Introduction to Photonics
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Lecture 04 – Interferometers

Welcome to yet another session in introduction to photonics. So we started the last couple of lectures trying to appreciate the science of light in terms of ray optics initially. We saw some examples of how ray optical principles can be used to design various elements like endoscopes and we were looking at what happens when light goes through a prism and so on. And then we said, ray optics can get us so far, but there are certain properties of light especially when it comes to explaining the wavelength and phase of light.

We are not able to capture that using ray optics, so we went on to looking at light waves as, sorry, light as a propagating in terms of waves. And once we realized it can propagate as waves and it can accumulate phase as it propagates. We said, two light waves can meet with each other and then you could have things like constructive and destructive interference.

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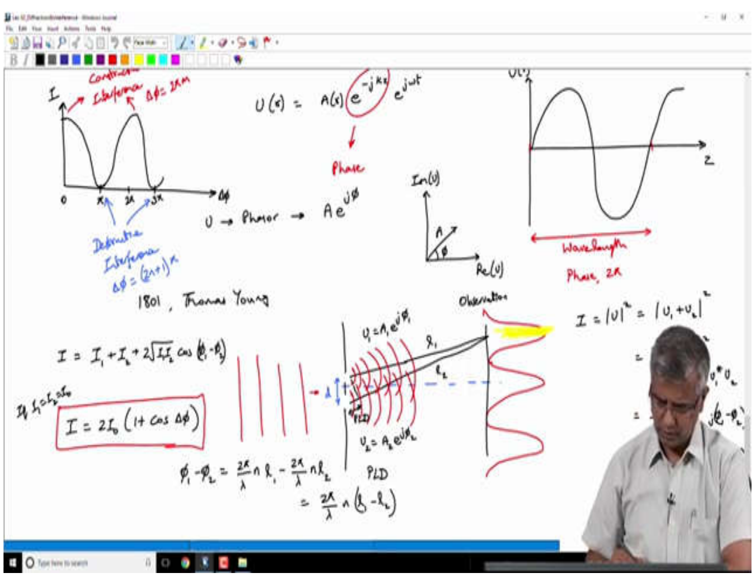

So we looked at the example of Young's double slit experiment through which we said, when two wavelets are generated, secondary wavelets generated through the slits which are size comparable to the wavelengths of light, then they could interfere in the far-field providing bright fringes at some points and dark fringes at some other points and so on. And you had the opportunity to work on this. Last week you did a hands-on experiment or some of you following this online you saw a demo corresponding to this, where you could see diffraction patterns and interference patterns from single slit and double slit and etc.

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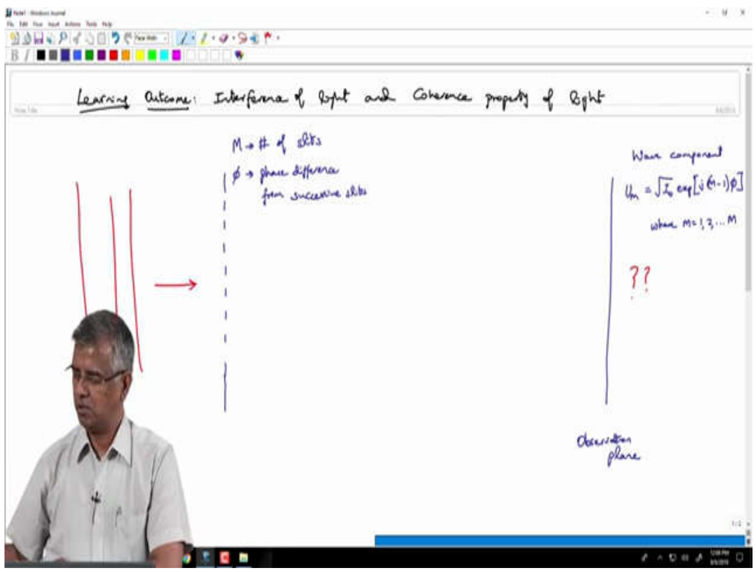

So what we are going to go towards this week is understand interference in a deeper sense. We are going to first of all extend our discussion from a double slit to multiple slits and see what is the effect of that. We will of course look at what is the practical example of a multiple slit type of interference and then extend whatever we have learned there to multiple configurations. So we said, to produce interference it could be as simple as Young's double slit experiment. But there could be more specialized configurations and there could be more specialized functionalities that you could realize out of these configurations.

