Introduction to Photonics
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Lecture No 03
Tutorial on Ray Optics and Wave Optics

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Let us get started. Good morning. Welcome to the third session of Introduction to Photonics. Today's session is going to be slightly different, as in we have dealt with a little bit of theory so far, now we will work out some practical problems, let's us try to design something, for example. We have already designed, endoscope, earlier this week and so let's try to see if there are other things, we could design based on the principles that we have learnt so far. So, what did we learn so far?

We learnt to identify that there are different approaches to photonics, the science of light. Essentially, we say that the basic approach is considering that light travels in straight lines which gives us ray optics and then what we have learnt in the last couple of lectures is that ray optics has some limitations in terms of describing the properties of light. Especially when we have light encountering feature sizes which are comparable to the wavelength of light, we are having to invoke the wave property of light. In the last lecture we saw how particular light wave can have a wavelength, phase and based on those properties how can we explain certain physical processes that are happening.

We took the example of Young's double slit experiment to show the process of interference: constructive and destructive, which obviously could not be described using just ray optics.

We will discuss couple of examples today, one related to just ray optics and another related to wave optics and let us see how we can work through that, whether we have enough knowledge to work out problems like this.

So, let me just take the first example.

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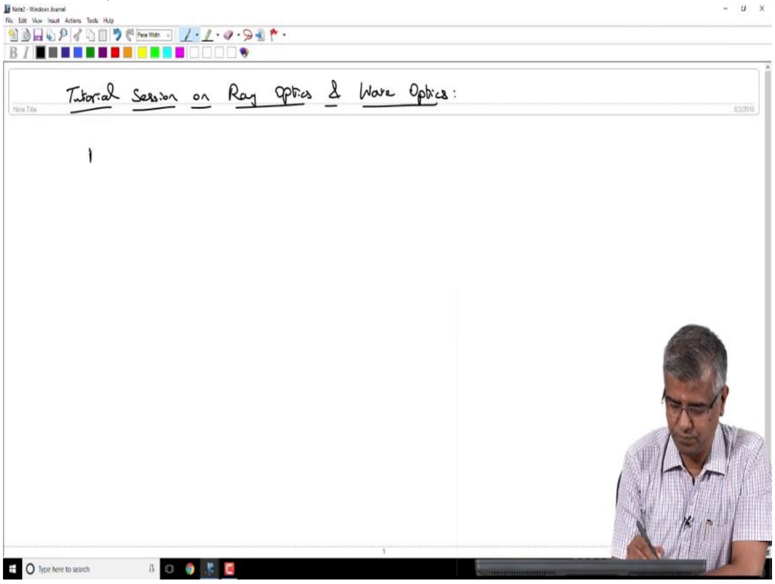

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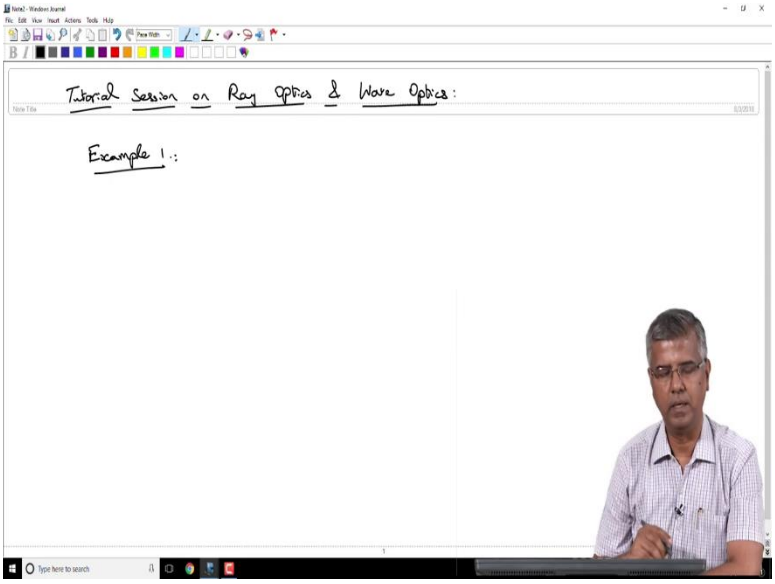

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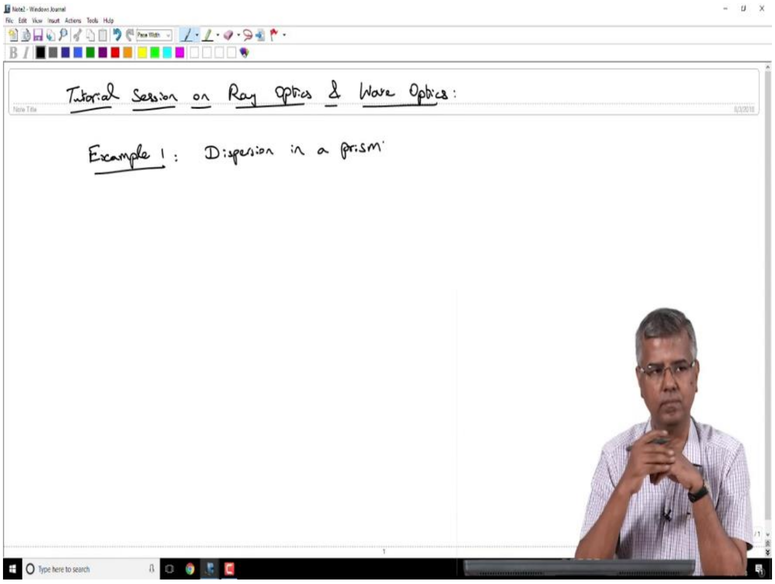

Example 1 is about dispersion in a prism,

So, how do we get dispersion in a prism?

So, what is dispersion?

(Professor – student conversation starts)

Student: Spreading of pulses (0:03:51.9)

Professor: Spreading of pulses; Well that is again something that we have not talked about. Maybe we will come back to that little later.

(Professor – student conversation ends)

There is a more fundamental physical process which, can represent dispersion.

(Professor – student conversation starts)

Student: Spreading of colors (0:04:09.9)

Professor: Spreading of colors, basically. Discriminating different colors. When light wave goes through a material, the material responds differently for different colors. As the material responds differently to different colors, then you get an effect wherein you are spreading all these colors. You can spatially separate out all these colors, Ok.

(Professor – student conversation ends)

Depression in a prism can explain separation of colors and also how a rainbow is formed. I will come back and re-qualify this statement a little later. But basically, each water drops can be modeled as a thin prism.

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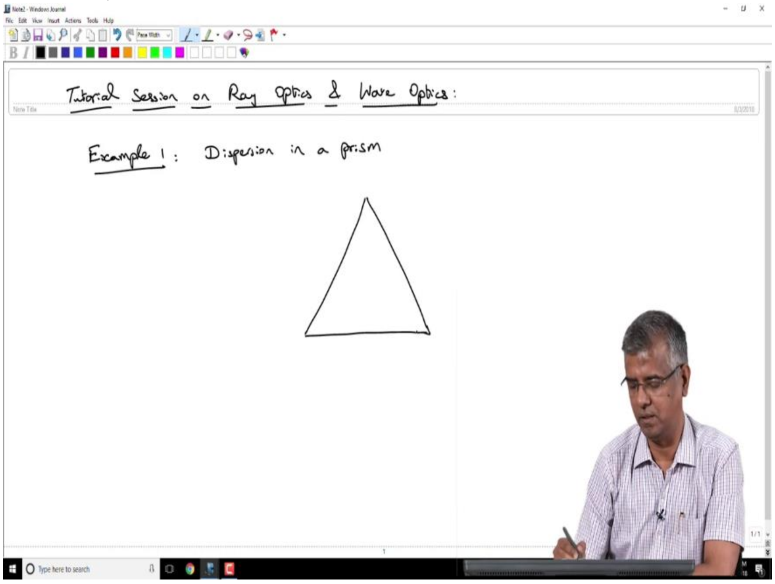

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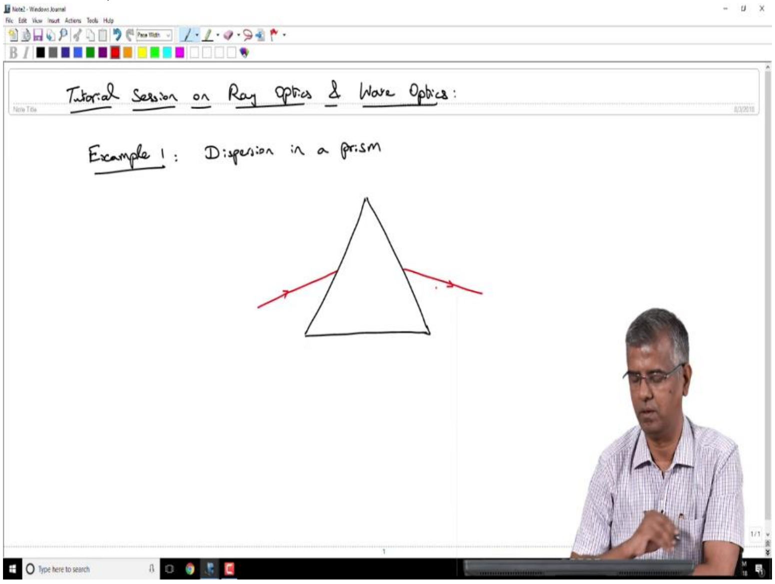

You can see a prism over here and a ray of light is coming in. As the prism is transparent the ray is going in and comes out after refraction as shown.

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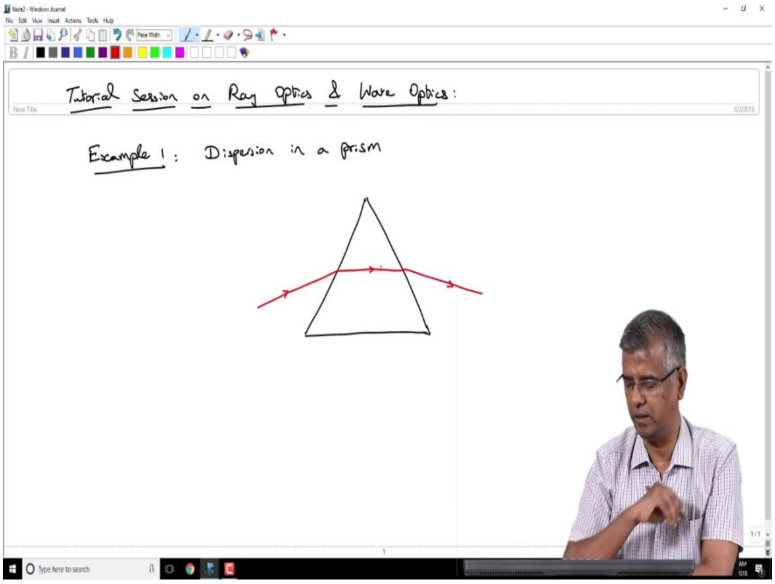

If the ray had gone straight it would have traced the black line, but instead the ray gets diffracted and comes out at angle δ, with respect to the initial direction.

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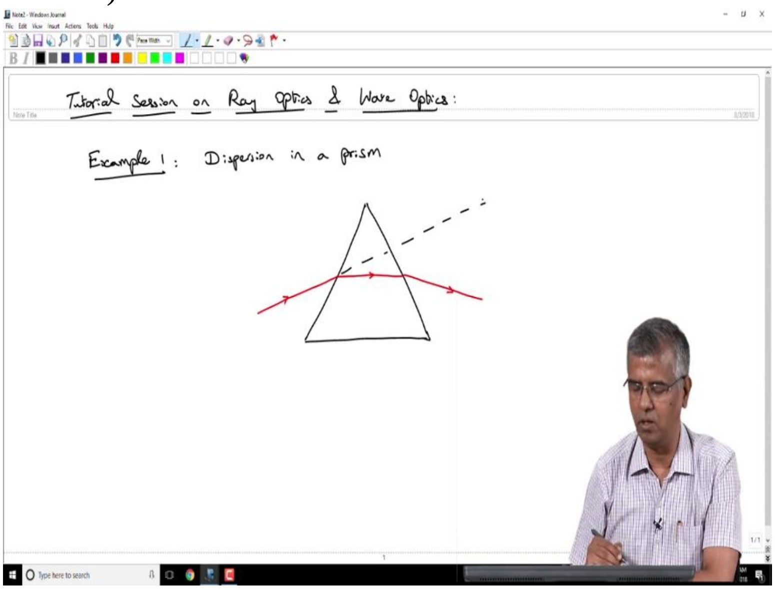

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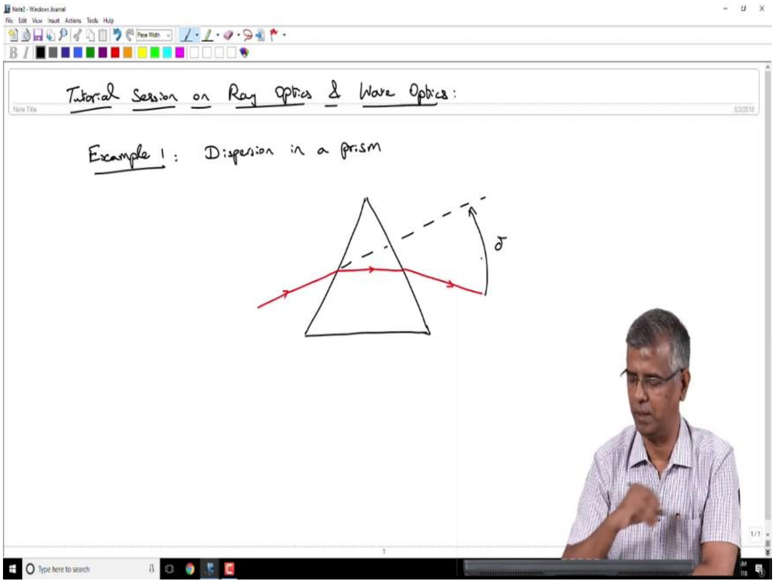

Here we want to figure out δ and specifically we want to know if delta is different for different colors. If it is different for different colors, then the colors spread out, giving you a rainbow. So, for doing that we have to define the refractive index of the material.

Let assume, we are modeling a water droplet. For blue color water has a refractive index of approximately 1.34 and for red color it exhibits a refractive index of 1.33. The other colors have refractive index between these values.

