

Introduction to Photonics
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Lasers Part 3

Okay, welcome to another session of Introduction to Photonics, so we had been talking about fundamental principles associated with a laser and we are trying to quantify its characteristics both in terms of the output power how it you know varies or how it increases as you increase the pump power but we also would also like to quantify the spectral characteristics ok.

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The whiteboard content includes the following elements:

- Objective:** L_0 : Identify the fundamental principles of laser & quantify their characteristics
- Diagram:** A schematic of a laser cavity. A central box labeled "Gain" is flanked by two mirrors: R_1 (High Reflector) on the left and R_2 (Output Coupler) on the right. The cavity length is L . An arrow labeled "Pump" points into the gain medium. An arrow labeled "Laser output" points out from R_2 . A feedback arrow points from R_2 back to R_1 .
- Equation 1 (Laser Oscillation Condition):**
$$E_0 e^{+j\frac{\omega}{2} \cdot 2L} e^{-\alpha_{int} \cdot 2L} \sqrt{R_1 R_2} \cdot e^{-j k_0 n \cdot 2L} = E_0$$
- Equation 2 (Round-trip Phase Condition):**
$$e^{+j\omega t} e^{-\alpha_{int} L} \sqrt{R_1 R_2} = 1 \quad \text{and} \quad k_0 \cdot n \cdot 2L = 2\pi m$$

So that is what we were at so we were actually starting from the basic conditions that you need to satisfy so that you have laser built and that essentially meant that on one side you are offsetting the loss in the cavity with the gain and on the other side you are also saying that only specific frequencies can exist within the cavity specific frequencies that corresponds to the constructive interference criteria of this fabry perot cavity.

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$$\textcircled{1} \Rightarrow \gamma = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \alpha_r$$

$$\textcircled{2} \Rightarrow \frac{2\pi}{\lambda} \cdot n \cdot 2L = 2\pi m$$

$$\nu = m \cdot \frac{c}{2nL}$$

For laser oscillation, $\gamma > \alpha_r$
 Assume $\sigma_1 = \sigma_2 = \sigma$, $\sigma N > \alpha_r$
 Threshold inversion, $N_{th} = \frac{\alpha_r}{\sigma}$

The graph shows intensity I versus frequency ν . A central peak is labeled $\frac{c}{2nL}$. Vertical lines represent longitudinal modes. The width of the peak is labeled FWHM. An arrow indicates that increasing the pumping rate leads to a narrower FWHM.

Those are the ones that will survive so we talked about case where you will have all this longitudinal modes corresponding to the case where the gain is greater than the loss those are the ones that will survive in the cavity.

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Output Power $P_{out} = h\nu_s \phi_{out}$

dominated by spontaneous emission \rightarrow P_{th}

dominated by stimulated emission \leftarrow

Output @ R_2
 $\phi_{out} \text{ at } \beta_s$
 $= \frac{\alpha_{m2}}{\alpha_r} \phi_s$

$$\frac{\gamma_0(\nu)}{1 + \beta_s(\nu)/\beta_{sat}} = \alpha_r \text{ at threshold}$$

And specifically we went on to look at the issue which includes saturation of the gain so we said as we build up these photons inside the cavity through stimulated emission you start reducing the gain also.

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To the point at when we were looking at what happens when you turn on a laser.

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I think we were talking about this particular picture, what happens when we turn on the laser is. We have initially very few photons at the signal wavelength and so you have a fairly high gain that is what we call as a small signal gain but as you build up more and more of this photons at the signal wavelength the gain starts saturating it may go to a point where we gain is actually saturated below the loss in the cavity and at which point the gain recovers ok and then it will go

through some oscillations before it stops at steady state and the key point that we noted was at steady state gain equals the loss.

So anymore pump photons coming into the cavity is getting converted into signal photons right anymore pump photons that are coming into the cavity beyond the point where you reach threshold beyond the point where the gain equals a loss right all those other photons will escape the cavity and so we have a conversion of the signal photons or the pump photons into signal photons.