So we could, we will go on to look at specific configurations that can produce interference and then we will go on to look at the coherence property of light. So that is what is coming up in the next couple of lectures. So first of all, let us look at this case where you have, we are extending this from a Young's double slit to multiple slits.

So you have multiple slits over here. And once again we have let us say a plane wave that is incident on these slits. And we are interested in figuring out, if you observe it in the far-field, so this is the observation plane. We want to know how is this light beam going to look when we observe it over here. So we want to know whether the locations of constructive and destructive interference is going to be the same and also one of the examples that we were looking at in last week was the specific example of how to discriminate different colors.

We said, as you go to higher and higher order of interference, you start separating out the colors more and more, because the interference phenomenon is a color dependent phenomenon. So you can separate out multiple colors through this. Now what happens to that property when you have multiple slits involved? So that is what we are interested in looking at. To do that, let us say we have, we are considering M as the number of slits and let us consider that ϕ is the phase difference at the observation plane from successive slits.

So you know that there are going to be, each of those slits are going to produce a secondary wavelet. And from successive wavelets, let us say the phase difference between wavelets from successive slits is ϕ. Then if you look at any of these fields over here, U_m, any of those waves coming from one of the slits, U_m can be written as, let us say the intensity of each of those wavelengths is I_0, so the amplitude corresponds to $\sqrt{I_0}$.

So $\sqrt{I_0}\exp[j(m-1)\phi]$ is, you would describe each one of those where m corresponds to, it can take values of 1, 2, all the way up to M. M is the total number of slits that we are considering. This is m over here and that is the index and that index can take values of 1 to M. So what we are saying is that if you take one of these slits as the reference, that has let us say phase of zero, then the next one would have accumulated a phase of ϕ. The next wavelength would have accumulated a phase of 2ϕ and so on.

So if I am looking at the total, yes, there is a question, so we are coming to that. This is, so the question is whether we would have to sum over multiple waves. So what I am describing is a single wave component, the component that we have from a single slit. Now what we are going to do, as somebody just pointed out, is to look at the total wave that reaches the observation plane. That is going to be summation of all these wavelets. So that is what we are coming to.

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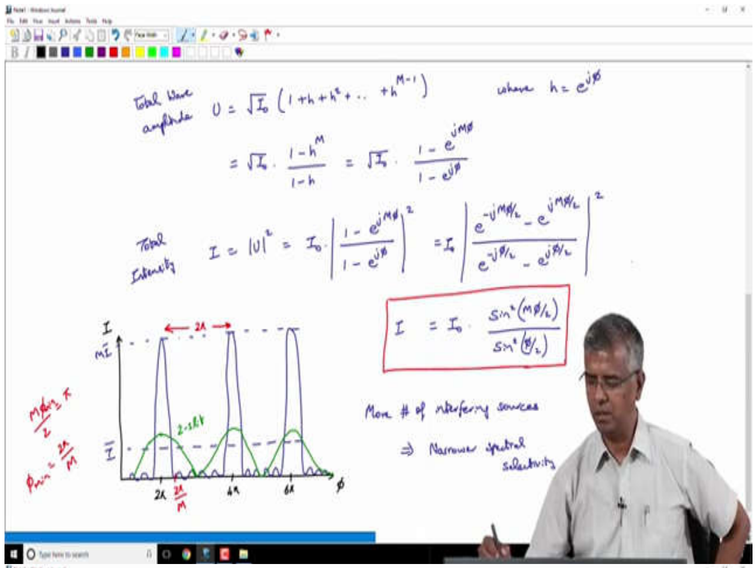

