

Introduction to Photonics
Professor Balaji Srinivasan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Non-linear optics-Kerr Effect

(Refer Slide Time: 0:14)

Nonlinear response of materials to light

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$= \epsilon_0 (1 + \kappa) \vec{E}$$

$$= \epsilon_0 \vec{E} + \epsilon_0 \kappa \vec{E}$$

$$P = \epsilon_0 (\kappa E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots)$$

$\chi^{(2)}$ → Second order susceptibility
 $\chi^{(3)}$ → Third order susceptibility

$E_1 e^{j\omega_1 t}$
 $E_2 e^{j\omega_2 t}$

$\chi^{(2)}$
 PNL

$2\omega_1$ → Second Harmonic Generation
 $\omega_1 + \omega_2$
 $\omega_1 - \omega_2$ → Difference frequency generation
 $2\omega_2$

Welcome back introduction to photonics, so we have been discussing this exciting topic of nonlinear optics and I should you know do a disclaimer here nonlinear optics is a ocean, it is has actually got lot of deep physics involved in it and a lot of nice concepts that evolve through that, not all of that we are able to do justice as far as this course is concerned it is not even the intention to do justice to the entire area of nonlinear optics. What we are doing here is only at a very basic level, very fundamental level, not going too much into the math just giving you a physical feel for what is happening and of course if you are interested in learning more about this there are other courses that go into the advanced topics one of such one of those courses is optical signal processing it is a fairly good course that deals with nonlinear optics in detail.

So we started yesterday by saying that all materials would start behaving non-linearly beyond a certain field applied to them and we characterize that nonlinear response through the second order susceptibility, third order susceptibility and so on and these terms are usually when it you know it is represented by this Chi 2 and Chi 3, Chi 2 and Chi 3 are usually very small values, we talked about Chi 2 which is in the order of 10 power minus 10, so it is fairly small so that is why you do not see effect of that at lower values of electric field but if your electric

field you know yesterday we were looking at an example where we had 5 volts across 10 microns, so that is 5 into 10 power 5 volt per meter.

So when you go to relatively large electric fields very close to the breakdown strength of the material itself, then you get to see this nonlinear properties show up. So it is not as if these things are not there and we have only linear response, you know all this Chi 2, Chi 3 are there but because your E value the magnitude of the electric field is low such that those all those terms are negligible so you can approximate their response as a linear response at relatively low electric fields.

So we looked at second order susceptibility yesterday and we said okay there are several things that are possible using this second order susceptibility, if you have a material that exhibits high level of Chi 2 which we later defined as materials that do not have a central symmetry, then you could have processes such as second harmonic generation, difference frequency generation.

(Refer Slide Time: 4:12)

Pockel's effect

Diagram: A crystal of length L with an electric field E applied across it. The voltage V is applied across the crystal.

Equation: $n(t) = n_0 - \frac{1}{2} r n_0^3 E$ (electro-optic coefficient)

Equation: $\phi(t) = \phi_0 - \frac{2\pi}{\lambda} \cdot \frac{1}{2} r n_0^3 E \cdot L$ ($\Delta\phi(t)$)

Materials and coefficients:

- LiNbO_3 , $r = 30 \text{ pm/V}$
- SiO_2 , $r = 0.01 \text{ pm/V}$

Diagram: Mach-Zehnder interferometer with a Phase Modulator. Input intensity I_0 , output intensity $I_{out} = \frac{I_0}{2} (1 + \cos \Delta\phi)$.

And then we looked at the Pockel's effect so all these are possible through the Chi 2 and through the Pockel's effect we said we could manipulate the phase of light using an external voltage external electric field and so you can essentially build a phase modulator and that phase modulator when it is incorporated inside a Mach–Zehnder interferometer can provide you intensity modulation.

