

Introduction to Photonics
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Lecture 24
Lasers Part 01

Welcome to another session of introduction to photonics, so we will start with a slightly different topic today so we have been discussing optical amplifiers till the last lecture and at even the last lecture I promised that we will get on to looking at fundamental principles of lasers we did not get an opportunity to do that in the last lecture but let us get started with that.

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Lo: Identify the fundamental principles of laser & quantify their characteristics

$$E_0 e^{+\gamma/2 \cdot 2L} e^{-\alpha_{int}/2 \cdot 2L} \sqrt{R_1 R_2} \cdot e^{-j k_0 n \cdot 2L} = E_0$$

$$e^{\gamma L} e^{-\alpha_{int} L} \sqrt{R_1 R_2} = 1 \quad k_0 \cdot n \cdot 2L = 2\pi m$$

Laser oscillation condition

$$\textcircled{1} \Rightarrow \gamma = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \alpha_r$$

$$\textcircled{2} \Rightarrow \frac{2\pi}{\lambda} \cdot n \cdot 2L = 2\pi m$$

$$\nu = m \cdot \frac{c}{2nL}$$

For laser oscillation, $\gamma > \alpha_r$
 Assume $\sigma_e = \sigma_a = \sigma$, $\sigma N > \alpha_r$
 Threshold inversion, $N_{th} = \frac{\alpha_r}{\sigma}$

The graph shows intensity I versus frequency ν . A central peak is labeled $\nu/2nL$. A dashed line indicates the Full Width at Half Maximum (FWHM). Vertical lines represent longitudinal modes. An arrow points to the peak with the text "Increasing pumping rate".

So when we talk about a laser what are the things that come to our mind? What do we associate a laser with? To the properties of the laser that coherence right both in terms of spatial as well as temporal coherence, so that actually leads to high directivity very narrow spectral with emission and so on. So how do we go about building a laser and then what are the characteristics of the laser is what we will look at now?

This part we already discussed briefly previously right, so laser is going to consist of laser is going to consists of a gain medium right, which you know it provides gain as long as you can provide this external excitation which we call as the pump right, the pump is required to take this medium above thermal equilibrium conditions and then we knew we know that it is going to excite the atoms in the gain medium to a higher energy level from which you can achieve stimulated emission and we said we will make a laser by providing feedback, so we put a mirror here and another mirror here right, let us call this mirror with reflectivity R_1 let us call this mirror with reflectivity R_2 .

So that those provide the feedback so when you have these you know excited ions initially providing spontaneous emission and then with feedback from either ends you are actually able to provide able to get stimulated emission and then we get laser output right that laser output could be from just one end or it could be from this end also right, it depends on what is the relative reflectivity of the two mirrors, ok.

Typically what we do in a laser as we provide we want to get directional radiation so we want everything to come out of one end so what we do is we try to make R_1 you know close to a 11 percent as close to 100 percent as possible, so this well little light coming from that side

and R_2 which is normally called as an output coupler is something that is a partially reflecting mirror which means that it will allow part of the light that is within the cavity to escape and then you can get the laser output from this end, right.

Now the question is what are the conditions that you need to satisfy, so that you can get that laser output, ok so one of the conditions that you need to satisfy is if you are looking at the fields corresponding to the radiation inside the cavity let us say we are starting with a field with amplitude E_0 right E_0 that field is going to as it propagates through the gain medium it is going to obviously get certain gain which we call as the gain coefficient as γ .

So since we are dealing with the fields instead of power values we say that gain (coeff) they are actually gain that you get is actually γL , if I was tracking power you know I would basically say P_0 into E power γ times the length right but since I am actually looking at tracking the field I am actually writing it down as γL why am I tracking the field? Because I not only want to match the amplitudes but also the phase I want to also track the phase right to track the phase you need to see what is happening to the fields, ok.

So if we say that the length of the cavity is L then within one round trip right it is traveling a distance of $2L$, I made it look like you know the cavity length here is equal to the length of the gain medium right so that is typically the case, ok. So although I have drawn it at with a certain gap between the gain medium and the mirrors typically you know you can consider that the length of the gain medium is actually the length of the cavity, ok.

