Introduction to Photonics
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Lecture 05 – Coherence

In the last couple of lectures, we have been looking at the wave properties of light and have defined the concept of phase of a light wave. Then based on the phase of the light wave we have said that the light waves go through interference phenomenon and by looking at constructive and destructive interference we have been looking at how light tends to be brighter at certain spots and not so bright at other spots. We took the example of Young's double split experiment and then we extended to multiple slits.

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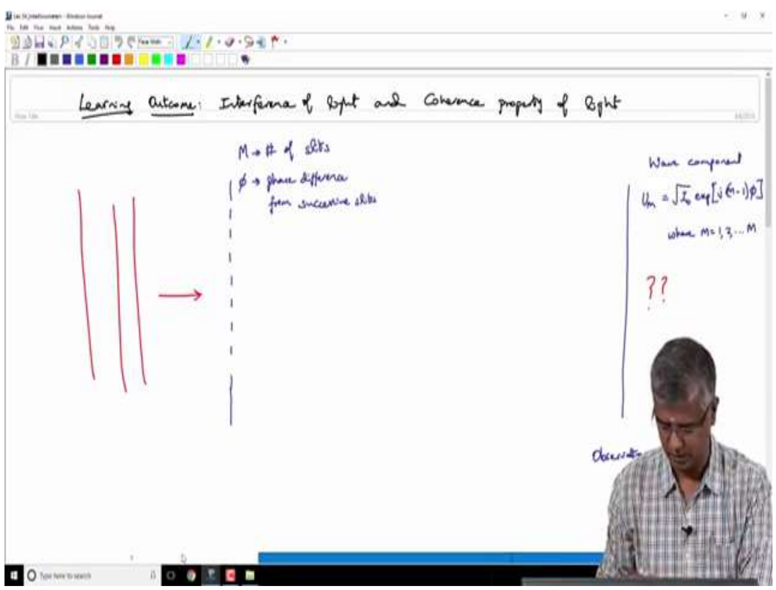

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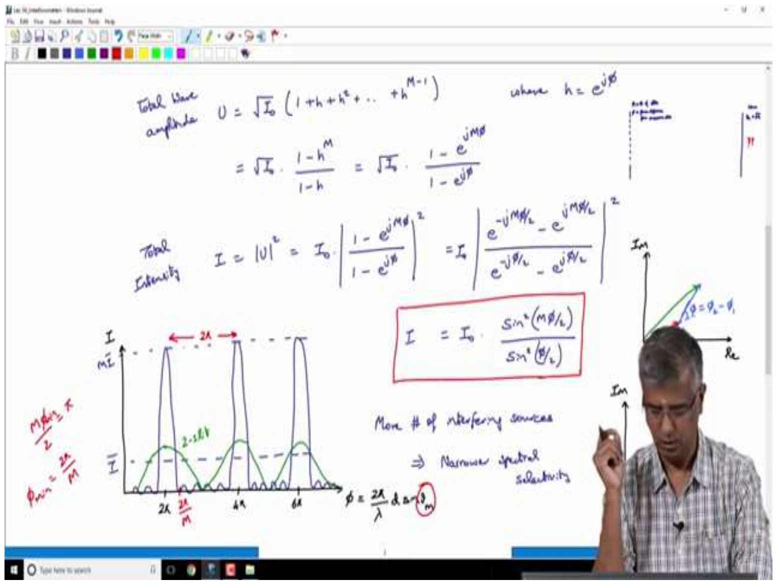

And then we came to the conclusion that when you go for multiple slits, we have more number of interfering sources, you have more constrained interference criteria which provides you much better selectivity in the response. And so we had taken the specific example of wavelength selectivity but in general,

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \tag{1}$$

So even for a constant wavelength you can say by looking at the interference pattern, the interference pattern is going to be sharper spatially when you consider multiple sources of interference.

In the last lecture we looked at some examples of this phenomenon. We had seen that the CD has multiple grooves which essentially provide the situation of an interference criteria. So you can separate out colors and that actually is a subset of larger class of components called diffractive components. So just the principle that we have looked here in terms of multiple slits can be extended to having optical components which are called diffractive components or diffractive optics, which essentially allow you to change the characteristics of light.

