Introduction to Photonics
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EDFA - Tutorial

So let us get started good morning, welcome to one more session of introduction to photonics. So what we want to do during this week is identify the fundamental principles of lasers and quantify their performance characteristics so that is what we want to get to an and we do understand that laser is essentially an amplifier with feedback, so we have been spending a lot of time trying to understand amplifiers, understand the process of amplification and we have some unfinished business from that perspective.

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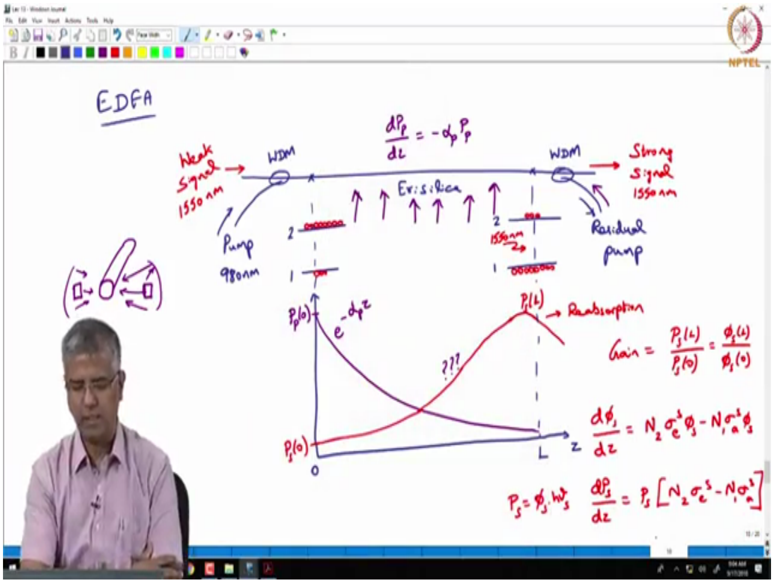

So in the last lecture we were trying to address a problem related to the design of a erbium doped fiber amplifier, so we said erbium doped fiber amplifier construction is typically like this where you excite the erbium doped fiber from one of the ends of the fiber because this lateral excitation is not practical and when we do that we started looking at how the pump power is getting absorbed along the length of the fiber and also how the signal power evolves along the length of the fiber and we needed to quantify how the signal power evolves so in order to do that we were defining the signal power along the length as dP s over dZ which was essentially dependent on this gain coefficient.

Now this gain coefficient itself one thing that we realized is its N 2 multiplied by sigma e as it is mentioned over here its N 2 multiplied by sigma e minus N 1 multiplied by sigma a, previously we had made a rough approximation saying sigma e is equal to sigma a but if you actually write it out the gain coefficient would depend on this factor in the square brackets and what that tells us is that it is not necessary that N 2 is greater than N 1.

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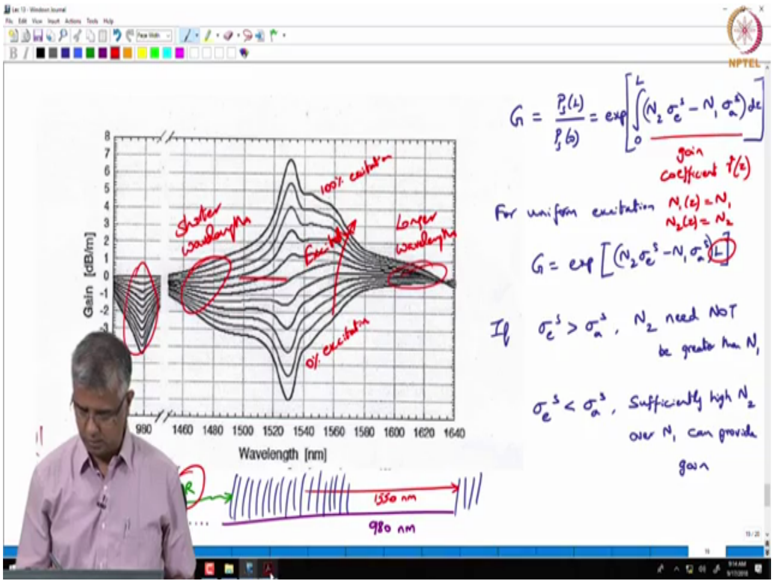