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The whiteboard contains the following content:

- Equation: $\frac{\dot{\nu}_o(x)}{1 + \beta_s(x)/\beta_{sat}} = \alpha_r \Rightarrow \phi_s(x) = \begin{cases} \phi_{sat} \left[\frac{\nu_o(x)}{\alpha_r} - 1 \right] & \nu_o(x) > \alpha_r \\ 0 & \nu_o(x) \leq \alpha_r \end{cases}$
- Equation: $N_m \cdot \sigma(x) = \alpha_r$
- Equation: $\phi_s(x) = \begin{cases} \phi_{sat} \left(\frac{N_o}{N_m} - 1 \right) & N_o > N_m \\ 0 & N_o \leq N_m \end{cases}$
- Graph: Photon flux ϕ_s vs Pumping rate N_o . The curve is zero until N_m , then increases linearly. A note says "At threshold, Slope = ?".
- Equation: $\phi_{s,r} P_{out} = h\nu P_{out}$

So in our last discussion we looked at photon flux right so this is the photon flux from the cavity and within the cavity and that photon flux we essentially came over the expression where we said it is phi sat multiplied by n knot over n threshold minus 1 where n knot corresponds to the small signal inversion and n threshold corresponds to the inversion required for achieving the lasing condition. So anymore inversion beyond the threshold inversion is going to result in the generation of this extra signal photons which will essentially escape the cavity.

So that is what we are projecting over here and we stopped at the point of saying you know this conversion happens at a with a particular efficiency and we wanted to quantify that efficiency right so that is where we stopped. So we said this would be look at phi s or you could also looked at it in terms of the output power, the output power as a function of the input pump power and this output power which will be given by h mu multiplied by the output signal flux ok that output

signal flux is actually a fraction of the ϕ_s corresponds to the signal photons that are generated in the cavity ok.

So what determines that fraction? What determines that fraction that escape the cavity? The mirrors basically the mirror reflectivity or the transmission over the mirror. So what we will find is first of all we will get an expression for that for ϕ_{out} and what we will find is that there is an optimum reflectivity ok for extracting this output power from the cavity so we will actually look at what is the optimum reflectivity, what is the expression for the optimum reflectivity ok. So let us continue over here.

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Output power, $P_{out} = \eta_e (P - P_{th})$

↓
slope efficiency

$\eta_e = \frac{\alpha_{m2}}{\alpha_r}$

$\alpha_r = \alpha_{m1} + \alpha_{m2} + \alpha_{int}$
 $= \frac{1}{c \tau_{ph}}$
 $\alpha_{m2} = \frac{1}{2L} \ln\left(\frac{1}{R_2}\right)$

$\eta_e = \frac{c \tau_{ph}}{2L} \ln\left(\frac{1}{R_2}\right)$

$\eta_e = \frac{P_{ph}}{P_{rt}} \left(\frac{T_2}{T_2}\right)$ Optimum?

If $T_2 = 1 - R_2 \ll 1$
 $\ln\left(\frac{1}{R_2}\right) = T_2$

So what we are saying is this output power P_{out} can be written as some efficiency let us call that the slope efficiency multiplied by the power that is available beyond threshold power. So the cavity is going to consume a certain amount of pump power just to reach threshold anymore power beyond that is getting converted to signal photons that are escaping the cavity right so that is the output power that we are looking at. So what we want to see is what is this quantity which we call as the slope efficiency right and that slope efficiency is just the slope of this output power versus input pump power right. Now we can actually say that the slope efficiency η_e so this slope efficiency can be expressed as so if we have a cavity let me just draw the cavity again so we have one mirror here another mirror here so this is let us say reflectivity R_1 this is reflectivity

R2 and of course we have the gain medium here and we are tracking the output power in this direction.

So as far as the slope efficiency is concerned with respect to the power that we are monitoring coming out of reflector with the second reflector we can say that corresponds to the fractional loss given by mirror 2 with respect to the overall loss right. So the overall loss is α_r , where α_r we said corresponds to α_{m1} plus α_{m2} plus the internal losses in the medium right inside the cavity. So you can write the slope efficiency as is corresponding to the fractional loss for mirror 2 and divided by the overall loss. Now overall loss we said is inversely proportional to what? We talked about the average photon lifetime inside the cavity we said it is inversely proportional to the average photon lifetime and we said α_r is given by C , where c is the speed of light in vacuum multiplied by the photon the average photon lifetime.

Ok and then what is α_{m2} ? α_{m2} corresponds to the mirror loss which is given by $1/2L$ (11:17) of $1/R_2$ right, where did we get that $1/2L$? That is just the $2L$ corresponds to the round trip propagation but we are normalizing the mirror loss with respect to $2L$ so that all of this is overall resonator loss can be you know represented as a per meter basis right α_{int} is distributed across the medium whereas the mirror losses are just lumped elements at the ends but when we are looking at α_r it is a per meter quantity so just so that we have everything equivalent we are basically taking a lumped quantity and making it like as if it is distributed across the cavity but (you know) physically you know that the mirror losses are just lumped quantity it is not a distributed (quantity) ok right.

