

Introduction to Photonics
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Non-linear Optics-Pockels Effect

Ok welcome to Introduction to Photonics, in one of the previous lectures we considered the response of a medium to an electromagnetic wave this case we are talking about a light wave and we were characterizing that mass spring sort of model and we were tracking the resonances that appear because of this we basically said that the response epsilon which is characterize through the epsilon is typically complex in nature right so it is got both real and imaginary parts and it is also frequency dependent ok so those are two properties that we looked at. Now in today's lecture we will consider a case where your excitation is so high that your response is not linear anymore ok.

So what happens if you work in a regime in which you are excitation in this case let us say in the mass spring model it is like the excitation corresponds to puling that mass stretching that spring you stretch it to such a large extent that the displacement is not you know is not changing linearly with respect to the excitation ok. So what happens in that case?

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Nonlinear response of materials to light

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$= \epsilon_0 (1 + \chi) \vec{E}$$

$$= \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E} \vec{P}$$

$$P = \epsilon_0 (\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots)$$

Second order susceptibility Third order susceptibility

$E_r e^{i\omega_1 t}$
 $E_s e^{i\omega_2 t}$

$\chi^{(2)}$
 P/N

$2\omega_1 \rightarrow$ Second Harmonic Generation
 $\omega_1 + \omega_2$
 $\omega_1 - \omega_2 \rightarrow$ Difference frequency generation
 $2\omega_2$

Graph: $|P|$ vs $|E|$.
Nonlinear response (solid red line)
 Linear response (dashed blue line)

Non-linear response of materials to light, so far we have characterize the response of a material to light through the displacement vector we said as dependent on the electric field applied to that

material and the specific thing that is changing is your epsilon R that is what is defining the response of the medium and that we said is equal to $1 + \chi$, χ is the susceptibility of the medium.

So we wrote this as $\epsilon_0 E + \epsilon_0 \chi E$ and this term we call the polarization response of the medium ok so that is actually representing the polarizability of the medium and that is not to be confused with the definition of polarization light polarization which is essentially how E you know the terminus of E changes as it propagates so this actually slightly different thing that we are talking about this is talking about polarizability of the medium. But nevertheless if you plot the magnitude of P with respect to the magnitude of E according to this expression you would say that you should get just a linear response like that right.

In reality what happens is it is linear but only upto a certain level of electric field ok. So beyond certain electric values it starts deviating from that linearity so it has non-linear response exhibits a non-linear response beyond certain value of E ok. So most of the applications including the communication application that we consider in an optical fibers we typically rely on the linear response of the medium and what we are saying is that linear response is valid the approximation of linear response is valid only upto a certain electric field. So what you really have to look at is when we are talking about the polarization we just drop the vector notation because we just interested in the magnitudes.

So you have $\epsilon_0 \chi E$ that is the first order effect but then you could have higher order terms $\chi^2 E^2$ that is $\chi^3 E^3$ and so on right. Now let us examine these higher order terms a little more closely ok so what is this higher order term mean? For example what is this mean? It says that it this response is proportional to E^2 right so that is essentially saying you can have two different electric fields interacting with each other ok to provide a net response.

So going back to the mass spring model we said the displacement is because of an applied electric field right that applied electric field we were what was that corresponding to when we were discussing that? That electric field was corresponding to the electromagnetic wave that was travelling through that material right that was the excitation that we were considering there. But now we are saying that if you have another field ok you could get a mixed response you could

have two fields maxing through this chi 2 term ok and similarly if you are talking about the chi 3 term which is called the 3rd order susceptibility.

So chi 2 is called the 2nd order susceptibility and chi 3 related effects is called the 3rd order susceptibility there you could have mixture of three different electric fields ok and quite interestingly these electric fields need not be at the same frequency you could have one field at one frequency and another field at another frequency both are exciting the same location both are exciting the same mass spring model, so what do you expect the response is going to be like? It will be a mixture of both those frequencies right. So, now when you invoke these 2nd order and 3rd order terms now we are starting to talk about mixing frequencies mixing multiple frequencies.

So just to give you an idea if we talk about 2nd order susceptibilities now I can come in with $E_1 E$ power $j \omega_1 t$ and then $E_2 E$ power $j \omega_2 d$ right and then I can go out if this medium has certain value of chi 2 then you can go out with multiple terms because this ω_1 and ω_2 beating with each other right so you could have two let us say two photons at ω_1 they could get mixed with each other and so you could actually get two ω_1 as one result you could have ω_1 plus ω_2 as a result.