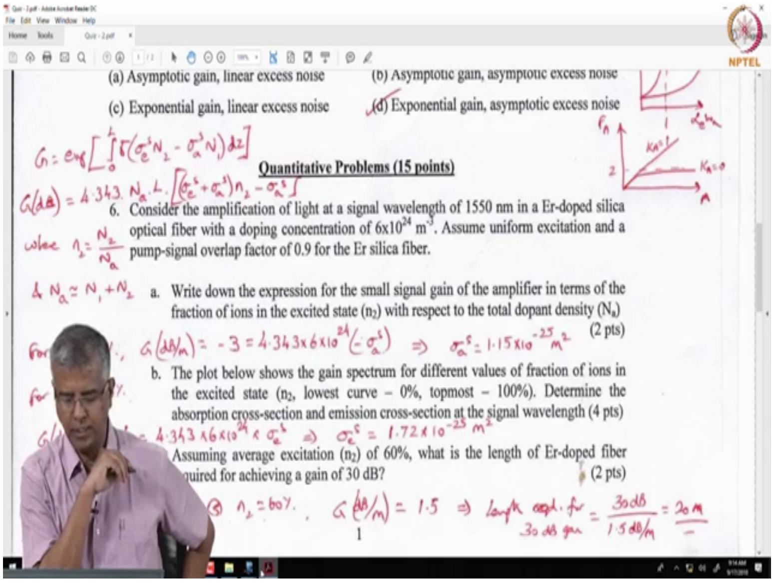
$$G = \frac{P_s(L)}{P_s(0)} = \exp\left[\int_0^L \underbrace{(N_2\sigma_e^s - N_1\sigma_a^s)}_{\text{gain coefficent } \gamma(z)} dz\right]$$

For uniform excitation $N_1(z) = N_1$, $N_2(z) = N_2$

$$G = \exp\left[(N_2\sigma_e^s - N_1\sigma_a^s)\boxed{L}\right]$$

If $\sigma_e^s > \sigma_a^s$, N_2 need NOT be greater than N_1

$\sigma_e^s < \sigma_a^s$, Sufficiently high N_2 over N_1 can provide gain



(a) Asymptotic gain, linear excess noise (b) Asymptotic gain, asymptotic excess noise

(c) Exponential gain, linear excess noise ✓(d) Exponential gain, asymptotic excess noise

$$G = \exp\left[\int_0^L \Gamma(\sigma_e^s N_2 - \sigma_a^s N_1)\,dz\right]$$

$$G(dB) = 4.343 \cdot N_a \cdot L \cdot \left[(\sigma_e^s + \sigma_a^s)n_2 - \sigma_a^s\right]$$

where $n_2 = \frac{N_2}{N_a}$

& $N_a \simeq N_1 + N_2$

Quantitative Problems (15 points)

6. Consider the amplification of light at a signal wavelength of 1550 nm in a Er-doped silica optical fiber with a doping concentration of $6\text{x}10^{24}$ m^{-3}. Assume uniform excitation and a pump-signal overlap factor of 0.9 for the Er silica fiber.

   a. Write down the expression for the small signal gain of the amplifier in terms of the fraction of ions in the excited state (n_2) with respect to the total dopant density (N_a) (2 pts)

   $G(dB/m) = -3 = 4.343 \times 6 \times 10^{24}(-\sigma_a^s) \Rightarrow \sigma_a^s = 1.15 \times 10^{-25}\,m^2$

   b. The plot below shows the gain spectrum for different values of fraction of ions in the excited state (n_2, lowest curve – 0%, topmost – 100%). Determine the absorption cross-section and emission cross-section at the signal wavelength (4 pts)

   $4.343 \times 6 \times 10^{24} \times \sigma_e^s \Rightarrow \sigma_e^s = 1.72 \times 10^{-25}\,m^2$

   Assuming average excitation (n_2) of 60%, what is the length of Er-doped fiber quired for achieving a gain of 30 dB? (2 pts)

   ③ $n_2 = 60\%$, $G(dB/m) = 1.5 \Rightarrow$ length reqd. for 30 dB gain $= \frac{30\,dB}{1.5\,dB/m} = \underline{20\,m}$

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So we looked at specific conditions where if sigma e is greater than sigma a then N 2 may not be greater than N 1 and conversely we said if sigma e is less than sigma a if you have sufficiently high N 2 over N 1 you can achieve gain. So we ended up at the point where we were pointing out to certain spectral regions where in the longer parts of the spectrum the longer wavelengths we find that sigma e is typically greater than sigma a and because of that for a very low levels of excitation you have gain, whereas the other shorter wavelengths over here we have the case where sigma a is greater than sigma e and it takes a high level of excitation so we were plotting this as a you know for different levels of excitation and we said it requires a fairly high level of excitation to get gain, gain is defined by this line so above that you have positive gain which means amplification and below that we have negative gain which essentially means loss.

So it just tells you that you have to look at the relative levels of sigma e and sigma a to and as well as the relative levels of N 2 and N 1 to determine you know what N 2 versus N 1 you need to achieve gain. The question is about the absorption coefficient which is essentially you know the number density multiplied by the transition cross section. So N 1 sigma can be looked upon as the absorption coefficient. So this is you can just say it is basically the emission coefficient versus the absorption coefficient.