(Refer Slide Time: 4:48)

$\Delta\phi(V) = \pi = \frac{2\pi}{\lambda} \cdot \frac{1}{R} \cdot r n_0^3 \frac{V_x}{d} \cdot L$
 $\phi(V) = \phi_0 - \pi \cdot \frac{V(t)}{V_x}$ → Voltage Controlled phase modulation
 $V_x = \frac{\lambda d}{r n_0^3 L}$
 Let's say we need $V_x = 5 \text{ V}$
 We know $n_0 = 2.2$
 $r = 30 \text{ pm/V}$
 $\lambda = 1.5 \text{ } \mu\text{m}$
 $d = 10 \text{ } \mu\text{m}$
 $\Rightarrow L = \frac{7.5 \times 10^{16} \times 1.5 \times 10^{-6}}{2 \times 3 \times 10^{11} \times (2.2)^3 \times 5} = 1 \text{ cm}$

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So we were trying to get an expression for V_{π} that is the voltage required to achieve a π phase change through this material and then we looked at the output intensity of the Mach-Zehnder we are saying that is actually dependent on the phase change that we are created which is actually a function of the voltage that we are applying. So we looked at biasing at different points and what is the response we get to see. If you want a purely linear response which is fairly important for communication applications, if you do not want to have any distortion of your signals you should operate at which point? At that $\pi/2$ bias point so you get a linear response.

But there could be applications where you need to suppress the carrier and achieve modulation in which case you can actually you know operate at the null point. So the

question is does the frequency remains the same you know whenever you are modulating this this electro-optic modulator using a particular voltage. So yes if it is linear then you do not have any distortion, so the exact replica of your modulating voltage would show up in the output, so in that case the frequency is exactly the same.

Now in this case itself if we had let us say the let us call this V_M the magnitude of that voltage if that V_M was closer to V_{π} , it approaches V_{π} then you are starting to access these regions. So then clearly it is not linear, so your waveform is going to get distorted the response that you get is going to have a distorted response, that distortion if you do a Fourier series analysis on that you will find that it is represented by multiple harmonics. So you start generating harmonics of that as you go away from that linear response region.

So in certain way and this is very widely used in signal processing and power electronics. You look at the harmonic content and you can tell whether you have linear response or not. If you have a large harmonic content you say the response of whatever system you are analysing is not behaving linearly.

Student is questioning: (8:03).

Professor:

Harmonics of that, yes so you do get that whatever frequency you are trying to achieve and of course if you are biasing around the null point you know it is straddling both sides and in which case you get twice there from that you have and of course you could get harmonics of that also. So that is so much we want to talk about second order susceptibility, I just brought this I am sure all of you have seen this you know this laser pointer you know it is producing this green light this is what I was talking about yesterday most of it is the battery actually this much of it is battery, there is a switch here which turns on which closes a circuit and that circuit is actually for the pump diode.

So the semiconductor pump diode is sitting somewhere over here and then there is a small crystal the (9:11) crystal which produces 1064 and beyond that there is actually another small crystal which is it could be KTP or some other nonlinear crystal and that crystal essentially takes photons at 1064 nanometers and doubles it, doubles it in frequency that means half in wavelength.

So that is how you get 532 nanometer radiation from this so it is amazing that they managed to put it all in such a compact manner.

(Refer Slide Time: 10:00)

Third order susceptibility (χ^3) $\rightarrow \chi^3 E^3$

Energy Conservation
 $\omega_1 + \omega_2 = \omega_3 + \omega_4$

Momentum Conservation
 $\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$

$E(\omega_1) \rightarrow$
 $E(\omega_2) \rightarrow$
 $E(\omega_3) \rightarrow$

χ^3

$\omega_1 + \omega_2 + \omega_3$
 $\omega_1 - \omega_2 - \omega_3$
 \vdots

Four-wave mixing
 Self Phase Modulation/
 Kerr effect
 Stimulated Raman Scattering
 Stimulated Brillouin Scattering

Wavelength Conversion
 Parametric Amplification

Okay, so that is so much of what I wanted to talk about in second order susceptibility. Now let us look at third order susceptibility, what are the nonlinear processes that are enabled through the third order susceptibility, which is represented as Chi 3. Now Chi 3 we understand is associated with an E cube term and just like we talked about for second order susceptibility E cube may mean that you have 3 electric fields interacting, which could be at 3 different frequencies.