If the light is going in a specific direction, it allows you to bend light or direct light at a particular direction. When we look at this interference criteria, it tells you is that you can direct light into multiple orders (refer the equation 1). The 0^{th} order would correspond to the same direction that the incident light had. But then first order, second order, third order and so on, is going to have different directions defined by this θ (refer equation 2).

$$\phi = \frac{2\pi}{\lambda} d \sin\theta_m \qquad \text{, 'm' is the order} \tag{2}$$

So different orders are going to have different directions. You could do some manipulation with the phase at the point of the slit, and as part of the tutorial, we have examples where you could direct light into a particular order. This is a general criterion where we say that the incident light which has a very specific direction is actually diffracted into multiple orders.

But you could also come up with manipulations at the point of the slits through which you can direct most of your light in a particular direction and that is similar to the concept that you might have encountered in antennas called phased array antennas. In this case manipulating the phase and the amplitude with each of the sub-antennas you can have a very specific radiation pattern. In fact, you have a radiation pattern that can scan across different directions just by manipulating the phase and the relative phase between the different antenna elements. So similar concepts can be possible here as well. Here also you could direct most of your energy in a particular direction and it is slightly more advanced concept. This similar concept will be seen some problems of the tutorials.

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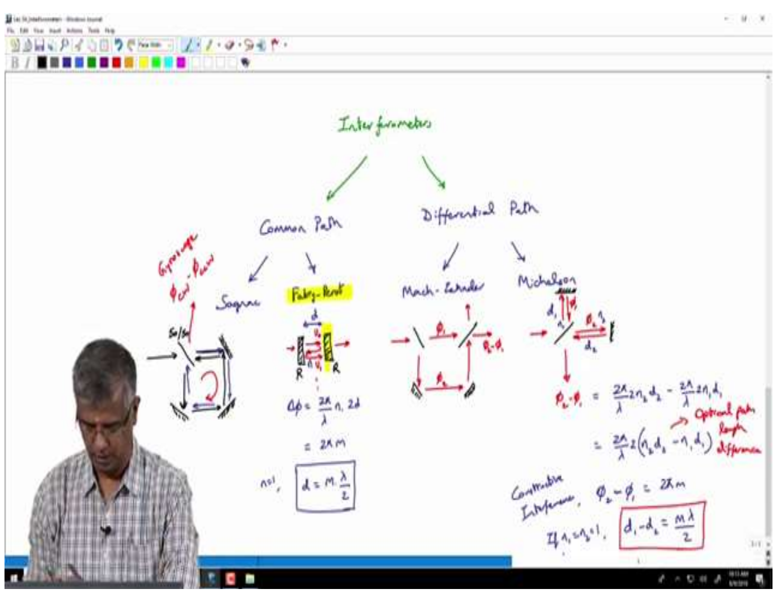

We have seen the specific configuration of the multiple slits. The similar principles can be extended to, when you consider Fabry- Pérot interferometer. In Fabry Pérot there are multiple reflections and all these reflected components are going to constructively interfere at a particular point (near the output coupler – refer lecture @ 6:00) which allows light to go through the mirror. So you could potentially extend the concept of multiple interference to this Fabry- Pérot case as well. Only difference will be that in a case of a Fabry- Pérot cavity, you would have certain reflectivity for the mirrors, so the reflectivity in terms of power, and if you want to look at it in terms of the amplitude it would be $\sqrt{R}$ for each of the mirrors. So the reflected components are going to have lesser and lesser amplitude. So all the interfering components are going to have lesser and lesser amplitude and that is different from the case when we were looking at multiple slits.

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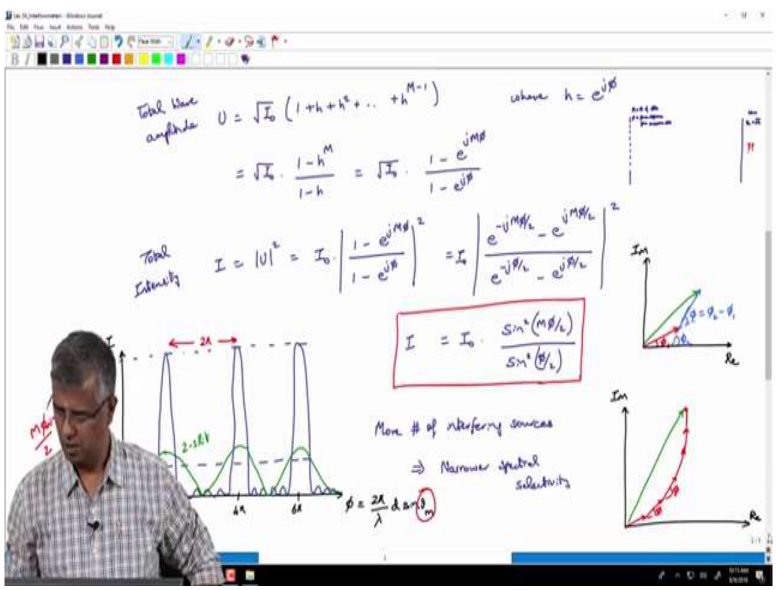