But just to you know visualize all of this or help the visualization I have actually drawn all the energy levels so it is such that the ground state is to the right side of this graph and as we go towards the left side you are increasing in energy. So if you are going from (98) if you are pumping from 980 nanometre photons you are going to that higher energy level which corresponds to the left most features and then from there you are nonradiatively relaxing to this other energy level which we call as energy level 2 which is not just the single energy level, I wanted to emphasize that it is actually a bunch of energy levels there.

So you could have transitions over a wide range of wavelengths, so essentially you can get gain over a wide range of wavelengths, okay. So one important thing is what is happening at this point, so it says that irrespective of your pumping level or irrespective of the fraction of ions of the excited state you cannot achieve gain at 980 nanometres, why is that? Well there are two aspects to it one is that we have a very strong nonradiative relaxation component.

So you are not actually achieving emission at 980 nanometres, but even if you end up getting emission it is essentially corresponding to a 2 level system. So there is equal probability of having the erbium ions on the excited state and as well as the ground state so unless you have a significant emission cross section you cannot achieve gain and in this case the emission cross section that is the first statement that we made is very low because of the fact that you know the nonradiative process is actually much more likely process than emission back to the ground level, do you understand this?

Okay, so let us actually go back and see if based on all this discussion whether we are able to solve this problem, this is where we started in the last lecture. So we are asked to consider the amplification of light at signal wavelength of 1550 nanometres you are given the total doping concentration N a, you are asked to assume uniform excitation, okay. So that is actually a key point over here so if you have uniform excitation what can you say? That the inversion is actually uniform across this entire length of the fiber you can make an assumption like that and because of that if we go back and look at this we can say that under uniform excitation you can say that N 1 of Z is just constant, N 2 of Z is a constant.

So there is no length dependence on the inversion and because of that that integral will just yield the value of L. So that is what is uniform excitation would correspond to and you are also told that you can assume a pump signal overlap factor of 0.9 for the erbium silica fiber so what does that correspond to?

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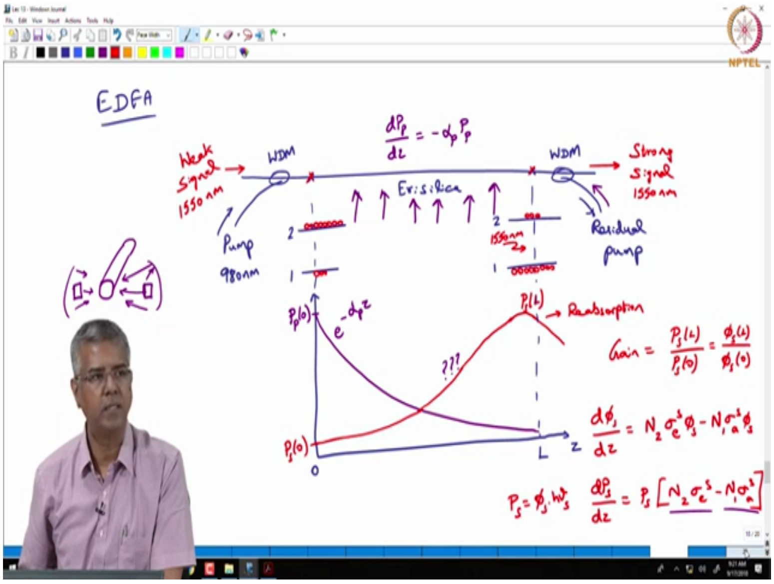
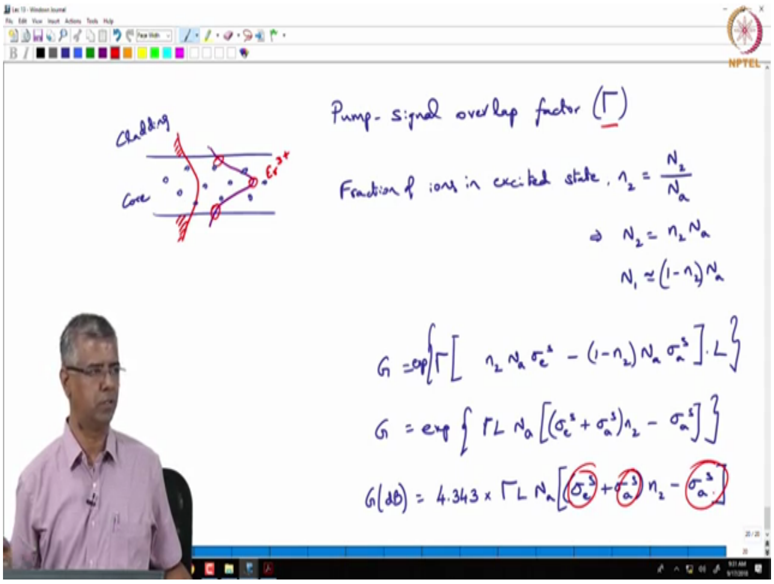