So it is going through a again according to E power γ times L but it might also go through certain losses in the cavity okay how do you represent the losses, now of course it could actually when light is like propagating through this medium there could be some scattering centers, there could be some spurious absorption centers, the certain ways that you can lose photons within the cavity, so that loss I would call it as the loss coefficient is α and once again since we are tracking fields I would say αL multiplied by $2L$ corresponds to the all this background losses and then you also have losses at the mirrors, right.

So only R_1 which corresponds to the reflection coefficient for mirror 1 and R_2 corresponds to the reflection coefficient for mirror two only that is sustained in the cavity right and then of course it accumulates phase as it is traveling within the cavity, so that phase is

accumulated as $e^{-\alpha L}$ power minus $j k$ nought with K nought is the wave number multiplied by once again if we say the refractive index of this medium we call it as n , so n times 2 times L is the face that you accumulated within a round trip.

Now laser oscillation is possible, so the condition for laser oscillation is that this equals to E nought essentially what we are saying in writing all of this down is you are able to sustain the fields upon round trips upon consecutive round trips right, such that the gain that you have is enough to offset all the losses within the cavity right and then you are able to sustain the fields within this structure.

So the question is we are tracking the initial field to the round trip field whether R_1 and R_2 are accounting for the losses yeah basically what we are saying is only a certain fraction of that field is going through the round trip right, so we are we are accounting in the round trip we are accounting for the losses because of the finite reflectivity. The question is how is this then equal to that, that is the condition we want to achieve we want to say that is actually sustaining the fields ok, so that is the you can call that as the threshold condition where the loss is met by the gain.

So the question is why should it be E nought well E nought is the original field without amplification but after amplification where the amplification is denoted by what is happening over here right, that is the amplification term after amplification the fields are certainly increasing right it is a higher value but it still has to overcome the losses within the cavity, right.

So we are basically saying this is the minimum condition to achieve laser oscillation, right. So this actually gives rise to two conditions, one where you can say that this the phase condition is going to be satisfied if k nought multiplied by n multiplied by $2L$ is integral multiples of 2π right, so that is basically the constructive interference condition, right. So the phase will be satisfied if k nought multiplied by n multiplied by $2L$ equal to $2\pi m$ and then the remaining parts the amplitudes are going to be satisfied when this exponential term a power γL multiplied by $e^{-\alpha L}$ multiplied by root of R_1, R_2 equal to 1, right so that is the other condition that we get.

So we can rewrite that so basically we can we are saying that the when you are looking at the amplitude you are saying $e^{-\alpha L} e^{-\alpha L}$ multiplied by root of $R_1 R_2$ should be equal to 1 right, so those are the two conditions that we get as a result of you

know making sure that the fields are consistent upon a round trip, ok. So I can take this expression on the left side and basically do a log of that and then I can get a slightly modified expression.

So let me call this as equation 1 and let us call this as equation 2 and equation 1 provides we just rearrange the terms what you get is γ is going to be equal to $\alpha \ln \frac{1}{R_1 R_2}$ ok. So where do I get this factor $2L$ well the L is straightforward right you had γL and αL so we are just taking that and loading it on this reflectivity term but where does the factor of 2 come into the picture the square root, right.

So since you are you have a square root there you have taken that half out right, So this is one important condition that needs to be satisfied essentially we are saying that the gain in the cavity has to be sufficient to overcome the losses in the cavity and that is when you get laser oscillation ok, so that is one important result and the other important result is what we are saying is $2\pi \frac{c}{\lambda} n L$ is equal to $2\pi m$ and λ you can write it as $\frac{c}{\nu}$ where ν is the frequency, so you based on this you get the result that ν has to be equal to $\frac{m c}{2 n L}$, right.

So there are only certain frequencies that can be sustained within that cavity ok, so there are certain frequencies which are given by you know $\frac{m c}{2 n L}$ where m is an integer right, so only those frequencies survive within the cavity. So if you put this all together if you say for example this entire loss you just load it on one term we call it as a resonator losses α_r ok, so we can visualize it like this, right.