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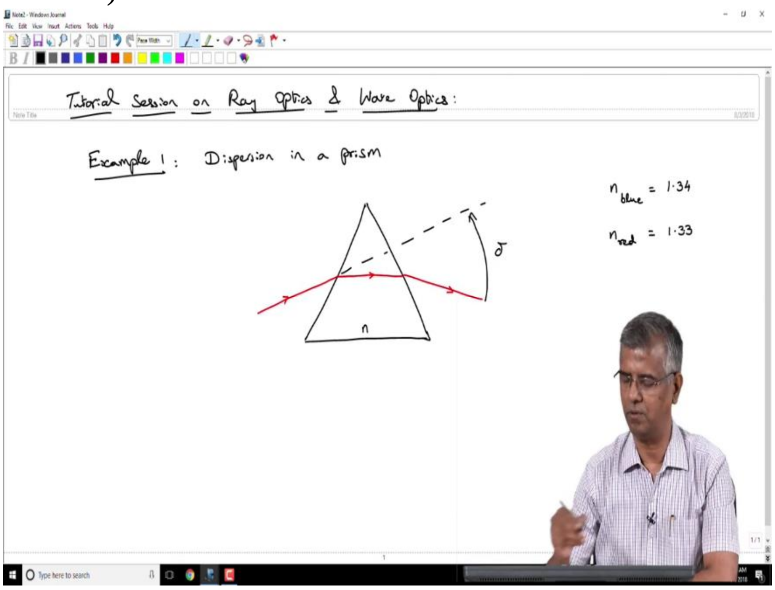
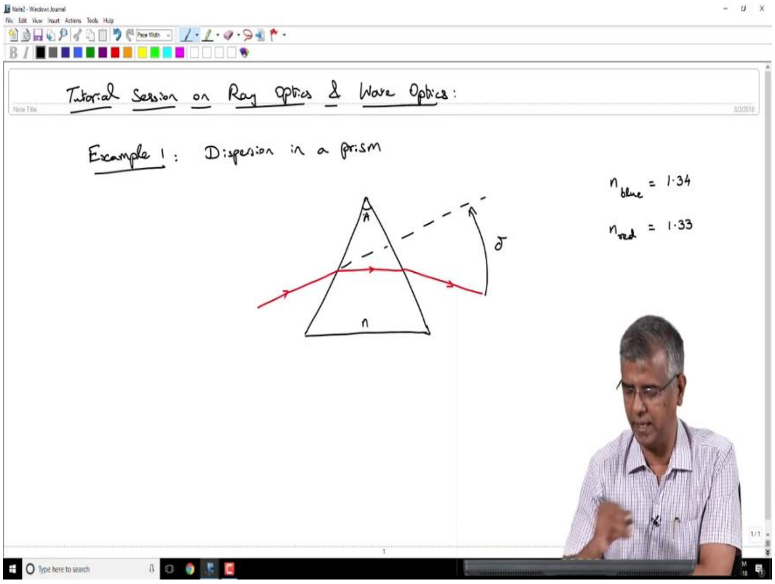


So, in this scenario can we find out the delta (the angular deviation) as a function of these two colors? I.e. weather the two colors separate out; how much do they separate out?

To do that, let us say this prism has an apex angle A. (Refer Slide Time: 08:24)



So, the idea is to represent the angular deviation (δ) as a function of apex angle (A) and refractive index. So how do we go about solving this problem?

Now to solve this problem, we need to define a few things. First, we need to define this

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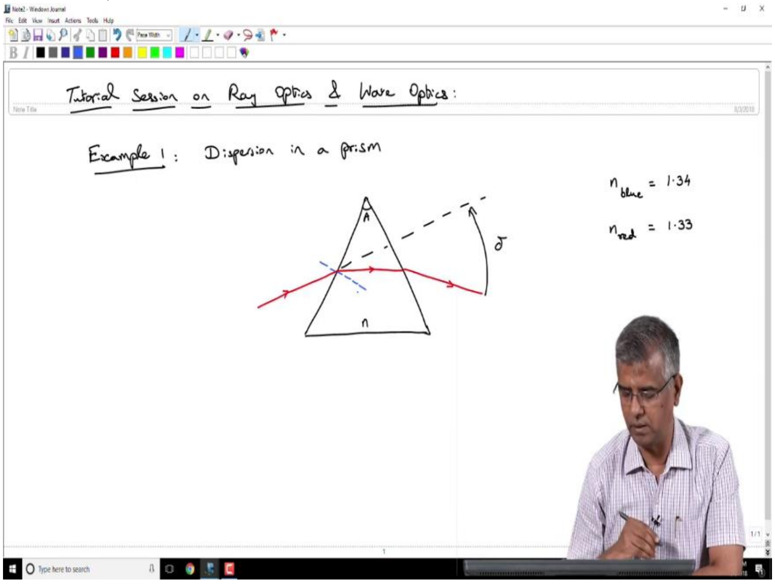

normal over here(the left surface) so that we can apply Snell's law at this interface, and we need to define the normal over here(the right surface),

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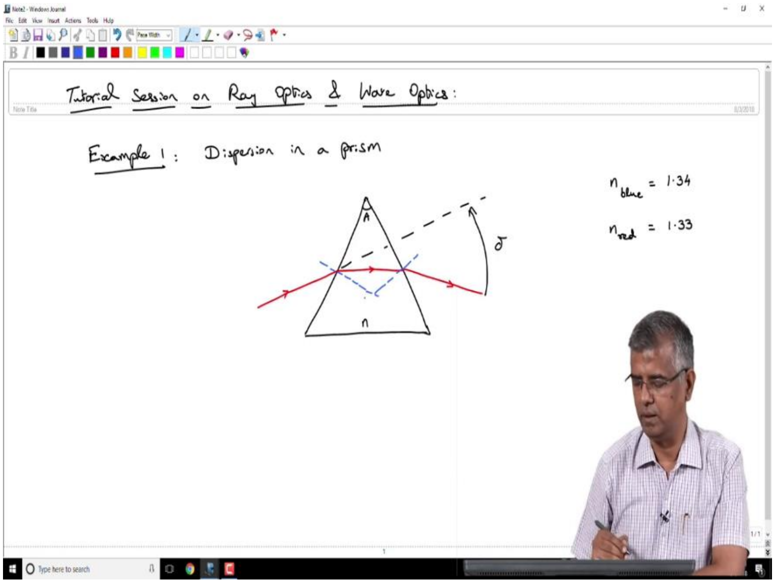

so that we can apply Snell's law at that other interface. These two normal intersects each other and makes an angle, let us say α. Lets also assume that the ray is incident with an angle θ1 and goes into at an angle θ2.

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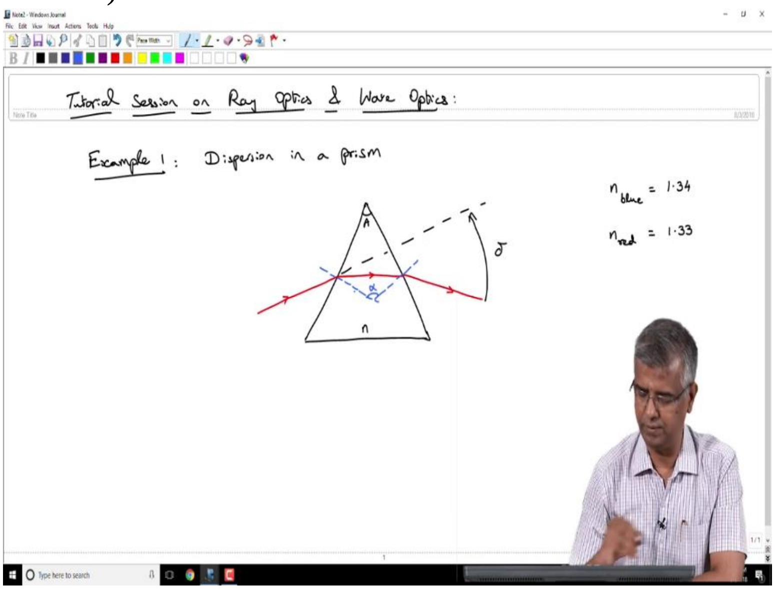

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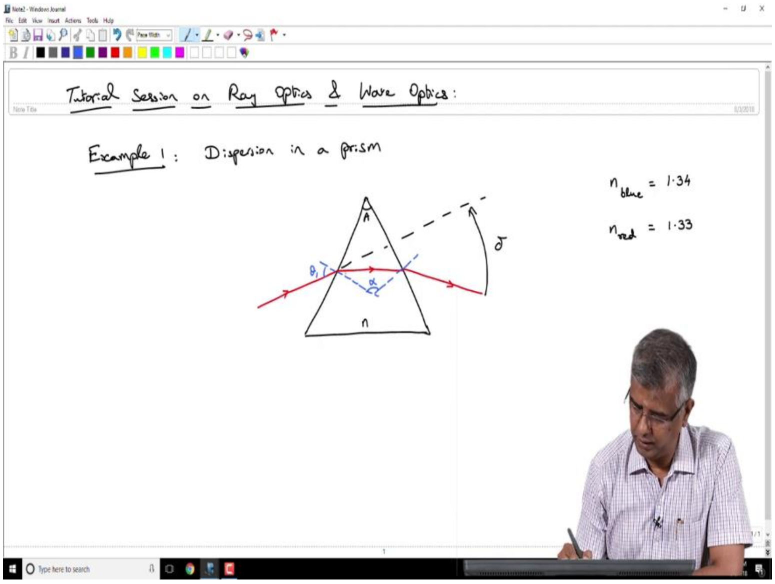

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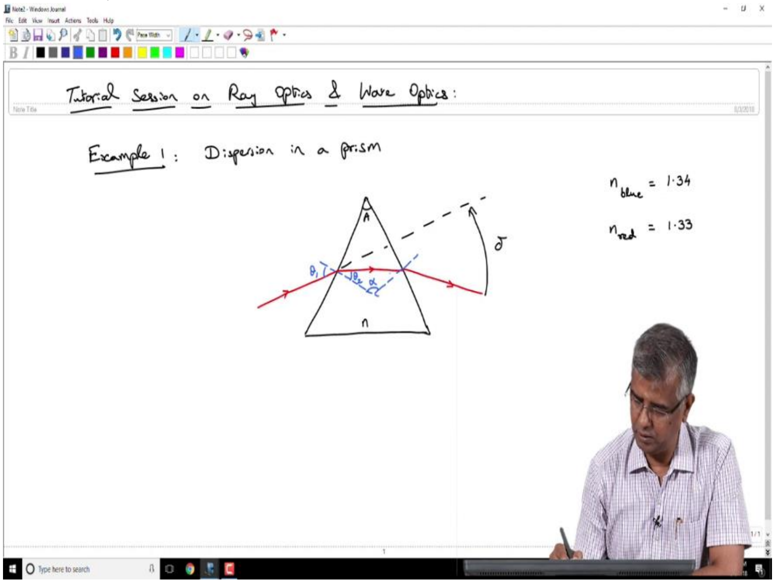

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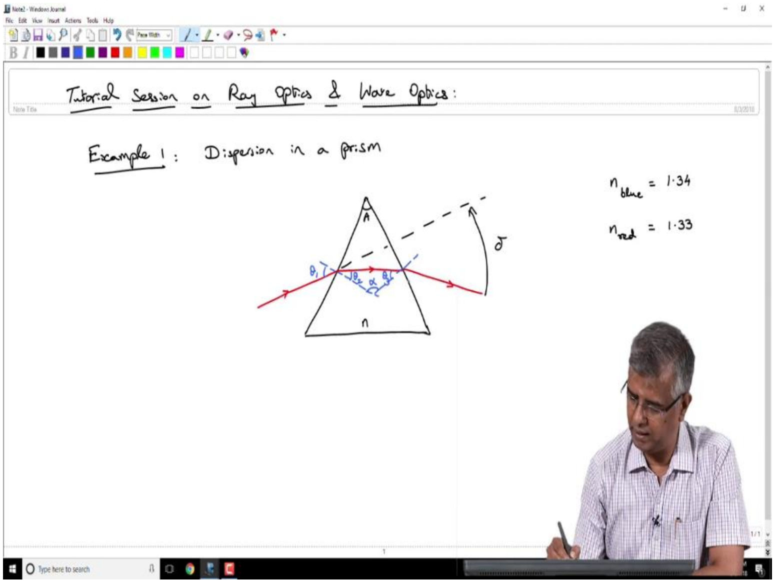

It is incident on the second interface with an angle θ3 and it goes out with an angle θ4. And let us, for simplicity consider this prism as a thin prism.

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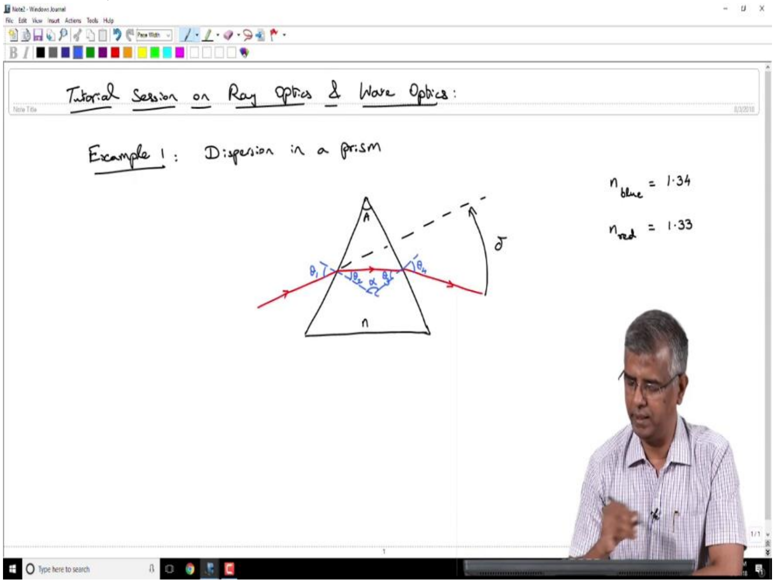

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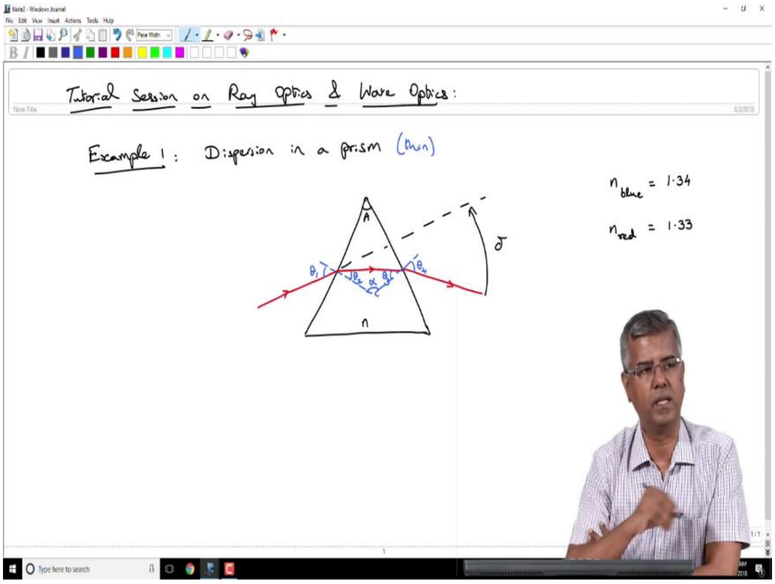

If it is a thin prism, then all these angles (θ1, θ2, θ3, θ4) are small and hence we can apply the paraxial approximation.

