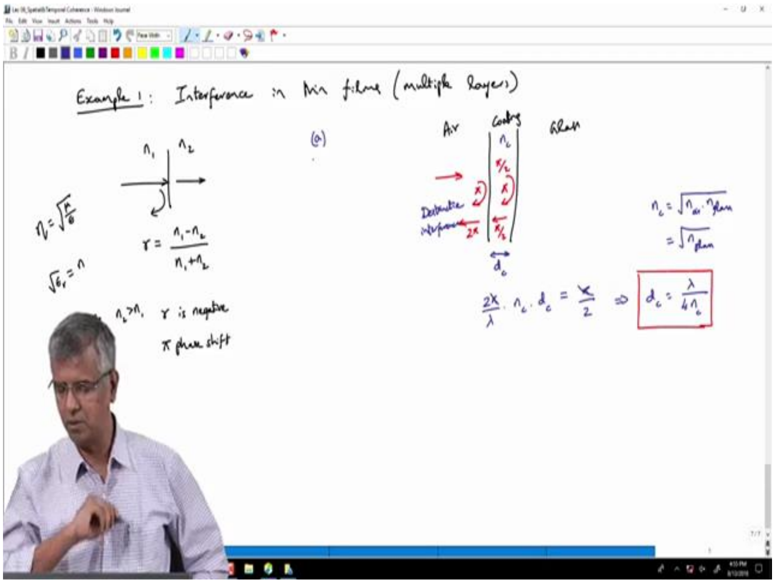
Introduction to Photonics
Tutorial on Wave Optics
Professor Balaji Srinivasan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Welcome to Introduction to Photonics.We have been talking about interference of light, we have been treating light propagation in terms of waves and recently we have been talking about the statistical property of light which provides this concept of temporal coherence and spatial coherence. In today’s session, we are going to see if we can use the knowledge that we have gained so far to actually design certain optical components.

Specifically, we will see how to design an anti-reflection coating, a high reflection mirror, a band pass filter using a Fabry-Perot Etalon and finally a Fibre Bragg Grating.

In the last session we looked at the interference in thin films. While we are considering interference in thin films, we noticed, that whenever light goes from one medium to another medium it is going to undergo certain reflection. We recognized that if you look at going from a rarer medium into a denser medium that is $n_2 > n_1$ then you have π phase shift at that interface.

(Refer Slide Time: 02:06)



Now based on this, we were asking the question whether it is possible to, have things like anti-reflection coating. So I mentioned that if you are going from air to glass - let us say you have this air medium and going into a glass medium, we expect to have a certain reflection at that interface and so if you are going in from here, you are going to have a reflection at that interface and that reflection is going to give you a π phase shift.

Is there any possibility that you can cancel this reflection using the kind of properties that we have talked about, is the question. And the answer turns out to be –yes.We can do that provided you have destructive interference between the incident wave and the reflected components. So, what are the reflected components we are talking about?

Now, assume that the incident wave encounters another medium before it hits the glass interface- let us say a coating on that glass surface. This coating has a higher refractive index when compared to air. So it is obvious that we have a π phase shift from this interface also.

So, if you want to cancel this wave component with a wave component that is coming from here and through this, the question is: what phase should it accumulate so that this cancellation happens?

So, we are talking about one wave component which has got a π phase shift and another wave component which has gone through this coating, hit that glass interface and come back. What should the accumulated phase be so that these two cancel each other? So, if it encounters a $\pi/2$ phase shift this way, and a $\pi/2$ phase shift this way, the total accumulated phase would have been 2 π. The relative phase shift between these two would have been π, in which case, this initial reflected component will be canceled by this other component.

So, if we have a structure, which has a coating whose refractive index is more than air but lesser than glass, and the thickness of the coating is such that it has a $\pi/2$ phase shift,then we see that these two reflected components will cancel each other. That is essentially saying that there would not be any reflection, for a particular wavelength.

Why a particular wavelength? Because if we consider this condition here within this region, let us say this certain thickness is d_c. Now, d_c is such that the phase that you accumulated in the single path, that is given by $2\pi/ \lambda * n_c d_c$ where n_c is the refractive index of the coating, will have to be equal to $\pi/2$ okay,

$$\mathbf{2\pi/ \lambda * n_c * d_c = \pi/2}$$

and we can go ahead and cancel this π with this π over here so this implies that

$$\mathbf{d_c = \lambda/4n_c}$$

So, if we have this condition, then if we have a thickness of coating that comes into the picture, such that you have a $\pi/2$ phase shift incorporated in one direction, there is a $\pi/2$ phase shift in the other direction as well and then there is actually a π phase shift happening because of reflection between the coating interface and the glass interface. Then, overall there is a 2π phase shift for this component that is coming through, then you can have destructive interference between these two components.

And that destructive interference means that any glare that you get normally, with a glass surface, you would have ended up cancelling that glare with this extra coating that we have provided. Does that make sense? Yeah, of course there is this thing about what the refractive index of the medium should be and it can be shown that this refractive index, (there is a parallel in terms of what you do with transmission lines or in electromagnetic theory) would have to be equal to the geometric mean of $n_{air} * n_{glass}$ and n_{air} is of course 1.

So, you would say that this will have to be equal to n_{glass}. So if I take a material which is having a refractive index corresponding to $\sqrt{n_{glass}}$ and which has a very specific thickness, so if you incorporate that, then you should be able to cancel the glare for a particular wavelength.