You can have ω_1 minus ω_2 as a result or you could have two ω_2 as a result ok. So interesting things happen you are generating new colors ok once you have this medium which exhibits chi 2 and ofcourse the value of chi 2 is very small right so it is in the order of let us say picometer per volt ok so because chi 2 and then it is interacting with electric field so that is why we are talking about picometer per volt so it is fairly small numbers so you would say that to really see effect of something like this you need to go to you know several orders of magnitude volt per meter right so you need to have very high electric fields to see this sort of effect but nevertheless there is a possibility that you can mix frequencies you can generate new frequencies ok.

So where do you see two ω_1 or two ω_2 becoming useful? That is actually what we call as 2nd harmonic generation right so clearly two ω_1 corresponds to the 2nd harmonic of ω_1 . So your 2nd harmonic generation so twice the frequency what would that mean in wavelength? Half the wavelength, so now you have the possibility of converting a laser let us say with 1 micron wavelength right that is going in going through this medium you can actually have

half a micron come out of that ok how it happens and all of that you know will go into details later on but that is what is possible through this medium.

Two photons, two electric fields at omega one they beat with each other and they create 2nd harmonic generation and that is what you have when your laser pointer that is why as asking about the laser pointer few minutes ago so if you look at green laser pointer so what exactly do you have? That green laser pointer has got neodymium yag crystal, yag yttrium aluminum garnet that is actually a glass crystal so neodymium is doped in that crystal and just like ytterbium neodymium has a transition at 1064 nanometers ok so you could make a 1064 nanometer laser out of a neodymium crystal.

If you can pump it with some radiation around 810 nanometers ok so in that laser pointer you have a 810 nanometer laser that is actually pumping a neodymium yag crystal which has got mirror coatings on either end so you can actually make a laser out of that so that laser is happening at 1064 nanometers and that output is made to pass through a crystal right KTP is one crystal potassium (KTP stands for does not matter) so there are crystals which have a large value of chi 2 so you send it through one of those crystals then you can get from 1064 you can get 532 nanometers and that is the green laser that you see from a green laser pointer.

It is not like a green semiconductor laser that is emitting that ok that is actually you know it is a very good example for 2nd harmonic generation and you could also do some frequency generation and more importantly difference frequency generation I say more importantly different frequency generation because normally it is very difficult to achieve lasers in that work in mid infrared region, mid infrared and far infrared region ok one of the reasons is when you look at the energy corresponding to those long wavelength region you find that the energy is very small, very small with respect to what it is comparable to thermal energy.

So when you have when you want to generate light corresponding to you know closer to thermal energy you look at your Boltzmann distribution it tells that it is very hard to maintain an inversion so that you can emit light with very low energy ok. So in those sort of cases if you want mid infrared radiation this is actually a very good way of producing that because you just come up with two lasers with the right spacing you can actually achieve you can generate

photons at a much longer wavelength corresponding to the difference between the two frequencies ok.

So interesting things can happen through 2nd order susceptibility I will come back and talk about 3rd order susceptibility also but will stay with the 2nd order susceptibility and see what else can be done with that, any questions after this point?

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Pockels effect

$r \propto \chi^{(2)}$

$n(E) = n_0 - \frac{1}{2} r n_0^3 E$ (Electro-optic coefficient)

$\text{LiNbO}_3, r = 30 \text{ pm/V}$
 $\text{SO}_2, r = 0.01 \text{ pm/V}$

$\phi(E) = \phi_0 - \frac{2\pi}{\lambda} \cdot \frac{1}{2} r n_0^3 E \cdot L = \Delta\phi(E)$

$I_{\text{out}} = \frac{I_0}{2} (1 + \cos \Delta\phi)$
 $= I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

Phase Modulator
 Intensity Modulation
 Mach-Zehnder
 LiNbO₃

Ok one of the other things that is possible with due to chi 2 is called the Pockels effect. So what is Pockels effect? When you have a material that exhibits a large chi 2 this sort of phenomenon is possible you could put an electrode across that material ok connect this to some voltage source right so you have a electric field across that material and through that electric field you can control the response of the medium.