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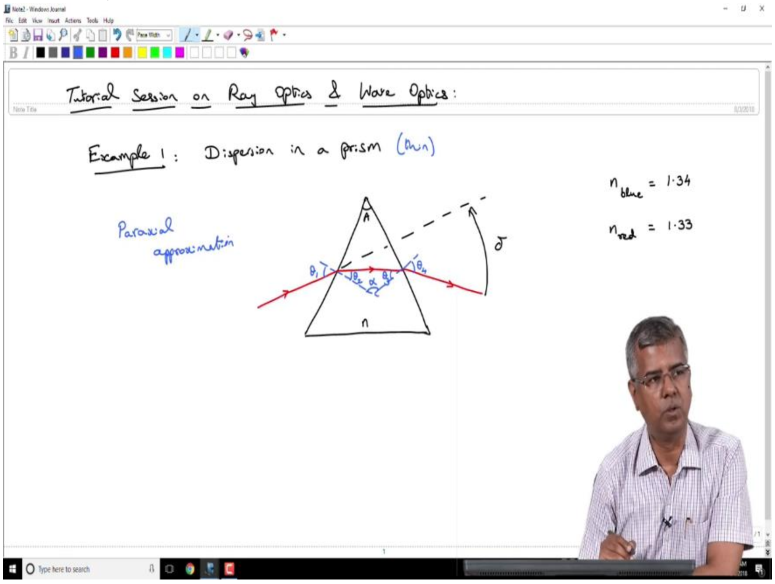

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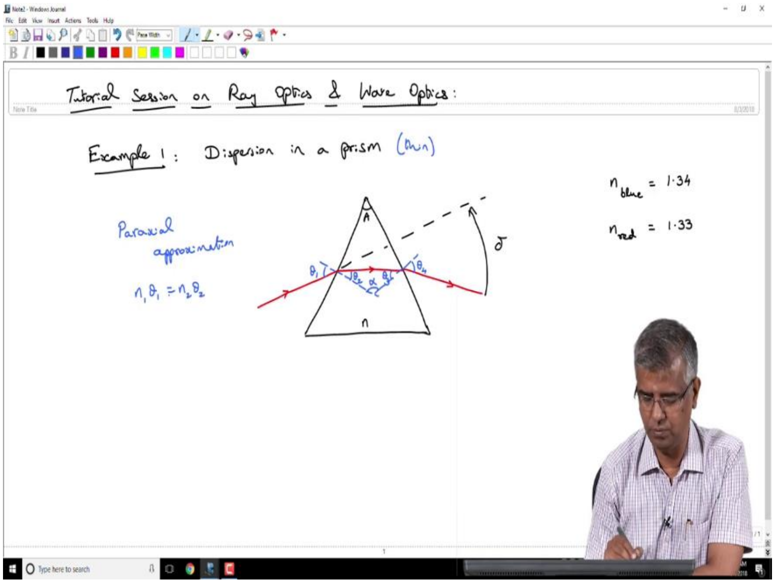

So, what is the advantage of using the paraxial approximation?

Of course, it makes our calculation simpler, so you have, $n_1\theta_1 = n_2\theta_2$. I.e. Snell's law becomes to compute.

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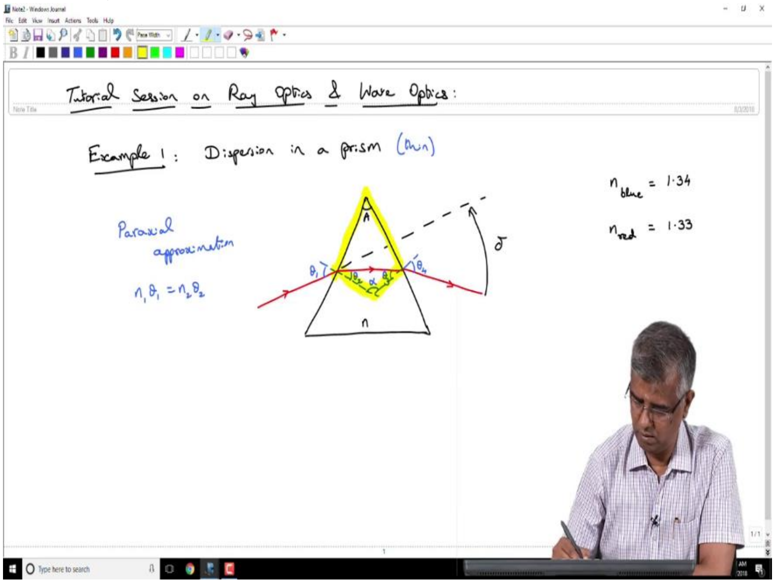

Now, consider that trapezium (highlighted with yellow color). It includes the apex angle A, α and the angles at the left and right interface.

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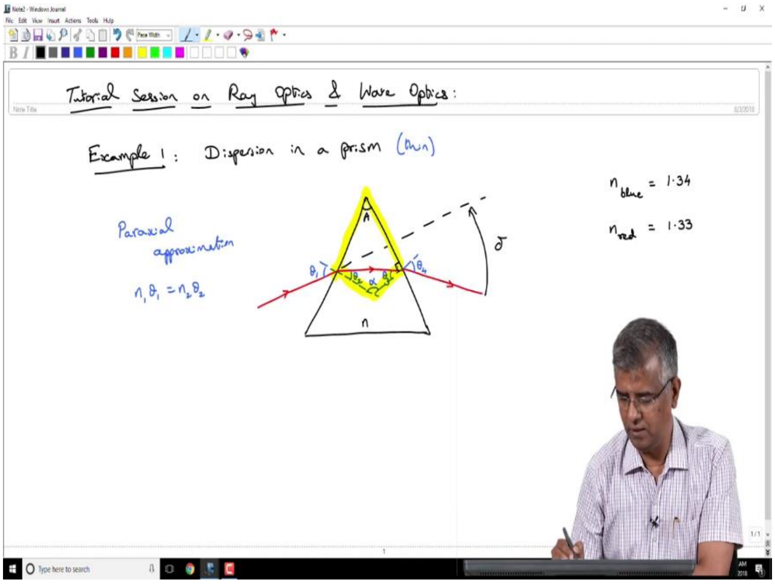

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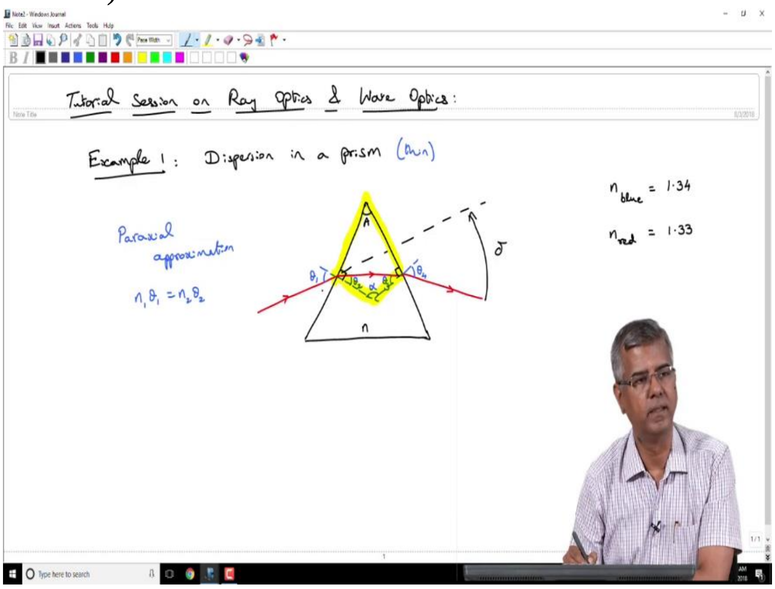

The angles at the left and right interface are both 90^0, because they are normal to the surface. The total internal angles must be equal to 360, and we know that two angles are 90^0. So, we have $\alpha + A = 180^0$.

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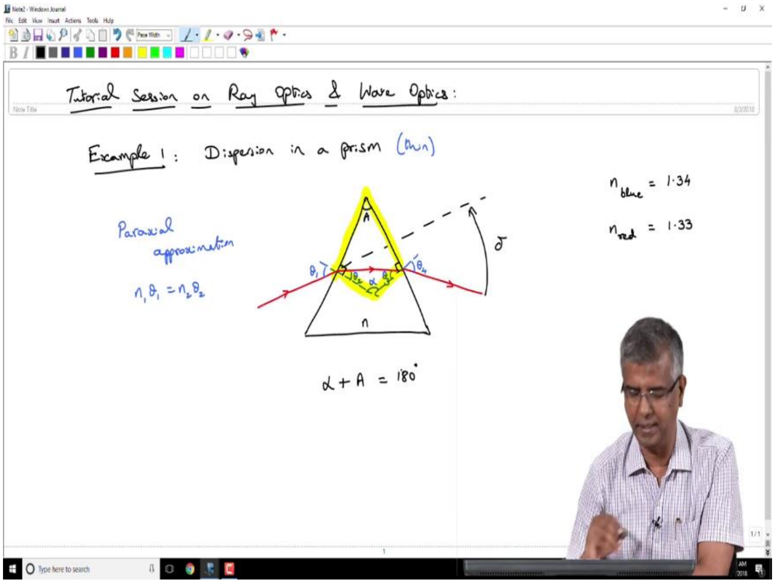

Now consider this triangle (see below, the one including the left, right interface and α)

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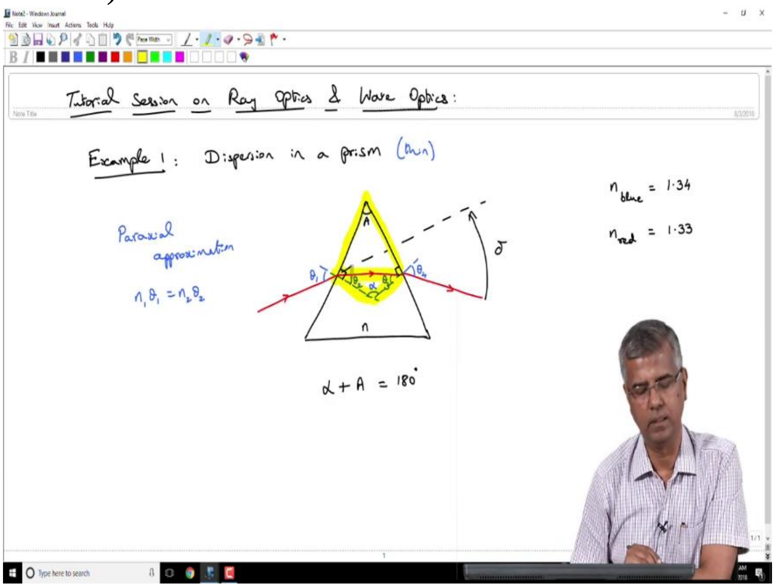

From this triangle, we can say that

$$\theta_2 + \theta_3 + \alpha = 180^0$$

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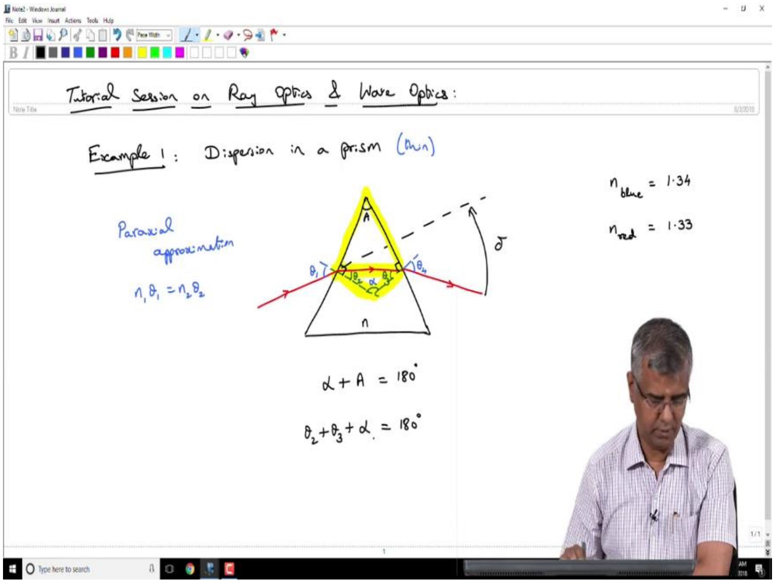

So, α common for both of these expressions. Using the two equations we get,

$$A = \theta_2 + \theta_3$$

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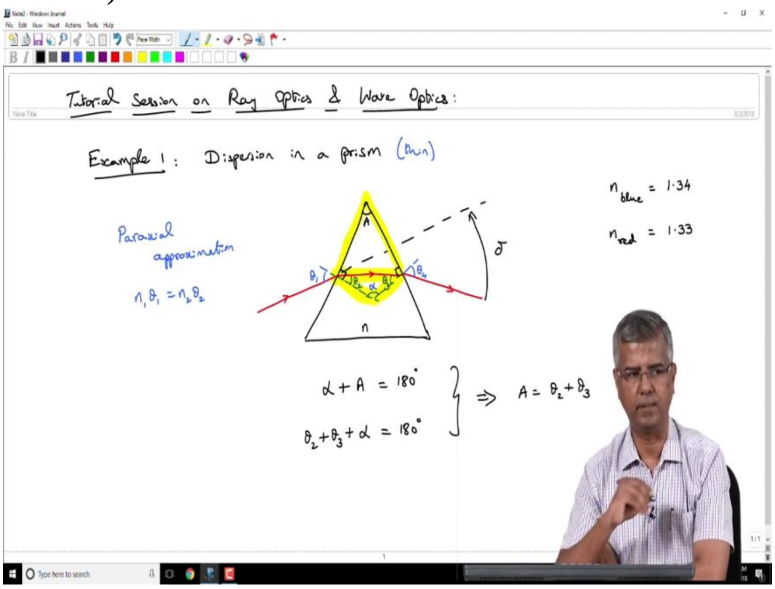

Now, consider the angle made by extending the incident ray (black) with that of the refracted ray inside the prism (red). Let’s call this angle as β (see Fig). Similarly, consider the angle made by extending the refracted beam outside with inside beam and call γ.