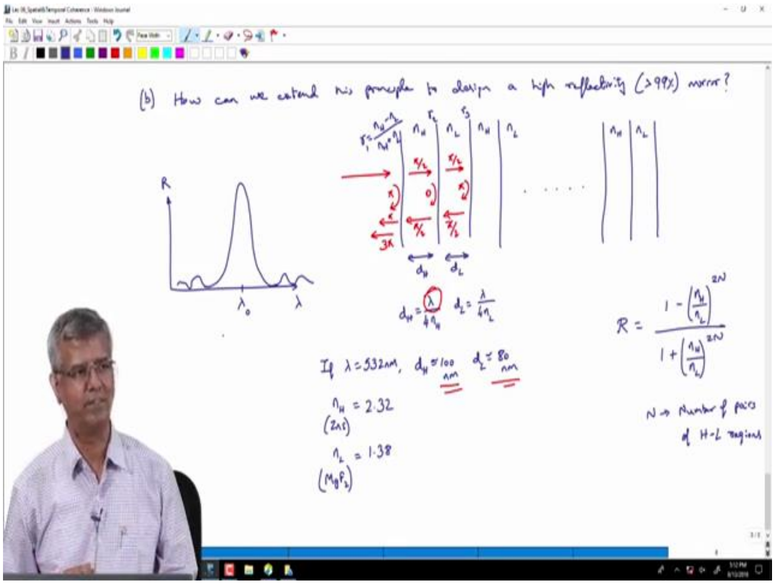
So it so happens that for the visible radiation for which we want to cancel this glare, we would have to choose one particular wavelength. So if that wavelength is green,

let us say $\lambda = 550$ nanometers and $n_c = \sqrt{n_{glass}}$, and $n_{glass} = 1.45$ or something like that, then you can figure out what should be the corresponding thickness and that thickness would be roughly in the order of 100 nanometers .

So if you are able to apply that coating on your glass, then you can make it an anti-reflection coating. So we are saying that there is no reflection of light, that there is destructive interference over here. So correspondingly, can we assume that all that light is going this way? That is exactly what we are saying right- it is cancelling all the reflection components so all the light is going straight through.

So the question is whether energy is remaining constant or getting lost over here. In this case the energy is conserved as we are moving in. As long as there is no loss in the propagation itself, we are saying the energy is conserved over here. So that is let us say the part A, actually this is one of the questions that is given as part of your tutorials, tutorial number 2 so let us say this is Part A.

(Refer Slide Time: 11:14)



Can we use this principle now to make a high reflectivity mirror? So that is the second part. Question is - how can we extend this principle to design high reflectivity? High reflectivity- let us say greater than 99 percent mirror. So, if that is what we want to do, how do we go about doing that?

So suppose you have - let us say material 1 with refractive index n_H followed by another material 2 with refractive index n_L and then followed by another material 3 with n_H and so on. Let us see how we can essentially get a mirror.

So for getting a mirror, we said we will have to have constructive interference of all the reflected components.

So let us say you have this incident wave over here hitting material 1, and of course it is encountering a high index medium which is a denser medium. So it is going to have π phase shift . Then whatever is reflected from the next interface if that is π or 3 π or whatever phase shift is accumulated then it would be constructively interfering right.

When you go from a denser medium to a rarer medium, you get no phase shift so, the phase shift is 0 . So there is no phase shift when it gets reflected from material 2 (with refractive indext n_L) to material 1 (refractive index n_H) over here.

So, in that case, if you wanted to get out with a π phase shift you have to make the thickness such that the phase accumulated here is $\pi/2$ and after reflection obviously it is going through the same thickness so that is also $\pi/2$. So the component that is reflected from the next interface that is in phase with the initial reflected component is adding together .

Now can we see if we can extend this to the next layer? So let us see the next interface - how much is the phase shift that you get upon reflection? π right. Now once again if you do $\pi/2$ shift here and $\pi/2$ over here, and count all these phases. So this is $\pi/2$, this is next and is $\pi/2$, so this is π and then there is a π phase shift upon reflection and then coming back, this is $\pi/2$, $\pi/2$, so the total that we get out of all of this is 3π and whether that is in phase with the other reflected components? Yes, it is just got a 2π phase shift between so that is going go to be a constructive interference.

And similarly you can go on to build over multiple layers so after this there will be a n_L and then you go on to n_H, n_L and so on. Then all of these things, as long as you are picking up only a $\pi/2$ phase shift, you are going to end up with constructive interference from all these layers, all these interfaces. So correspondingly what is d_H and d_L, just based on what we saw in the previous case we can say

$d_H= \lambda/4\ n_H$ and $d_L= \lambda/4\ n_L$

And if we say λ in this case we want to reflect green light , let us say green light if $\lambda= 532$ nanometer, then you can calculate what the corresponding thickness is,

now what we do not know is n_H and n_L

what we are not considering here the other important consideration is, if we want all of this to add up, you want good reflection from each of those interfaces right.

So how will you pick n_H and n_L so that you have the minimum number of layers in your mirror? What should be the relative values of n_H and n_L so that you have minimum number of layers? The relative quantities should be large with respect to each other since the reflection co-efficient depends on the index contrast.

So from each reflection you are getting the reflection co-efficient is

$$\frac{(n_H - n_L)}{(n_H + n_L)}$$

So if $n_H >> n_L$ you have a larger reflection co-efficient, so you need lesser number of layers. So in the industry Zinc Sulfide with a refractive index of 2.32 is taken for n_H, and for n_L, the industry standard is Magnesium Fluoride which has a refractive index of 1.38 .
So if we use these two material then, so the corresponding d_H if you calculate that corresponds that will be 532/4*2.32 that will be roughly about 100 nanometers.

I do not have a calculator in front of me, you can do that calculation and d_L would be slightly larger than that because the n_L is lower than n_H so d_L in this case will be roughly in the order of 80 nanometers so you can do the exact calculation later to figure out those numbers.

But we have done relatively advanced design of an optical component by tracking the phase and using this principle of constructive interference. We have just managed to construct a mirror like this, now this seems to suggest that I look at this thing as r1 and then I look at r2, r3 and so on, I can just add all of these reflection coefficients .

And from that I can find out the overall reflectivity. Does that make sense? Can I just add all the reflectivities from the individual layers and based on that can I find out what the total reflectivity is? No, why not? Because at every reflection you are losing some intensity in the backward direction and it is constructively interfering, that means that energy is lost .

So every reflection you have a certain loss of energy- that means that for the subsequent reflection, the effective reflection co-efficient is lesser so you have lesser and lesser contributions from the further points of further interfaces of reflection. So it is such that you get certain reflection from the first few layers and to get more and more reflection it is going to take more and more layers right, to improve the reflectivity beyond the first few layers it is going to take more and more layers .