Remember when we talked about that mass spring model we said that material in that case the electronic response is to an applied electric field we did not have that point quantify or qualify what that electric field source is it could actually be an external electric field source right. So even that can change the response of the medium ok so much so that if you are (talking) if you are looking at light that is propagating through that medium that light actually sees a different refractive index ok that refractive index which we can call as n of E is given by n knot which is the refractive index that it has originally minus one half of r n knot Q multiplied by E ok, where

n corresponds to the refractive index of the medium, E is the field applied across the medium and r is known as the electro-optic coefficient.

That electro-optic coefficient is slightly different form of χ^2 so r is actually proportional to χ^2 the 2nd order susceptibility. So what we are saying here is through the 2nd order susceptibility and an applied electric field you can change the response of the medium you know such that the refractive index that this light wave sees as it propagates through the medium is changed ok. To give you a idea for this electro-optic coefficient for lithium niobate which is one of the very popular crystals lithium niobate has r of 30 picometer per volt right.

You can see where the picometer per volt unit comes from so that multiplied by electric field which has volt per meter is going to give you dimensionless quantity right so r for lithium niobate is 30 picometer per volt for silica that r is very low it is 0.01 picometer per volt ok it is negligible so you can say that χ^2 for silica is negligible if you look closely as why χ^2 is negligible for silica? It so happens that all centrosymmetric material right all centrosymmetric material have very low χ^2 and that you know also means that if you are silica which is highly amorphous if you are talking about few silica that is amorphous that amorphous material has a high level of centrosymmetry we won't go into the details of what that means I mean it is basically crystallographic notation.

But lithium niobate which is actually a crystal does not have that centrosymmetry and that can exhibit a large electro-optic coefficient part of my PHD research was on this can we break the centrosymmetry of silica so that we can get a increased electro-optic coefficient and should say we were successful in doing that and we were able to enhance the electro-optic coefficient but I won't bore you those details right now. The point is if you have a large electro-optic coefficient if you have a large χ^2 essentially you can actually create a large change in the refractive index of the medium through an applied electric field ok.

So why is that important? Because if you change the electric field what else changes? The phase of light that goes through this medium that changes. So the phase is given by you know you have some $\phi = \frac{2\pi}{\lambda} (n - n_0) L$ the refractive index which is the refractive index change is given by $\frac{1}{2} r n_0 E$ multiplied by the propagation length so in this case let us say the propagation length is given by L so multiplied by L right. So what we are saying is the phase of

light can be manipulated using an external electric field. So if you can manipulate the phase of light what is possible?

You can you know interfere this light wave with another light wave or let us say you take a light you slit it into two ok one part goes through this phase modulator the other part goes you know without actually accumulating much phase and then you mix them together then the relative phase difference determines whether you have constructive interference or destructive interference right and that relative phase difference now is control by this external voltage, you see where I am getting at getting to so let us say you have a structure like this you have managed to split the light two ways one part going this way one part going that way and on this part you have incorporated two electrodes right and you have field applied electric field applied across that ok and these two come together over here and interfere and you go out ok.

Now are we saying is by applying this electric field I decide whether I get constructive or destructive interference over here I can make light appear over here or I can kill that light. So I can do an intensity modulation with this right an voltage controlled intensity modulation. So if I want to communicate data I have my data I am speaking now and let us say this is actually transmitted live somewhere you are digitizing the voice that you (())(28:20) the video you communicate that as data that data is available as a voltage.

Now you want to send this over a long distance so you going to send this you are going to try to out this onto an optical signal and send it through an optical fiber communication system right. So you need some device which can convert this voltage changes into changes in the intensity of light that will be this, this sort of a device ok. So essentially what we have here is a phase modulator so this section corresponds to a phase modulator but since that phase modulator is within the structure which is like a Mach-Zehnder configuration right, Mach-Zehnder interferometer configuration because of it is in the structure this phase modulation is converted to intensity modulation your essentially getting light all through this V of t that you are applying to this electrode and that is provided this waveguide is actually made with this material which has a high electro-optic coefficient.

So if this entire structure is made of lithium niobate right then I can actually have this phase modulation and through the phase modulation you can do intensity modulation ok, any question

about this? Sorry both the (π) (30:36) not necessarily, question is whether both the arms has to be made out of lithium niobate, not necessarily it could be some other material right but it is easy to make it on lithium niobate because you can just define all those waveguides in one material if you have to do it on some other material let us say glass right so you have to have a glass waveguide else everywhere else and then corresponding to the phase modulation region you need to take lithium niobate and stick in there and make sure it is all aligned and integrated all the which is a very tough thing to do ok.