When we were talking about a phasor diagram in case of a multiple slits, we have seen equal intensity for each of these phasors or equal amplitude for each of these phasors. But if you have a configuration like a Fabry- Pérot where you have a reflectivity of mirrors which is less than 100% each time it goes through a bounce it is going to lose some of the light. So the phasors are not going to be equal, they are going to be diminishing in terms of their amplitude (refer lecture @ 7:11). So correspondingly you have a slightly different result. But nevertheless most of the other principles in terms of the number of interfering sources defining the interference criteria, the constraint in the interference criteria, all those things would be same as multiple slits.

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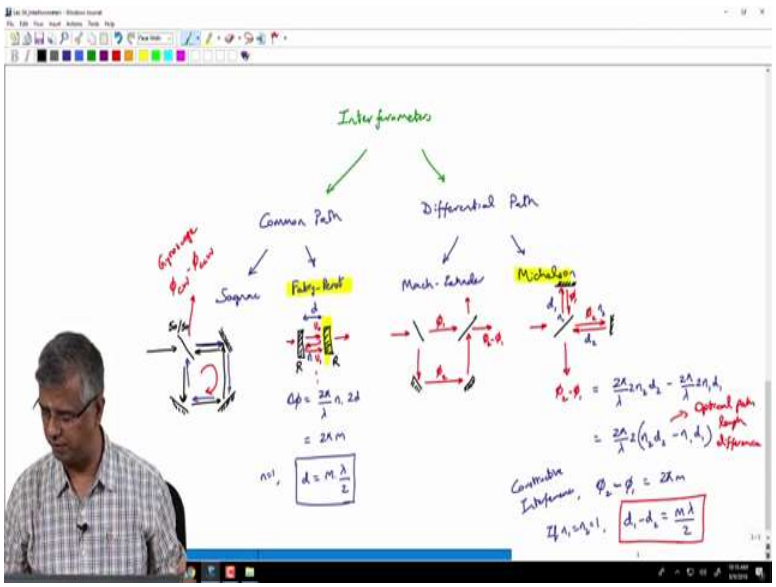

At the end of the last lecture we were looking at the Michelson interferometer. The Michelson interferometer is this configuration over here (refer lecture @ 8:36). Here we could basically split the light into two arms. And in the two arms you put a mirror and deflect the light back and if you are observing the interference output, that output would depend on the relative phase that the light waves in the two arms have accumulated. The constructive interference criteria when the path length difference,

$$2(d_1 - d_2) = m\lambda$$
$$\Rightarrow d_1 - d_2 = \frac{m\lambda}{2} \tag{3}$$

The factor 2 in the equation (3) is due to the light is taking a round trip. In case of the multi-slits case we said the path length difference has to be $= m\lambda$.

We extend the concept of Michelson interferometer to introduce the coherence of light. To give you an idea of what coherence is all about, so far in all our examples we have made two assumptions, at least in the case of Michelson one of those assumptions was that we have considered monochromatic light. Because we have been looking at all these interference conditions with respect to only one wavelength, so this is actually for a monochromatic wave.

And when we were talking about double slit, we said we have a plane wave that's incident on this double slit. But in reality there is no such thing as a perfect monochromatic source and a perfect plane wave source. You may have some light source that is very close to that but not exactly perfect monochromatic plane wave. In reality these light waves are polychromatic and these does not have absolutely planar wavefronts.

So what is the effect of these wavefronts on the interference and how do you quantify that? That quantification what we are going to see, is through the coherence what we call as the temporal coherence or spatial coherence. So let us go ahead and try to define all these terms.