So it is such that, I do not have the time to to derive this but if you take all those into consideration, you can come up with this expression that this is going to be given by

$$\frac{(n_H/n_L)^{2n}}{(1+n_H/n_L)^{2n}}$$

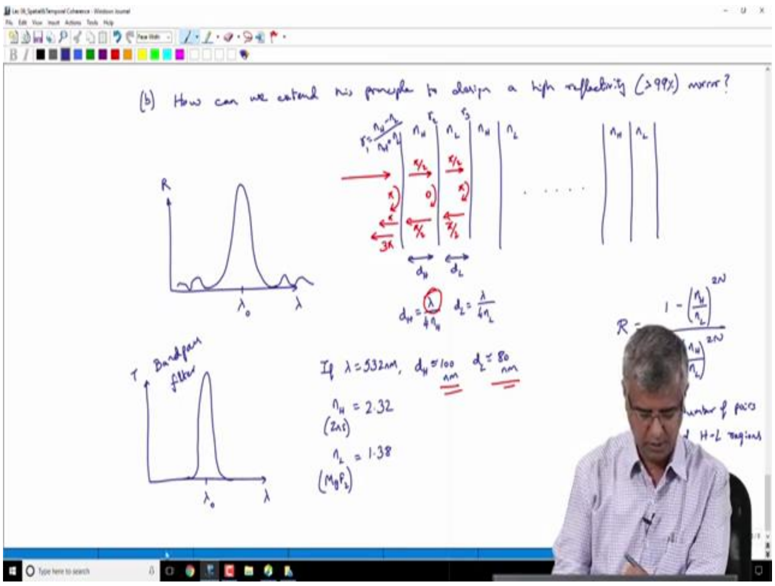
where n corresponds to the number of pairs of high and low index regions. So you can basically calculate the reflectivity that you can get and what you will find is reflectivity of greater than 99 percent is possible if you go for n in the order of 10 or 15 or something like that for the given pair of n_H and n_L values.

But on the other hand, when we were looking at the Michelson interferometer you had a 50-50 beam splitter - 50 percent went one way,50 percent got transmitted, 50 percent got reflected, so what does that mean? You have lesser number of these layers, so that you can achieve that 50 percent so you plug in R equal to 50. n_H and n_L we know, so we plug that in and then if you find value of n, n has to be an integer obviously right?

So you put that value then you can find out how many layers you need to get that 50 %. That is how that particular component was made and of course one of the key things to note here is everything is dependent on this value λ. What does that mean? You are designing this mirror for one particular value of λ. So, for all the other values of λ it is not actually meeting this constructive interference criteria. So you may not be able to actually add up as much, so the reflectivity will be a function of λ.

So if you look at the overall reflectivity as a function of λ, you will find that it will be like this and then there will be side bands also but, that will be all for one particular value for which it is designed so that reflectivity will be dependent on wavelength. Okay, so let us extend this even further.

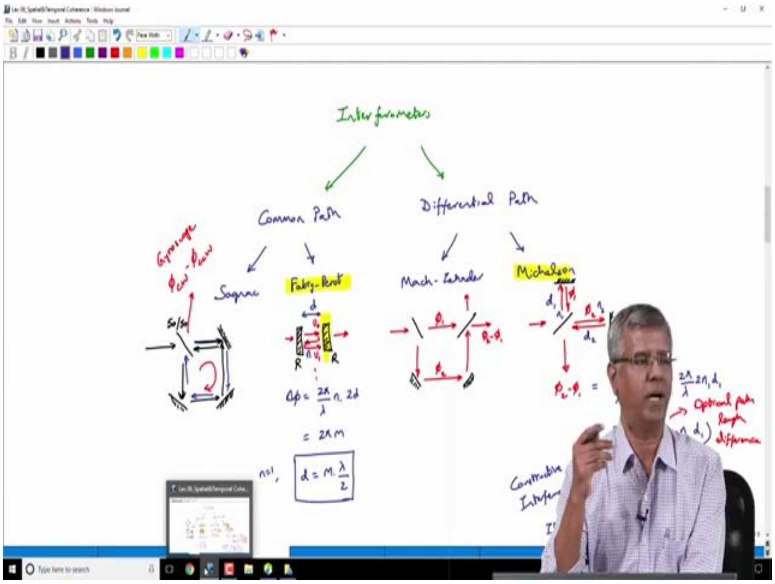
(Refer Slide Time: 26:30)



Can I do some trick over here with this structure, with this principle such that - instead of getting a reflection like this, I get a transmission over a narrow wavelength range around a particular λ_0? Okay, is it possible to have something like this? So what type of filter would you call this? Band pass filter right. So can I extend this concept and find a way to get a band pass filter?

This is basically a mirror, which is basically a band stop filter- it reflects a particular band but transmits everything else but if I want to change this to a band pass filter is that possible?

(Refer Slide Time: 27:54)

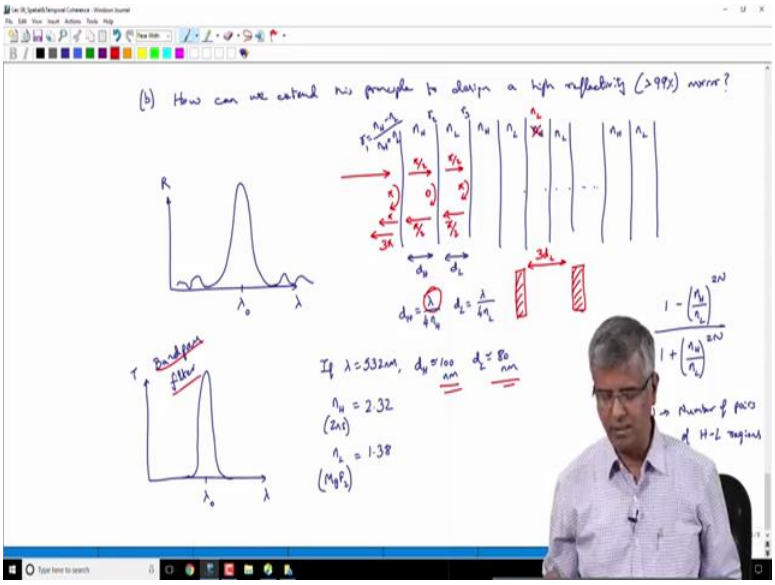


So turns out that it is possible. Let us go back and look at one of these configurations that we were discussing earlier. For example, can you say which one of these interferometer configurations I should use to achieve a band pass filter condition? You see something here which we can use as for making a band pass filter?