So you have a material that is how you know providing a large Chi 3 and you could have E of Omega 1, E of Omega 2, E of Omega 3, multiple frequencies that are exciting this medium and then you can get an output which is a combination of this Omega 1 plus Omega 2 plus Omega 3, Omega 1 minus Omega 2 minus Omega 3 and so on you can get all possible combinations. You could get things like 2 Omega 1 minus Omega 2 2 Omega 1 minus Omega 3 all sorts of things are possible.

So this process through which you have three waves interacting and creating a new wave because when you talk about these frequencies these frequencies are representing a new wave you know a different frequency. So there are actually four waves that are involved here so this sort of process is called four-wave mixing. So Chi 3 enables a four-wave mixing, wherein like I said you could come in with even two frequencies and you can generate two other frequencies.

So you can say you have Ω_1 and Ω_2 going in let us say the power in the Ω_1 frequency is higher than Ω_2 , so you could have 2 photons of Ω_1 you know interacting with 1 photon of Ω_2 so you get can get $2\Omega_1 - \Omega_2$ type of process possible. So essentially you have what is called four-wave mixing happening that is one other process that could happen.

But the key thing to note here is you need to have energy conservation which may say that $\Omega_1 + \Omega_2$ should be equal to $\Omega_3 + \Omega_4$ that sort of thing you need to do so corresponding frequency you can write down, so you need to do energy conservation. But you also need to do momentum conservation, so the \mathbf{K} vectors I am just giving you I am just throwing out an example but in reality that could be some other combination but point is that momentum also would have to be conserved in these processes, so that only when you have energy and momentum conservation you have maximum conversion happening from one set of frequencies into another set of frequencies.

But nevertheless we are starting to talk about as in the case of four-wave mixing when you look at it in terms of frequency you could have two pumps at Ω_1 and Ω_2 and then you could generate new frequencies. So what is mentioned in the red is actually the input and what is you know put down as the green is actually the new frequencies that are generated. So you in this case in typical four-wave mixing this is called the signal which is the desirable component but you may also have an idler which is a natural by-product whenever you have these two frequencies beating with each other.

So you could have processes like this happening, so this is in this case these are essentially (())(15:57) the pump, these are the excitation frequencies and signal and the idler are the by-products of that. So in a normal linear medium in a medium with very low χ_3 if you send in those pumps only those pump frequencies are going to come out, there will not be any extra frequencies. But through χ_3 you are able to achieve mixing between multiple frequencies and you can generate new frequencies, so that is what we call as four wave mixing.

Where is it useful? A lot of applications obviously which require new frequencies to be generated one example of that may be in communications you are using multiple frequencies to transmit information in multiple channels essentially, so each of these frequencies are carrying different information. And at some point you find you get to a node and you find that

the fiber in which the signals have to propagate is already carrying this frequency, so I cannot insert new information at this frequency.

So whatever I have in this frequency I need to transfer to some other frequency, some other wavelength so that is actually called wavelength conversion. So you essentially through this process you can take information in the pump and you can actually offload it onto the signal. So you can do these things called wavelength conversion through you can take information from one wavelength and put it on some other wavelength, so then all of them can go together in the same fiber, you can actually build an amplifier with this.

So I can start with something where you know in terms of frequency so I still have my pump like this and let us say I have a very small signal coming in and I want to amplify that signal so I can go through a medium like this provided this energy and momentum conservation is done, I could amplify that signal okay and of course I would also generate an idler in the process but I could achieve signal amplification through this.

So this process is called parametric amplification this you could have done in three-wave mixing also through Chi 2 also, so you could you could actually get when we talking about frequency up conversion or frequency down conversion, difference frequency generation there also you could have done this you come in with some weak photons at a particular frequency provided you have the appropriate pump frequencies you can actually amplify those photons, you are transferring essentially in this process what you are doing is you are transferring energy from pump to the signal.