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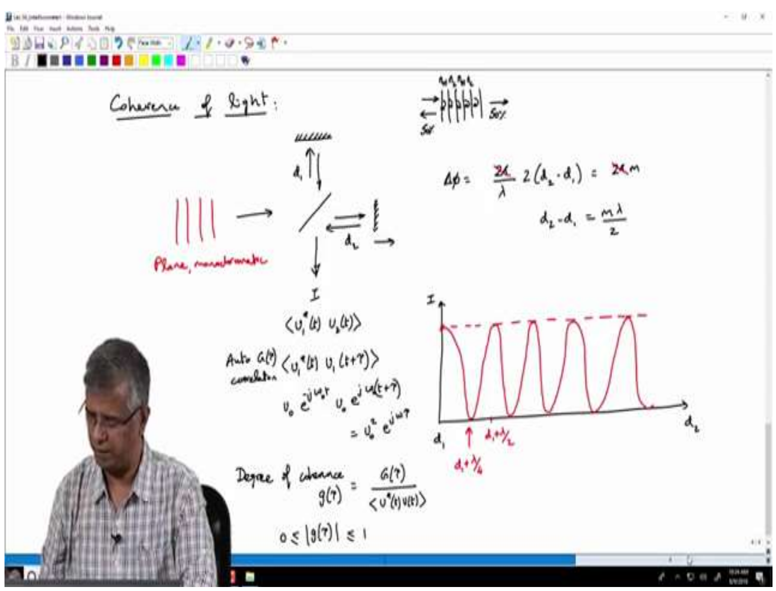

So let us start by looking at the coherence property of light. And I should give a disclaimer here, this coherence property of light which actually leads you towards the quantum optical principles of light is a fairly deep subject. As far as this course is concerned we are just skimming through the topic just to give you a basic feel for what this is all about. If you are interested, you can go into deeper reading of this topic.

Let us look at the coherence by taking the example of Michelson interferometer. We have a beam splitter here which splits light, let's say in two ways. So you have a light beam coming in that is getting split and you put a mirror over here and a mirror over there. It bounces back, this also bounce back and then we are observing over here. (Refer lecture @13:00)

We have not defined many of these optical components and mirror so far. You can say it corresponds to a metal coated substrate, so that it can act like a mirror. But there are other mirrors called dielectric mirrors. Similarly, with a beam splitter if you want to achieve a perfect 50-50 split, what you use is known as a dielectric coated mirror. The principles of handling phase come into the picture while discussing about these mirrors. We need to track phase as it goes through the multiple layers. All these dielectric mirrors will have two materials, n_H (high index material) and n_L (low index material). If you have alternating layers of these materials, then if a wave is incident on it, it is going through reflections at each of those interfaces where the refractive index is different. This is what you call as Fresnel reflection. And all these reflected components are in phase, they will all add up together and you will get perfect reflection. All the light that is going forward, it is all going to get reflected for 100% reflection. Or you could have lesser number of layers through which you don't reflect all of the light, you reflect part of the light, let's say 50% of the light and the other 50% is transmitted. That will be a partially reflecting mirror.

So you can make mirrors using some of the principles that we have talked about so far. What you have to do is essentially look at what is the relative phase of all these components that are getting reflected from the different interfaces. We have a tutorial problem on this concept as well, so you can appreciate some of the details of this. You could go to the extent of connecting this principle to the Bragg diffraction theory which we had learned in high school.

As you know if you have a periodic arrangement of atoms, if you come in with x-ray radiation, then you have these x-rays diffracted and they all constructively interfere in a particular direction. Through that you can possibly tell what is the period of this atomic spacing. So that principle you know is not very different from what we are learning here about multiple slit interferometry or in this case multiple layer interference. So you can say that this multiple layer interference is actually one-dimensional Bragg reflector in that sense. So I am just connecting this to some of the concepts you might, we are already familiar with.