So let us actually track the gain as a function of frequency ok, we have a certain for a given cavity we have a certain loss that is given by α_r so that is the level that we have, so both γ and α are in terms of inverse centimeters or you can think of it as inverse meters right when we look at the overall gain what we will find is for a given gain medium you may have at low powers you may have something like this and as you increase the power level or maybe I can just put this in dotted lines over here as you increase your pumping rate ok your gain is going to pick up right and beyond a certain point you are going to have sufficient gain to overcome the losses.

In the cavity right, but that still does not mean that all these frequencies exists in the cavity what the second condition is telling us is there are certain longitudinal modes of the cavity only certain frequencies these frequencies are spaced by $\frac{c}{2 n L}$ ok, only those

frequencies survive inside the cavity. So what this actually say is any of these lasers that you look at ok it is going to consist of multiple longitudinal modes, we are saying longitudinal modes because these is a mode that is consistent with the cavity along the axis this is to be differentiated from other type of modes that we have been looking at when we are looking at optical fibers and other waveguides right those are called transverse modes, those are transverse to the direction of propagation right.

The transverse distribution is what we call as transverse modes but the this distribution which is because of the mirrors at the end is right and in the longitudinal direction these are giving rise to what is called longitudinal modes, so when somebody tells you we have several modes in the laser you are given a laser with several modes you first want to ask the question is it you know longitudinal modes or transverse modes, right.

So the transverse modes of obviously you know you can have as far as the fiber is concerned you can have this LP 01, LP 11, LP 02 modes and all that and this longitudinal modes are essentially comb of frequencies that are supported by the laser, ok. so this is not something that is very obvious when people say laser if they say ok it is occupying this much of the spectrum and they look at it as a continuum, the spectrum as a continuum but if you look at the output of the laser with enough resolution, enough spectral resolution you will actually be able to make out that there are multiple longitudinal modes with equal spacing in the laser spectrum, right.

So is that is that clear? Yes is question yes. So the overall spectrum will be determined by those modes that are meeting the threshold condition so the envelope of that you look at the envelope of that and you look at the full width half maximum of that that is what people look at it as a spectral width that is the total width that is occupied by all the longitudinal modes, right.

So you have what is called full width half maximum, so half from the maximum but the complete width that is called the full width half maximum, so when people refer to the line width of a laser they talk about the full width half maximum of this and within the line width there could be multiple modes. Now of course under some very special conditions you can have only one mode oscillating in the cavity as well ok we will come back and look at that special case later on but if there is only one longitudinal mode operating the laser that will

correspond to a very narrow line width ok and those would be called a single longitudinal mode laser ok but how we get that and all details that we can come back to later on, ok.

So what we are saying in other words is for laser oscillation we need to have the gain coefficient greater than α_r ok, we need to have the gain coefficient greater than the resonator losses, ok. Now if we try to simplify the picture let us assume that α_e equal to α_a equal sorry σ_e equal to σ_a equal to σ which is the emission cross section, if it is equal to the absorption cross section we know that is a very special condition but if we assume that then we can just say that σ multiplied by N where N is what N is the in inversion, right.

So σ multiplied by N has got to be greater than α_r or N ok, there is a there is a threshold N right there is a there is a minimum inversion that you need to have which we can call as the threshold inversion, inversion there is a threshold population inversion which is given by α_r over σ , a σ is the transition cross section ok, just looking at a very simplistic picture.

So your inversion has to be greater than that so that you can achieve laser oscillation and what we also are saying is that as you so this is actually as a function of pumping rate right, so what we are looking at is increasing pumping rate as we have increasing pumping rate you get over this condition where you get over the loss inside the cavity and now let us see if we can express this in terms of a slightly different quantity in rather than looking at α_r which is the resonator losses let us look at it from perspective of the average lifetime of a photon, ok.

So what do I mean by the average lifetime of the photon? It basically the photon bounces around for a certain time before it escapes the cavity ok, so let us look at for example the equation number 1 in little more detail what is R_1 , so to get them to get a very low threshold or to get to the threshold condition with very low pumping rate what do you want to have in your cavity? Certainly you want to have very low losses background losses so you start with a material gain material which does not have very many scattering centers, which does not have very many background absorption and all of that.

