

**Introduction to Photonics**  
**Professor Balaji Srinivasan**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**  
**EDFA - Introduction**

(Refer Slide Time: 0:52)

Three-level system:

Energy levels:  $N_3$  (short  $\tau$ ),  $N_2$  (long  $\tau$ ),  $N_1$

Transitions:  $R_1$  (pump),  $T_{21}$  (spontaneous decay),  $T_{32}$  (fast decay)

Rate equation ( $\varnothing$  steady state)  $\frac{dN_2}{dt} = 0$

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}} - N_2 W_i + N_1 W_i = 0$$

$$(N_2 - N_1) W_i = R - \frac{N_2}{\tau_{21}}$$

$$N W_i = R - \frac{1}{2\tau_{21}} (N + N_a)$$

Total number of quanta:  $N_a = N_1 + N_2 + N_3$

$$N = N_2 - N_1$$

$$= 2N_2 - N_a \Rightarrow N_2 = \frac{1}{2}(N + N_a)$$

Four-level system

Energy levels:  $N_3$  (short-lived),  $N_2$  (long-lived),  $N_1$  (short-lived),  $N_0$

Transitions:  $R$  (pump), non-radiative decay,  $1064\text{ nm}$  (laser transition), non-radiative decay

Rate equations:

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i$$

$$\frac{dN_1}{dt} = -\frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i$$

Under steady state conditions,  $\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$

$$N = \frac{N_0}{1 + \tau_2 W_i}$$

$$N_0 = N_a \cdot \frac{\tau_{sp} W}{1 + \tau_{sp} W} \rightarrow \text{Pump function probability } (R = W N_0)$$

$$\tau_2 = \frac{\tau_{sp}}{1 + \tau_{sp} W}$$

For weak pumping  $W \ll 1/\tau_{sp}$

$$N_0 = N_a \tau_{sp} W$$

$$\tau_2 = \tau_{sp}$$

Okay, good morning welcome to yet another session of intro to photonics, we started this particular module several lectures ago we said we are going to identify the fundamental principles of photon interaction with atoms and especially analyse light generation and amplification. So as part of this what we have been considering is amplification of light and then you know with respect to the amplification we said okay we would typically deal with a

three-level system or a four-level system, right like this and then in terms of analysing such systems we said we will first of all start with the rate equations you learnt how to write rate equations and then we will assume a certain condition what condition?

Steady state condition because that is when that is the condition we are typically interested in and we said we assumed steady state condition when  $dN_2$  over  $dt$  and  $dN_1$  over  $dt$  equated to 0, so that allows us to come up with a relatively simplified expression for the inversion and we are trying to get that inversion as a function of the pumping rate, as well as you know as a function of the amount of signal photons that are going through the amplifier, okay.

So what we will, what we are going towards beyond this is you know we start from an amplifier and then how can we build a laser from this? So we are going to start incorporating feedback and then we will say how to get a laser based on this amplifier maybe that is what we will start doing from the next lecture onwards but for today I thought we could just see how well equipped we are to take up a practical problem, okay.

(Refer Slide Time: 2:54)

The screenshot shows a video lecture interface. At the top, it says "EE 5411 – Introduction to Photonics" and "Quiz II, 20 marks, 50 minutes". Below this, there is a "Remember..." section with a bullet point: "Make reasonable assumptions wherever necessary, but you should show all steps and justify assumptions for full credit. Use of figures may attract bonus points." This is followed by "Objective Type Questions (5 x 1 pt = 5 points)". The main question is: "distribution that provides atomic population at an excited state under thermal equilibrium". The options are: (a) Boltzmann, (b) Fermi-Dirac, (c) Poisson, and (d) Bose-Einstein. Handwritten notes in red ink show  $p(\epsilon_n) = \frac{e^{-\epsilon_n/kT}}{Z}$  next to the options. Below the question, there is another question: "flux density for a gain medium with excited lifetime of 1 ms and emission cross section of  $10^{-22} \text{ m}^2$  has a value of". The options are: (a)  $10^{22}$ , (b)  $10^{28}$ , (c)  $10^{28}$ , and (d)  $10^{22}$ . Handwritten notes in red ink show  $\phi_{\text{sat}} = \frac{1}{\sigma \tau}$  next to the options. The NPTEL logo is visible in the top right corner of the slide.