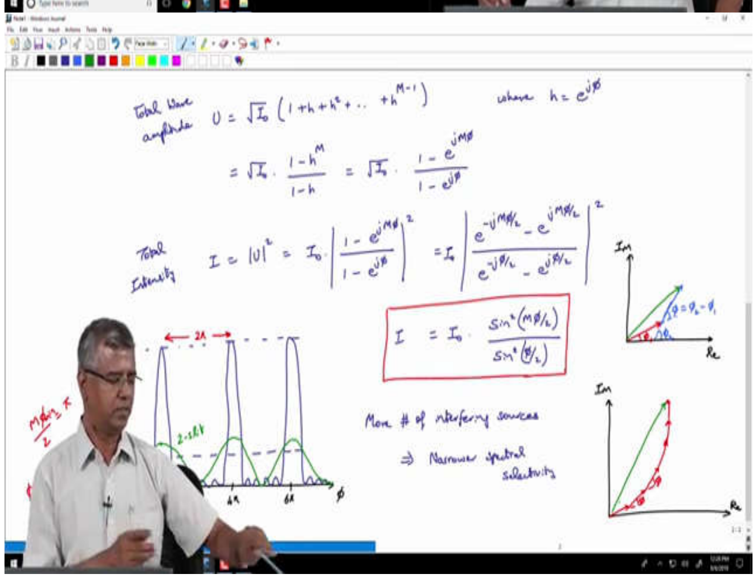

And so we can write the total wave amplitude, the complex wave amplitude as $U=\sqrt{I_0}\left(1+h+h^2+...+h^{M-1}\right)$, provided where h is nothing but $e^{j\phi}$. So we are considering that each of those wavelets has an equal intensity that corresponds to I_0. But they have different phase. So that is why we wrote them as $1+e^{j\phi}+e^{j2\phi}+...$ and so on. So we substituted $e^{j\phi}$ with h for simplicity, and then we have this expression.

Now if you simplify this, you are getting an expression which is $\frac{1-h^M}{1-h}$. That is just the series, whose simplification would yield this term. And now I can go back and substitute what h means.

So this is $\sqrt{I_0}\left(\dfrac{1-e^{jM\phi}}{1-e^{j\phi}}\right)$. Now what we are interested in is the total intensity, that is what we are observing at that observation plane.

So the total intensity I is what? It is magnitude of U^2. And if I do a $|U|^2$, that I_0 term will fall off. And what I will be left with is $\left|\dfrac{1-e^{jM\phi}}{1-e^{j\phi}}\right|^2$. So I am going to do a small trick over here. So what I will do is I will take $e^{jM\frac{\phi}{2}}$ outside from the numerator as a common term. And similarly I will take a term $e^{j\frac{\phi}{2}}$ as a common term in the denominator.

So then what I will have is $\left|\dfrac{e^{-jM\frac{\phi}{2}}-e^{jM\frac{\phi}{2}}}{e^{-j\frac{\phi}{2}}-e^{j\frac{\phi}{2}}}\right|^2$. And I would have $e^{jM\frac{\phi}{2}}$ term outside the numerator, and $e^{j\frac{\phi}{2}}$ term in the denominator. But when I am taking a magnitude of that, that will be 1. So I can just write this as I_0 of this. So if I then expand these two terms in terms of sine and cos, so what I am going to get finally is $I_0\dfrac{\sin^2\left(M\dfrac{\phi}{2}\right)}{\sin^2\left(\dfrac{\phi}{2}\right)}$. So that is my total intensity.

So this is the intensity that I am going to see in the observation plane, that is in the far-field. So now let us go ahead and plot this function and try to get some physical idea of what this represents. So let me plot I as a function of ϕ. So what is ϕ here? ϕ here represents the phase difference between successive slits, wavelets created by successive slits. So I can plot this as 2π, 4π, 6π and so on.

So clearly this function will peak whenever we have this 2π, 4π, 6π and so on. So the overall the wave function is going to look like this. So it is going to actually take a peak value which is going to correspond to, let us say the average intensity value is $\bar{I}$, this peak will correspond to $M\bar{I}$. So that is going to be the peak intensity of these, of the combined wave pattern that we are going to see.

So essentially light is getting scooped out of certain areas and collected in certain other areas. So we have constructive interference and destructive interference is happening over here. And one interesting aspect, well of course, one thing to say is that between two of these peaks you have 2π as the phase difference. So you would, you will have to accumulate 2π to get from one peak to another peak. But the thing of interest to us is this point where it goes to the first minimum. If you look at where it goes to the first minimum, why is that of interest to us?

