Introduction to Photonics
Spatial and Temporal Coherence

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Hey welcome to another session of Intro to Photonics so the last couple of lectures we been looking at some more examples of wave optics from specific properties in wave optics, including multiple slit interference and then the last lecture we have just introduced the concept of coherence, right so we looked at what temporal coherence is all about.

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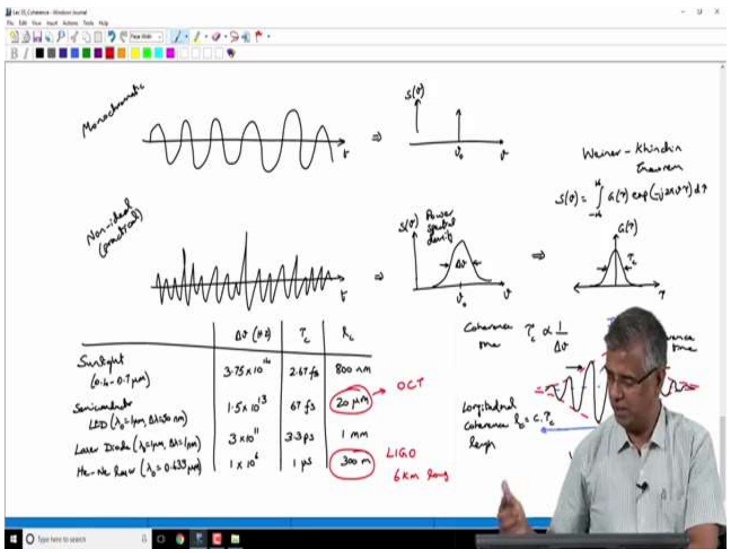

At the end of the last lecture we were examining certain examples for sources and we said sunlight is a source. But when you look at the spectral width of sunlight and specially, we are just looking at picking up the visible region and that is quite broad, so the result at the coherence time or the coherence length is quite short right. As supposed to helium neon laser which has a single, so called single frequency operation but the line-width can be in the order of megahertz in which case we found that the coherence length is rather high.

As you know what the uses of coherence are, something like this. If you are building an interferometer. In general, you would say what do you want longer or shorter coherence length? Shorter, why would we need shorter coherence length? There is one good example of that I will come back to that but in general you want to have longer coherence length so that you do not have to match the path lengths so accurately.

So, for example with LED we are saying the coherence length is 20 microns so that means both the path have to be match to 20 micron accuracy, so that you can have interference and that is very very hard to do, whereas if you are trying to build an interferometer with a 300 meter coherence length you could roughly put the two mirrors at some certain distance and chances are that you will get good interference.

So, if you want to build an interferometer, typically you are looking for long coherence length and that is what is behind we heard of the LIGO. So, what is LIGO? LIGO is basically a project which is looking at detecting gravitational waves. You heard of gravitational waves recently? The Nobel Prize recently was given for the detection of gravitational waves and believe it or not, that detection was through one of these interferometers, the Michelson interferometer configuration.

And there since the gravitational wave, the wavelength is fairly long, you need to have very long interferometer. How long is long? So, one of the interferometers that I know of is 6-kilometer-long arms. To minimize the any external perturbations effect lot of precautions taken. The entire thing is in a maintained under vacuum conditions and buried underground.

So, but the point is if you are making a 6-kilometer-long arm, so you have two arms, each of those 6 kilometers long, you would want to have here and there some leeway in terms of positioning the mirrors. So clearly in those sorts of application it helps to have a highly coherence source, highly temporally coherence source. So, if you have a fairly long coherence length that certainly helps in those sorts of cases.

But somebody mentioned maybe you want to have short coherence length and that was still quite useful in application called optical coherence tomography or in short OCT. So how would this help in optical coherence tomography? Well what you are trying to get using optical coherence tomography? OCT gives a fairly accurate investigation of what is happening let us say in the eye. In the eye you want to look at every 10 microns slices and see what is happening.