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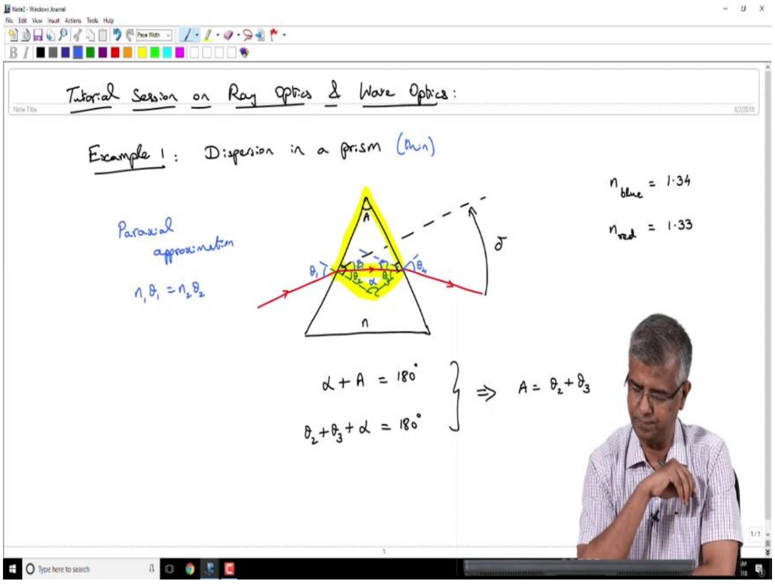

Ok.

Now, we try to represent the angular deviation, δ, with respect to α and β. The total angle δ is the sum of β and the angle between the red line in the prism extended outside the prism through the right interface and the refracted ray (red, outside). But this angle is same as γ (vertical angles). So, we can express δ as,

$$\delta = \beta + \gamma$$

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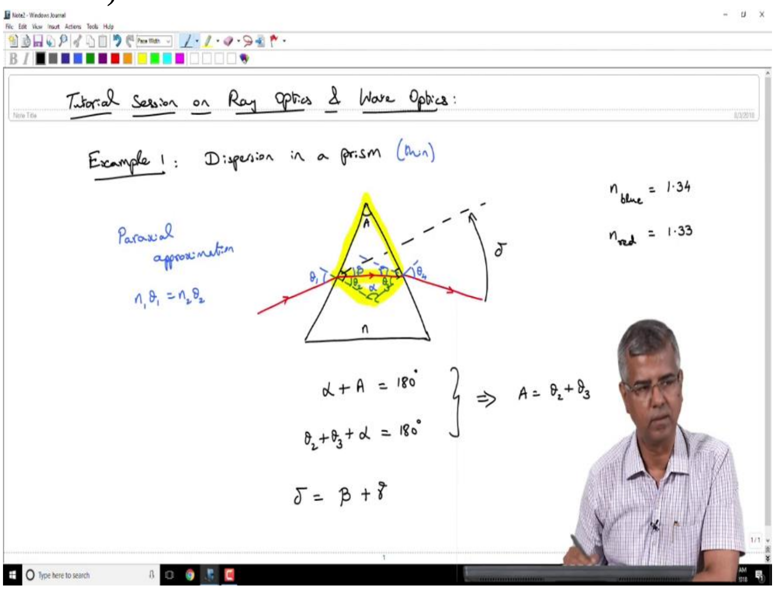

Notice that $\theta_1 = \theta_2 + \beta$. Hence β can be expressed in terms of θ_1 and θ_2 as

$$\beta = \theta_1 - \theta_2$$

Similarly, $\theta_4 = \gamma + \theta_3$. So

$$\gamma = \theta_4 - \theta_3$$

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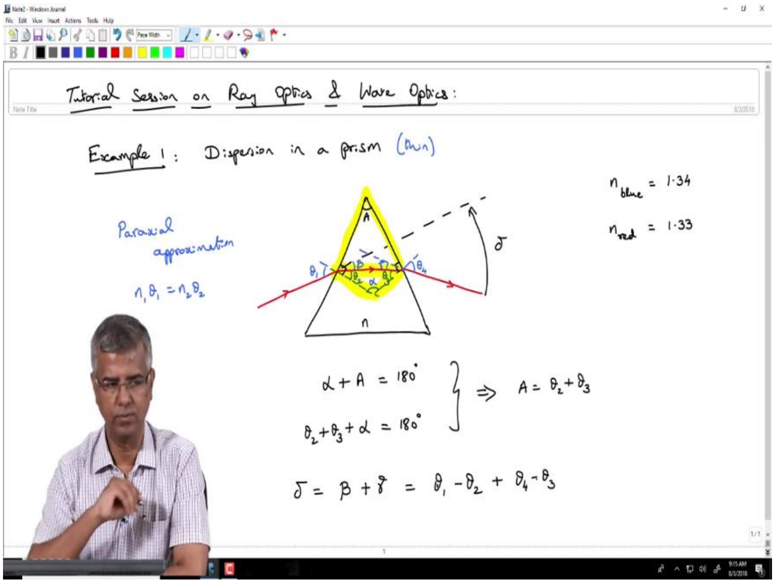

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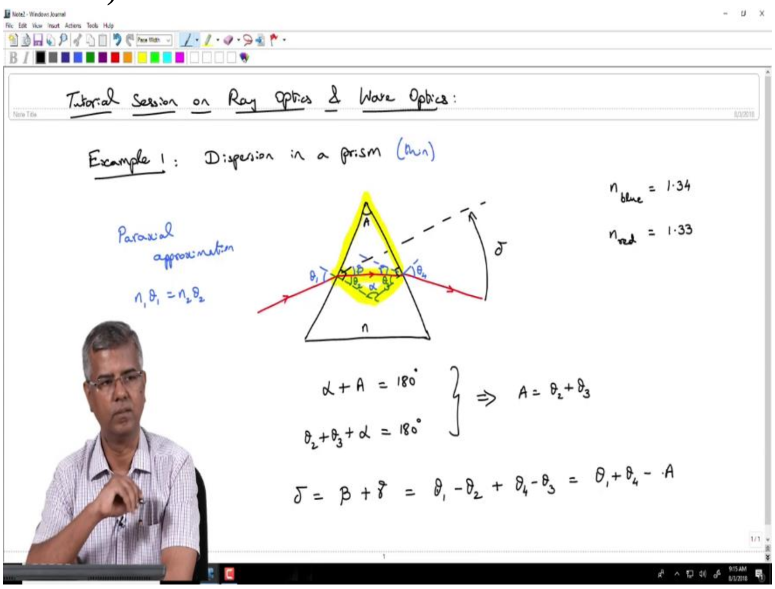

Substituting, β and γ in the equation for δ and remembering that $A = \theta_2 + \theta_3$, we get,

$$\delta = \beta + \gamma = \theta_1 - \theta_2 + \theta_4 - \theta_3 = \theta_1 + \theta_4 - A$$

So, we have an equation that connects the angular deviation δ with the apex angle A. Now, we can apply Snell's law at the interface.

Let us say, we are going from air to a medium (in this case, water) with refractive index, n. If this is the case, we can rewrite θ_1 and θ_4 from above as $n.sin\theta_2\ and\ n.sin\theta_3$.

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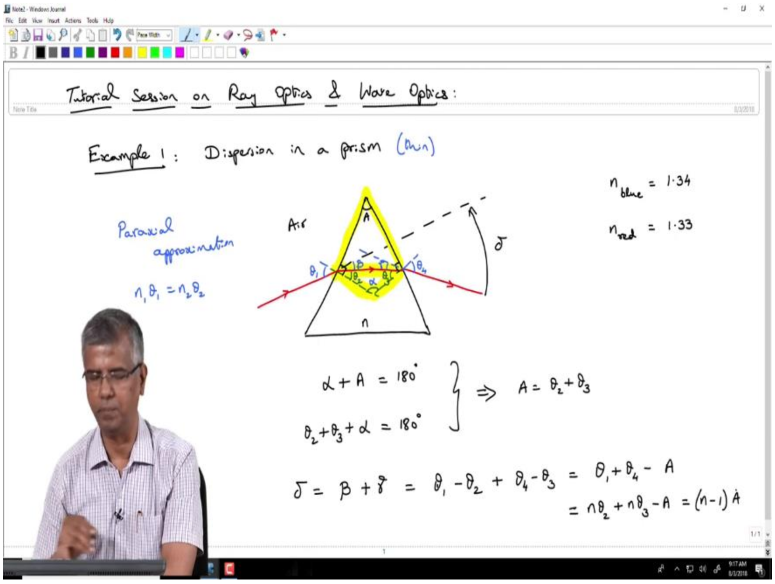

So,

$$
\begin{aligned}
\delta &= n\theta_2 + n\theta_3 - A \\
&= n(\theta_2 + \theta_3) - A \\
&= nA - A \qquad\qquad (A = \theta_2 + \theta_3) \\
&= (n-1)A
\end{aligned}
$$

Remember that this expression is an approximation reached using the paraxial approximation, some geometrical principles and Snell’s law.

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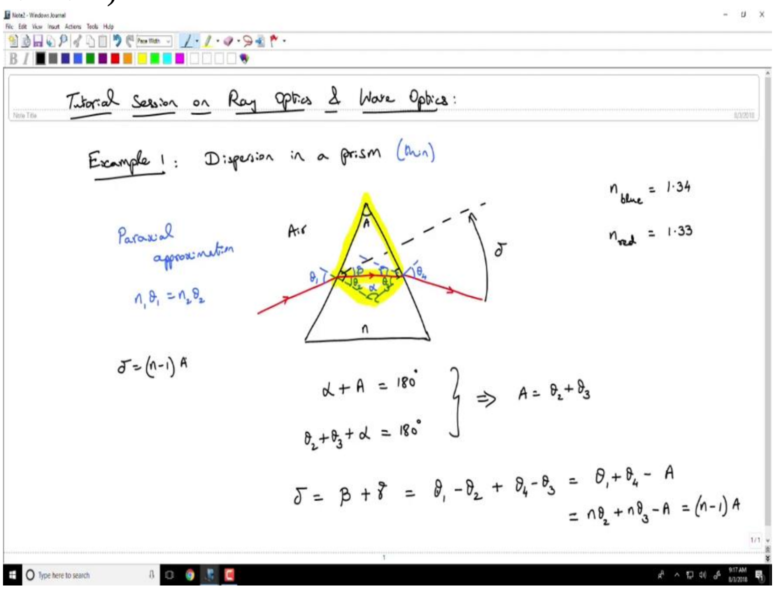

So, if you want to extend this to, any arbitrary angle of incidence or for a thick prism etc., then you just go ahead and use the precise values, like sinθ. And then you get a slightly more complicated or complex expression. But nevertheless, this approach gives you some sense of what you can expect.

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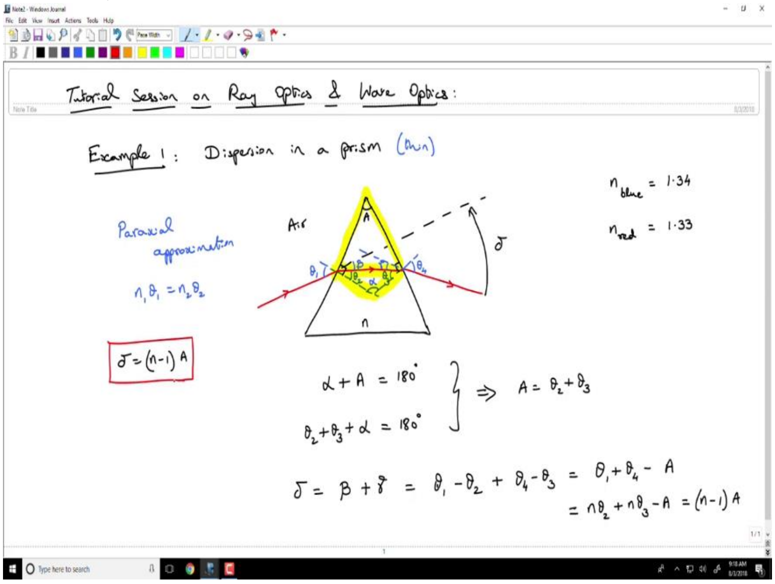

Now does this convey the idea that we are going to have dispersion in the prism? When you look at expression, what part of that expression tells you that there is going to be dispersion or separation of colors? That angle of deviation, Yes! But what specifically gives you different angle of deviation for different colors?

(Professor – student conversation starts)

Student: Refractive Index

Professor: Yes, The refractive index.

(Professor – student conversation ends)

So, δ is dependent on the refractive index. So, if I substitute these values (of n) for different colors then I get a different angle of deviation. So, let us just do that.

Let's find δ for red color, let us say A equal to 10^0. So, δ for red color is going to be,

$$\delta = (1.33-1)x10$$

Which gives., 3.3^0.