Because that first minimum defines the width over which you have maximum intensity. So if you look at that wave, so if you equate this to zero, the total intensity to zero, and you try to figure out what is the value of ϕ where this is going to zero, you will find that happens when $M\frac{\phi}{2}$ equals to π and in other words M is, or $\phi_{\min}$ is what I should say, $\phi_{\min}$ would correspond to $\frac{2\pi}{M}$. $\frac{2\pi}{M}$ is where that first minimum happens.

So what does that tell you? If I have just two slits, where would the first minima happen? π. In fact if I were to use a different color and show you how the two slit interference would have, what the two slit interference would have yielded, so I am just going to normalize and show that the intensity would have been much lesser or maybe I can show it in that way. So the maximum would have happened here, the same location.

But at π phase shift, it would have gone to a minimum and then maximum, minimum, maximum, minimum and so on. So this would have been for the 2-slit case. And going to more number of slits what have we done is the width over which that constructive interference happens is much more narrower. Now what is the effect of this?

When we bring this discussion that we had last week, if you bring that into perspective, what were we talking about? What can we, on Friday what were we talking about? We said we could get color selection using this interferometer. And that color selection for each of those colors it would have been slightly shifted over here. So we were saying if you want to discriminate between one color and another color, you put a very small slit that corresponds to the maximum of the color that you are wanting to pick out, and then you can pick that out.

But there is always, the neighboring color also, some part of that coming through the slit. Now you say, no, no, my application I cannot have that. I want to pick one particular color, one particular shade of a color very precisely and I should not want any interference from any neighboring shades. Then what this tells you is you go for a multiple component interference, so more number of sources that are interfering, the resolution with which you are picking up a particular color is going to be much higher.

Or in general, I will just write this out because it is an important aspect to keep in mind. More number of interfering sources, you have narrower spectral selectivity or finer spectral selectivity. More number of interfering sources, more constrained that interference condition, and hence even if you are off by a fraction of a nanometer in terms of the wavelength of light, you drop off from the peak very rapidly. So you can resolve different colors with very high precision.

I am saying colors but you can think of it in terms of spatial quantities as well. So it becomes very fine, you can pick up things with very fine resolution when you have multiple slits participating in your interference process. There is actually a different way of viewing this and that is actually, if you go back and in our previous discussion we said we can represent each of these waves as phasors and that is the representation we have used here in all these calculations.

But let us look at it from a phasor perspective. So this is the real and imaginary parts. So this is the complex plane in which you are representing these phasors. When you are considering two waves that are interfering, you say, there is one wave which has ϕ_1 as the phase, and then there is another wave which has ϕ_2 as its phase. That is, both those things are adding together to the point that when we look at the overall or what is the angle with respect to the first wave? That would be ϕ which is given by $\phi_2 - \phi_1$ is the phase difference.

And then of course, you can say the resultant phasor is like this and in this case you can represent constructive interference as the condition where.. What happens? ϕ_2 and ϕ_1 are the same or integral multiple of 2π. Integral multiple of 2π is just the one rotation in this phasor, this complex plane and it would come at the same point. And if ϕ_2 is equal to ϕ_1, then you have constructive interference which means that both these phasors are adding. And that is giving you the largest resultant phasor.

On the other hand, if ϕ is, actually $\phi_2 - \phi_1 = \pi$, it would have pointed exactly, the blue phasor would have pointed exactly in the opposite direction as the red phasor. If both of them have the same magnitude, then you are looking at zero as the resultant. That corresponds to destructive interference. So in a phasor notation, you can see how constructive and destructive interference is looking like.

And if we extend this to this multiple sources that are interfering, then you say each one of them is going to be slightly off from the previous one. Each one of them is going to have an angle ϕ with respect to the previous one. And when all of them add, you get a resultant phasor like this. But in this case, even for a very small change in ϕ the result is going to be drastically changed. So if ϕ is zero, or 2π or so on, then all of them are going to line up. So you are going to get one large phasor.