Fabry-Perot, very good! So Fabry-Perot Etalon can be made as a band pass filter, why?

Because we are saying that light actually enters Fabry-Perot structure and goes through multiple bounces and we are looking at the relative phase between all these bounces and one of the mirrors if they are all constructively interfering, then that particular wavelength at which constructive interference is happening, is going to escape that cavity right. So if we are able to make a Fabry-Perot structure with two mirrors - you can possibly get a band pass filter.

(Refer Slide Time: 29:37)

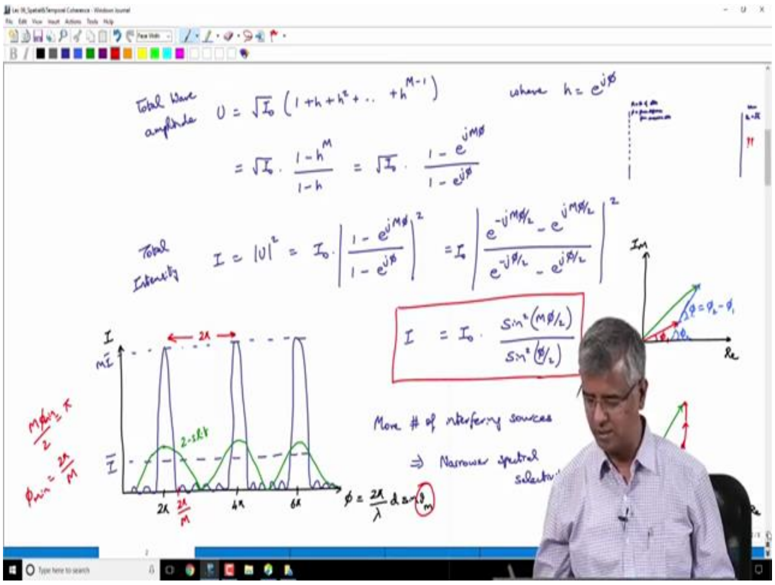


So let us see how we can do that over here in this case. So you know how to make a mirror. Can we make two mirrors out of this structure - parallel to each other? So what do you have to do to get something like that? So if you were to extend this to n_H, n_L and so on, now let us say at one particular layer instead of going to n_H, I just have n_L. So to give you a perspective of how to make this structures.

What you do is - you start with a substrate with a glass substrate and you evaporate this material, you put Magnesium Fluoride on that, you monitor the thickness for 80 nanometers and then you stop that evaporation. Next, you start evaporating Zinc Sulfide and let it build up on atomic layer by atomic layer. You can do that and you can get this 100 nanometer thickness and then go back to Magnesium Fluoride and so on. So you can stack everything up layer by layer right.

And in that process you can choose to say after certain stack is built, after let us say 10 pairs are built, I can choose to say now I will just have the low index material, right. That is going for instead of 80 nanometers I do it for 240 nanometers and then I can go back to my other routine of alternating layers- then what do I have? If we are able to do this, this corresponds to one mirror on this side and beyond this inner layer I have another mirror. - and these are separated by a thickness of 3 times d_L .

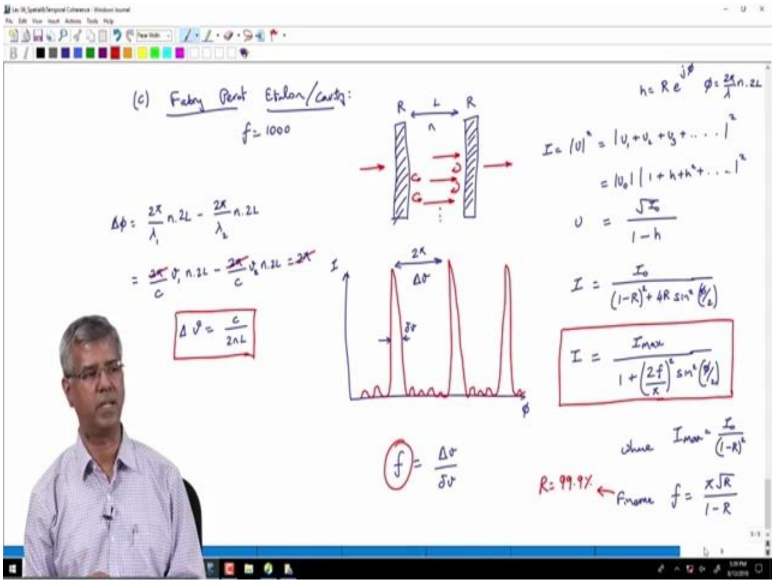
(Refer Slide Time: 32:50)



So I can make, in the same structure I can make a Fabry-Perot Etalon just by doing this. So let us go ahead and try to see how this structure can give this bandpass filter characteristic. So, to analyze this structure what do we have to rely on? We go back to this sort of an analysis we did previously - where we have multiple beam interference.

So you have multiple beam interference and in this case it is coming from different layers right, different interfaces. So let us just extend this multiple beam interference to the case of a Fabry-Perot structure

(Refer Slide Time: 33:31)



So overall what we are set to do is a Fabry-Perot Etalon or Fabry-Perot Cavity using this thin film interference concept. So what we have said is - we have assumed that we are going to be able to make one mirror here and another mirror here and light that is getting into the cavity is coming here. It is going through reflection, it is coming back reflection and it is coming here and then that is undergoing one more reflection and coming back and so on right.

So if 'I' represents the total intensity that is coming out of this, 'I' will correspond to the (magnitude of the wave)^{2}and that will correspond to multiple components that are coming together. And what will each of those components represent? So you can actually write this as some component U_0 , something you take out as common and then magnitude of that and then you have $1+h+h^2$and so on - that is an infinite series right.