So you might as well just define the entire structure in lithium niobate. So r is proportional to χ the proportionality factor is some $3\pi/2$ or something like that it is just directly proportional to χ , χ_2 , r is proportional to we are saying r is proportional to χ_2 ok so let us go into the details let us actually say this part let us call this $\Delta\phi$ of E ok so let us figure out the voltage at which you can get $\Delta\phi$ of π , why is that important π ? Because π phase difference can mean that you are going from constructive to destructive interference so I am turning the light on to off ok.

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So let us quantify that voltage so what we want is $\Delta\phi$ of E to be π and $\Delta\phi$ of E is given by $2\pi/\lambda$ multiplied by $1/2$ of $r n_0^3$ this particular voltage that gives me π phase shift I call that V_π so that electric field is V over d what is d ? d is the thickness of this material so d is the distance between the electrodes multiplied by L ok so you can rearrange

terms of course you can cancel this you can cancel π over here and if you rearrange terms what you get is an expression for V_{π} , V_{π} is going to be given by λ multiplied by d divided by $R n^3$ multiplied by L ok.

So let us see if it makes sense so V_{π} is smaller when d is smaller makes sense right because for the same voltage if d is smaller that corresponds to a higher electric field right higher the electric field that means more the modulation so more the phase change ok, V_{π} is inversely proportional to R once again that makes sense because R is representative χ^2 so larger R larger the electro-optic coefficient, larger the electro-optic coefficient more the phase change for a given electric field right and it is also inversely proportional to L because phase is accumulating over distance so longer length of that interaction region more will be the phase change right.

So let us actually throw in some realistic numbers to this so let us say we need V_{π} equal to 5 volt, why do you want V_{π} equal to 5 volt? 5 volt corresponds to TTL ok now of course V_{π} if you want it to be comparable to CMOS logic you will say 3.3 volt you can you want a relatively small number (with) through which you can control this amplitude modulator ok so let us pick V_{π} equal to 5 volt we know that for lithium niobate n equal to 2.2 and R for a given particular configuration of electrodes it is given by 30 picometer per volt.

Let us say λ we are working at 1.5 microns, what else is left? D , let us say D is 10 microns the separation between the electrodes is 10 microns. So question is what value of length is required so that you can get V_{π} of 5 volts right so this would imply that length is given by λ multiplied by d which is 1.5×10^{-6} multiplied by 10^{-5} divided by R which is 30×10^{-12} n^3 2.2 the whole cube multiplied by V_{π} which is 5 right and right off you can see 10^{-6} 10^{-5} and 10^{-12} so those cancel and 10 goes into 3 as 30 and 1.5×10 is 15 goes in 30 as 2 so it is $\frac{1}{2}$, $\frac{1}{10}$ into 2.2 the whole cube.

2.2 whole cube is roughly about 10 (38:17) somebody has a calculator you can check that so what is the final value you get? It is roughly about 1 centimeter, 9.89 millimeter right so that is roughly 1 centimeter ok so with just 1 centimeter of lithium niobate right provided you have a 10 micron separation between electrodes you can achieve a V_{π} of 5 volts ok so you can make reasonable modulator with this. Now so this is in terms of so if I have this expression for V_{π} I

can actually plug that into this expression over here. So I can tell that ϕ of E in terms of V_{π} is going to correspond to $\phi_{\text{knot}} - \pi$ into V of t divided by V_{π} .

Because V_{π} is given by this expression and E is nothing but V over d so that is where you can see that is in the denominator is $\lambda d R n_{\text{knot}}^3 L$ is all there so you can put it up like this and the other thing that you get to see is when you are looking at the output intensity right let us call it I_{out} , I_{out} over here let us say what is coming in is I_{knot} and you managed to send I_{knot} over 2 there I_{knot} over 2 here and then it is going through relative phase change so I_{knot} can be or I sorry I_{out} you will find that if you do the math it will be like I_{knot} over 2 multiplied by $1 + \cos$ of $\Delta\phi$ or you can just write it as $I_{\text{knot}} \cos^2 \Delta\phi$ by 2.