But the moment you have a small change in ϕ, it is all going to wrap around to the point that it can make all the way back over here for a certain value of ϕ. And that value we found is $\frac{2\pi}{M}$, where M is the number of interfering sources. So the resultant, the green would have been right at zero if ϕ is $\frac{2\pi}{M}$. So since we have multiple sources adding, the interference criteria is that much more constrained and that much more sensitive.

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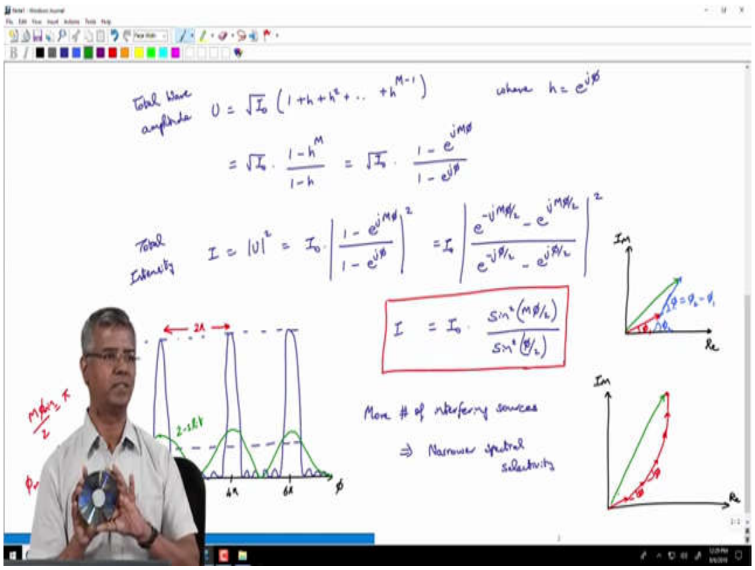

And now of course, you can see practical examples of that. So what do I have here? CD. And what do you see? As I move this, you can see all these different colors separating out. So what is this due to? Exactly the same thing that we are talking about. What are the multiple slits in this case? All the different tracks that has been written in this CD. All these tracks are essentially like notches and essentially like slits, except that these are all reflecting. The back plane is reflecting.

So it is a reflective configuration. But you have all these multiple things that are interfering together and they are producing all these, they are separating out the colors very nicely. You can see all the colors corresponding to the white light. So it is very good for discriminating between the different colors. Very simple spectrometer you can build with this. And as we go along, we will see more and more examples of this. But the key point to note is more number of interfering sources, more constrained is your interference condition.

And hence better the selectivity when it comes, if you are trying to select different, one color from different colors. You are getting a very narrow band filter or filter with a very high Q as they say in electronic, electric circuits high-Q circuits. You can build high-Q circuits when you have more number of interfering sources. So what are these, is this the only interference configuration that you can possibly have? And it turns out, that is not true. You have so many different configurations that are possible where this interference criteria can be realized.

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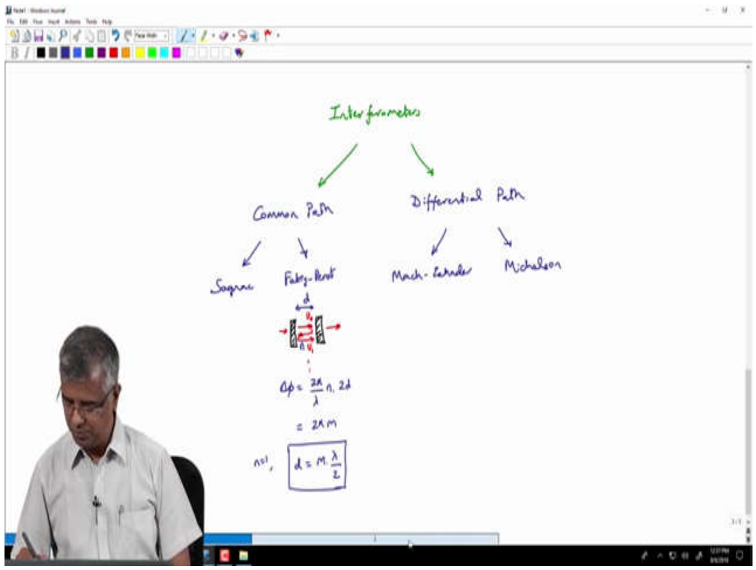

And when we, so let us just spend a few minutes talking about that. When we talk about interferometers, there are basically two classes of interferometers. One is called common path interferometer and another is called differential path interferometer. So common examples for differential path interferometers are one configuration which is called Mach–Zehnder interferometer, another configuration which is called Michelson interferometer.