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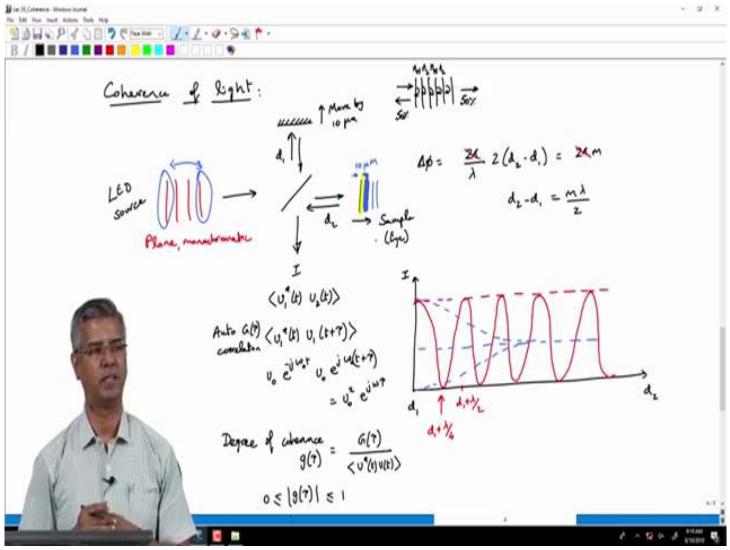

So, how do you localize this investigation to 10 microns? Well you can essentially let us say we have a Michelson interferometer as shown configuration. In this case, what I have is a sample where I need to get 10 micron resolution. I want to characterize this sample with 10 micron resolution, so what do I do? I initially start with d_1 and d_2 being equal.

So, I start with for this example, marked layer over here. So, I will position my mirror so that I am essentially looking at only this layer, why? Because anything of this layer has got a different path length, I don't have the coherence to support the interference. I am getting interference fringes only from that layer at any particular time. Now how do I go to the next layer? What can I do to go the next layer? Change d_1, so I would basically move this mirror by 10 microns and then what am I looking at in that case, if I increase that path length by 10 microns?

I am looking at this next layer and then I move by another 10 microns I am looking at the next layer and so on. So, I can limit my interference to only 10 microns at a time provided. What should be the nature of my source? It should have very low coherence length. What does that mean? Low coherence length is supported by a source with very large spectral width.

So, if I use LED source, a broadband source like that then I can get information from each slice of the sample. So, this my sample like I said it could be your eye for example you are going for an eye testing. The doctor wants to see layer by layer how your eye is, they can put you through an OCT and get all that information okay and it is actually a commercial device equipment right now but something once again as advanced as this as a very fundamental principle based on interference of light specifically building this Michelson interferometer, specifically using a low coherence source and then you can do something as accurately as this.

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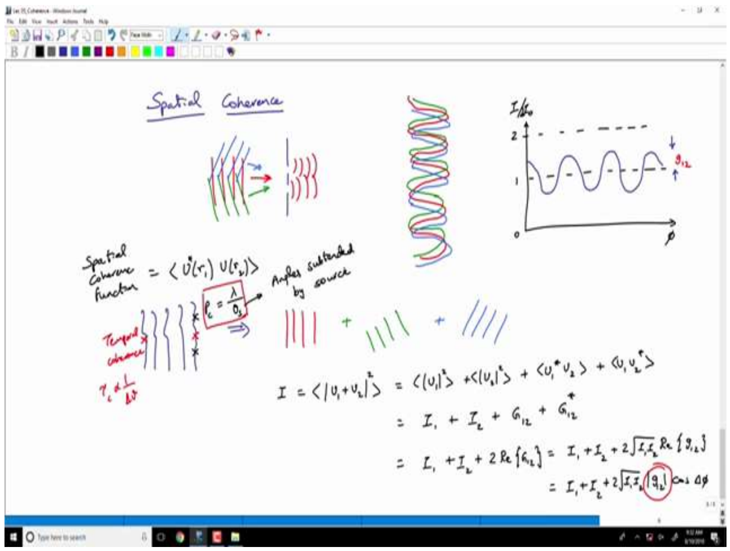