$$\frac{dP_p}{dz} = -\alpha_p P_p$$

$$\text{Gain} = \frac{P_s(L)}{P_s(0)} = \frac{\phi_s(L)}{\phi_s(0)}$$

$$\frac{d\phi_s}{dz} = N_2 \sigma_e^s \phi_s - N_1 \sigma_a^s \phi_s$$

$$P_s = \phi_s \cdot h\nu_s \quad \frac{dP_s}{dz} = P_s \left[\underline{N_2 \sigma_e^s} - \underline{N_1 \sigma_a^s} \right]$$



Pump-signal overlap factor (Γ)

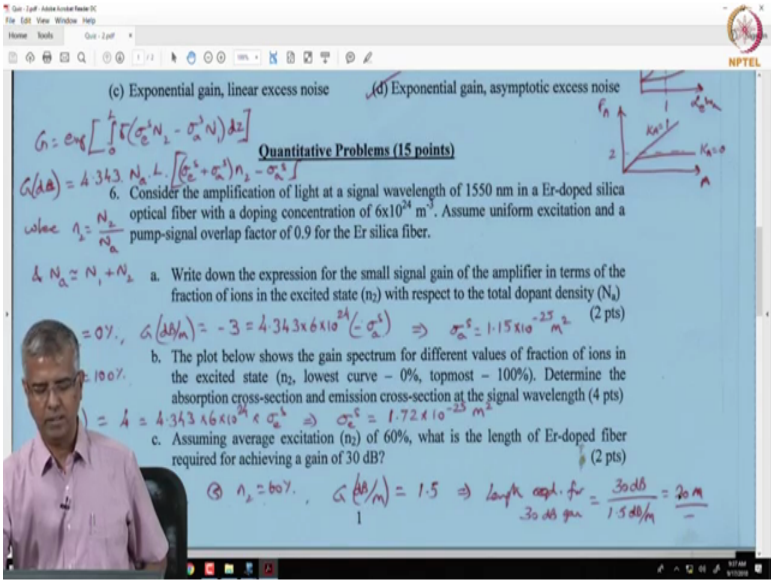
Fraction of ions in excited state, $n_2 = \frac{N_2}{N_a}$

$$\Rightarrow N_2 = n_2 N_a$$
$$N_1 \simeq (1-n_2) N_a$$

$$G = \exp\left\{\Gamma\left[\; n_2 N_a \sigma_e^s - (1-n_2) N_a \sigma_a^s\right]\cdot L\right\}$$

$$G = \exp\left\{\Gamma L N_a\left[(\sigma_e^s + \sigma_a^s) n_2 - \sigma_a^s\right]\right\}$$

$$G(dB) = 4.343 \times \Gamma L N_a\left[(\sigma_e^s + \sigma_a^s) n_2 - \sigma_a^s\right]$$



(c) Exponential gain, linear excess noise ✓(d) Exponential gain, asymptotic excess noise

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   a. Write down the expression for the small signal gain of the amplifier in terms of the fraction of ions in the excited state (n_2) with respect to the total dopant density (N_a) (2 pts)

   $= 0\%.,\; G(dB/m) = -3 = 4.343\times6\times10^{24}(-\sigma_a^s) \Rightarrow \sigma_a^s = 1.15\times10^{-25}\,m^2$

   b. The plot below shows the gain spectrum for different values of fraction of ions in the excited state (n_2, lowest curve – 0%, topmost – 100%). Determine the absorption cross-section and emission cross-section at the signal wavelength (4 pts)

   $= 100\%$

   $) = 4 = 4.343\times6\times10^{24}\times\sigma_e^s \Rightarrow \sigma_e^s = 1.72\times10^{-25}\,m^2$

   c. Assuming average excitation (n_2) of 60%, what is the length of Er-doped fiber required for achieving a gain of 30 dB? (2 pts)

   ③ $n_2 = 60\%$, $G(dB/m) = 1.5 \Rightarrow$ length reqd. for 30 dB gain $= \frac{30\,dB}{1.5\,dB/m} = 20\,m$