Michelson interferometer is something that you are going to be building yourself this week. And for those of you online, you will be shown a demo of how the Michelson interferometer is constructed. But you will look into that little more detailed later on this week. When it comes to common path interferometers, one of the most popular common path interferometers is what is called Sagnac interferometer. And another one that can be put in this category is a Fabry-Perot interferometer.

So Fabry-Perot interferometer is probably the simplest that you can think of. It just consists of two mirrors that are held parallel to each other. Let us say these are like plane mirrors that are held parallel with respect to each other. And of course, what is the interference that we are talking about? What we are talking about is light that goes into this. It goes this way, it takes a bounce, goes this way and that takes a bounce and goes this way and so on.

It just goes back and forth. So what are the sources? What are the multiple interfering sources? It will correspond to let us say this is U_0, U_1, U_2 and so on. So you can have multiple bounces and all of these are coming to this other mirror where you can actually have an interference between all those waves. So that is what typically you are looking at as far as the Fabry-Perot interferometer. And in this case it is easy to analyze what is happening in the structure. When you look at $\Delta\phi$ which corresponds to the phase difference between two successive bounces, two successive waves, it is going to be given by $\frac{2\pi}{\lambda}n$, if you say this medium in between is characterized with a refractive index n. And let us say the thickness between the two mirrors, the space between the two mirrors is d, so you have n multiplied by 2d, is the phase difference between successive bounces. And if this is got to be, so for constructive interference at this point, you will have this outgoing wave.

So for constructive interference to happen, this has to be equal to integral multiple of 2π, that is what we have been looking at. And let us say n equal to 1, let us say this mirror pair is actually just having air in between. If $n = 1$, then that just gives the simple thing that d is going to be an integral multiple of $\frac{\lambda}{2}$. So if you have integral multiple of $\frac{\lambda}{2}$ as the spacing between the two mirrors, then you can have constructive interference and light will go out in the other side of that Fabry-Perot interferometer.

So here again, we see it is dependent on wavelength. So you could potentially have only one wavelength go through and some of the other wavelengths are not allowed to go through that interferometer. And of course, if we are talking about spectral selectivity, what you need to have here if you want to have high spectral selectivity? More number of this thing. So what enables more number of interfering components here in this configuration? There should be very low loss.

So the two mirrors would have to be very highly reflective, high quality mirrors that does not distort the wave, there is nothing, there is no dust particles or any other scattering components inside the cavity. So if you have a very what is called a high finesse cavity, then that corresponds to more number of waves that are supported inside the cavity. And hence much higher spectral selectivity. So if you are trying to build a filter, very narrow band filter using a Fabry-Perot interferometer configuration, you will make the mirrors highly reflective and make sure that the losses within that etalon is what is called that cavity, the losses within that cavity is very very low. Actually that is an interesting point.

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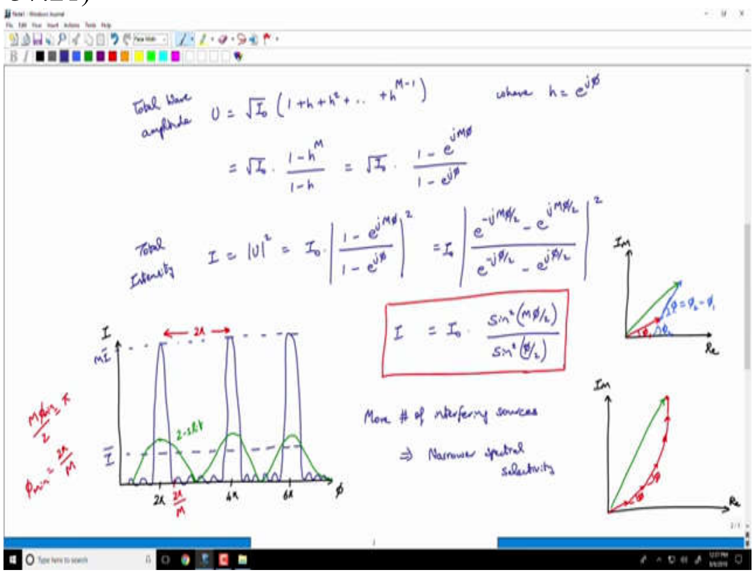