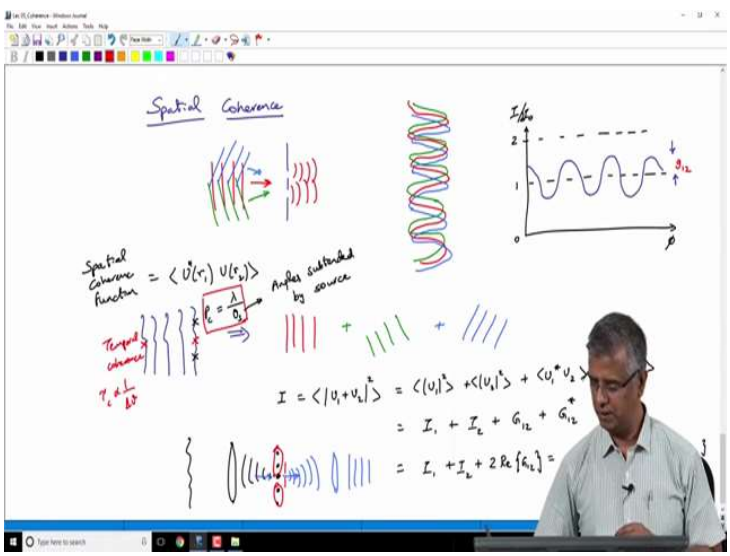

So what I want to actually do is, time permitting work out a couple of problems related to the concept that we have looked at so far but before I go to that I want to give you an idea of what Spatial coherence is all about? To understand this let us just go back to your Young's Double Slit Experiment wherein we said, you have these two wavelets that are generated when you have this plain wave and that is incidence on it and then in the distant observation point you are actually seeing constructive and destructive interference.

Now suppose I have this source coming at a slight angle over here, it is not normal to the pinhole it is coming at a slight angle where do you expect the interference fringes to be and what will happen to the interference fringes? It will get dislocated, it will get shifted so now the interference fringes will be like this, similarly I can come in with, if I come in with something like this then it will be shifted down so the corresponding interference fringes will look something like this.

So effectively if your source has multiple wave vectors, wave directions you get an interference fringe that is effectively smudged or in other words you are basically looking at some average value of interference fringe and previously you were going from 0. Let us say we are plotting $\frac{I}{2I_0}$ right, we are plotting over 0 to 2, it was going from maxima to minima and so on but now, so this is as a function of the path length difference or phi(ϕ).

But now what happens is you do not have that much of clarity. You cannot go to maxima and minima. Because, if you have a single wave vector the maxima and minima clearly defined like its further red but if it has got multiple wave vectors then that maxima and minima definition is changed so effectively what you see is something like this. So, the maxima are not the highest you can get, and minima is not the lowest you can get so effectively your visibility, the contrast of your fringes reduce.

So, the contrast essentially is getting reduced. So, why am I talking about all of this? Because when you look at practical light sources, most of the light sources that we have will essentially not be absolutely planer. So, you would have wave front like this and this wave front if you decompose will consists of multiple wave fronts like this wave front plus this wave front plus this wave front.

So, it will consist of all these different wave vectors and if it has all those wave vectors then your visibility of the fringes is reduced. So, effectively what we are defining here is another coherence term which dep ends on the spatial properties of the light that we are looking at. So, previously we were looking at for temporal coherence. You were looking at $U(t)$, $U(t+\tau)$ which is correlation between a point in one wave front with another wave front, a delayed wave front. So, you were looking between the correlation between those two wave fronts.

But here you are looking at the correlation between two points in the same wave front. So, you are looking at basically$U(r_1)$, $U(r_2)$, a correlation between those two points. That is actually the spatial coherence function and defined by $\langle U^*(r_1)U(r_2)\rangle$.

So, in general when we are looking at partially coherence sources, you say $I(intensity)$ is nothing but $I = \langle |U_1 + U_2|^2 \rangle$ and that we can write as,

I= $\langle |U_1|^2 \rangle + \langle |U_2|^2 \rangle + \langle U_1^* U_2 \rangle + \langle U_1 U_2^* \rangle$

Where, U^* is conjugate. And $G_{12} = \langle U_1^* U_2 \rangle$ is defined as coherence function. So again above expression we can express as,

$$I = I_1 + I_2 + G_{12} + G_{12}^*$$

Any complex value and its conjugate if you add them together that is going to give to you,

$$I = I_1 + I_2 + 2Re\{G_{12}\} .$$

This is same as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} Re\{g_{12}\}$$

Where g_{12} degree of coherence.

And this is what you can write out as:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} |g_{12}| Cos\Delta\phi$$

Where $\Delta\phi$ is phase difference between the wave fronts.

So, what we have done here is with respect to our previous discussion, what is the new thing here? So, we have g_{12} factor also coming to the picture and this factor if you look closely at it, that will be fringe contrast and the degree of coherence provides this fringe contrast.

We have been looking at the ideal case before, the ideal case of plane monochromatic wave and in which case g_{12} was equal to 1 so we had these constructive and destructive interference yielding maxima and minima. Which is the absolute maximum and absolute minimum which is 0. So, that is what we were going but now we are bringing into the discussion a property of light called coherence of light. One part of that is spatial coherence which we have defined over here and the other part of it is temporal coherence which is related to looking at the relative auto correlation right between two wave fronts that are delayed with respect to each other.