**Quantitative Problems (15 points)**

6. Consider the amplification of light at a signal wavelength of 1550 nm in a Er-doped silica optical fiber with a doping concentration of  $6 \times 10^{24} \text{ m}^{-3}$ . Assume uniform excitation and a pump-signal overlap factor of 0.9 for the Er silica fiber.

where  $n_2 = \frac{N_2}{N_a}$

$N_a = N_1 + N_2$

a. Write down the expression for the small signal gain of the amplifier in terms of the fraction of ions in the excited state ( $n_2$ ) with respect to the total dopant density ( $N_a$ )

for  $n_2 = 0\%$ ,  $G(\text{dB}) = -3 = 4.343 \times 6 \times 10^{24} (\sigma_a^e) \Rightarrow \sigma_a^e = 1.15 \times 10^{-25} \text{ m}^2$  (2 pts)

b. The plot below shows the gain spectrum for different values of fraction of ions in the excited state ( $n_2$ , lowest curve - 0%, topmost - 100%). Determine the absorption cross-section and emission cross-section at the signal wavelength (4 pts)

for  $n_2 = 100\%$ ,  $G(\text{dB/m}) = 4.343 \times 6 \times 10^{24} \times \sigma_e^e \Rightarrow \sigma_e^e = 1.72 \times 10^{-23} \text{ m}^2$

c. Assuming average excitation ( $n_2$ ) of 60%, what is the length of Er-doped fiber required for achieving a gain of 30 dB? (2 pts)

for  $n_2 = 60\%$ ,  $G(\text{dB/m}) = 1.5 \Rightarrow \text{length req. for } \frac{30 \text{ dB}}{30 \text{ dB gain}} = \frac{30 \text{ dB}}{1.5 \text{ dB/m}} = 20 \text{ m}$

So for this I thought it be a good idea to go back and consider this quiz problem this is actually quiz 2 last year or the year before in 2016, right. So this is quiz 2 paper and then from that paper there is this one question on amplification using a erbium doped silica fiber. So let us see if we are equipped to handle such a question, okay so let us take this example and see how we can do this, okay.

(Refer Slide Time: 3:42)

**Four-level system**

**Neodymium**

For weak pumping  $W \ll 1/\tau_{sp}$

$N_0 = N_a \tau_{sp} W$

$\tau_3 = \tau_{sp}$

$N_a = N_0 + N_1 + N_2 + N_3$

$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_2} - N_2 N_1 + N_3 W$

$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 N_1 - N_3 W$

Under steady state condition,  $\frac{dN_i}{dt} = \frac{dN_j}{dt} = 0$

$N = \frac{N_0}{1 + \tau_2 W}$

$N_0 = N_a \cdot \frac{\tau_{sp} W}{1 + \tau_{sp} W} \rightarrow \text{Pump transition probability } (R = W N_3)$

$\tau_3 = \frac{\tau_{sp}}{1 + \tau_{sp} W}$

### EDFA

$\frac{dP_s}{dz} = -\alpha_p P_s$   
 $\frac{dP_s}{dz} = N_2 \sigma_e^s P_s - N_1 \sigma_a^s P_s$   
 $P_s = P_s^0 \exp\left[\int_0^z (N_2 \sigma_e^s - N_1 \sigma_a^s) dz\right]$   
 $Gain = \frac{P_s(L)}{P_s(0)} = \frac{\phi(L)}{\phi(0)}$

Weak Signal 1550 nm, Pump 980 nm, WDM, Erbium, Residual pump, Strong Signal 1550 nm, WDM, Reabsorption.

Optical Density (a.u.) vs Wavelength (nm). Peaks at 796 nm, 972 nm, 1529 nm.

Cross-section (cm<sup>2</sup>) vs Wavelength (nm). Shows Absorption and Emission curves.