So you start with a very pure gain medium but what can you say about the you know reflectivity of the mirrors where will you get very no threshold under what condition and at what reflectivity is you get very low threshold? Reflectivity closer to one right, so that is 100

percent reflectivity, so you if you have 100 percent reflectivity theoretically right then you get the you know the lowest losses right and then then you it is easier to achieve that inversion, ok.

So if you have very high reflectivity is what can you say about the lifetime of a photon assume that you have zero losses that alpha int zero, so the photon is going to keep bouncing around for a very long time so higher the reflectivity when we are talking about Fabry-Perot interferometer we were talking about finesse of the Fabry-Perot cavity, so this is actually a Fabry-Perot cavity right so if you have very high finesse that means the photon spends a large amount of time a relatively long time bouncing around, so the life time is larger.

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Avg. Photon lifetime, $\tau_{ph} \propto \frac{1}{\alpha_r}$
 $\tau_{ph} = \frac{1}{c \cdot \alpha_r} \Rightarrow \alpha_r = \frac{1}{c \cdot \tau_{ph}}$
 $\Rightarrow N_{th} = \frac{1}{c \cdot \tau_{ph} \cdot \sigma}$

Graph 1: Loss coefficient α_r vs. Pump power P_s . The curve starts at a high value and decreases as pump power increases, with a point labeled "Start" and a dashed line indicating a transition.

Graph 2: Output vs. time. The curve shows a sharp rise followed by a steady state with "Relaxation oscillations" indicated by small oscillations on the line.

Graph: Output vs. Pump power. The curve is zero until a threshold pump power P_{th} is reached, after which it increases linearly. The region before P_{th} is labeled "dominated by spontaneous emission" and the region after is "dominated by stimulated emission".

$\frac{\gamma_o(\omega)}{1 + \beta_o(\omega) \rho_{sat}} = \alpha_r$ at threshold

$$① \Rightarrow \gamma = \alpha_{int} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \alpha_r$$

$$② \Rightarrow \frac{2\pi}{\lambda} \cdot n \cdot 2L = 2\pi m$$

$$\gamma = m \cdot \frac{c}{2nL}$$

For laser oscillation, $\gamma > \alpha_r$
 Assume $\sigma_e = \sigma_a = \sigma$, $\sigma N > \alpha_r$
 Threshold inversion, $N_{th} = \frac{\alpha_r}{\sigma}$

The graph shows intensity I versus frequency ν . The peak is at $\nu/2nL$. The FWHM is indicated. A note says "Increasing pumping rate" with an arrow pointing to the right. The x-axis is labeled "Longitudinal modes".

So I can say that if I am expressing in terms of the photon lifetime and I would that this is the average photon lifetime because this could be statistical just like anything that we have discussed so far as far as photons concerned you can only talk in terms of an average value that average photon lifetime if I calling that tau photon tau photon is going to be inversely proportional to alpha r, if the resonator losses are very low then I am going to have a long photon lifetime and the other on the contrary if the resonator losses are very high the photon is going to either escape the cavity very quickly or it the resonator losses could be hide because of lot of background absorption in which case the photon vanishes because of the background absorption, right.

So the average lifetime photon lifetime is going to be inversely proportional to alpha r and if we were to get to a since we are talking about photon that proportionality constant has to be equal to the velocity right, so you can just write it as c multiplied by alpha r where c is the velocity of light right, the velocity of which the photon is traveling ok. Now of course we say c in general as if with is a free space cavity but if we were to be technical about it if it is actually a cavity with refractive index n then c over n would correspond to the actual velocity of the photon, ok.

So tau photon is given by this which obviously you can write it you know alpha r is equal to 1 over c into tau photon and this we can now you know substitute in this expression n th equal to alpha over alpha r over sigma, so this implies that int the threshold inversion is going to be given by 1 over c times tau photon multiplied by sigma. It is just a different way of writing it

but nevertheless this is the inversion with respect to the average lifetime of the photon inside the cavity.

So now I think we have enough material to see what happens when you turn on a laser ok, so can we just try to visualize it and to visualize it what we actually need to look at is the saturation of the gain, so the gain as a function of the signal photon flux right which we know is going to be like this right and in this I would say that let us say this corresponds to the level of losses in the in the cavity, ok.