Coming back to the Michelson interferometer, we define this distance we called as d_1 , this we called as d_2 (refer lecture @17:18) and we were looking at

$$\triangle\phi = \frac{2\pi}{\lambda} 2(d_2 - d_1)$$

For constructive interference

$$\begin{aligned} &\frac{2\pi}{\lambda} 2(d_2 - d_1) = 2m\pi \\ &\Rightarrow (d_2 - d_1) = \frac{m\lambda}{2} \end{aligned} \tag{3}$$

So suppose I am looking at the intensity of light at the output and I move one of these mirrors. Let's plot d_2 versus intensity of light. Initially let us say I have this condition that $d_1 = d_2$ and then I start moving one of these mirrors. At this point it is maximum because the phase difference is zero and as d_2 increases, the plot goes through series of maxima, minima, maxima, minima and so on (Refer lecture @19:16). So of course, you can define the next maxima to be happening at $d_2 = d_1 + \frac{\lambda}{2}$. The minima would correspond to $d_2 = d_1 + \frac{\lambda}{4}$ and so on.

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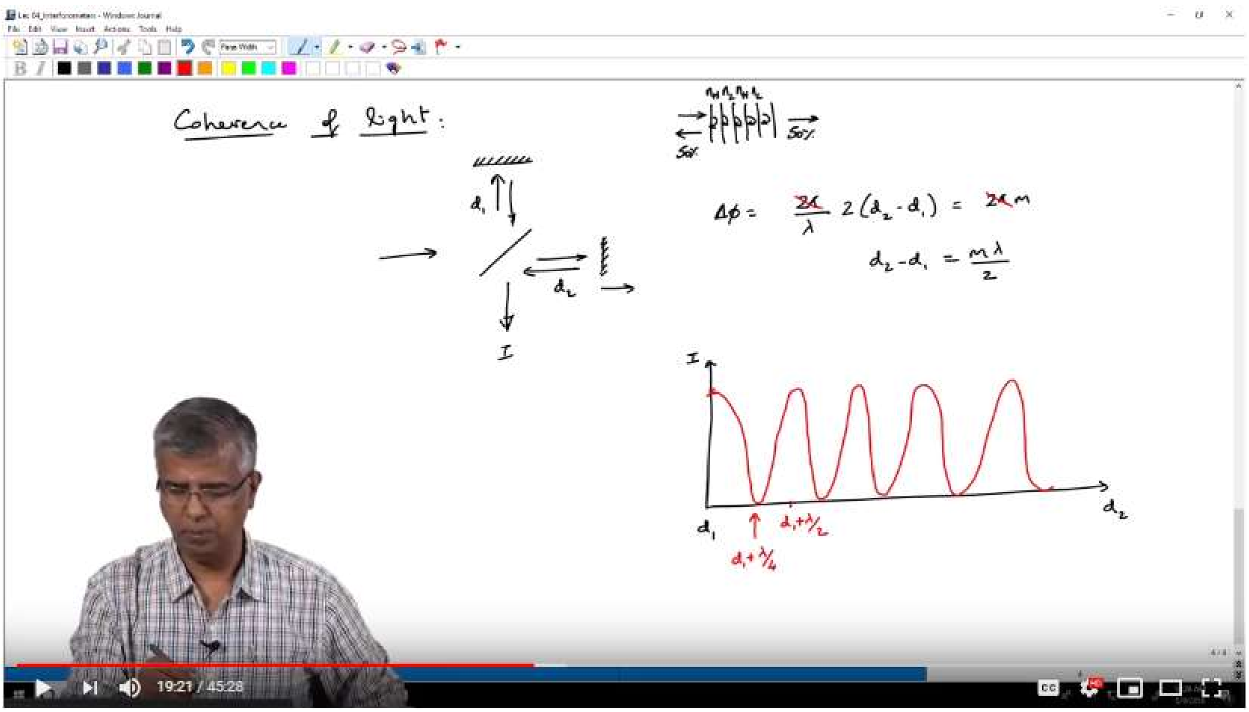

But the key point here is that we are saying that the intensity is going to be constant irrespective of whatever the value of d_2. Now we will question this particular assumption. Is this constant for whatever is the value of d_2?