So this is temporal coherence which is dependent on what? The temporal coherence time is inversely proportional to the spectral width and in this case with spatial coherence we will define a coherence length, a coherence distance (ρ_c) which is given by

$$\rho_c = \frac{\lambda}{\theta_s}$$

Where,

ρ_c = Coherence length,

θ_s = Angle subtended by the source

λ = Wavelength of light

If it is absolute plane wave, what is the angle subtended? the angle subtended is 0 but if you have a non-planer wave front that wave front can be decomposed into superposition of multiple plane wave fronts with different angles, that angle subtension is what we are looking at is θ_s . So, if you have more angle subtension, let us take the example of sunlight.

You know it is not only temporally incoherent but spatially also. It subtends a large angle or LED for example, LED I mean why do we use LEDs for these lights here, because the emission subtends over a large angle right so that means you have multiple wave vectors and hence this LED will be spatially incoherent also. It is not just temporally incoherent, temporally incoherent comes from the perspective of large spectral width but spatially incoherent because of it.

Now, let us say I take a very poor spatially coherent light, and can I improve the spatial coherence? Yes, no, maybe not sure? It turns out yes you can. So, what happens when you take a source like this and put it through a lens, so you are actually looking at a spatially incoherent source and you are putting it through a lens then what you will see and of course I am not giving you all the details to appreciate this but what you will see is that it will come to a focus but it will not come to a focus at one particular point.

If it was a planer wave front you would have light come to a focus at one particular point but if it is a non-planer wave front it will not only have only particular focal point here but also have here and there and so on. It will have multiple spots because, like I said you can decompose it into multiple plane waves that are slightly at a different angle with respect to each and with respect to that angle when you are focusing you will get different points at which light is focusing.

Now if put a spatial filter, I put a pin hole and such that it allows only that central point that central point constitutes a wave vector that is going in a particular direction so wave vector will go through a particular direction. Now of course you would say that it is not, it is going a particular direction, but it is not a plain wave front, it is a spherical wave front.

Because what is coming out of this pin hole you would say through diffraction it will go like spherical wave front but any well-defined spherical wavelength, I can always find some way of compensating for that wave front, what will I do? I will put a lens here that compensates for this divergence and what I get out of here is a plain wave front. Do you understand this? So if you have picked one particular wave vector, even though it is diverging it will have nice spherical wave fronts which can be corrected if you can put a lens in front of spherical wave front which straightens out all those wave fronts. Which delays spherical central part of the wave with respect to end part and then you can straighten out the wave front.

So, what I get out of this is nice plain wave fronts. What do I loose in the process? I made the source spatially coherent but what do I loose in the process? Intensity, all the power that is present in these other wave vectors I am loosing those. So, that is what you have to do so to clean up the beam so you can take sunlight for example and you want to make sunlight spatially coherent. So, you can put one of the spatial filters in place and take out all these other higher spatial frequencies and you get one particular spatial frequency out through a pin hole.

And that pin hole, what should be the size of that pin hole? That size must be comparable to the wavelength of light because that spot size will be comparable to the wavelength of light. So, it is got to be a very very narrow pinhole in the order of microns and then only you can do this. Now can I improve the temporal coherence of light? What do you think?

Yes, I mean if I can find a filter to do the spatial coherence, I can find a filter to improve temporal coherence also. All I have to do that, I have all these multiple frequencies, I put a filter and I get only one frequency out or only very narrow band of frequencies out. When you look at that narrow band of frequency that is going to be temporally coherent.

So you get sources which are not spatially coherent, not temporally coherent but if you want to achieve particular functionality you could do things to improve the coherence of light. But if you know why or if somebody tells you it is not spatially coherent then you say what is the angle subtended by this and I can find a way, I can put a spatial filter, I can improve the spatial coherence or I can put a spectral filter, I can improve the temporal coherence.

So I do not have a whole lot time but in this session but what I can quickly do is give you a feel for how this principles that we have learnt so far we looked at multiple beam interference and then this properties of temporal and spatial coherence, how these principles can be used to design optical elements, optical instruments.