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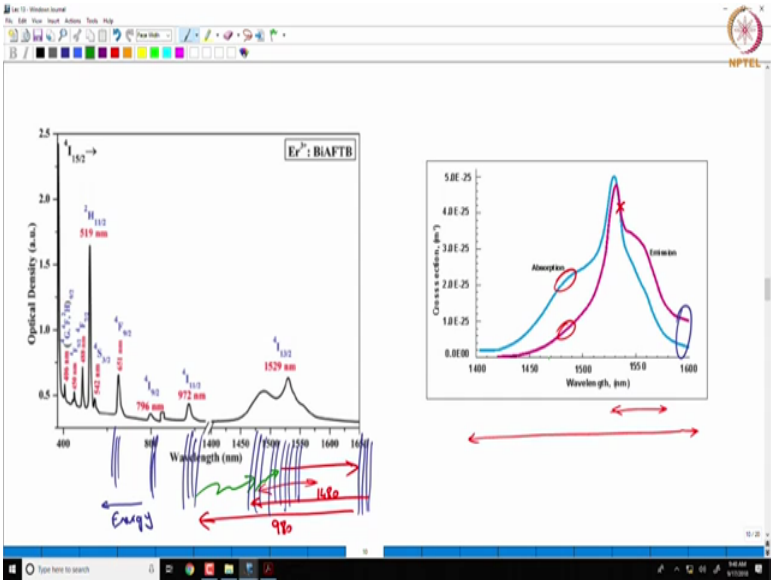

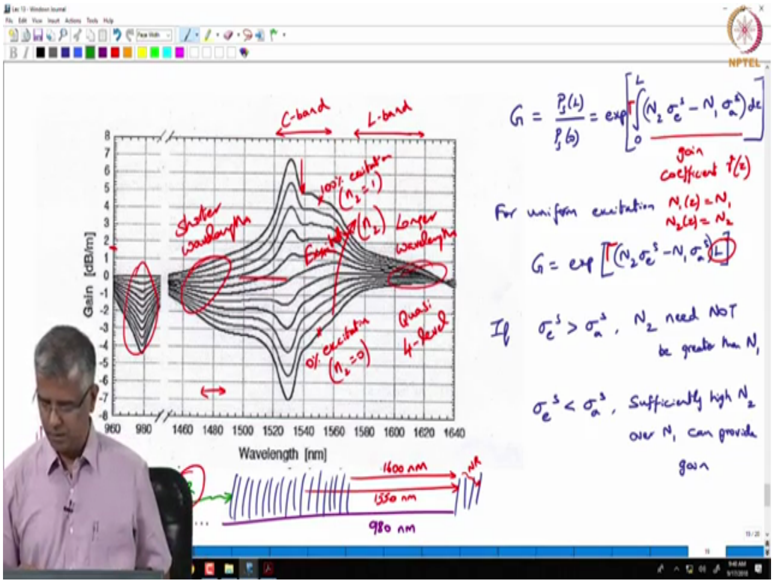

$$G = \frac{P_s(L)}{P_s(0)} = \exp\left[\Gamma\int_0^L \underbrace{(N_2\sigma_e^s - N_1\sigma_a^s)}_{\text{gain coefficient } \gamma(z)}dz\right]$$

For uniform excitation $N_1(z) = N_1$, $N_2(z) = N_2$

$$G = \exp\left[\Gamma(N_2\sigma_e^s - N_1\sigma_a^s)\boxed{L}\right]$$

If $\sigma_e^s > \sigma_a^s$, N_2 need NOT be greater than N_1

$\sigma_e^s < \sigma_a^s$, Sufficiently high N_2 over N_1 can provide gain

So I have drawn the cross section of the erbium fiber okay so we are looking from the side of the fiber and so we know that you have a core and a cladding and the core essentially consist of all this erbium ions which is actually like I said in the last lecture it is strictly ionized so erbium 3 plus ions but what can we say about our light distribution especially the fundamental mode.

When you talk about the signal fundamental mode that mode is not completely confined in the core. So there is certain parts of it that is in the core or most of it is in the core but there is a certain fraction of that signal which is propagating in the cladding, so what can we say about that as far as the amplification process is concerned? That part is not actually undergoing gain, only the part that is going through the core is undergoing gain.

And the other aspect is that you have a pump and a signal your gain would depend on what is the overlap of the pump and the signal, why? Because if your pump is like this which has got a peak you know corresponding to the centre of the core region and over the periphery the amount of pump intensity available is lower than the inversion is lower in those areas the inversion is very high in the central area the inversion is very low at the other side.

So your gain once again actually depends on what is the pump profile or especially what is the overlap of the pump with the signal? So we are taking care of all of this through this pump signal overlap factor which we will denote as capital gamma and this overlap factor is such that when we are defining gain, if we go back and look at this when we are defining gain over here or when we are defining the power itself you have to multiply that gain coefficient by that pump signal overlap factor, right that overlap factor is a number between 0 and 1, typically on the higher side because your signal you know if you designed your fiber correct your signal will have a fairly good overlap with your fiber and the pump will also will design the fiber such that pump has such (())(13:52) overlap with the signal as well, okay. So you have to multiply that gain coefficient by that factor.