We will find by examining this closely that you are observing the beating of $U_1(t)$ with $U_2(t)$. In fact, $U_1(t)$ and $U_2(t)$ are complex quantities, so the beating would correspond to $U_1^*(t)U_2(t)$ and then $U_2^*(t)U_1(t)$. We look closely at what $U_2(t)$ means. In this case is $U_1(t)$ and $U_2(t)$ different? It is different from $U_1(t)$, as it is going through a different arm and then the distance is different. But otherwise is it different? Not really. It came from the same source. So, I can basically write

$$U_1^*(t)U_2(t) = U_1^*(t)U_1(t+\tau)$$

When we are making a measurement, we are not making an instantaneous measurement. If you have to make the instantaneous measurement, the light frequency at 1 micron corresponds to $10^{14}\,Hz$ .So when you talk about the time scales, the light, the period with which this oscillation happens that period corresponds to some femtoseconds. So we are not doing any measurement in femtoseconds. So what we are doing in general is a time average measurement i.e.

$$\begin{aligned}&\left\langle U_2^*(t)U_1(t)\right\rangle\\&=\left\langle U_1^*(t)U_1(t+\tau)\right\rangle\end{aligned}$$

This relation $\left\langle U_1^*(t)U_1(t+\tau)\right\rangle$ reminds us a correlation. In this case it is the same source, so it is basically autocorrelation function.

So through the Mach-Zehnder what you are really observing is the autocorrelation function. If you have a plane monochromatic wave incident on it, then that light wave is going to have a certain amplitude. Let us say $U_1 = U_0 e^{i\omega_0 t}$. Then we can represent the autocorrelation function as $G(\tau)$ such that

$$\begin{aligned}G(\tau) &= U_1^*(t)U_1(t+\tau)\\&=(U_0 e^{i\omega_0 t})^* U_0 e^{i\omega_0(t+\tau)}\\&=U_0^* e^{-i\omega_0 t} U_0 e^{i\omega_0(t+\tau)}\\&\Rightarrow G(\tau) = U_0^2 e^{i\omega_0 \tau}\end{aligned} \tag{4}$$

You can now define another quantity called degree of coherence, $g(\tau)$ which is given by

$$g(\tau) = \frac{G(\tau)}{\left\langle U^*(t)U(t)\right\rangle} \tag{5}$$

The denominator represents the total intensity. The numerator $G(\tau)$ is called as the temporal coherence function and we are normalizing $G(\tau)$ with the intensity. The degree of coherence $g(\tau)$ is a normalized coherence representation. Let's see the magnitude of $g(\tau)$ i.e. $|g(\tau)|$ for a monochromatic wave.

$$g(\tau) = \frac{G(\tau)}{\left\langle U^*(t)U(t)\right\rangle} = \frac{U_0^2 e^{i\omega_0 \tau}}{U_0^2} = e^{i\omega_0 \tau} \tag{6}$$

$\therefore |g(\tau)| = 1$

So for a monochromatic wave $|g(\tau)| = 1$. For a totally uncorrelated source of light, $|g(\tau)| = 0$.

So we can write $0 \leq |g(\tau)| \leq 1$

$|g(\tau)| = 1$, perfect monochromatic source

$|g(\tau)| = 0$, perfect incoherent source.

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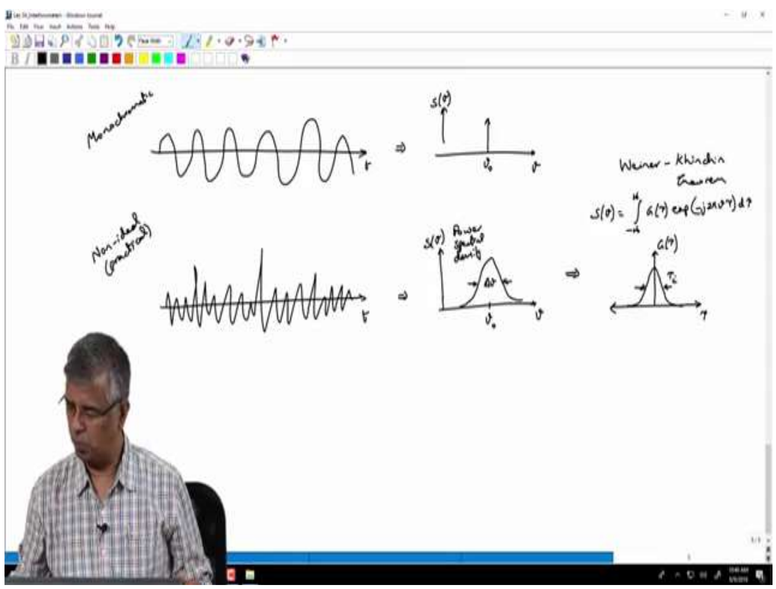

If we see the time and frequency domain representation of a monochromatic wave. In time domain, it will be nice periodic function with frequency of oscillation as ν .In frequency domain, it will be a delta function at ν (refer lecture @28:42). The perfect monochromatic or ideal source look like in this representation.