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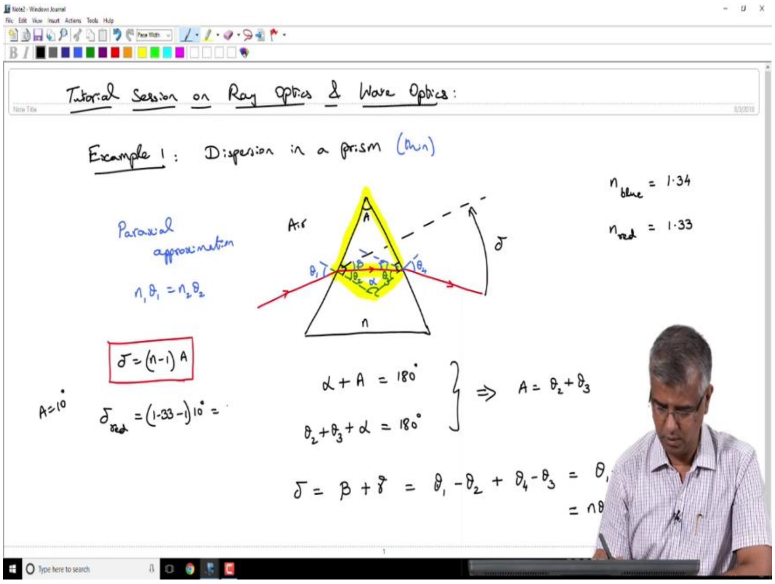

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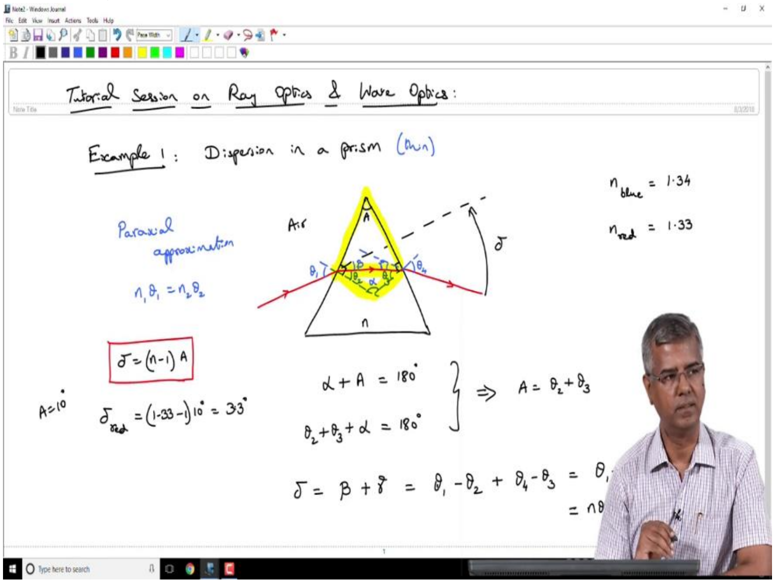

Similarly, δ for blue,

$$\delta = (1.34\text{-}1)\text{x}10$$

$$= 3.4^0$$

So effectively between the two colors, there is only 0.1^0 of extra dispersion., which may seem very small. But when you project it over long distance, this gives significantly larger spatial separation between the colors. So, there is going to be a dispersion.

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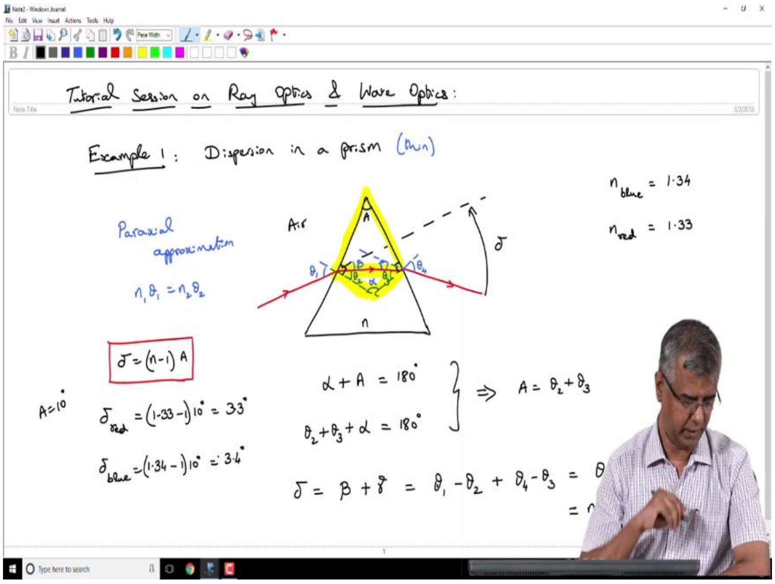

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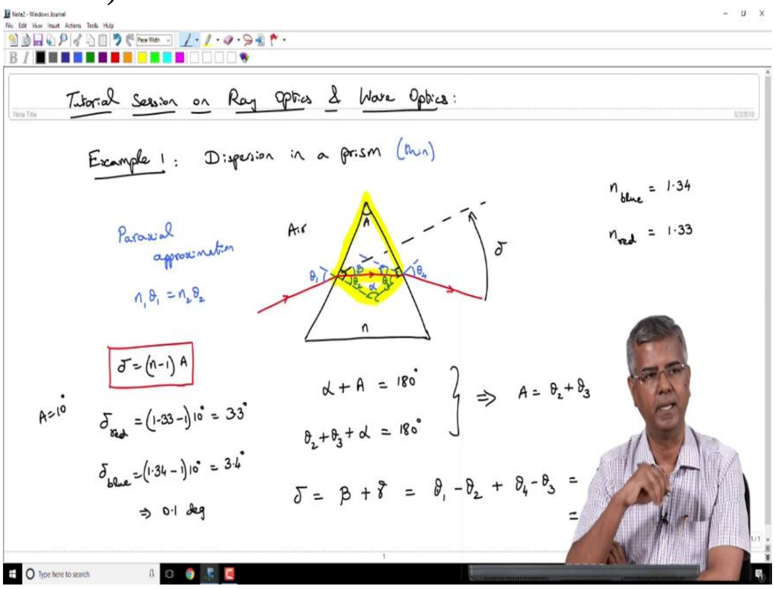

So, if I were to draw this blue light also in the same graph, we can see that it takes a slightly different path because of the slight extra deviation. The figure is an exaggerating of the effect, though, but gives you an idea of how dispersion happens.

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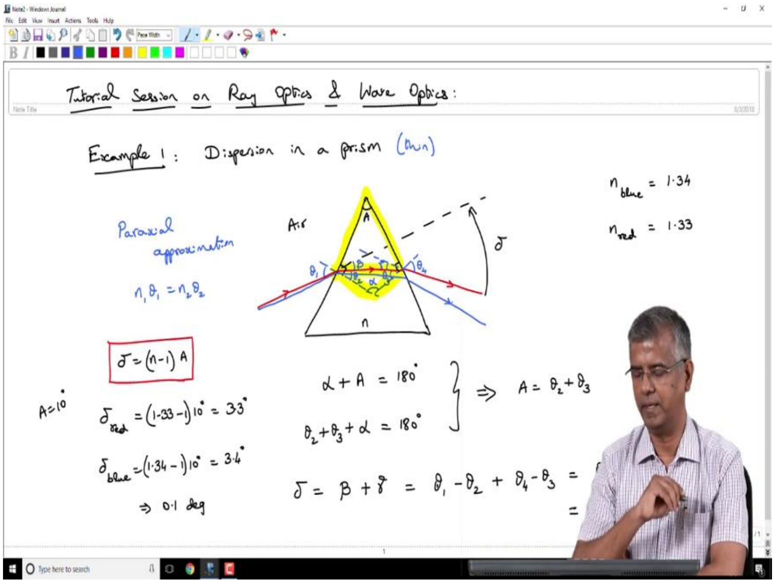

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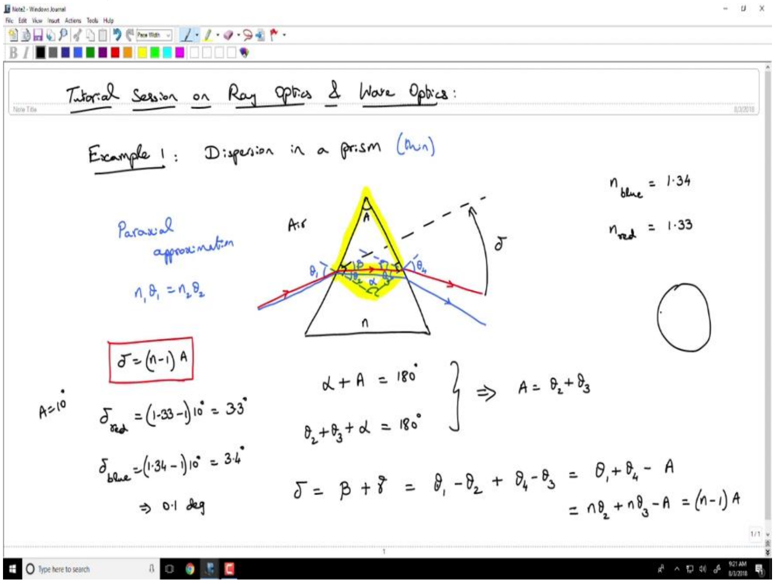

But this is not exactly how the rainbow is formed. The water droplets are slightly spherical in shape and the sunlight goes through it, undergoes total internal reflection, comes out and scatters out this way (see image below).

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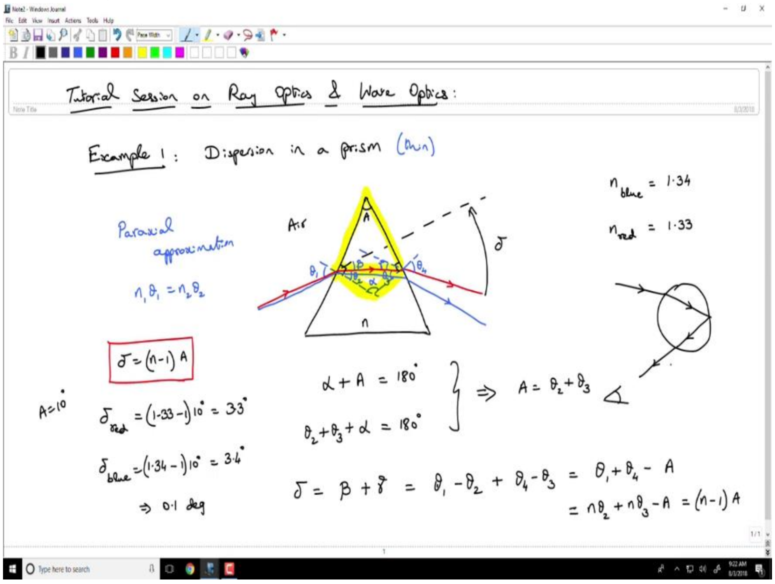

But the dispersion principle applies in a similar way here as different colors are going to go through different angles of refraction within this water droplet and the separation is going to be very similar. So, red, blue light and all the other colors separate out nicely and that is how you see a rainbow.

What happens in an actual rainbow situation is that, it is not only that we are looking at it from a distance but also the fact that this dispersed light can go through multiple successive dispersion in other droplets, so you can essentially get an effect that is magnified in terms of dispersion.

I do want to let you know that this is a powerful concept to utilize. You can use this for a spectrometer. For example, when Sir C. V. Raman looked at the separation of colors, he used a spectrometer to say that there is Raman scattering happening and that gives a different color from what was incident on the medium and that spectrometer could be something like this. It could be based on a prism which separates out the colors and what you do is you put a slit over here (image below: at the end of blue line through the prism) if you want

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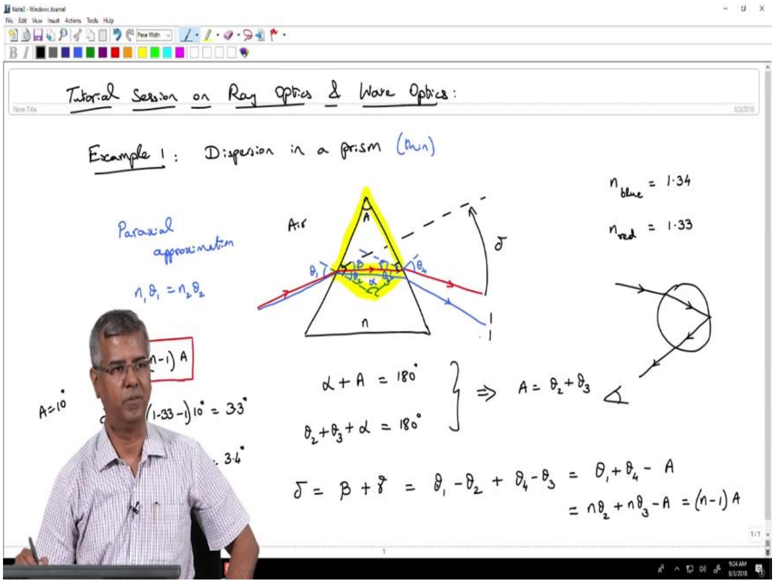

to see a particular color. If you move that slit, you can separate out different colors. Of course, in modern day, we put a CCD array over here.

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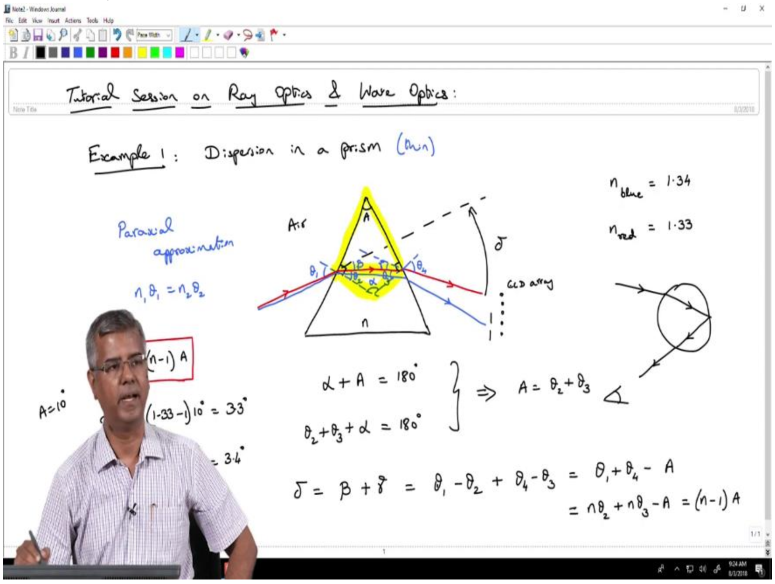
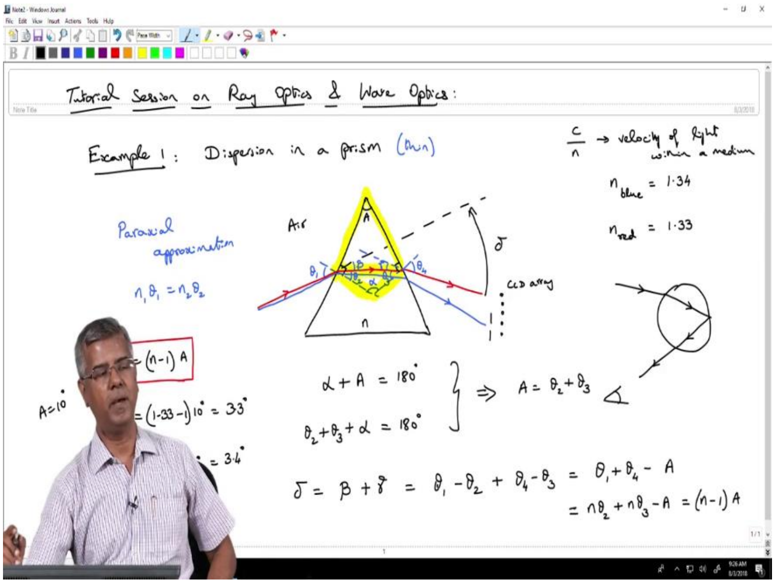


If you put a CCD array each pixel is actually picking up a different color. So modern day spectrometers could be differentiating colors by using some dispersing element. It could be a prism, or it could be a grating also (which we have not talk about, but the depth of the corrugation in the surface is actually periodic and that can give a similar effect).