So I turn on the laser right when I turn on the laser I am pumping some photons which are getting absorbed in the gain medium and that absorption results in the emission of certain photons spontaneously ok and those photons upon feedback are going to start stimulating certain transitions ok and you start you know building up the stimulated emission emitted photons, ok.

So what we are doing in that case is you may start here, so this is your starting point but as you build up the photons inside the cavity you are going to saturate that gain medium right, so the gain is actually going to decrease as you build up the photons inside the cavity and it may decrease to a point even below αr right at which point the stimulated emission you know it is not as much.

So you are you are basically having spontaneously emitted photons and those photons are going to be very few right because of which you go back to the small signal regime right, so you start going past here and then you realize oh no that we have very few photons coming around so the gain actually corresponds to a small signal gain, so then you go up right and you may go past this point also but then as you build up photons you are coming back to this point, ok.

So from the point you start the laser when you turn on the laser this is what happens yes you are immediately getting some spawned I mean the moment you start pumping it you start getting some spontaneously emitted photons those photons are going to bounce around right and they are going to stimulate certain, so they are going to create stimulated emission so the stimulated emitted photons are going to build up but I even as they are building up they are starting to saturate this the gain medium and you may saturate to a fairly high level in which case you have very few photons in the cavity and so you are picking up on the gain right and so you are going towards a small signal gain and then finally.

So finally after all that you started there and you would end up over here ok you would end up at the point where the gain equals the loss in the cavity, so you can understand this a little more a little more easier if we look at it as a function of time and we are looking at let us say the output of the laser the output power of the laser so this is the time at which I decided to flip the switch I turn on the pump right then I am going to have a high output and then as you have the high output you starting to saturate the gain medium, so you are going to come down and then when you come down you have fewer photons so you can actually pick up on the gain so you are going to go up you are going to do this and finally reach a steady state value, ok.

So these sorts of oscillations are what you call as relaxation oscillations right, so you have relaxation oscillations corresponding to this gain saturation that you are having and gain saturation and then gain recovery saturation, recovery that is what is happening and that is what causing those oscillations. So the question is can we plot against output as a function of pump power, so output as a function of pump power is what we are looking at over here as you increase the pump power ok or maybe I can based on this I can draw one more picture.

So let us just say output as a function of pump power right, so what we are saying is that output is going to be fairly low beyond a certain threshold it is going to you know pick up in terms of the output power and so what exactly is happening over here, so this region you are essentially it is dominated by what process? It is very spontaneous emission and as you build towards the threshold right you are actually building up the stimulated emission and what you can say is this region is dominated by stimulated emission.

So the relaxation oscillations that we are showing is suppose I am turning on with this much pump power right and I am turning on at this much pump power and this is actually showing that it is above the threshold right, so it is above the threshold condition as far as your gain is concerned but still from a perspective of photons the photons have to build up, so there is a finite time over which the photons build up and as they build up to high enough value they start saturating the gain right and because of that the photon you know the number of photons are decreasing and then you say ok because the number of photons are decreased it is not as much saturated the gain medium is not as saturated, so the gain recovers ok.

So if you look at the microscopic picture the what is happening from the photon perspective it is you can actually track all these processes really well ok. So that is what we have as far as

the laser emission is concerned ok. So now what we actually need to do is we need to consider this gain saturation so we are basically saying the gain saturation is a very integral part of this this laser emission.

So we need to you know when we were talking about gain has to be greater than α_r that gain has to now account for the saturation as well ok, so now I need to modify this expression a little bit. So how do I modify that basically I would say that the actual gain would correspond to the small signal gain divided by $1 + \frac{\text{signal flux}}{\text{saturation flux}}$ right and the signal flux is actually is a function of frequency as well right.

So this has got to be equal to α_r at threshold ok and I say it has to be equal to α_r at threshold but in the picture that we showed even though you are pumping at a higher rate even though you actually have your small signal gain greater than α_r right through the process of saturation you are going to get eventually clamped at a point where your gain your saturated gain is equal to α_r you are just meeting that condition.

So anymore pump photons beyond that condition is actually converted to signal photons you see what is happening so I am actually having lot of pump photons go into the medium right and it is creating some signal photons there are certain signal photons that are required to meet the losses in the cavity right anymore pump photons that I put beyond that, that is actually going to get you know leaked out as your laser emission, as far as the medium is concerned the laser gain medium is concerned the gain is actually clamped at that α_r value, the resonator loss value.