So I can put all of this together and get an expression for the slope efficiency which is going to be C over C multiplied by τ_{photon} , it is a photon lifetime divided by $2L$ (12:59) of $1/R_2$ ok so does this make sense? What are we saying here maybe it will make sense if I represented slightly differently $2L$ over C what do they correspond to? That corresponds to some time right so what is that? That corresponds to a round trip time round trip time for a photon inside the cavity right. So I can write this as τ_{photon} over τ_{Rt} where τ_{Rt} corresponds to the round trip time inside the cavity right and (14:13) of $1/R_2$ if R_2 is my reflectivity let me say that T or may be T_2 , T_2 corresponds to the transmission of this mirror so I can basically say that R_2 plus T_2 has to be equal to 1 for energy conservation and so R_2 can be represented as $1 - T_2$ right and if P_2 which corresponds to $1 - R_2$ is far-far less than 1 if it is very low

transmission then I can basically say that $(1 - T_2)$ of $1/R_2$ is approximately equal to T_2 right.

So $(1 - T_2)$ of $1/R_2$ can be approximated as that number itself right and so I can write this as T_2 ok this I should say is approximately this is valid only when we have a very low transmission right. So what does that tell you I am just writing it so that we can get a little bit of insight into this so we want if we want high slope efficiency if we want high conversion efficiency for the pump to signal photons beyond the threshold right so you lose certain energy to overcome the losses right that is your threshold pump power so forget about that but beyond that threshold if we want to convert the pump photon to signal photon with higher efficiency I need to have a long photon lifetime.

Long photon lifetime will allow me to you know keep multiplying the signal photons within the cavity right but that by itself does not help ok you also need to extract this power from the cavity so you need to have transmission you need to have sufficiently high transmission so that you can extract power from the cavity right. So this are contradicting requirements to have long photon lifetime what do you do? You make a very low loss cavity which means that you need very high reflecting mirrors but it does not help that you have so high reflecting mirrors that all the power remains within the cavity you need to extract some power from the cavity right.

So you need to leak out a little bit but then the question is how much do you leak out? Is there an optimum value for T_2 right where you leak out without comprising the buildup of power within the cavity right. So you need enough signal photons to keep the stimulated emission going you cannot compromise that if you go for very high extraction very high value of transmission for this mirror 2 then you are very high loss if you have very high loss not only is your threshold going to be higher but the buildup is also not very good right so do you understand the contradiction? On one hand you can actually get to very low thresholds right or in this case you can also say you can give very high slope efficiency by making highly reflecting mirrors for both R_1 and R_2 basically high value.

But in that case we are not extracting much power from the cavity and but on the other hand if you try to extract a lot of power from the cavity and by making one of the mirrors low reflecting which corresponds to high transmission right in that case you may not have enough photons to

sustain the laser or solution so sustain stimulated emission. So we need to there is an optimum value for T2 which you need to figure out right so let us try to figure that out, any questions after this point?

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Optimization of Output Coupling.

Assume $R_1 = R_2$. $\phi_{out} = \frac{\phi_s}{2} \cdot T_2$

$\phi_{out} = \frac{\phi_s T_2}{2} \left(\frac{r_0}{\alpha_r} - 1 \right)$

$\phi_{out} = \frac{\phi_s T_2}{2} \left[\frac{r_0 \cdot 2L}{L_{ex} - 2L(1-T_2)} - 1 \right]$

$\frac{d\phi_{out}}{dT_2} = 0$ when $T_2 \ll 1$

$2L(1-T_2) = -T_2$

$L_{ex} = 2L(\alpha_r + \alpha_m)$

$T_{op} = \left(\frac{g_0 L}{L_{ex}} \right)^{1/2} - L_{ex}$

$g_0 = r_0 \cdot 2L$
Gain factor

Graph parameters: $g_0 = 0.5$, $L_{ex} = 0.02$, $T_{op} = 8\%$

So let us say optimization of output coupling right so let us assume R1 equal to R2 ofcourse you can change that assumption also I will just come back to that in a minute. But if you assume R1 equal to R2 then what you are getting the photon flux that you are getting out is corresponding to the flux that you have built in the cavity divide by 2 multiplied by T let us say T2 (in this case) right if we are extracting if we are looking at a light out of mirror 2 I said divided by 2 because there is a similar extraction happening from mirror 1 also right. Infact I do not want that, what is the point of having a laser like this and the light going this way, light going this way, right.

You are going to utilize the laser to do something in one particular directions so you do not want like going from here. So what do you do you make R1 as close to one is possible ok so that everything escapes out of R2 and if you have that condition then actor of 2 will go away because you are extracting everything out of one end ok, you understand that. So I am just being consistent with what is giving in (())(22:13) textbook. Now phi S I have an expression for phi S is it here? Will be here so phi s I have expressed in terms of phi sat and then the small signal gain and the total losses ok so I can use that expression over here so I can write this as phi sat

multiplied by T_2 divided by 2γ which is the small signal gain divided by αR minus 1 right.