So if you go back and look at all of these things all of these things should have that factor capital gamma in that which corresponds to pump signal overlap. So the question is whether gamma will be constant throughout the fiber, yes you can assume that right because for a given fiber the mode profile does not change as along the length of the fiber, so the gamma can be assumed as a constant along the length of the fiber.

So the question is you know the fiber is designed to be single mode for a particular wavelength and in some ways does that whole good for a pump also? So that is certainly a good question because we do know that the mode profile depends on the mode field diameter depends on the wavelength. So you have for longer wavelengths you have larger mode field diameter, okay.

So typically you do design such that it is single moded for both the pump as well as the signal and if you are single moded for the pump that means the signal at the signal wavelength you are still single moded because anything longer than that you are still single moded but more energy will be in the cladding region will be propagating in the cladding region. But at the end of the day you do have to when you are designing an amplifier like this you are making these splices at the ends of the fiber and the splices are with regular telecommunication grade single mode fiber and that fiber has a certain mode field diameter so you need to maintain that mode field diameter in this fiber itself also right to the this erbium silica fiber also has to be mode matched with that with the telecom fiber.

And that is a challenge because if you are mode matched then essentially either your pump because your regular telecommunication grade fiber has a cut off wavelength around 1200 nanometres, okay. Now if you use the same or a similar type of fiber you will end up having slightly multimode for the 980 nanometres you have one higher order mode also possible and that might affect the pump signal overlap factor, okay.

So those are all certain design considerations that you have when you make this erbium silica fiber but yeah I just wanted to make you aware of this we are designing the fiber itself at this point (())(17:13). So how do we you know include this pump signal overlap factor we said okay we are actually multiplying this by gamma and so that is what is given as 0.9 in our question but the.

So when we look at part a it says basically write down the expression for the small signal gain of the amplifier in terms of fraction of ions in the excited state with respect to the total doping density. So what we are asked to do is to write this expression in terms of the fraction of ions, so the fraction of ions in excited state is n 2, so n 2 is such that n 2 equals to N 2 over N a, where N a is the total number of ions in the fiber.

So or in other words N 2 can be written as n 2 multiplied by N a and N 1 what can we say about N 1? So that is going to be 1 minus N 2 whatever is not in the excited state have to be in the ground state, so 1 minus N 2 multiplied by N a, I should say it is approximately because this is under the approximation that N 3 equals to 0, right. So instead of N 1 and N 2 I can substitute this expression and so what you get for what we considered here is G is going to be given by gamma into (N 2 well instead of N 2 I can write as) 1 minus n 2 N a sigma e s minus (no this is sorry) n 2 N a sigma e s minus 1 minus n 2 N a sigma a s multiplied by L and the entire thing is under an exponential, okay.

So I can simplify this and I can just write this as G equals exponential of gamma L capital gamma L N a sigma e s plus sigma a s multiplied by n 2 minus sigma a s, I just rearranged terms. So that is a little more convenient to use right basically the gain it just now you know you have a certain gamma for a given fiber the gamma is given you know what is the length of your fiber you know what is your N a the total number density of ions in the fiber erbium ions in the fiber and then you have your cross sections. So in this one the only variable is N 2 so you can now calculate G for different values of N 2.

And especially if you want to do this in dB scale the gain in dB scale it has got to be so it is basically a 10 log 10 of exponential all of this, so what is that going to give you? Is basically it is going to give you 10 log 10 e multiplied by this factor that is in the (())(22:59). So 10 log base 10 e is what? What is the value of that? It is a tabulated value, right? 4.343, so multiplied by this entire factor over here so this is going to be given multiplied by gamma L N a sigma e s plus sigma a s into minus sigma a s.

So and if you of course want to take out the length dependence you can express this gain per metre and then the rest of that stuff will remain as it is and this is what we have represented over here, in this plot what we are represented is gain in dB per metre as a function of wavelength and as a function of this excitation level which is N 2, so excitation level N 2 equal to 0 which corresponds to 0 pump power launched into the fiber to excitation of 100 percent where N 2 is equal to (())(24:41).So those are the different cases that are being represented over here, do you understand this?

So for the given problem we can actually use this graph to find out certain values, so what are the values that are being asked? First of all we are asked to come up with this expression so we have done that we have looked at the gain expression as a function of N 2, okay and then when we go back the second part is the plot below actually the plot that I showed just now shows the gain spectrum for different values of fraction of ions in the excited state lowest curve corresponding to 0 percent and top most corresponding to 100 percent, determine the absorption cross section and emission cross section at the signal wavelength.