$G = \frac{P_s(L)}{P_s(0)} = \exp\left[\int_0^L (N_2 \sigma_e^s - N_1 \sigma_a^s) dz\right]$   
 gain coefficient  $g(z)$

For uniform excitation  
 $G = \exp[(N_2 \sigma_e^s - N_1 \sigma_a^s)L]$

If  $\sigma_e^s > \sigma_a^s$ ,  $N_2$  need NOT be greater than  $N_1$   
 If  $\sigma_e^s < \sigma_a^s$ , Sufficiently high  $N_2$  over  $N_1$  can provide gain

And before we get to that I just want to give you a general idea of a construction of an EDFA, so some of you did an experiment with EDFA you constructed an EDFA and you even demonstrated that you could get light amplification using this erbium doped fiber amplifier, but let us just look at this in a little more detail than what we have done so far. So what we need as far as a EDFA is concern is clearly the erbium doped fiber so let us say I have an erbium doped fiber that is plies with some other fiber with regular un doped fiber, so this is a erbium doped in silica, okay.

So to make an amplifier with such fiber what all do we need? We need to send in the pump, so you have the pump radiation coupled into this launched into this erbium doped fiber along with your weak signal radiation and the pump is getting absorbed along the length of the fiber and maybe some of the pump still remains after this length of the fiber you may want to remove that so you use one more fiber coupler these fiber couplers are called in short WDM's it stands for wavelength, division multiplexer, okay.

So in this case it is actually on the input side it is combining two different wavelengths the pump radiation at 980 nanometre is combined with the signal radiation let us say at 1550 nanometre, okay and the output side you have strong amplified signal at 1550 nanometre and then you may have some residual pump radiation that you can probably remove using another wavelength division multiplexer which is actually doing a demultiplexing operation it is separating out two wavelengths.

We will not go into much detail on the wavelength division multiplexer right now it is you know we can come back and look in that in more detail later on but let us just focus on the amplification process let us try to understand what exactly is going on in this structure and how light gets amplified. So let us say this is the length of this amplification region and let us try to figure out what all is happening over here, so this is 0 and let us say the length of the amplifier corresponds to L, so let us say this is the propagation direction which let us call that as Z direction.

So let us first track how this pump radiation is going to be distributed in this. So the pump radiation is launched from this point let us say  $P_P(0)$  corresponds to the pump power that is launched into the fiber and within that fiber it is getting absorbed that pump radiation is getting absorbed. So you can say that  $dP_P$  over  $dZ$  as it propagates is going to go through a certain absorption at the rate let us say  $\alpha_P$  let us say absorption coefficient multiplied by  $P_P$ . So it is actually getting absorbed at a constant rate.

So what is the solution you expect? So it should be the solution for this is an exponential, so you would basically say that this is getting exponentially decaying so that is given by that rate at which it decays it's given by exponential of minus alpha P multiplied by Z. So does that mean? What does that mean as far as your population inversion is concerned? Let us think about that for a second.

So what we have let me just for simplicity just consider this two energy levels, we do have a third energy level in erbium, so we are not paying attention to that. So you have a very high pump power at the beginning of that fiber, so what can you say about the inversion that you have there? So there will be a lot of ions these erbium ions that are you know going to the higher energy level compared to what we have at the lower energy level.

Now what can you say about what is happening over here? Here the pump is actually much lower so correspondingly the inversion that you expect is going to be much lower, so here you may have very few erbium atoms or erbium atoms in silica actually are sitting in an ionized form it is a triply ionized form. So if you want to be really rigorous about it, you would actually mention them as erbium 3 plus, okay that is how (11:07) it is common to say that what we are considering are erbium ions instead of erbium atoms.

But nevertheless you what it tells us is this inversion is not constant in a real amplifier, if you want a constant inversion what would you do what would you have done? Pump from the side, so if you actually sent in pump radiation from the side of the fiber if you pumped it from here you could have possibly maintained constant inversion uniform excitation that is not possible or you should not say it is not possible it is highly challenging to implement for an optical fiber because the optical fiber itself you know the telecommunication fiber that we use is 125 microns in diameter out of which the core region in which the erbium atoms are doped into those correspond to only a few microns. So you are not going to be able to efficiently focus all the pump light so that they are absorbed in the erbium fiber.