So I can just substitute instead of ϕ_s I can substitute you know ϕ_{sat} and this other part. Now what do we want to optimize? We want to optimize T_2 but T_2 if you look closely is not just sitting in the numerator it is sitting in the denominator also, where is it sitting in the denominator? As part of αR right so αR which I say is $\alpha_{int} + \alpha_{m1} + \alpha_{m2}$ right I can I want to isolate α_{m2} right because that is what I want to optimize. So I can just basically say this is $\alpha_{int} + \alpha_{m1}$ is α_L and α_{m2} is $1/2L$ (24:16) of $1/2R$ right and $1/2R$ I can write as you know R_2 I can write as $1 - T_2$ so I can just write this as this equals α_L minus $1/2L$ (24:42) of $1 - T_2$.

Why do I get a minus sign over here? Because it is $1/1 - T_2$ I will just you know take a $1/1 - T_2$ in which case there is a minus sign comes out ok and this I can further write I can take $1/2L$ outside ok as a common factor and then what I have is some total loss let me call that L_x right that is extra loss minus (25:30) of $1 - T_2$ where L_x corresponds to $2L$ multiplied by α_L , α_L corresponds to $\alpha_{int} + \alpha_{m1}$ ok, why I am doing all of that till the obvious in a minute so just bear with me so I am just going to substitute that expression for αR over here so I will have $\phi_{sat} T_2/2$ and here I am going to have γ multiplied by $2L$ divided by this loss factor L_x minus (26:40) of $1 - T_2$ minus 1 ok.

So here lies the contradiction so you can say that when T_2 is very small right then (27:11) of $1 - T_2$ can be written as you know plus T_2 so that will be L_x plus T_2 sort of thing in the denominator and T_2 is very small you know that is the assumption that we started with T_2 is very small than that factor essentially is very small with respect to the extra loss factor so T_2 has very little effect in the denominator for small values of T_2 it is dominated by what is in the numerator ok but for larger values of T_2 you start having the denominator coming to the picture so then it will essentially you know start playing a role in terms of the power extraction.

So if I were to plot this function in general right let me just say the output flux normalize it with respect to the saturation flux because saturation flux we know is a constant for a given gain medium that is a, what is this saturation flux depend on? That is the excited state lifetime as well

as the emission cross section ok. So that will be a constant for a given medium so I can just take that out of the equation that is not a variable that I am interested in, what I am interested in, in the X axis is T_2 ok so if I plot this I will get some function like this right at small values of T_2 the whatever is in the denominator is negligible right T_2 does not have that part you can neglect and so it is proportional to T_2 right the output flux is proportional to T_2 and then it is you know then as you increase T_2 you will have increase in the output flux and also the output power.

But as you increase T_2 further you start having the denominator coming to the picture so then that will end up reducing ϕ_{out} ok so we are looking at what is this optimum value, what can you say about this point, that is a point of inflection what happens at the point of inflection? The slope is zero so the derivative of ϕ_{out} with respect to T_2 that is zero at that point right so what I can do is you know apply that I can take a derivative of this entire expression corresponding to ϕ_{out} with respect to T_2 and also under specific conditions so if I am looking at $d\phi_{out}/dT_2 = 0$ and when T_2 is far-far less than 1 such that $(1 - T_2) \approx 1$ of 1 minus T_2 is approximately equal to minus T_2 right if I have those two incorporated then I will come up with a final expression which says that the optimum transmission is going to be given by G_{knot} into L_x to the power of $R - L_x$.

L_x corresponds to the extra loss so this is what you get as the optimum transmission for this cavity. This cavity where I should also mention where G_{knot} corresponds to γ_{knot} multiplied by $2L$ right that is the gain factor this is the gain factor for cavity and L_x corresponds to the extra loss factor inside the cavity ok. If you look at specific values you know this values are used by your book if we say G_{knot} is 0.5 and L_x corresponds to 2% which is 0.02 then in that case what we find is this is about 0.1, 0.2, 0.3 and so on that is 10%, 20%, 30% transmission and this is 0.1 (0.2 is way up there) so on right.

So we find that for this values you find that the optimum transmission is roughly about 8% right. So and if you have higher, so what this tells you is if you have higher gain right if you have higher gain inside your cavity while you freeze all other losses then you can afford to push this optimum point to higher values then you can afford to get use higher transmission extract more power from the cavity ok. So that is the physical $(34:35)$ so let me stop at this point.