed to achieve the translation of the sampling rate.

So this is an important element which then says that okay if I could do this, possibly there are several advantages and you will find that over the course of this or during the course of this study, you will find that these are very substantial advantages in many of the applications that we will encounter in communications in everyday. So this is a motivation for us to look at the reason for multirate signal processing, okay.

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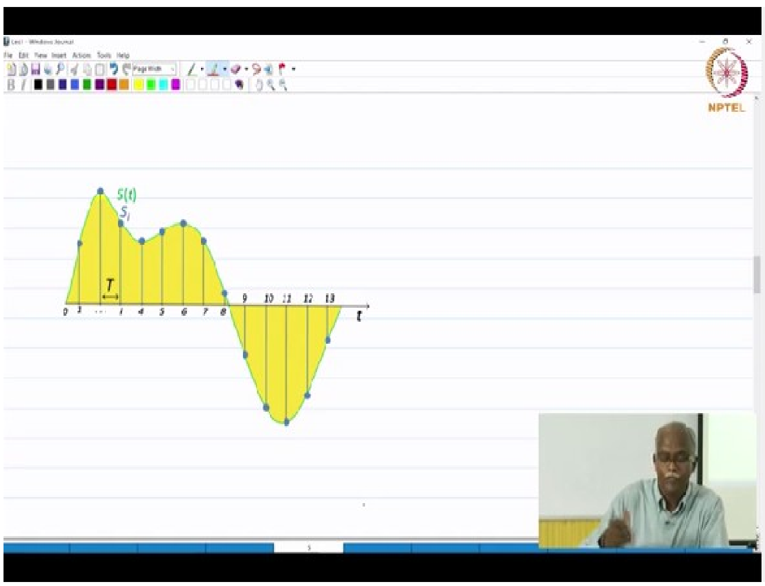

Now as a first step for us in the context of, is the notion of sampling. So we have a continuous time signal. So you can see that there is a continuous time signal which is then sampled at regular intervals which is called periodic sampling and in order for us to begin to enter this framework of a sampled representation, we need to have the understanding of when is sampling an acceptable representation without loss of information of a continuous time signal.

Again, that is a well known concept both in communications and signal processing. But nevertheless, I would like to introduce it as part of this discussion as well.