$I = |U|^2 = |U_1+U_2+U_3 ....|^2$

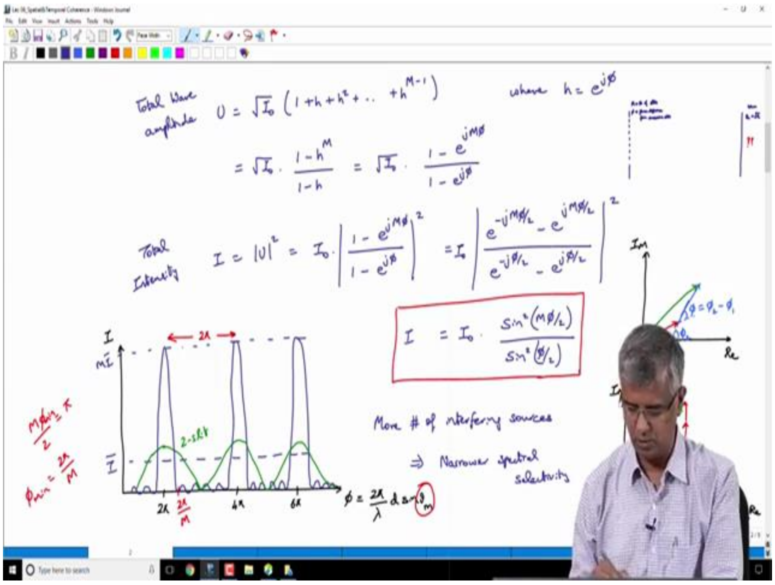
$= |U0|\ |1+h+ h^2+ ....|^2$

So what is h over here? h will correspond to - if each one of those have a reflectivity R, right as far as the wave amplitude is concerned the reflection co-efficient corresponds to $\sqrt{R}$, so it is undergoing$\sqrt{R}$ reflection over here multiplied by $\sqrt{R}$ reflection over here, so overall it undergoes R as the total reflection and then, what is the total phase? $e^{j\varphi}$ where φ will correspond to $2\pi/\lambda$.

Let us say this has a refractive index n and let us say this is corresponding to distance l so n * 2l would be the round trip phase.

So in this case what we will find is - this is an infinite series, so the total U is going to be given by $\sqrt{I_0}$ /(1-h) and where h is given by r*$e^{j\varphi}$, so the total I is going to be magnitude of that the whole square and that is going to be given, by I_0 divided by - it is the same thing that we did over here that we are going to follow right.

(Refer Slide Time: 37:35)



What we did over here is - we just expanded it out and had this $e^{j\varphi}$ /2 and minus, we took a$e^{j\varphi}$ /2 outside and we got to this form and then we were able to convert that to a $\sin^2$ form. So we are going to do the same trick over here and when we do that, what we find is

$$I = \frac{I_0}{(1-R)^2 + 4R\sin^2(\varphi/2)}$$

If you divide the numerator and denominator by 1-R^2 factor, you can write this as
$I = I_{max}/ (1+ (2F/\pi)^2\sin^2(\varphi/2))$ where $I_{max} = I_0 /(1-R)^2$ and the Finesse, F is given by $\frac{\pi\sqrt{R}}{1-R}$

So, the key thing is with respect to φ we have a $\sin^2$ function which is similar to what we had before. So if we were to plot this function, we have basically will have multiple peaks that are going to show up over here and this is actually as a function of φ. So, between two peaks how much φ do we have to go?

What is the phase difference between two consecutive peaks? 2π right, it is the same thing that we saw over here. So that is going to correspond to 2 π and what is important here in this case, is only specific wavelengths or specific frequencies are going to be passing through this cavity. So if you want to make this as a band pass type of filter, what you are trying to see is weather one of those peaks can be used to show the band pass characteristic.

And to do that, one of the key aspects is what is this FWHM (Full width Half Maximum) width that you have. And that FWHM width is actually something that you can find out by looking at the finesse. It is similar to the electronics circuits, the cue of the cavity, so that finesse is going to be given by $\Delta\nu$ where $\Delta\nu$ corresponds to the frequency difference between two successive peaks divided by this $\delta\nu$

$$\mathbf{f = \Delta\nu / \delta\nu}$$

So larger is your finesse - to get large finesse - what do you have to do? To get large finesse, the reflectivity has to be very high. So, to get large finesse - you need to have more number of interfering beams and you will have more number of interfering beams if you make your cavity with mirrors of high reflectivity.

So suppose you want to design a finesse of 1000. If you plug that in over here, what you find is - to get a finesse of 1000, your reflectivity would have to be 99.9% to achieve that. This is because it is primarily determined by that 1-R factor so you can almost write it as $\pi/(1-R)$ for very high values or reflectivity. And if you say f =1000 then R would have to be something around 99.9 percent to achieve that.

So, how do we make this structure now? To get that mirror what do we do? We have that high and low refractive index material with which we make those dielectric stacks. So if you have enough number of stacks you can achieve this 99.9 percent reflectivity. And then we have that gap that we talked about with a low index region and then another stack and so with that we have essentially made a Fabry-Perot Etalon.

So there are some interesting things over here which I would like to go into little more detail. So what is this $\Delta\nu$ mean? So what is the expression for $\Delta\nu$? To look at that let us look at $\Delta\varphi$. So what is phase difference between two successive peaks and that is going to be given by

$$(2\pi/\lambda_1) * n*2L - (2\pi/\lambda_2) * n * 2L$$

See in this case, what I want to emphasize is that the Fabry-Perot Etalon, let us say the length of the Etalon which is length of the cavity is fixed, then you can actually see what is the spectral response provided by that Fabry-Perot Cavity. So that it is going to have constructive interference for one wavelength, not so good interference for some of the other neighboring wavelengths and then it is going to be constructive interference for another wavelength.

So it is going to be like a periodic series of peaks and now what you are trying to find out is - when is the next, what is the wavelength corresponding to the next constructive interference? Where is that going to happen, and specifically what is the frequency corresponding to that? So, we can write this as $2\pi/c\lambda_1$ you can write λ_1 as c/ν_1 that is in free space frequency.