So in this picture over here what are we saying as we increase the pump power we are getting to the point where we have a threshold right we have a threshold pump power beyond that you are essentially whatever pump photons that you put is actually getting converted to signal photons, so the output of the laser you any increase in this pump photon you know in the pump power corresponds to an increase in the signal power right.

But in that process what we are saying is beyond this it does not mean that the gain is you know increasing the small signal gain is increasing as a function of pump power but the saturated gain gets clamped at this α_r value right. So you just meeting that condition and whatever other pump photons you have is actually escaping the cavity as signal photons, you understand that ok.

So if you have this condition then you can rewrite this expression in terms of ϕ s ok, so let me just grab this the question is what happens to the signal photons that we are giving into the system, for an amplifier you have some weak signal photons coming in you are getting amplification right laser you are not giving any signal photons to start with it is actually self-seeding through the spontaneous emission of the system.

Now the spontaneous emission acid goes through these reflections it has to get in tune with the cavity right only certain frequencies are allowed, so the spontaneous emission may be broad but only certain frequencies are allowed by the cavity and at the photons at those frequencies will actually go through stimulated emission and that is what is giving you that comb structure as far as the spectrum is concerned ok.

So there are no external signal photons that are giving you just providing pump photons to the system I should just say pump energy to the system because we are talking about a case where we are using light energy to get this laser going we were going to go into semiconductor lasers where we are giving electrical energy to the system and through that we are getting light emission ok we will we will look at that later but yeah in general you threw that external energy we are initially getting spontaneous emission and from that you are building up your refining and you are building up stimulated emission at specific frequencies that corresponds to the longitudinal modes other ok.

An interesting thought here ok let us see how well you know about your mathematical you know signals, now if I have a comb of frequencies right if I have a comb of frequencies like this if I take inverse fourier transform and look in the time domain what should I see if I have a comb of frequencies if I do an inverse fourier transform or should I seen in time, pulses you should see pulses right.

So if the laser was working perfectly if you have these comb of frequencies right corresponding to that if you look in the time domain you should actually have a train of pulses you do not see that when you take a laser and normally when you when you operate the laser you look at the output it looks like it is continuous output why? You will get pulses if all of these longitudinal modes are locked in faced if they are all oscillating synchronously you will get pulses but when you look about when you any think about practical lasers they are operating at some ambient conditions you have basically the medium is subjected to

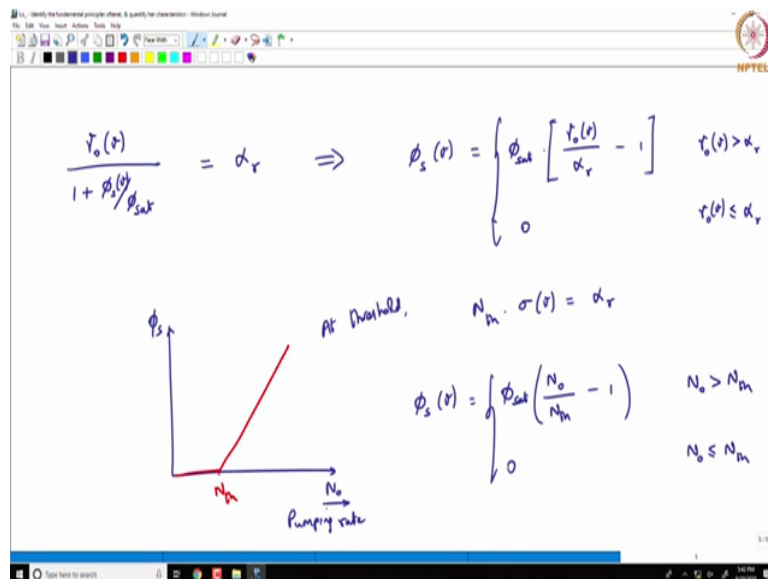
certain temperature you know certain perturbations that you do not have a good phase relationship between the longitudinal modes, ok.