So you can you can achieve amplification through this, like I said I am not going to the detail there is a lot of very interesting details in these things but this is from going into those okay because the idea is to give you a overview of what all is possible, what all is possible the other thing that is possible is what is called self-phase modulation which is through what is called the Kerr effect.

So just like you know with Chi 2 process we were talking about the Pockel's effect, we have something associated the Chi 3 process where it is actually what is called the Kerr effect, let us see if we can guess this. In the Chi 2 process the Pockel's effect was enabled by an electric field an external electric field which was modulating the electric field corresponding to your light wave.

What do you think is happening in a Kerr effect? It is a Chi 3 process, so what do you think could be happening in the Kerr effect? So you have a medium which is exhibiting Chi 3 and you are now sending a light wave through that medium and that medium's response can now be controlled by something else what can you control it with? In Pockel's effect we were controlling with electric field, in Kerr effect what do you think you can control with?

(No) so you have one electric field component corresponding to the wave that is going in right so that is one E, what is left is? E square, so you could you have an opportunity of using E square to modulate that light, what is the E square correspond to? Intensity, so with intensity that is in this case light intensity you can illuminate this the Chi 3 material with some external light and using that in the intensity of the external light you can modify the response of the medium, you can change the refractive index of the medium, so that is what Kerr effect is all about.

So with Kerr effect what we are saying maybe before I go into the specifics of those, let me just finish this list. The other things that are possible are stimulated Raman scattering and stimulated Brillouin scattering, I will come back to those details later but let us just focus on the Kerr effect for now.

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Kerr effect

Free Space → [Optical Fiber: $\chi^{(3)}$] → [Kerr effect: $\chi^{(3)}$, Intensity (E^2)]

$$n(E) = n_0 - \frac{1}{2} S n_0^3 E^2 \quad S \propto \chi^{(3)}$$

where $n_2 \rightarrow$ Kerr index
 $n_2 = 10^{-10} \text{ cm}^2/\text{W}$ in SiO_2

$$n(I) = n_0 + n_2 I$$

$$\Delta\phi(I) = \frac{2\pi}{\lambda} n_2 \cdot \frac{P}{A} \cdot L$$

$$I = \frac{\text{Power}}{\text{Area}} = \frac{1 \text{ mW}}{\pi (4 \times 10^{-4} \text{ m})^2} = \frac{1 \text{ mW}}{\pi (16 \times 10^{-8} \text{ m}^2)} = 20 \text{ MW/m}^2$$

So in the Kerr effect what we have is once again this Chi 3 medium you have a light going through the medium but the properties of the medium can be controlled by external source and in this case what matters is the light intensity of the external source because in intensity

is proportional to the magnitude of the electric field square of the magnitude of the electric field.

So light controlling light, so what is the advantage of this? Now I did not quite go into the details of the Pockel's effect one of the advantages of the Pockel's effect is that the response time of the medium to an applied electric field, it corresponds to moving some charges and that could be in a very very short time scale, it could you know moving an electronic charge you know around its orbital essentially can happen in femtosecond time scales.

So that means you can use that effect to modulate information at the rate of if you say femtoseconds that is 10^{15} seconds, so you can possibly think about modulating at the rate of hundreds of terahertz, but you cannot do that in practice, why? Because you have to apply your electric field through some electrodes and there is a finite time taken by the electrical wave to go through that material go across that material and that would essentially limit the modulation frequency.

So you can achieve with smart way of you know figuring out your electrodes you know you go for a sort of what is called a traveling wave configuration and you pattern your electrodes such that the field can propagate across that at fairly high speeds with all done we can probably achieve tens of gigahertz of modulation frequency, that is because you cannot do faster than that in with your electrode configuration, your light frequency and the frequency of the RF wave are essentially going to one is much faster than the other. So the actual interaction happens only over a short time, short length, short region.

So there you are limited by the electronics in terms of the speed at which you can do the modulation, but now you are talking about doing it with light intensity here you do not have those constraints, I can actually come in with a femtosecond pulse, I can come in with a femtosecond pulse and I can modify the refractive index of the medium within that femtosecond for that time duration the refractive index is different.