So, for constructive interference for successive peaks. $\Delta\varphi$ should be 2π

That is

$$\boldsymbol{\Delta\varphi = (2\pi/c)\,\nu_1 * n * 2L - (2\pi/c)\,\nu_2 * n * 2L = 2\pi}$$

So this 2π we can cancel across over here and that will give you the expression that

$$\boldsymbol{\Delta\nu = \frac{c}{2nL}}$$

So, $\Delta\nu$ which is the separation of frequencies is going to be given by $\frac{c}{2nL}$.

So in a Fabry-Perot Cavity, this is a generic sort of solution that we get and we are going to revisit this later on, when we go to the topic of lasers also. But essentially a Fabry-Perot Cavity allows only certain light frequencies to past through it.

But it is going to allow frequencies, that are spaced periodically. And between two successive frequencies which have the highest transmission you find that the spacing is going to correspond to $\frac{c}{2nL}$. So n is in this case a given.

n we have picked as n_L so that is a fixed quantity, by changing L (the distance) you can actually figure out.

If it is a very small L then $\Delta\nu$ (separation of frequency) is very large. That is probably what you want for a band pass filter. You want to pass a particular frequency but all the other neighboring frequencies, a broad range of neighboring frequencies you do not want to pass.

That means the next peak should happen at a much higher frequency - that you can ensure by picking a very small cavity. So those are the considerations that you go with and then of course you are also worried about $\Delta\nu$ - so if you want to get a very small $\Delta\nu$, the key thing that you work with is - try to achieve a high finesse. High finesse will correspond to a very small full width half maxima of the peak.

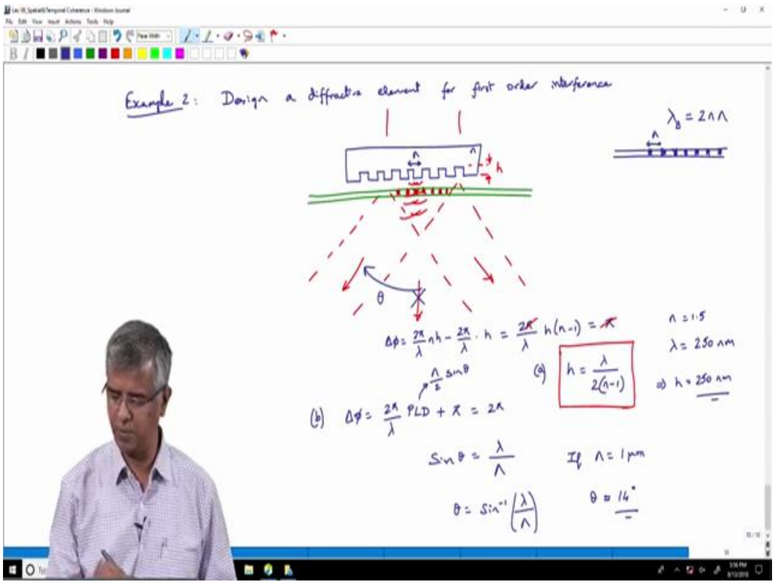
So you can pass one particular wavelength and reject all the neighboring wavelengths. I think in one of the tutorial problems, you were asked to crunch some numbers related to this. So you can go and try that but this is essentially the principle of this Fabry-Perot Etalon. So we have used the concept of interference in multiple layers as in the case of thin films.

We have shown one example where you can get destructive interference - and that is, important from the perspective of anti-reflection coatings. But you can turn it around to multiple layers and have constructive interference criteria- which can be useful for mirrors. And then, we have extended the concept to stack two mirrors and form a cavity and if you form that cavity then you can pass only a selected frequency or selected range of frequencies.You can choose them from other frequencies or certain colors you want to transmit and all the other colors you want to reject then this would be the case.

So this is actually very interesting because one of the projects, research projects that we do is establishing free space optical communication. It is supposed to work in broad sunlight and for this particular link we are using green light source, a green laser. So we are shooting a green laser from here to several kilometers away and we have to capture that information.

But can you guess, what is the biggest problem in this link? It is the receiver- which is looking for green light. This is also seeing all sorts of solar radiation during the day. How do I make sure that it is solar blind and can only allow this green light to go through? I use a filter like this. I basically say-I am going to make a Fabry-Perot Etalon which is allowing me only to transmit one particular frequency - one particular color and it is rejecting all the other neighboring color. So all the solar background radiation is rejected and I get good transmission of this color that I want.

(Refer Slide Time: 53:05)



So let us look at another example corresponding to a multi slit interference. So example 2 is once again one of the problems that is given to you in the tutorial. You are asked to design a diffractive element for first order interference. So this is also a problem that is inspired by something that we do in the lab - there are these devices called fiber Bragg gratings, which have a periodic change in the refractive index of an optical fiber.

If I make a periodic change in the refractive index of an optical fiber, what we saw in multiple layer interference holds good. So I can make a mirror at a particular wavelength right? So that is what a Fiber Bragg grating is. So it just consists of an optical fiber in which I have modified the refractive index in a periodic manner. So reflection from all of these interfaces will constructively interfere at one particular wavelength and that wavelength is called the Bragg wavelength which will be corresponding to 2 n Λ where Λ is the period of this high and low refractive index region .