$\Delta\phi$ essentially is the phase difference introduced by the phase modulator ok so if you plot I_{out} as a function of $\Delta\phi$ that is basically saying it is a \cos^2 function so it is going to be something like this so when $\Delta\phi = 0$ that is going to be the maximum and that corresponds to I_{knot} and then when $\Delta\phi$ equals to π right $\cos^2 \pi$ by 2 is zero so you go to zero and then when it is 2π you go to a maximum and then 3π you come back to zero and so on right. So key thing is that this $\Delta\phi$ is a function of the voltage that is applied right so by changing the voltage you can actually change the output of that modulator so you can make the output go from maximum to minimum and the corresponding voltage here is what we are calling as V_{π} so the voltage corresponding to maximum to minimum is what we are calling as V_{π} ok.

So if you have something like this then what you can afford to do is you can afford to bias it the for a \cos^2 function the best place to bias it would be at quadrature so it is $\pi/2$ so if you have a bias phase bias of $\pi/2$ between the two arms then if you have some voltage waveform like this you see a corresponding intensity waveform at the output ok so you can essentially (()) (44:14) your modulation which is in voltage so this is basically v of t and correspondingly you have an output intensity which is changing with respect to time ok. So you can so this is what we call as you know intensity modulation using this Pockels Effect ok.

Any questions about this? So this is voltage controlled phase modulation (I have) voltage control phase modulation which when we incorporate into Mach-Zehnder configuration you have a voltage controlled intensity modulation ok ofcourse there are applications like in modern day

communications you play around with a phase you have not you not turning light on and off any more you are actually trying to change encode the information and phase going from zero to pi or zero to pi by 2 or whatever right. So if you are interested in phase modulation you can just use the lithium niobate modulator as it is you do not need to put inside a Mach-Zehnder interferometer ok so it can achieve direct phase modulation.

So this is what you are going to be doing yourself in this week laboratory session you will first you will given a lithium niobate modulator so you will actually connect it up and apply an external voltage you apply a DC voltage and you connect the lithium niobate modulator to a light source and that side to a detector and then you would for applied DC voltage you will see whether the power that is detected whether it is going through this curve this Cos square curve and once you are verified that then you will apply a bias appropriate bias to your modulator and so that it comes to that part where you have a linear in this point you have sort of a linear response so you bias at the center of that and then you apply some voltage modulation then you can see the corresponding change in the intensity at the output ok.

So you will send it to a photodiode and then take the output of photo receiver to an oscilloscope to see whatever waveform that you sent in as voltage to that lithium niobate modulator is the same waveform that you are seeing in terms of intensity as well ok so what happens if you bias at this point? If you bias at that null point and you apply some voltage there, voltage swing there what you expect to see? So that basically says DC is 0 but you have a modulation so corresponding to that voltage you will have light go through right you are letting so if you do this so let us actually put the modulation point somewhere over here right and if you do this what do you expect to see at the output?

So corresponding to the this zero over here in terms of voltage you have zero intensity but when the voltage goes to the positive cycle what do you get to see? There would be a corresponding positive cycle and then it goes to the negative cycle you still are letting light through so you are actually having for the negative cycle also you have so you essentially have twice the modulation frequency so if your frequency of modulation is F_m over here you get $2 F_m$ over here right and that is over a bias of you know with zero intensity so that is actually what you call as carrier suppressed modulation because your carrier is essentially light carrier is at null point ok.

So what you get here is carrier suppressed modulation ok so you have your modulation side bands but your carrier is suppressed in this case ok so let me stop at this point and ofcourse we can look at other examples as an 2nd harmonic generation, difference frequency generation and all of that I will desist from doing that all I can say there is this picture is valid so you can go in with photons at two different frequencies if we go through a chi 2 material you can get all this other components you know difference between the two frequencies you can get double the frequencies and so on right so you can achieve frequency conversion through non-linear medium ok and in this case what we are saying is we are modulating the phase of light through this Pockels Effect through this external.

In this case we said this could be interaction between two electric fields, one electric field in this case is the applied electric field and the other electric field is the one that is going through the material the corresponding to the light wave ok. So the that is the interaction that we are seeing here electrically controlled phase modulation ok so let us stop here and tomorrow what I would like to do is to jump on to the third order effects third order susceptibility effects what can be done with those sort of materials, material that exhibits a large chi 3.