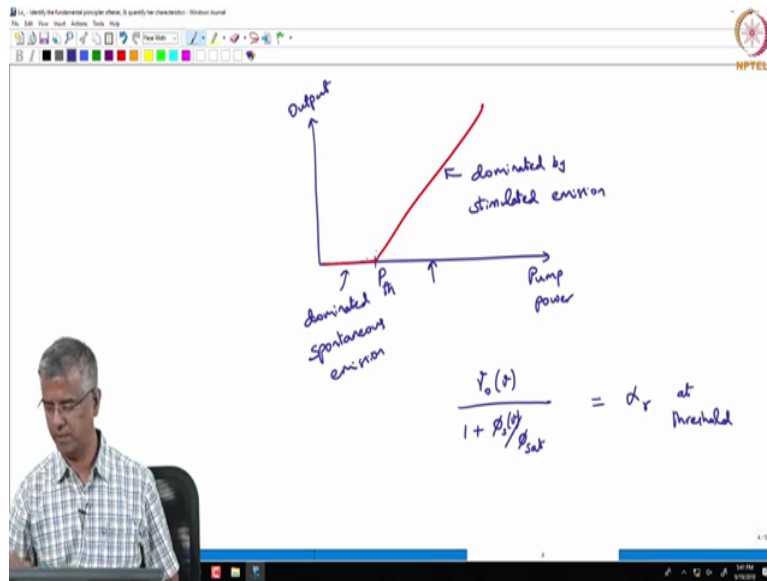
So you basically each of those longitudinal modes is oscillating around the at center and they are oscillating at different rates, so if you do not have you know all of them oscillating synchronously you will actually have continuous emission ok. So that actually gives you another idea it says that if I am able to synchronize all these longitudinal modes then I can get then I can produce short pulses from the laser how short? That depends on how wide my spectrum is, if I have a large gain bandwidth and I am able to sustain oscillation across a large gain bandwidth larger the gain bandwidth shorter the pulse ok and that is a very important concept neat view maybe we will come back and look at it a little more detail later on but we have something called mode lock lasers ok.

All the longitudinal modes are locked when you go through you know certain procedures to make sure that all the longitudinal modes are locked then you can actually produce short pulse lasers how short? People have already done femtosecond pulses there have been demonstration of attosecond, hundreds of attosecond pulses also attosecond is 10^{18} power minus 18 right so people have demonstrated that also ok.

So that is an important concept to understand although I show all this in in spectrum as equally spaced lines and all of that in reality these lines are not face locked because of which you actually have a continuous output as a function of time ok.

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So come back to this if I write this in terms of the signal flux I can rearrange terms and I can say that this is going to be equal to we do not have to say in terms of μ so that is in terms of ϕ_s is going to be given by ϕ_{sat} multiplied by γ_0 of μ divided by α_r minus 1 ok but this is valid only when this small signal gain is greater than α_r without that what do you have you essentially have zero signal photons, if you have less than an equal to the small signal gain coefficient is less than or equal to the resonator loss coefficient ok.

So now I can write this slightly differently since I know that we already seen that at the threshold the threshold inversion multiplied by σ of μ equals to α_r I can just write this as ϕ_s of μ equal to ϕ_{sat} γ_0 can be written as N nought multiplied by σ but in the numerator in the denominator you have n threshold multiplied by σ so the σ can would get cancelled minus 1 for N nought greater than N threshold and 0 otherwise right.

So we are we are basically saying that there is going to be a sudden change in the signal photon flux around that threshold once you reach that threshold condition you would have a sudden change and that is what we are showing over here, so all that action happens around this threshold over here right, so below that threshold you do not have I am saying zero but technically there are still some spontaneously emitted photons amplified spontaneously emitted photons and all of that but there is a sudden change when you go beyond that threshold inversion, ok.

So we are out of time maybe I can just graphically represent this for we break off, so ϕ_s as a function of N nought is showing something like this, so ϕ_s is zero below N threshold and

ϕ s you know start increasing rapidly as you go to higher N nought which is actually representing the pumping rate right. So how do you get higher and higher N nought by going for higher and higher pump power, right.

So that is the picture that we have as far as the laser is concerned around threshold, so let us stop at this point and we will pick up from this tomorrow we will look into specific characteristics of the laser.