If you have loss, how would this look like? So it will turn around like this, but each of these arrows will be smaller and smaller, upon successive bounces it becomes smaller and smaller. So your interference criteria may not be complete. When you are talking about destructive interference, it may not be complete in that case. So that is Fabry-Perot.

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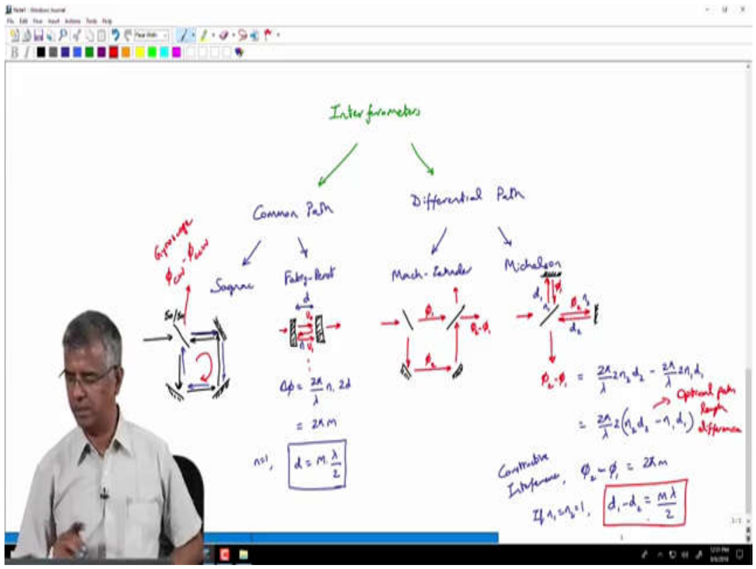

Now let us look at one more configuration and let us look at what a Sagnac configuration is about. Sagnac configuration essentially consists of wave that is incident on a partially reflecting mirror. So this would typically be like a 50-50 split. So 50% of the light goes this way and the other 50% goes this way. And here you put a mirror, here also you put a mirror at 45 degrees so that it comes here. And here you put another mirror. So it retraces the path.

Let me use a slightly different color to denote this one. So in the other direction it goes like this, like this and comes back here. So if you are looked at the net output in this direction, you have interference between what and what? You have one wave that is going in the clockwise direction around this series of mirrors and the other wave which is going in the anticlockwise direction. And both of them, some part of them go this way also towards the source. But if you look at the other part that is coming this way, that is going to have an interference component which is dependent on the ϕ of clockwise, anticlockwise depending on the phase difference.

An interesting aspect about this is since they are a common path interferometer they share the same path. So there is essentially no phase difference normally unless there is one condition. What is that condition? Unless you rotate this entire apparatus, so if you rotate that entire apparatus, then one path becomes shorter compared to the other path and then corresponding to that there will be a difference in the phase between the two waves.

So you may have some of the light going this way because that corresponds to the rotation, because they are seeing slightly different phase shifts. So Sagnac inherently is sensitive only to rotation. Any other external perturbations do not make a difference.

[Professor-Student conversation starts]

Professor: There is a question in the back.

Student: (())(41:19).

Professor: So the question is the optical path length, is that the same in this case? So what we are doing is we are saying both of them are interfering at this 50-50 interferometer. But by changing the position, suddenly you are changing where that 50-50 interference happens. I mean where that is located.

So effectively if you go in the clockwise direction, the counterclockwise beam will reach that 50-50 splitter first compared to the clockwise beam. For the clockwise it is actually slightly longer path. So it is actually more easily realized in a fiber form where you take optical fiber and you loop it around and the 50-50 splitter is actually a fiber optic coupler and so on. That is the typical realization that you have. This is what is called, this principle is what you call as a gyroscope.