So emission and absorption cross section are not given you are just given this graph by the manufacturer, okay. Now can you estimate the emission and absorption cross section at a particular wavelength, we have been asked to consider 1550 nanometre as the signal wavelength, okay can we get that? How do we get that? You need to essentially estimate these values given a graph of G versus G over L gain in dB per metre that is given as a function of N 2, any ideas?

So first of all let us substitute N 2 equal to 0, okay if N 2 equal to 0 then whatever the gain value it represents only this factor this factor so and everything else is known gamma is given as 0.9, L we do not have to worry about because G over L is what we are computing, okay and N a is given and then only unknown is sigma a s so we can find out sigma a s from that. So that just corresponds to at 1550 this point, so at that point the gain is minus 3 dB per metre.

So I will just go back to this expression and say for N 2 equal to 0 percent the gain is given from the graph you can read from the graph is minus 3 dB per metre and that just corresponds to this 4.343 multiplied by 6 into 10 power 24 and if so happens that in this solution I have missed out this overlap factor, so you do have to multiply 0.9 to this multiplied by minus sigma a s and then from that you can actually figure out what is your sigma a s.

So this number is slightly off, so this has to be multiplied or divided by 0.9 is the final value, do you understand what we did? So now that we know sigma a s now we need to find out sigma e s and for that we look at the case where N 2 equals to 100 percent because when N 2 equals to 100 percent we go back and look at this this factor is 1, so sigma a and a these two factors will cancel and then what will be left out is sigma e s.

So what is the corresponding gain value? I just go back, go up in this and say the corresponding gain value is about here for N 2 so this 100 percent excitation corresponds to N 2 equal to 1 this is N 2 equal to 0. So that is roughly about a factor of 4.5 so once again I equate I put it around as 4 but that should be 4.5 I equate that to this expression and from that we can find out sigma e s.

So 0 percent excitation corresponds to no pump power, so without any pump power the erbium silica fiber acts like a absorber all the ions are in the you know the ground state, so any 1550 nanometre photon that comes by will get absorbed and that is what you saw in your experiments the other day when you were actually doing this experiment on the EDFA when there was no pump power you actually saw that the EDFA the so called amplifier is actually providing loss, so you needed to pump it to a certain level before you can recover back to the original signal level that you were sending in and then you have to pump beyond that to achieve amplification, okay.

So the other part is about assuming an average excitation along the length of the fiber as 60 percent, what is the length of erbium doped fiber required for achieving a gain of 30 dB? So for that you go back and look at this graph, so 60 percent excitation corresponds to this is 0, this is 10 percent, 20, 30, 40, 50, 60 so this is 60 percent over here this curve corresponds to 60 percent.

So at 60 percent the corresponding gain per metre that you have is 1.5 dB per metre, sorry which point? So this is basically 0, 10, 20, 30, 40, 50, 60 so 60 corresponds to this curve over here and the corresponding point at 1550 is over here that corresponds to 1.5 dB per metre that is the gain that you have. So the question is for that gain so gain is 1.5, what is the length required to achieve 30 dB gain you basically say 30 dB divided by 1.5 dB per metre and so you need a length of 20 metres.

So this is typically the as if you are designing an erbium doped fiber amplifier this is what you have to deal with, you are given this graph gain versus gain per unit length versus wavelength and then based on that you are supposed to figure out what is the excitation level required, what is the length required to achieve a particular gain and all that and this is of course for the simple case of uniform excitation but if you are doing this for real you know there is going to be a certain pump profile depending upon the pumping configuration and based on that you will figure out what is the N 2 and N 1 at different points along the length of the fiber and based on that you will figure out the overall gain.

Now a couple of points before we jump to the next topic one is that you know if you look at the gain for a erbium doped fiber amplifier what this tells you is that you can achieve gain over this region 1520 to about 1560 nanometre which is called the C-band from the optical communication perspective C-band corresponds to the conventional band so you can achieve amplification and one of the things you see is that the gain is not uniform across that spectrum, so if you have communication if you are doing communication you have multiple wavelength channels which is carrying information they are all coming into this amplifier and you want to have constant gain or uniform gain across for all the wavelengths.

So what you normally have to do is to put in some loss element spectrally dependent loss element which actually provides the provides loss such that these points the gain is actually reduced, okay. So if that gain is reduced only for these wavelengths then you would have a relatively flatter gain for all the wavelengths so that is the process called gain equalization that is typically carried out from this in this amplifier.

But the other interesting part is that when you consider this band from about 1570 to about 1620 you have very low gain but you do have gain and that actually is what you call as an L-band amplifier it is called the long wavelength band, but when you look at the long wavelength band it is typically flat the gain spectrum is typically flat so that means you can achieve uniform amplification over a wider range of wavelengths, okay.