But when you look at non-ideal sources or practical sources, in time domain, it will be a non-periodic function. (Refer lecture 28:56). If you take a Fourier transform, the power spectral density is going to look something like this. It has the center frequency in ν_0 having a width $\Delta\nu$ (refer lecture @29:21). When we have a mixture of multiple frequencies, you do not have a perfect periodic sort of pattern, you have non-descriptive pattern as far as the temporal behavior is concerned (in time domain).

What we are looking at fourier domain is actually the power spectral density $S(\upsilon)$. (Refer lecture @20:22). The Wiener–Khinchin theorem (used in communications or signal processing usually) says that the power spectral density and the autocorrelation function are Fourier transform pairs.

$$S(\upsilon) = \int_{-\infty}^{+\infty} G(\tau) e^{-i2\pi\upsilon\tau} d\tau \tag{7}$$

This relation state that the power spectral density and the autocorrelation function are connected with each other. So knowing the autocorrelation function you can compute the power spectral density. Or conversely, knowing the power spectral density you can compute the autocorrelation function. So if you do this inverse Fourier transform, so what you will find is $G(\tau)$ (Refer 32:33)

But when we look at with respect to the delay that we are imposing, you would essentially see that this coherence function means that it will actually be maximum when the delay is 0. But it starts falling down once you start having a delay. This concept is relevant if we look at the interference pattern again. (Refer lecture @33:06)

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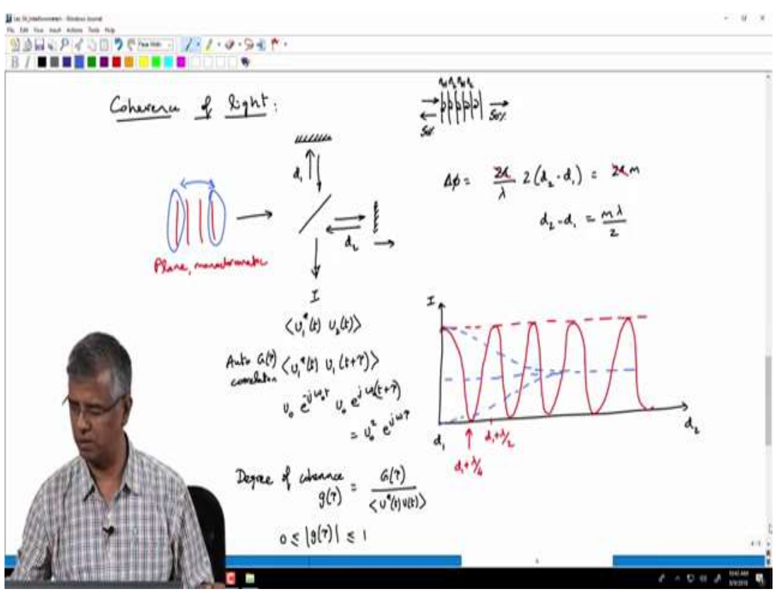

Here we were saying that as you move one of the mirrors, we are trying to figure out how the response will be. And for a monochromatic wave the response will be like this (Refer lecture @33:33). That, if you go to the maximum, to the same point as you move $d_2 \to \infty$.

But for non-ideal sources which are polychromatic in nature when you have frequency spread, then the temporal coherence limits your interference. The interference pattern gets modified, the envelope of the fringe pattern will look like this (refer lecture @34:48). The maxima and minima intensity will not be achieved as we obtained in case of monochromatic wave. This means full intensity will not be obtained in maxima, and there will some intensity is obtained in minima instead of zero intensity. At some point it will just be the, as if the two beams are not interfering at all. In this process, we are actually looking at the autocorrelation between two of the incoming wavefronts (Refer lecture 34:53).So as we move the distance d_2, you are like looking at further and further out in space or in time. So these two are reversible, so you can go back in time or in space and you take different components of the waves from different wavefronts from the same source and you are looking at how well they are correlated. And what we are saying is if it is a polychromatic source, that correlation holds good only for a certain time or a certain distance. And beyond that it will start reducing in terms of the overall contrast between the maxima and minima.