So if you have your light wave going across within that duration within that duration things change the medium that the light wave sees is slightly different, it is modified by the intensity of that femtosecond pulse. So you can actually get very fast modulation achieved using this process, do you understand that? The speed at which you can do modulation the Pockel's effect is limited by what you have with all your electrode configuration and all that, the bandwidth of your electrodes bandwidth of that circuit there, but as far as Kerr effect is

concerned since you are doing it with optical intensity you can come in with a very narrow pulse of light and during that pulse you know you can change the properties of the material.

So the specific way you change that is once again when you look at the refractive index change in this case you are the refractive index change depends on E^2 can be written as $n = n_0 - \frac{1}{2} S n_0^3 E^2$. So this is similar to what we wrote yesterday for the Pockel's effect we instead of S we add R that was the electro optic coefficient and here is the Kerr coefficient and we had n_0^3 when we had E there and here we have E^2 , it is because it is an intensity based effect.

So where S once again is proportional to χ^3 the specific expression is $\frac{12}{\epsilon_0 n_0^4} \chi^3$ does not matter that S is a proportional to χ^3 , basically through χ^3 and some external intensity you are changing the refractive index of the medium. So I can write this as $n(I) = n_0 + n_2 I$ because my E^2 is essentially representing intensity.

So through that external intensity you are changing the refractive index of the medium, where n_2 is called the Kerr index because it is the index that represents the Kerr effect and give you an idea it is in the order of 10^{-16} m² / watt, so that unit comes about just because of fact that your intensity is watt per centimetre square. So this is the n_2 is equal to this in fused silica SiO₂, so that just give you a idea of what order that is.

And of course in things like organic material χ^3 is larger so this n_2 value could be larger so same in semiconductors also you can have larger χ^3 , in fact that is very interesting because in a semiconductor you know the χ^3 can be several orders of magnitude larger so you can achieve this four-wave mixing, you can achieve this Kerr effect all of those things with a relatively small structure, relatively short structure.

So but whereas if you are doing it in silica that is in fused silica, you need a much longer length to achieve the same effect, so that actually talks about the through changing the refractive index you are changing the phase of the light. So your $\Delta\phi$ as a function of I that just looking at the Kerr index component, so $\Delta\phi$ is going to be given by $\frac{2\pi}{\lambda} n_2 I L$ but n in this case is corresponding to this Kerr index. so n_2 multiplied by intensity can be written as power over area multiplied by the length of propagation.

So you can achieve a change in the phase of light, basically you can make a phase modulator. In this case what you are modulating is the what you are exciting it with is the intensity of light, so you have some modulation of the of this source intensity then you can achieve a corresponding modulation in the face of another light beam that is going through the material. To observe all these effects it works out that you know your optical fiber is an excellent medium, it is an excellent platform.

So why do I say that? Because as far as an optical fiber is concerned you have a core and a cladding and your light is actually focused let us say we are talking about a single-mode fiber it is focused in the core region. Normally if you are trying to see these you know these effects nonlinear effects what you would do is you would take a laser beam use a lens to focus it down to as small as pot as possible.

But in this case you see that your nonlinear interactions are confined to only this region because that is where it is maximum intensity, but what are we doing as far as an optical fiber is concerned? The optical fiber you are just extending this interaction to how much our long distance you want to have, so that is the beauty of you know doing nonlinear optics within an optical fiber because you can get extended interactions and these interactions can go on over kilometres of fiber if you if you want it to you know if you design it that way.

So that is the beauty of that and of course what matters like we say in this case is the intensity of the light which is power over area and area let us say the power is only 1 milli watt, the area corresponds to what? How do you quantify the area? Is that the core area? As far as our optical fiber is concerned we realized that you have light it is propagating both in the core and the cladding so what makes more sense is not the physical core radius but mode field radius, thank you.