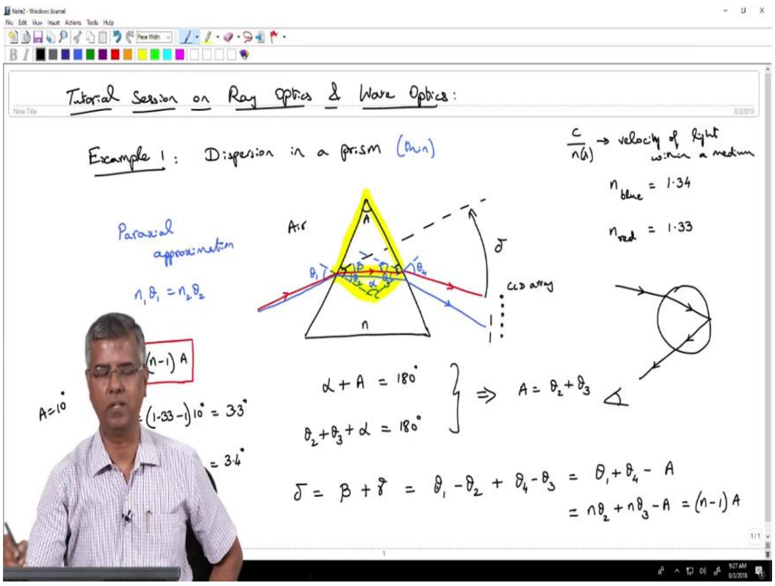
If you get a chance, maybe we will go back and look at that next week but that can also separate out colors in a certain way.So you could have a grating based spectrometer or you could have a prism based spectrometer. Grating based spectrometers are typically more preferred because it can give you more separation of the colors and we will probably come back and look at why that is the case, at a later point.

But this prism can also help in dispersion of pulses. So, you can imagine that when you are sending a pulse of light which consists of multiple colors through a medium and if the medium responds in a different way to each of those colors (meaning it gives a different refractive index for each of those colors) then dispersion follows.

The speed at which light is propagating in vacuum or free space is c but when it is propagating within a medium you represent the velocity as c/n. (Refer Slide Time: 26:31)



So, you can imagine that when light is going through this medium and medium has dispersion, meaning the refractive index is a function of wavelength, then the different colors are traveling at different speeds essentially. (Refer Slide Time: 26:43)



So, if you have a very narrow pulse of light and we look in its Fourier domain, what does that represent? It represents a spread in frequency, spread in wavelengths.

So different wavelengths are now traveling as different colors, so your pulse essentially gets spread out. You started with a very narrow pulse but because of the response of the medium you are starting to have a spread just because all these different colors spread out. And that is a major problem in optical communications. Light is going through a glass medium and that glass medium provides different refractive index for different colors. So, if you have multiple colors propagating over hundreds of kilometers of fiber, then all these colors separate out and the pulse which started as a narrow pulse, but after going through this optical fiber it becomes a broad pulse.

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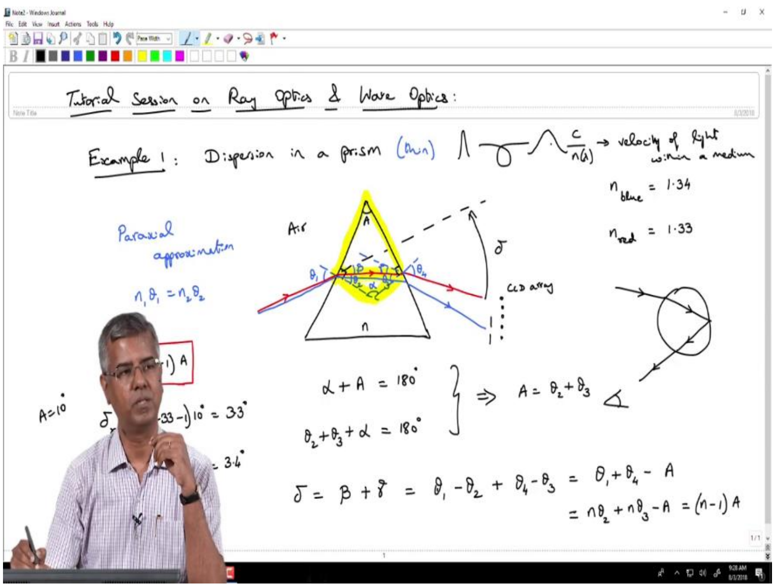

And all these colors separate out and they arrive at different times after they go through this fiber. So, if you have dispersion like this, as velocity is c/n (that is the velocity of a color) shorter wavelengths travel slower (also n is larger for shorter wavelengths) compared to longer wavelengths. So, here you can see (image in top) that when you are looking at the pulse spread, the blue lags and the red color leads (see the second figure below, right top corner)

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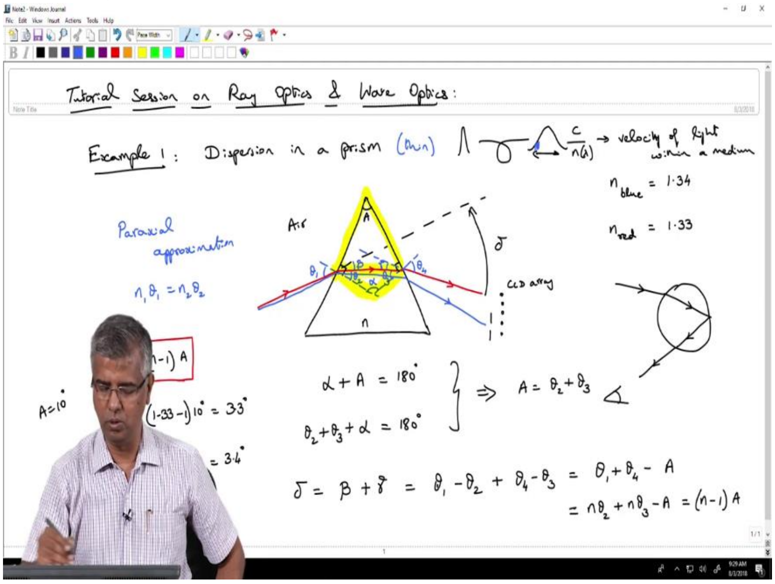

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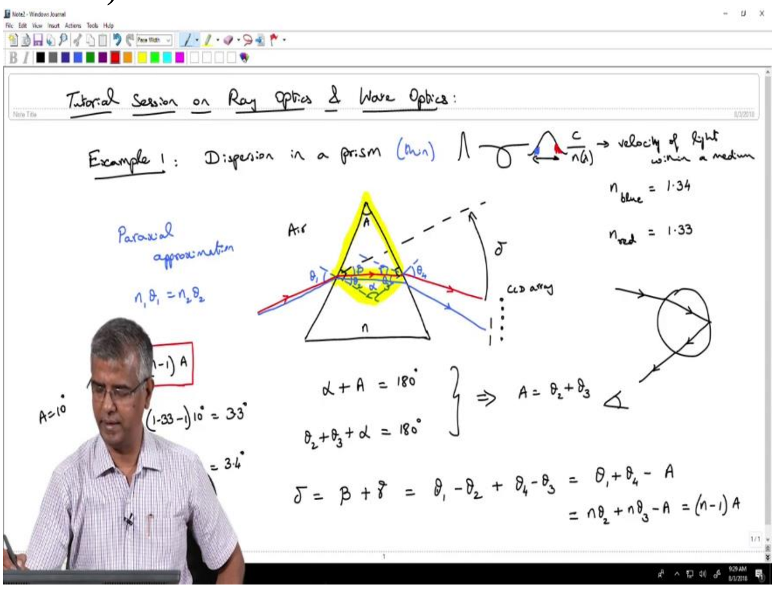

Now can you do something about this pulse spreading? Can you recompress it back to the original pulse? Yes, because all these colors are nicely sequenced, so if you find an element that can delay the red with respect to the blue, then you can get the pulse back. This is a process called dispersion compensation. So, you could possibly use a prism-based technique to do dispersion compensation. We are talking about some fairly advanced concepts, but if you know the fundamentals you can actually dare to dream of taking care of those things.

And it is amazing that, however far you go, however more advanced you go in photonics you always have to come back to the fundamentals, and you start thinking about some fundamental concept and think, 'hey that is a cool way of taking care of this problem' you are finding in this advance system. So, you will find that as you mature in photonics - as you take more and more advance courses in photonics, you will find that you always come back and revisit some of these fundamentals and there is value in doing that.

Let us take another example and curiously enough, this example is also going to be based on the separation of color of light. Let's go back to the last lecture for a bit.

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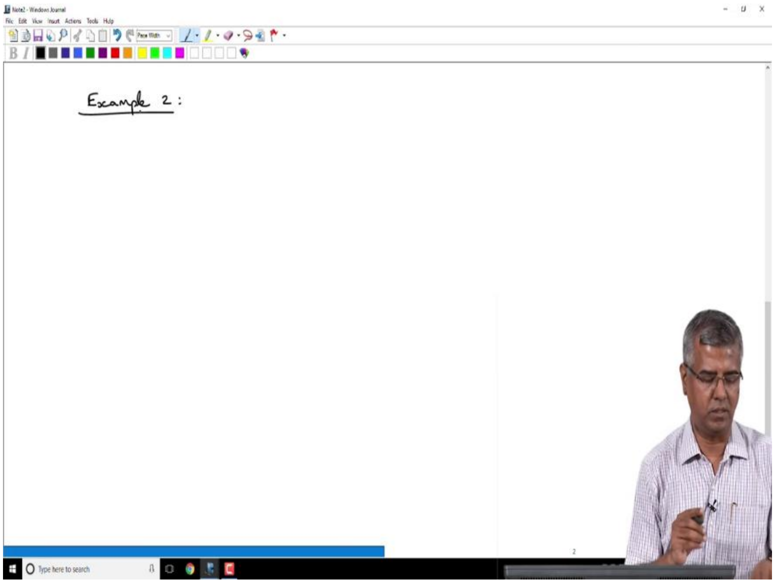

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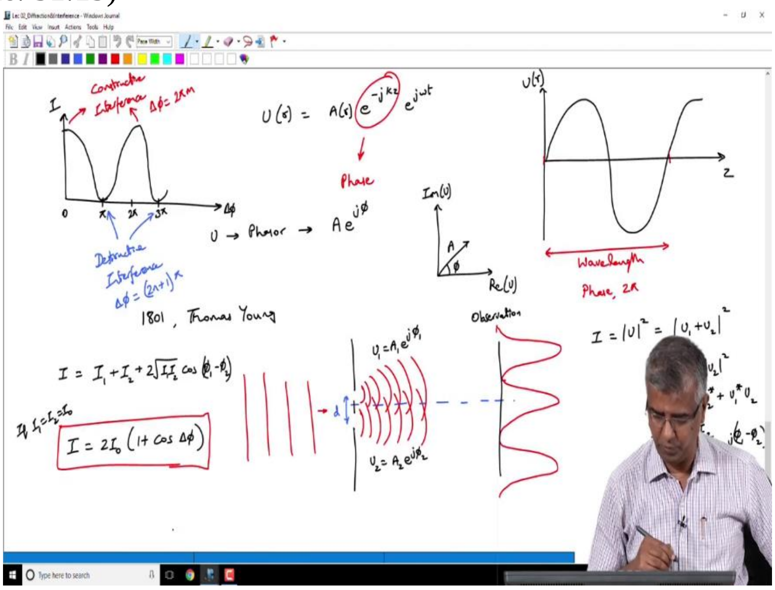
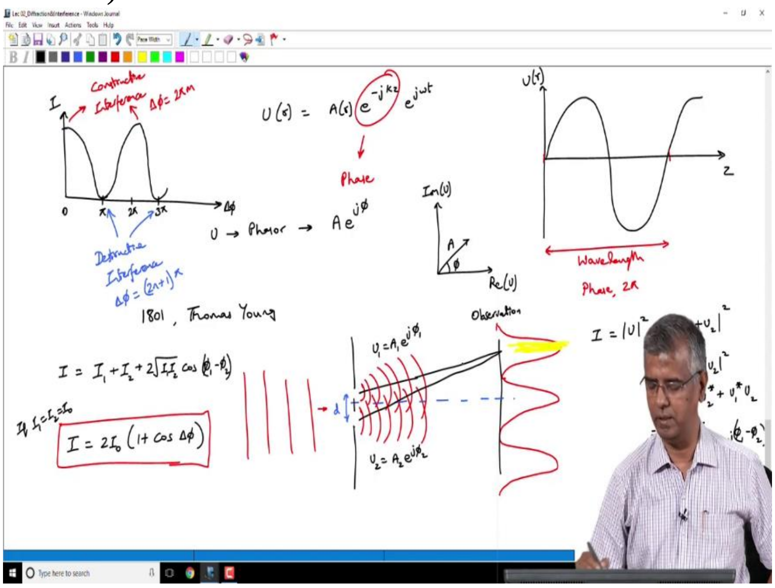


We had investigated (see image above) Young's double slit experiment and realized that we have constructive and destructive interference happening, due to this double slit with each of those slits having size comparable to the wavelength.

If you see this interference pattern closely, looking at the intensity of light as a function of position, we can see a maxima here (see the yellow patch in the image below) because of constructive interference from the rays (shown in black) leading to that point. As you remember, the constructive interference happens when the phase difference caused by the Path Length Difference (PLD) is an integral multiple of 2π. (ie $\frac{2\pi}{\lambda}.PLD = m.2\pi$). We will see it in a bit more detail now.