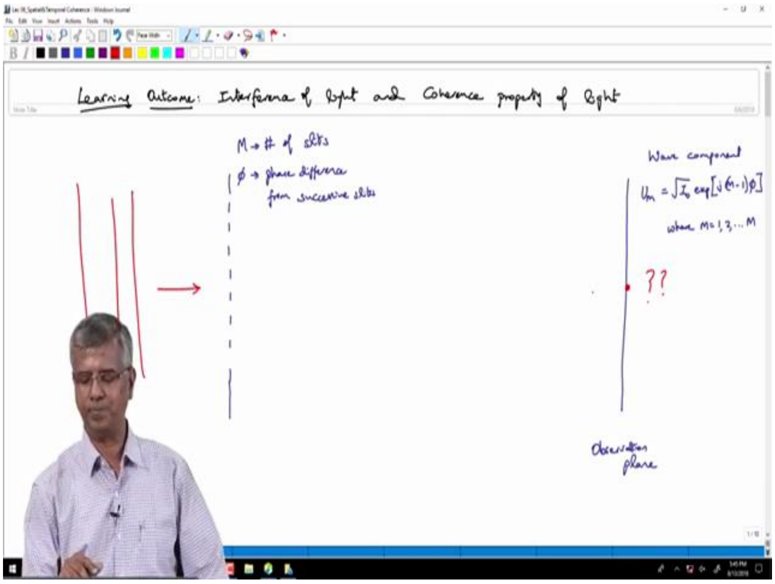
Bragg wavelength = 2 n Λ where Λ is the period of this high and low refractive index

So I am trying to make this structure and I am told that I can modify refractive index of this material if I expose this to ultraviolet radiation. If ultraviolet radiation falls on this fiber it will change the refractive index. Now the question is- how do I create this periodic high and low intensity ultraviolet radiation? So I want to have high and low intensity ultraviolet radiation so that the in the high intensity regions there is a change in refractive index, low intensity regions there is no change in the refractive index and so on. So then I can make this structure.

So how can I do that? Turns out that I can do that, by using an element that looks like this (ridged structure shown in figure). What this element should do is - if I come in with this ultraviolet radiation, it is a large beam that comes in, then part of the beam should go in one direction and part of the beam should go in the other direction, so that there is overlapping of these two beams.

So one beam is going this direction, another beam is going in this direction. If I am able to split the two beams, then in the near field what do I have? I have interference between these two beams and corresponding to the interference I have periodic and I mean I have high and low intensity structures. So what I would do is, I would put my Fiber Bragg grating over here. And because you have this periodic intensity changes, then I can make my fiber Bragg grating. And this is what we do in our laboratory. The biggest culprit as far as getting good grating is concerned, is this radiation that is going straight through.

(Refer Slide Time: 58:25)



Radiation that is going straight through would correspond to the zero order beam. So let me just go back and show you this thing over here. So there is actually a wave that is coming in here, setting out secondary wavelets and they are all interfering constructively at the axial point over here.

So if I have the same situation over here - that entire beam be such a uniform beam that it will wash out all these fringes. That will be like a DC shift in the refractive index change that I get. Any of this beam that is going through, which we call as a zero order interference will be a uniform exposure of this fiber which will wash out all this fringes. So in my case I do not want this zero order thing to go through. I want to find a way to nullify this and only have this plus 1 and minus 1 order go across and they interfere and they give me this pattern .

So what would be the solution for something like this? What would be the design of this element such that I can essentially cancel out any beam that is going straight down, but make these beams go through the first order interference? So to cancel something, what do you need to happen? Destructive interference- so there should be a phase shift of π.

π between what and what? There should be 2 components that you have here- so which are the two components? Well those are actually the diffracted components right, so those corresponds to constructive interference. But constructive interference between what and what? What are we considering here? This is actually a transparent element. Do you see two different features which can give you two different wavelets? So I can clearly get a wavelet from these regions right but I can also get a wavelet from this other region, this ridge over here .

So I can have constructive or destructive interference based on the relative path lengths between the two regions. So what is my destructive interference criteria? This ridge should be of such height that these two beams are π phase shifted with respect to each other correct? So, if I write that criteria for this to happen, $\Delta\varphi$ would have to be $2\pi/\lambda$.

In one case it is going through the glass of refractive index let us say n. So I would say n * h. In the other case, what is the phase that is accumulated? $2\pi/\lambda$ multiplied by- the ridge case that is also going through the height but in what medium? In air -okay , which has a refractive index of 1. So I would say that is accumulating a phase shift corresponding to $(2\pi/\lambda) * h$

So that corresponds to $(\pi/\lambda)\, h(n-1)$ and that should be equal to π phase shift - we want destructive interference so that is π phase shift.

$\Delta\varphi = (2\pi/\lambda)nh - (2\pi/\lambda)*1*h$ (since refractive index =1 for air)

$= (2\pi/\lambda)h(n-1)$

So, for Destructive interference

$$\boldsymbol{\Delta\varphi = (2\pi/\lambda)h(n-1) = \pi}$$

So I would cancel π and then that would give me the ridge height.

So ridge height **$h= \lambda/(2n-1)$**.

If my ridge height is **$\lambda/(2n-1)$** then I can achieve this. I can achieve a null for the zero order interference because if it is satisfying that condition at 1 point, it is satisfying the condition across the entire diffractive element.

So the beam that is going straight through, the components that are coming from these crest regions are cancelled by the component that is coming from the ridge region. So I do not have any light going straight through. And of course you can put some numbers to this

n is roughly 1.5 and λ is typically 250 nanometers for UV radiation

So if n equals to 1.5 let us say then using the formula, h will be the same number 250 nanometers. If you make this ridge height as 250 nanometers, then you can essentially make sure that the central part is cancelled and then you have interference between these other two regions. So then the question is what will be this angle θ. So there is an angle θ at which constructive interference happens.

So how do you find that value θ? Let us say this corresponds to Λ - the period of this diffractive element corresponds to Λ. So in terms of Λ can we find this value of θ? So for first order interference:

$\Delta\varphi= 2\pi/\lambda * PLD = 2\pi$ where PLD is the path length difference.

That is the constructive interference criteria for the first order right?