So if you say mode field radius corresponds to w naught then πw naught square is and that is actually very small for a single mode fiber it can be as small as say 4 microns, so 4 into 10 power minus 6 the whole square. So if you do this math you will find that even for 1 milli watt of power coupled into this fiber you have something like 20 megawatt per meter square as your light intensity, so that is a fairly high value and not only that, you are able to maintain that intensity over a long region and that is valuable when you consider something like this where you are doing the phase modulation because that phase modulation happens over a length L so it can maintain that intensity over a long region.

So then I can get more phase modulation do you understand this? So in some ways it is easier to observe a lot of these nonlinear effects in an optical fiber than if you do it in like what we say in free space, so free spaces this picture whereas in an optical fiber that interaction can be over much longer distances. Okay, so what about this? So what exactly do we achieve with this?

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Kerr effect

Free Space

Optical Fiber

Intensity (I^2)

$$n(I) = n_0 - \frac{1}{2} n_2 I^2 \quad S \propto I^2$$

$$n(I) = n_0 + n_2 I$$

where $n_2 \rightarrow$ Kerr index
 $n_2 = 10^{-10} \text{ cm}^2/\text{W}$ in SiO_2

$$\Delta\phi(I) = \frac{2\pi}{\lambda} n_2 \cdot \frac{P}{A} \cdot L$$

Power/Area = $\frac{1 \text{ mW}}{4 \times 10^{-4} \text{ m}^2} = 20 \text{ MW/m}^2$

Self Phase Modulation

Low intensity

High intensity

Delay

Dispersion

Chirping ($\frac{d\phi}{dt}$)

\Rightarrow Compensation

We can achieve what is called self-phase modulation, so what is self-phase modulation? What we have in this is let us say you have a long length of optical fiber let us say single-mode fiber and you have light going in and you have light coming out here. If you are sending in let us say a pulse of light like this, so what you have in this case is depending upon the intensity of the light you have a slightly different response of the medium. So what you can say is if it

is low intensity you have a linear response, so clearly these regions where there is low intensity the medium responds linearly, whereas these regions where you have high intensity they are going to see a different response.

So even as the pulse propagates down the fiber it is accumulating a differential phase, the low intensity regions just accumulate a linear phase, whereas the high intensity regions accumulate this non-linear component also. So if we go back and look at this the low intensity regions are accumulating phase corresponding to n naught, whereas the high intensity regions accumulate a phase corresponding n not plus $n^2 I$.

So what do you think is going to happen to that output pulse? Is it going to remain the same? No it is going to get distorted and how does it get distorted? Your high intensity part of your pulse right is actually seeing a larger refractive index because it is corresponding to n not plus $n^2 I$, so because of the larger refractive index it is going to undergo a phase delay. So your output pulse is not going to look like this rather I am just going to exaggerate it but it is going to have some high frequency components and some low frequency components it is essentially going to go through some phase distortion and that distortion now is actually generating new components (we are running out of time so I will just stop with this thought).

If you look at the frequency domain let us say we started with just this frequency but now because of this distortion I have actually spread out that frequency. So now know what I have and let us say I am looking at the delay as a function of frequency, now what I have is something like this I generate new frequencies not only do I generate new frequencies the higher frequencies are delayed with respect to the lower frequencies and this is actually what we call as chirping of your pulse. So you introduce a chirp in your pulse in other words it is basically a $d\Phi/dt$ term that you get there is a differential phase that you get with respect to time.

Okay, so it is introducing new frequencies but also that it gets chirped it is different frequencies are delayed with respect to each other. The idea is this chirping may not actually be bad from a communication perspective because we know normally in communications we have frequency as a function of delay, you have what is called chromatic dispersion and chromatic dispersion works out in such a way that the delay is like this with respect to frequencies it has got a negative slope.

So this is dispersion, so this dispersion plus chirping can provide compensation for your dispersion. So normally with your dispersion you would say your pulse is going to get wider and wider but in the presence of self-phase modulation you can actually you know offset dispersion and you can actually keep your pulse shape intact do you understand that? So chirping in this case can actually be a good thing that it can be used to offset dispersion in fibers. So we are out of time, let me stop at that point but we will pick up on this, any questions related to this before we move on to other topics? Thank you.