(Refer Slide Time: 32:41)



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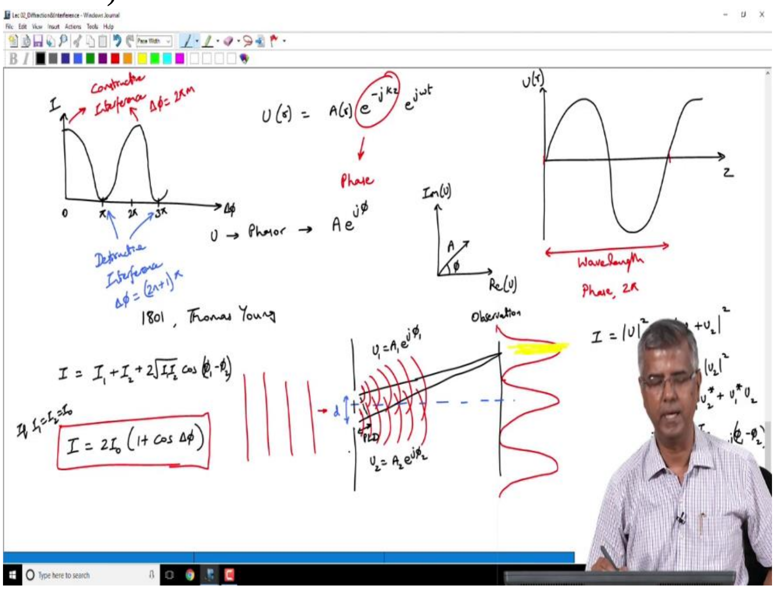

Let’s see the interference in a bit more of detail. Make constructions from the edges of the slit to the yellow marked region and names them l_1 a l_2. Let the phase accumulated along l1 and l2 be, $\phi_1 and\ \phi_2$. The phase difference at the end (yellow region) would be given by,

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda}.n.l_1 - \frac{2\pi}{\lambda}.n.l_2$$
$$= \frac{2\pi}{\lambda}.n.(l_1 - l_2)$$

Note that, PLD is l_1-l_2. We can see that PLD is wavelength dependent.

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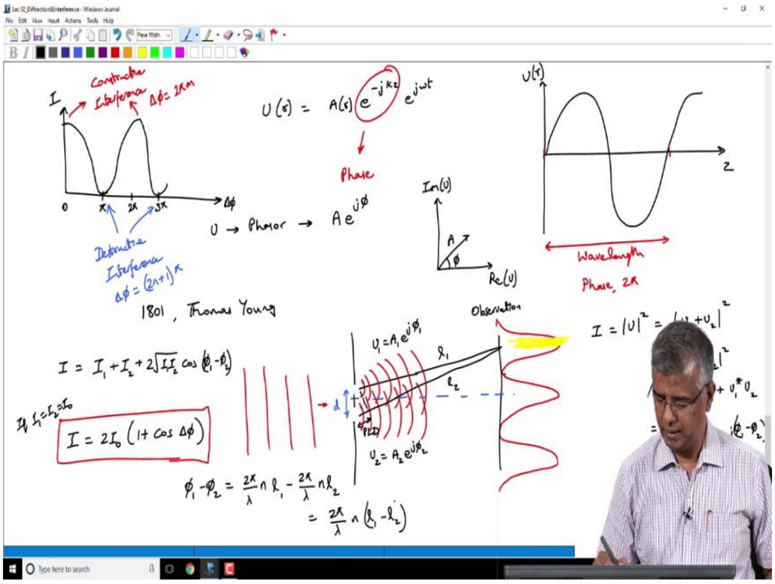

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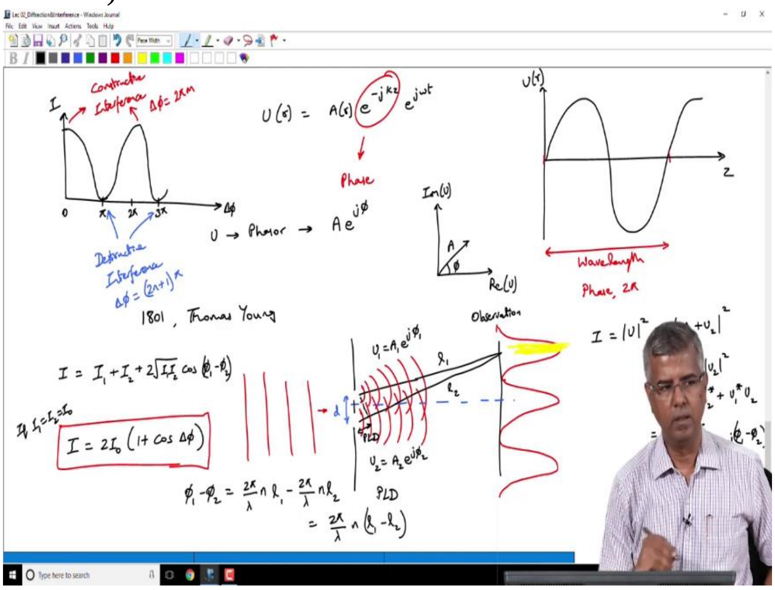

So, the path length difference (PLD) required to get that constructive interference is actually a function of the wavelength. So, the question now is, can you discriminate different colors using Young's double slit apparatus?

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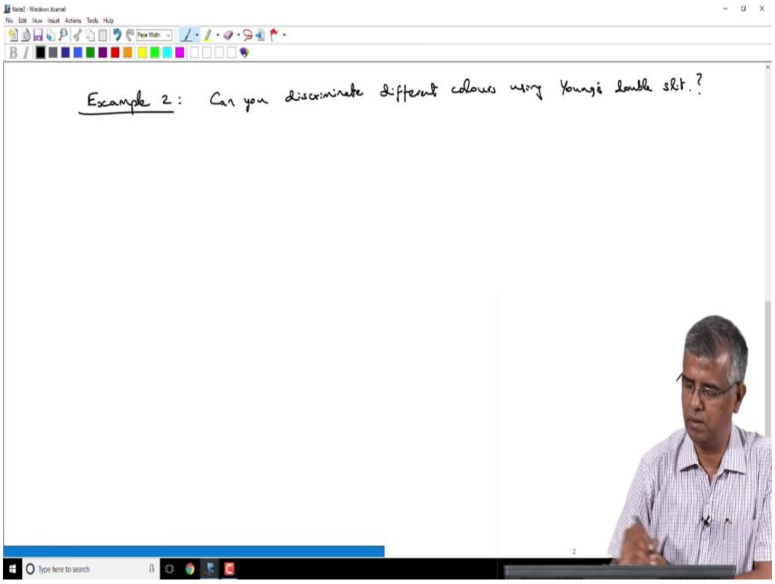

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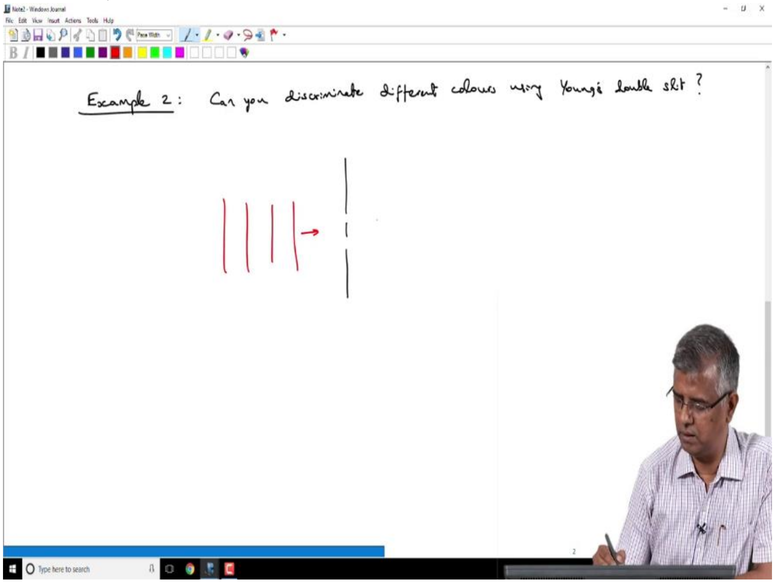

Let's redraw the apparatus here (see fig above and below). Light waves are incident on the slit, setting up secondary waves from the slits, these interfere at a distance point constructively and destructively leading to maxima and minima respectively in the observational plane.

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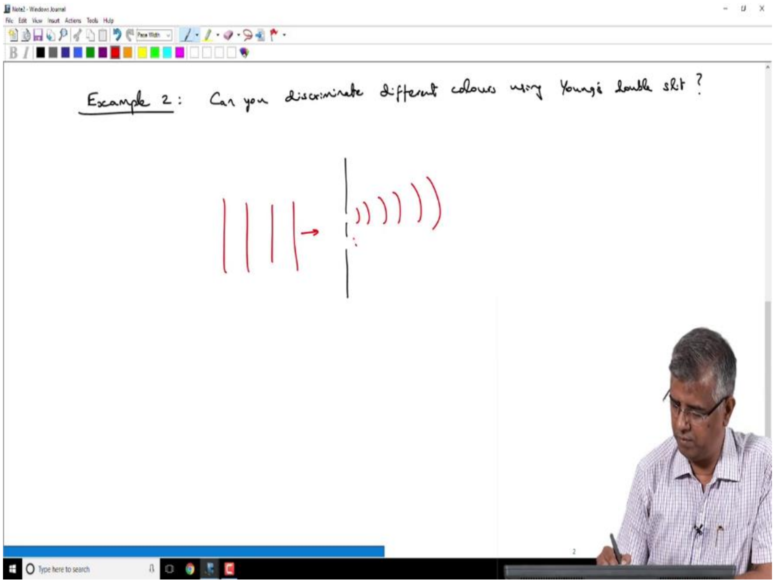

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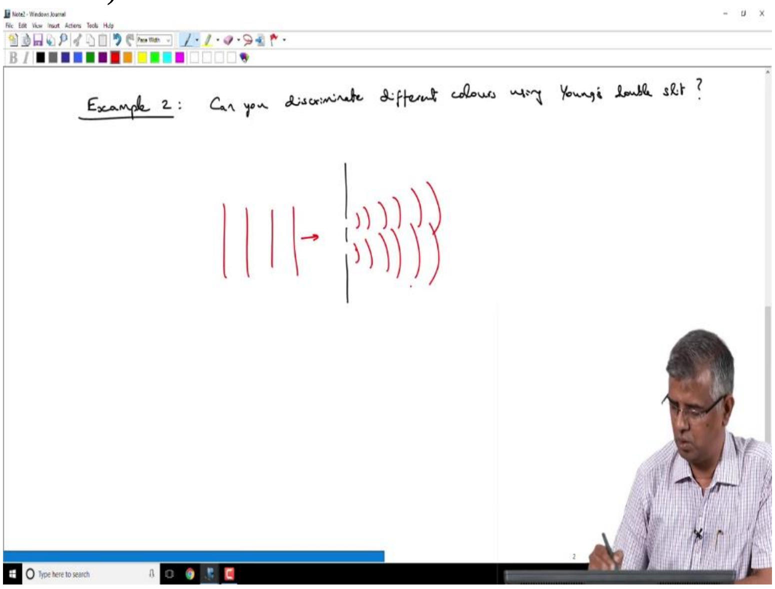

What we are interested in is finding where does this first maxima (x_1) happen? (see the fig below)

Assume that the distance from the diffraction element to screen, D is given as 1.5m and the separation between both the slits, d is given by 0.8mm. We want to find out whether we can separate out different colors using this setup. In other words, we want to see if we can separate out red color from orange color. $\lambda_{red} = 650nm$ and $\lambda_{\text{orange}} = 600nm$.(600 nm is in fact yellow color). Let us see, if we can separate out these two colors.

Let us say, let us see if we can separate out these, these two colors.

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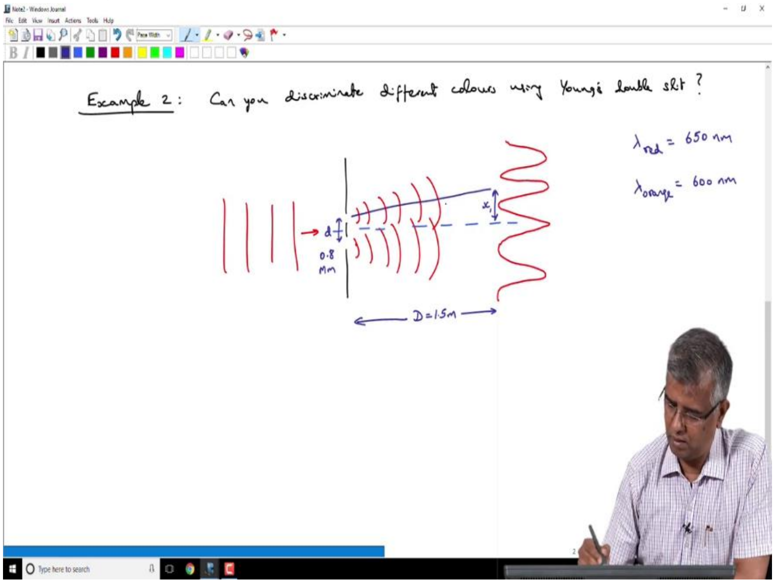

So, we need to work out that path length difference that I mentioned before.

For that let's drop a perpendicular from the top line (l_1 previously) to the bottom one (l_2)(see the dotted lines in the image below). Then this path length difference (PLD, see fig), can be expressed in terms of d.

To do that what we need to find the approximate angle both beams makes with the optical axis. For this we construct another dotted line from the center of the slit to the point in screen. The angle which this line makes with the optical axis (dotted horizontal line), θ, is approximately same as that made by the both the line above and below it.