So and quite interestingly if you think about it if you think about what does this long wavelength band means the wavelength will correspond to really short transitions basically low energy transitions. So this is basically you know something around 1600 nanometre. So when we consider a pumping scheme like this or amplifier like this it is typically ending up as a transition, the transition is ending up over here and you have nonradiative relaxation that is taking you to the ground state.

So the longer at the longer wavelengths it is acting as if it is a 4-level system, okay. So it may be looked upon as a Quasi 4-level amplifier at the longer wavelength and one of the properties of the 4-level system is that even for very low excitation levels you have gain and that is what you see even for very low excitation levels you have gain the gain is not very high because the probability of those transitions is very low the emission cross section is very low, okay. But nevertheless because it almost acts like a 4-level system you can achieve gain at relatively low levels of excitation.

So a graph like this can teach you a lot of things you know if you just pay a little more attention and look at this this can you can learn a lot about amplifiers from the graph like this and so we have spent a lot of time looking at this graph so hopefully you know has thought you something. So what we are saying is the shorter energy transitions are likely to end up at the higher energy levels of this ground state manifold and from that you would actually have a nonradiative relaxation to the actual balance level, okay.

So from that is similar to what you have in a 4-level system, we will not define 4 distinct levels but it actually looks like it acts like 4-level system because you are accessing only the top energy levels of your ground state and I want to actually end this with one more point so you see that around this wavelength 1480 nanometre (so where was this, is it here or I am trying to look for this yeah here) so around 1480 nanometres you have a significant absorption cross section.

So you can essentially pump at those wavelengths also you do not necessarily need to pump at 980 nanometres, you can pump you can choose to pump at 1480 nanometres also because it has got a very high you know absorption cross section and what does that corresponds to? So instead of going all the way over here in the case of 980 what you are doing is you are doing this for 1480 nanometres.

And what you see is that is okay because the emission cross section is relatively low so it is not a 2-level system, why? Because any photon any atom that is pumped at 1480 nanometres will correspond to a transition from one of the lower energy levels of your ground state to one of the higher energy levels of energy level e 2 from which you will have nonradiative relaxation before you try to achieve this gain.

So that is still a 3-level system and so you are not likely to achieve a lot of gain at 1480 itself because that will correspond to a 2-level system but if you are looking for gain at longer wavelengths gain at these wavelengths in the conventional band or the L band 1480 nanometre pumping would also be useful, it is not as useful or as efficient as 980 nanometre pumping because at 980 nanometre the emission cross section is 0 virtually 0, so it only contributes to absorption there is no emission, whereas at 1480 there is a finite emission cross section so part of the light can actually come back as 1480 itself, okay.

So it steals some of your inversion through that process and because of that you will not you do not achieve as much gain for the same level of pumping compared to 980 nanometre, do you see that point? So coming back over here at 1480 1480 that wavelength itself the gain is very low but you can use 1480 as the pump to achieve gain at the C-band or the L-band. So you may see commercial amplifiers with 1480 nanometre pumps also.

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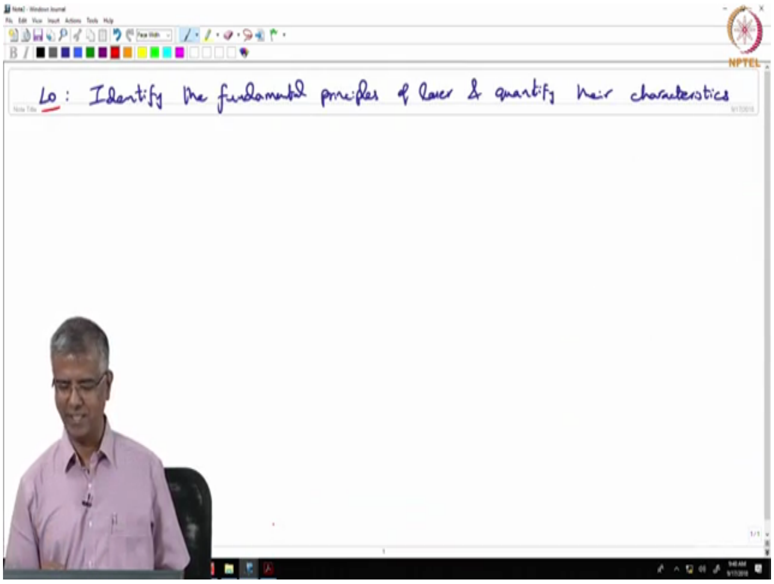

Lo : Identify the fundamental principles of laser & quantify their characteristics

Okay, I see that I have run out of time so we are not getting into this part but that is okay we can actually take care of that in the next lecture.