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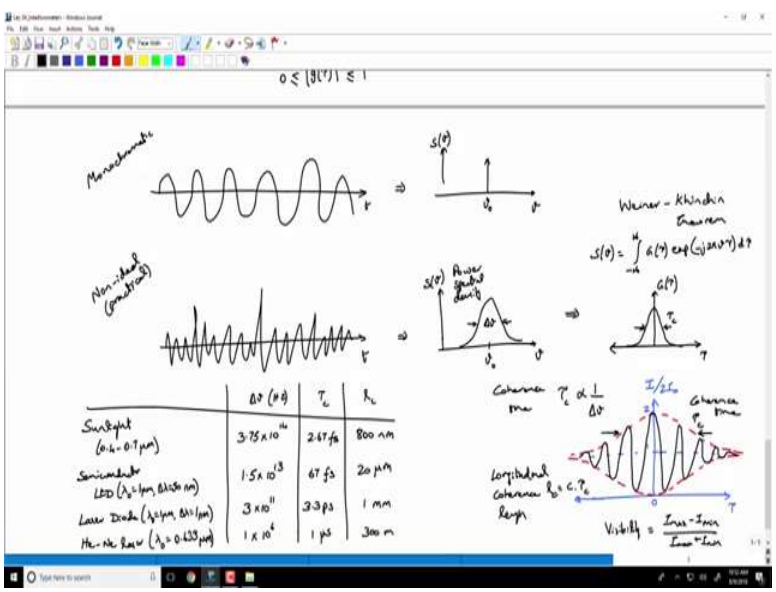

And that reduction is essentially defined by the envelope of this autocorrelation function (refer lecture @36:06). So effectively what we are saying is when we are looking at the Mach-Zehnder interferometer output, let us say we are looking at the plot (refer lecture @ 38:24). Within the envelope we have interference pattern. As we increase the d_2, the path difference increases, the contrast between the dark and bright fringes changes. The contrast is quantified with visibility which is given by

$$visibility = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{7}$$

The width over which the contrast goes to half is the coherence time (in the autocorrelation plot i.e. the envelope) (Refer lecture at 38:30). The coherence time is inversely proportional to the spectral width.

$$\tau_c \propto \frac{1}{\Delta \nu} \tag{8}$$

The proportionality constant depends on the shape of the power spectral density, so it could be a fraction but $\tau_c = \frac{1}{\Delta \nu}$. So larger the spectral width of your source, smaller will be τ_c .

τ_c is coherence time and in terms of spatial quantities you can define what you can call as longitudinal coherence length l_c, which is given by

$$l_c = c\tau_c \tag{9}$$

where c is the velocity of light in vacuum. So in terms of length it is just directly proportional to τ_c. Let's look at these values of some sources.

	$\Delta\nu$ (Hz)	τ_c	l_c
Sunlight (0.4 - 0.7 μm)	3.75×10^{14}	2.67 fs	800 nm
Semiconductor LED ($\lambda_0 = 1\mu m$, $\Delta\lambda = 50$ nm)	1.5×10^{13}	67 fs	20 μm
Laser Diode ($\lambda_0 = 1\mu m$, $\Delta\lambda = 1$ nm)	3×10^{11}	3.3 ps	1 mm
He-Ne Laser ($\lambda_0 = 0.633\mu m$)	1×10^{6}	1 μs	300 m

As we can see, for the sunlight $d_2 = d_1$ you get to see some fringes, you get to see some maxima and minima. But the moment you move even fraction of a micron, you lose all your fringes. So it is very hard to make interference fringes with the sunlight. Whereas with the laser diode it is $3\text{x}10^{11}$Hz which corresponds to 3.3 picoseconds. Here the coherence length is in the order of a millimeter. In case of Helium Neon (HeNe) laser the linewidth is 1MHz which will correspond to coherence time of 1 microsecond. For this laser if you take d_2 to 300m, you still have some visibility, you can still see some 50% visibility.

Let me stop at this point. So what we have just now started is quantifying the coherence of light and we say based on the fact that it is typically a non-monochromatic source that we are dealing with in practice, the coherence or the range of delays over which we can see interference between these two constituent waves is limited and of course what we are saying is that coherence time or the coherence length is inversely proportional to the width of the source spectrum.