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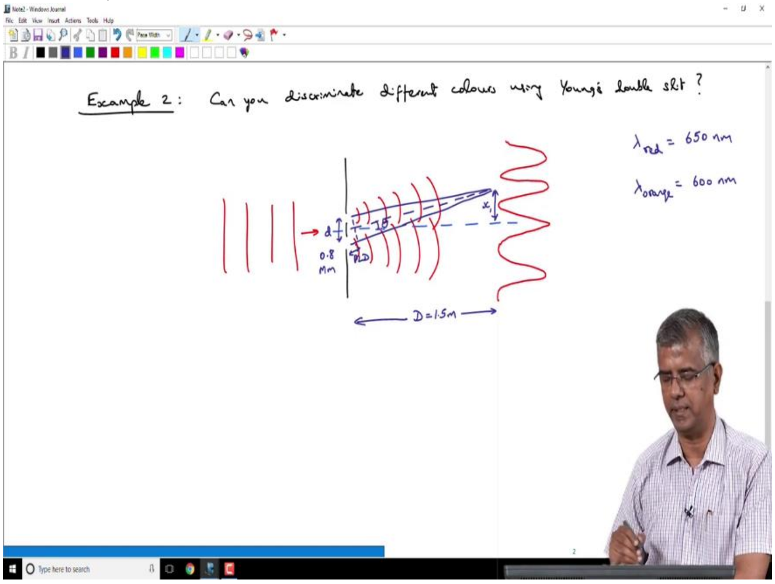

From geometry, the vertical angle between the slit line and the dropped perpendicular (dotted one) will also be θ (drawn again, separately towards the bottom left for clarity). So by using trigonometry, we have

$$sin\theta = \frac{PLD}{d}$$

$$PLD = d\sin\theta$$

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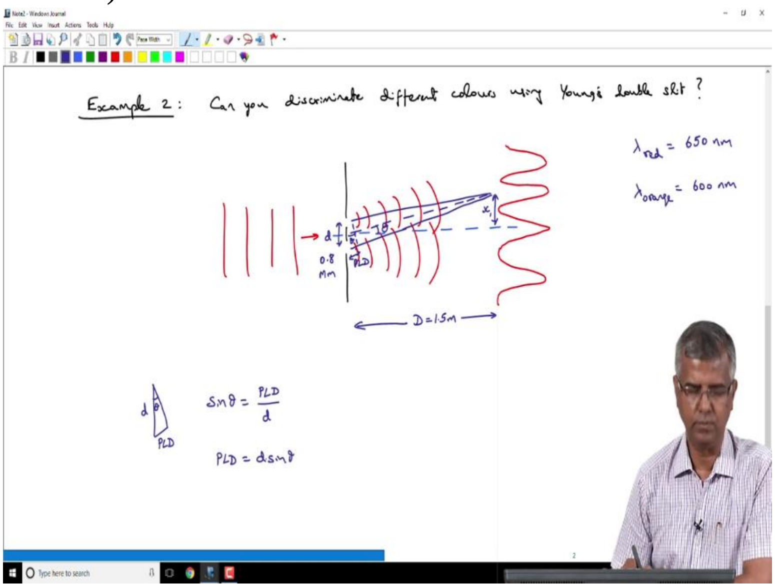

Now we can apply this to that constructive interference criteria which says,

$$\phi_1 - \phi_2 = 2\pi m$$

$$\frac{2\pi}{\lambda}.n.PLD = 2\pi.m$$

$$\frac{2\pi}{\lambda}.n.dsin\theta_m = 2\pi.m$$

The 2π gets cancelled from both the sides.

$$\frac{1}{\lambda}.n.dsin\theta_m = m$$

Here is the subscript m stands for different orders. But here we are interested in the first maxima, so m=1. Also, we are doing all this in air, so refractive Index, n=1.

Then θ_1 can be found from, the above equation (m=1),

$$\theta_1 = sin^{-1}\frac{\lambda}{d}$$

This is the condition for maxima, i.e. constructive interference. $2\pi.m$ is the condition for constructive interference.

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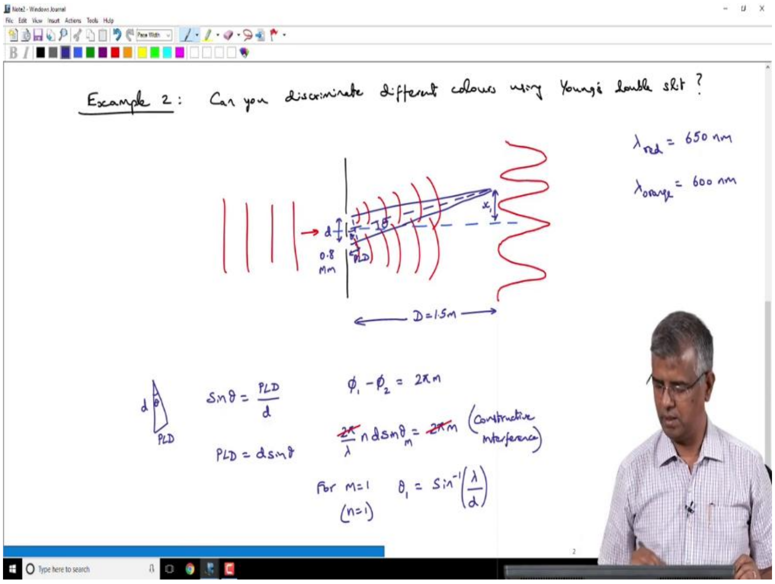

Now, we can substitute the values. Let do for red first.

$$\theta_1 = sin^{-1}\frac{\lambda_r}{d} = \theta_1 = sin^{-1}\frac{0.6x10^{-6}}{0.8x10^{-3}} = 0.8\, mrad$$

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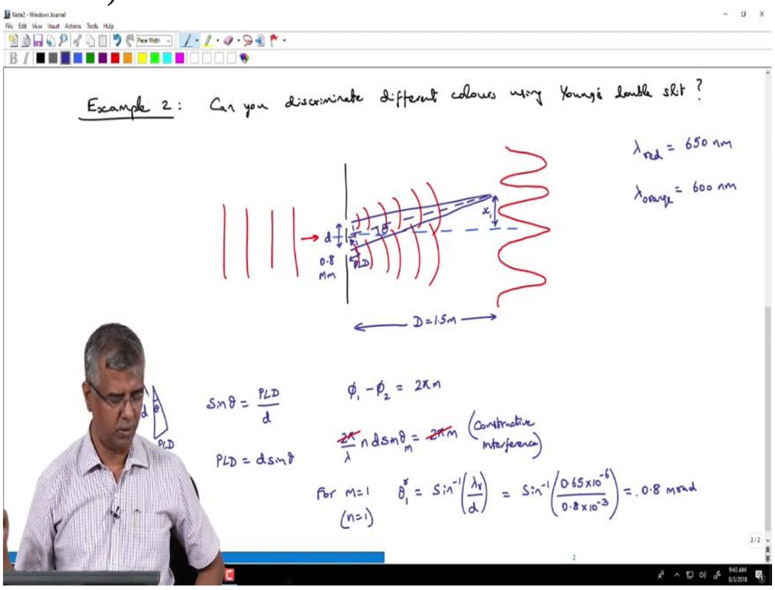

But now from this I want to find out x_1.

$$x_1^r = D.tan(\theta_r)$$

This is because $\tan\theta_r = x^1/d$. As D = 1.5m. Substituting both values we get,

$$x_1^r = 1.218\, mm$$

So the first maxima for the red color appears at a distance of 1.218 mm from the optical axis.

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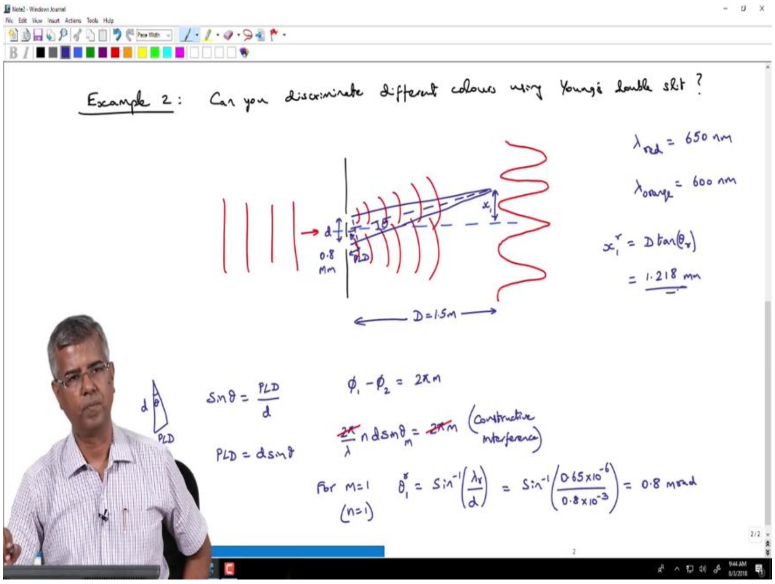

Now if I do the same thing for orange color, I get to find out θ_1 for orange and θ_1 for orange is,

$$\theta_1^{orange} = sin^{-1}(\frac{\lambda_{\text{orange}}}{d})$$

So, if I substitute orange color wavelength as 600 and d will work out to θ_1^{orange}= 0.75 mrad. If I substitute that in the above equation (for x_1), then I get the position of the first maxima for orange color for a wavelength of 600 micron is given by 1.125 mm.

$$x_1^o = 1.125\ mm$$

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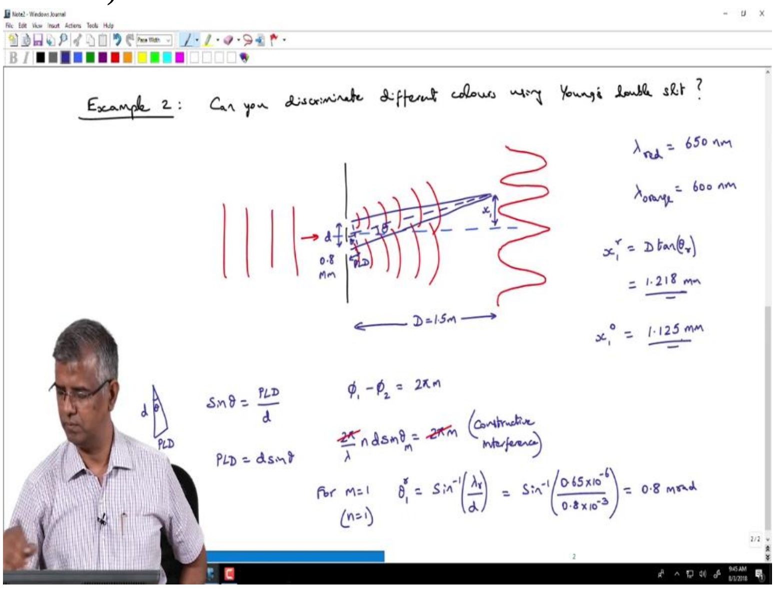

So, the difference between those two, ie. x_1 the position at which the red maxima happen and the position at which the orange maxima happens, is now 93 micrometers. $x_1^r - x_1^o = 93\ mm$

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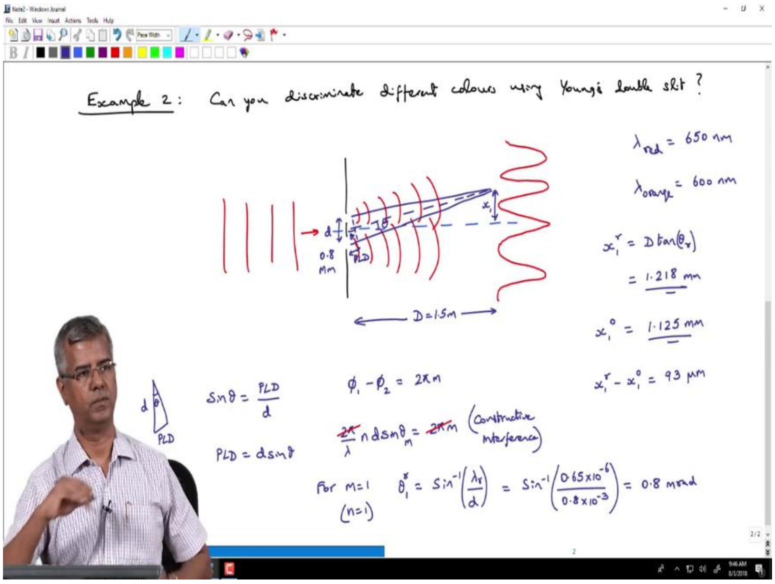

Ok. So, you see the maximum for the red happening at a slightly different location from the maximum of the orange. So, if you want to discriminate between these two, i.e. if you want to pick red and not orange what would you do?

Just put a pin hole, just put a slit over there. If I put a slit over there, then I can see the red color from the orange color. So, what should be the width of the slit?

That depends on the discrimination you want to get. Let's you are Ok with half of the orange coming into the red and so on. So, the slit width to discriminate the two is basically,

$$slitwidth = \frac{x_1^r - x_1^o}{2} = 46.5\mu m$$

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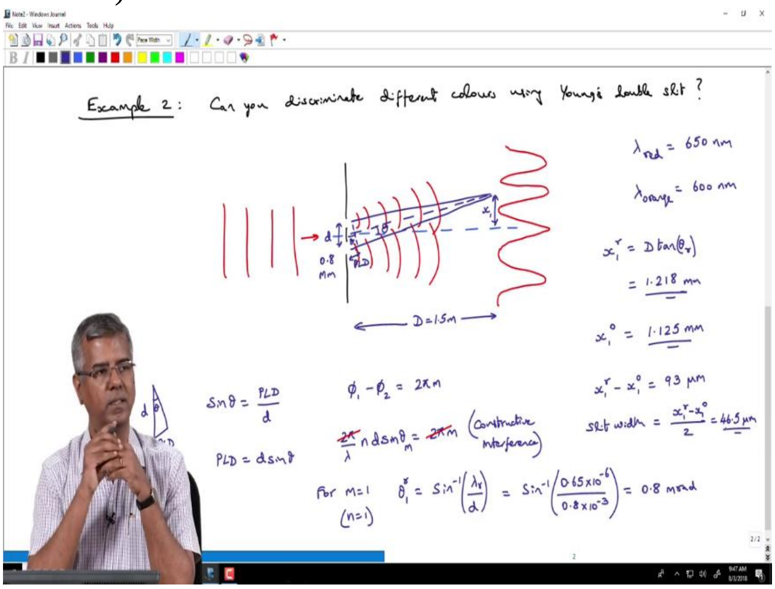

So, you get predominantly red or predominantly orange depending upon the slit position. And of course, like I said in the previous case, today if you want to do that, you put a CCD array. And in the CCD array typically the pixel spacing is less than 10 μm. So, for example if a particular pixel is picking up red predominantly and you move almost 10 pixels away, you are picking up orange.

But essentially what we are seeing here is that whenever we talk about interference of light, that interference is wavelength based. So, you can use that interferometer as, what? Spectrometer. A spectrometer gives you what is the intensity at different colors. As this interference phenomenon depends on wavelength you can use that as a nice way of picking out different colors.

Remember we saw some other thing in ray optics where with a simple prism you can get all these colors separated out. You can pick out these colors. But that is based on, what is called material dispersion because the material responds in a different way for different colors, but here there is no material involved. It is just a structure that you have, that provides interference and that interference is actually dependent on the color and that actually gives you an opportunity to discriminate between different colors.

So that’s just a quick example of some of the concepts that we learnt so far, there are more tutorial problems. So, I would encourage you to go back and try to solve all the other problems as well. Ok so with that let me stop this session. Thank you.