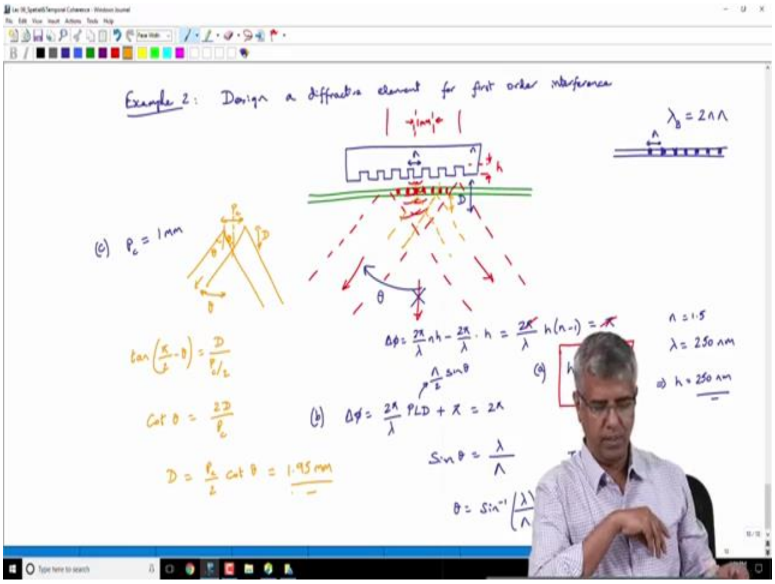
But in this case I am already starting with a π phase shift between the two components. At this plane itself they are π phase shifted.

So you have to take care of the constructive interference criteria. I can write it in a slightly different manner. So let me just write this as do this as **$2\pi/\lambda * PLD + \pi$** right, that phase shift you already encountered right at the diffractive element, that is equal to 2 π. In other words you can take the π to the other side and do that manipulation.

What we will find is this PLD, will correspond to $\Lambda/2 \sin\theta$. So if you do all this substitution what we will find is $\sin\theta=\lambda/\Lambda$. And θ will be $\sin^{-1}(\lambda/\Lambda)$,

Now, λ= 250nm and if we are given Λ = I micron; then on substitution θ will be 14^0

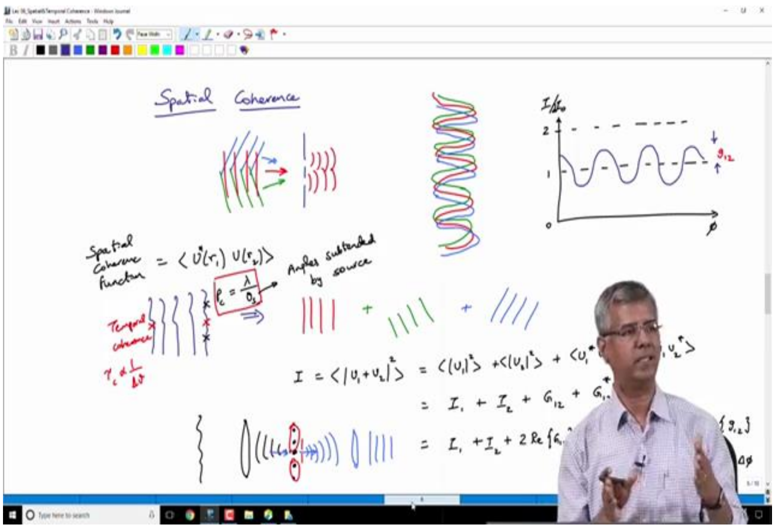
(Refer Slide Time: 69:48)



So that is the other part, one last note and then I will stop at this. In the tutorial the other thing that is asked is in the, so this is actually the A part of the problem, this is the B part of the problem. In the C part, you are given that this incoming beam has got a spatial coherence. That UV laser beam has got a spatial coherence of 1 millimeter and we are asked to find out what is the depth over which this interference happens. What is that longitudinal depth over which that interference happens.

Why is that important? Because in my FBG fabrication case I need to know how close I need to keep my fiber so that I can get this periodic structure. So I need to know what is the value of D, so how can we find this? Well if it is a spatial coherence of 1 millimeter what does that mean?

(Refer Slide Time: 71:14)



You know that only part of the section of the beam which is within 1 millimeter, they are going to have interference right. That is what we saw when we were looking at spatial coherence somewhere over here. We say spatial coherence means that only if the spacing between the two slits is within 1 mm I can get interference. If it is greater than that then the coherence actually falls off rapidly.

So essentially what we are saying is within this, you have a 1 mm section of the beam over which this is going to support this interference. So corresponding to that 1 mm, (I have used a different color - orange here) you are going to have this interference happening. So this depth over here is where you are going to get interference fringes. Although the beams are crossing each other all across the entire element, only over that 1 millimeter region the beams are going to have that memory that it came from the same source .

So only that will be correlated and will actually give constructive interference. So you can actually say there are 1 millimeter neighboring sections, each of which is interfering and giving you that interference pattern. So that interference pattern can in the lateral side can be across that entire beam size, but in terms of the depth that you get, that depth will be determined by these two beams - two rays that are crossing across within that 1 millimeter.

So if we do that, we will find that, if you say this is θ_1 we can actually draw this again so we have 1 beam going this way and another beam going this way. And this is at an angle θ_1 which is what we have determined over here or you can just call it θ right. So this angle is θ, so you can say this angle is also θ. And if we want to find out "D" over here you can write it as $\tan(\pi/2) - \theta$ right, is going to be given by this D over base of this is actually, that corresponds to ρ_c right but we are only taking half the triangle over here, so that will correspond to $D/\rho_c/2$.

Or in other words $\tan(\pi/2) - \theta$ which is nothing but $\cot\theta$,

So, $\cot\theta = 2D/\rho_c$ or in other words what we want to find out is d.

$D= (\rho_c/2)\cot\theta$ and θ we already found out is 14^0. So if you substitute ρ_c=1mm

D will correspond to 1.95mm

So you find that the depth of that interference region is 1.95 mm. I have to place my fiber within that region, preferably closer to the phase mask but certainly within that 1.95 mm region to have good contrast interference fringes.

If I move it away from that, the contrast in my interference fringes will fade, because the interfering beams are not spatially coherent. That means they are not going to give this nice constructive and destructive interference. So let me stop at this point. We have learnt some things about wave optics, and coherence of light and in coherence of light we were introduced to the statistical property of light.

And based on this, we will go back and see where this statistics comes from. You will see that it is coming from the emission characteristics of light and that actually brings about the particle nature of light. So far we have been looking at only the wave nature of light but we will go on to looking at the particle nature of light in the next session.

Thank you.