The gyroscope is something that measures rotation. So it is most easily realized in a fiber form but nevertheless the point is that where it interferes, that is shifted within the time of propagation of the light. It takes a finite time for the light to propagate. So within that propagation time it is shifted ever so likely. And that causes a phase shift between the clockwise and the counterclockwise beams. Any other questions?

[Professor-Student conversation ends]

So let us now jump to the differential path interferometer and see how things work here. So in a differential path interferometer, you have this incoming beam which I will note in red. And what it encounters is once again a 50-50 splitter and this 50% goes through a path consisting of two mirrors. And then it comes to another 50-50 splitter. So what happens is one part of the beam goes this way, the other part goes this way and both of those interfere.

It need not just interfere in this direction, it can interfere in this direction also but what it corresponds to is actually a relative phase shift between. So this is the phase accumulated. If this is ϕ_1, this is ϕ_2, because there are going through physically different paths the phase shift that it accumulates during the propagation is different and then you have $\phi_2 - \phi_1$ causing a change in the interference.

The interesting part here is if it is constructive interference in this arm it corresponds to destructive interference in this arm, or in other words if it is destructive interference, that is light is missing over here, you would see that it actually appears over here. Why that is so? I am not explaining at this point but maybe we will come back and look at that. But basically the Mach–Zehnder interferometer has two different paths through which light can propagate.

And depending on the phase difference between those two paths, you have constructive or destructive interference when you observe in one of ports. Clearly this can be used to see if there are any small changes in the path length. Suppose you move that distance between the mirrors, even if you move from, move by a fraction of a wavelength, you will see a sudden change in the intensity at the output.

So it is very sensitive to changes in the relative path lengths and very sensitive to changes in the refractive index of the medium in the two paths. So suppose you want to find the refractive index of some unknown material, you can put that unknown material over here and based on that you can change the constructive or destructive interference criteria and based on that you can actually find out what is the refractive index of the medium.

We will go into little more detail later on the week when we look at that Michelson interferometer experiment or demonstration. But those sort of things are possible. Precise measurements of distance, precise measurement of refracting index is possible with this Mach-Zehnder. Now the Michelson configuration is similar to the Mach-Zehnder except, it is similar in the sense that it has the 50-50 splitter but in this case what happens is you put a mirror that reflects straight back.

And similarly in this arm you put another mirror that is reflecting straight back. So if this is my incoming beam, 50% of it goes this way and it is reflected by. Similarly the other 50% goes this way and it is reflected back. And if you observe this, you have differential phase shift. Let us call this ϕ_1 and let us call this ϕ_2. And depending upon $\phi_2 - \phi_1$, you can have constructive or destructive interference.

So let us just write it out. Let us say this has, d_1 as the distance between your beam splitter and the mirror and similarly d_2 is the distance between the beam splitter and this other mirror. Then $\phi_2 - \phi_1$ can be written as $\frac{2\pi}{\lambda} n_2 d_2$. Let us say n_2 is the refractive index of this medium and n_1 is the refractive index of the other medium. So $\frac{2\pi}{\lambda} n_2 d_2 - \frac{2\pi}{\lambda} n_1 d_1$.

So this is $\frac{2\pi}{\lambda}(n_2 d_2 - n_1 d_1)$, which is known as the optical path length difference. The physical path length difference is just $d_2 - d_1$, but the optical path length difference is refractive index multiplied by the distance. And of course, I am missing a factor of 2 over here because d_1 is actually the single pass, it goes and comes back. So there should be a factor of 2 in all of these, so that factor gets carried over here.

So clearly you get constructive interference when $\phi_2 - \phi_1$ is $2\pi m$ and that you can equate against this other expression over here. And if we say, if $n_1 = n_2 = 1$, meaning this entire setup is constructed in air, then you get the condition that $d_1 - d_2 = m\frac{\lambda}{2}$. So the physical path length difference has to be $m\frac{\lambda}{2}$ for constructive interference.

And of course you can also work out the corresponding expression for destructive interference. And this is fairly simple to observe, simple to construct and observe and we will do this as hands-on experiment later in the week. Thank you.