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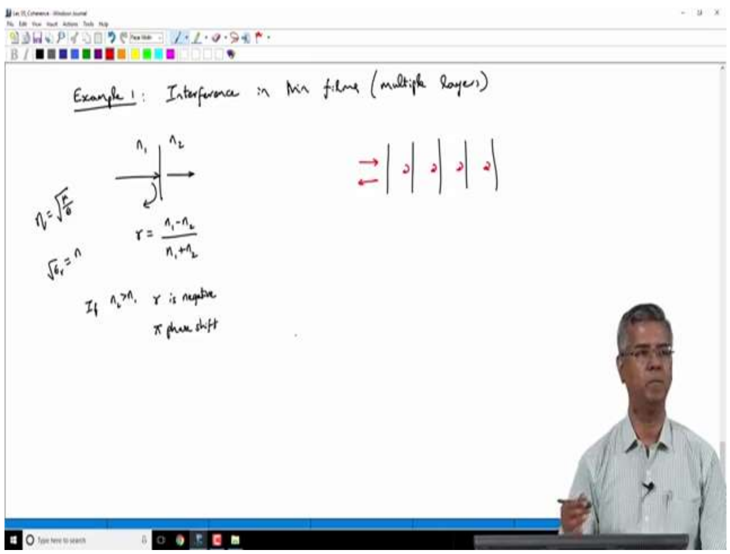

So, let us start with example 1 and I think we will have to continue this later on but we will at least get started. Let us start with example of interference in thin films which actually constitutes multiple layers. So, what is this based on? Now whenever light goes from one medium to another, let us say a nice plane wave is coming and hitting this interface, what do you expect to happen?

So, it can undergo refraction but, in this case, normal incidence so where do you think it will be going to go, going to the same direction? It is going to go straight down, is that the only thing that happens? There will be a reflection also right, so that reflection if you want to quantify that reflection co-efficient (r), what is that going to be based on, that is going to be based on the relative refractive indexes of the two media.

If you are talking in terms of electromagnetics, you learnt in electromagnetics that the reflection co-efficient (r)

$$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

where, η_2 corresponded to the impedance of the second medium that is the medium that is going into and η_1 corresponds to the medium where the wave is originating right but η the wave impedance as they call it in electromagnetics is nothing but

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

So, it is inversely related to $\sqrt{\epsilon}$ and I would say that $\sqrt{\epsilon_r}$ (where ϵ_r the relative permittivity) is nothing but the refractive index of a medium.

So maybe you have not heard that before but just tell you that root of that relative permittivity is nothing but the refractive index of the medium so we are going from whatever we learned in electromagnetics to what we want to do in wave optics but if you substitute all these terms what you will basically find is if μ is the same for both regions then you get,

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

Where n_2, n_1 are the refractive index of the medium 2 and 1. Whatever we learnt in terms of wave impedance we just bringing it into in terms of the refractive index. So, what does it tell you? Especially if n_2 is greater than n_1, you are going from a rarer medium into a denser medium, what does the r become? r is negative, what is the negative reflection co-efficient mean? You have a wave that is coming and hitting this interface which is giving you a negative reflection co-efficient, so what does that mean?

The wave essentially gets flipped or you have π phase shift. You have a π phase shift whenever a wave is going from a rarer medium into a denser medium and of course, if it is going from a denser medium to a rare medium there is no phase shift.

So, we are going to track as light goes through multiple layers, we are going to track how that light wave is going to accumulate phase and we will also be going to track how the reflected components when we look at reflection from multiple layers. If you have multiple layers how these reflected components? Whether they are going to come in phase over here or outer phase over there and so on.

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In certain examples you need them to be out of phase, one popular example is what I am wearing here, these glasses they are anti-reflection coated, I see some of you are wearing glasses when you go to the guy that makes the glass he asks whether you want an anti-reflection coating and you say yes. So, what goes into putting that coating on this? Anti-reflection basically says whenever light goes from one medium to another there is always reflection.

So, if I am putting my glasses you will see reflected light you would not see my eyes. So, you want to have anti-glare filter so that that reflection is frustrated, so how do you frustrate that reflection, destructive interference? In a way we are talking about frustrating something you is looking at destructive interference, making that reflection component 0.

So you may have applications like that where you are looking for destructive interference because of reflection from interfaces, you have applications where you want to reflect light like when you are doing this experiment, the Michelson interferometer you put these two mirrors, what are these mirrors? They could be dielectric mirrors, they could be metal coated mirrors but they could also be dielectric mirrors, in that case you want all the reflected components to be doing what, constructive interference.

It should have constructive interference so they all add up and it give you a very high intensity light coming back so we are going to look at those examples, we will do that in the next session because we do not have lot of time here. Thank you.