So because of those reasons you chose as far as fiber form is concern you chose to pump from one end. Now the situation may be very different if you are talking about some erbium crystal, erbium actually doped into crystal of glass a crystal of bulk glass, so you can probably have a cylindrical glass rod for example in which this (13:16) ion is doped and you can excite it from the sites, in fact you can put it as part of cavities some mirror around it and then you put some your pump on you know on the side of your rod and you can you can

pump it so that it either directly is incident on the rod or it goes and hits the mirror and then it is absorbed by the rod.

So you could have something like this, this is your let us say your erbium crystal and then you could have mirrors over here on either side of that and then you could have your pump from this side so it either goes directly into this or if the pump is going this way it is reflected back into the rod you know. So there are different configurations which you can achieve this but the idea here is that you would have to go to some crystal or bulk glass sort of configuration or form so that you can achieve uniform excitation, if you are doing fiber based amplifiers you invariably have to pump it from one of the ends of the fiber, okay.

I said one of the ends of the fiber, so you could possibly launch the pump from the output side and take out the residual pump from this side from the side where you are launching the 1550, so that would actually be called counter pumping sort of configuration. So the question is how would that help? Actually you know that is probably a preferred configuration, why? Maybe I will just hold on to that question hold on to the thought for a minute, let me just complete what I was going to say here as far as the signal is concerned and then it will become obvious.

What do we have as the signal? So just one more question, yeah the fiber is only 125 microns thin so that is the size of your hair and within that the erbium core is only a few microns diameter so if I launch light if I have the fiber here what are the chances you are going to have that light absorbed? So of course you can focus that light you put a lens and you focus that light and then you have to focus it into a very thin line, you have to hold your fiber straight and you have to have a very long focus line all of that.

So when you actually try to do that you find it is quite cumbersome. Suppose to a rod this rod can be a few millimetres thick in that case and it is relatively short rod so you can actually have these pump diodes stacked along the rod and they are all pumping like this so in this case we are saying that the rod in 3 dimension will look like this and so you could you could have these mirrors on either side and then the row of pump diodes which are directly pumping the rods, so you would actually have fairly good efficiency with which you are absorbing all the pump radiation.

So the question is so are not we actually having difficulty in focusing light within that optical fiber itself? And that is certainly a valid question that is actually saying you know if you can

do that, cannot we do that side pumping, it is not the same because if you are doing launching into launching light into the fiber you are doing it only at one point, you are not trying to keep the fiber straight and maintain that focus all along the length of the fiber.

So you are launching it only one point and how do you launch light into a single mode fiber? You have already done that, so you know how to launch light into a single mode fiber so it is that is quite straight forward. And I would also want you to consider this other thing which is light you know when you are pumping at 980 nanometre where does that light come from? That has to be laser radiation, so you have because your absorption peak is relatively narrow it is not a very broad absorption peak so you have to come in with a laser and you know excite these erbium atoms.

And when you consider what type of source you will use, you will find that you are typically using a semiconductor laser diode and we will the next few weeks we will actually go into the details or the construction of a semiconductor laser diode and what you will find is that the you know the aperture from which light is emitted in a semiconductor laser diode is in the order of a few microns.

So you have the possibility of taking the fiber if this is my semiconductor laser I take my fiber and just directly (( ))(19:30) it against my semiconductor diode. So I just hold it right against that aperture so I can couple light directly I do not even have to think about you know taking the light out and having some focusing optics and coupling it because the packaging that would be a mess.

So instead what people do is take this fiber directly you know hold it against that output aperture of the semiconductor laser and then you glue it and all that and you put it into a hermetically sealed package and you are done and that can stay for you tens of thousands of hours there is no problem. So it is much easier to come in with that pump radiation in a fiber form and couple into it but like I said this is what happens.

Now let us go back and see what happens to the signal what do you think is going to happen to the signal, signal is going to increase and it is going to probably go through an exponential increase to the point that you have fairly good inversion. In the case of erbium if you do not have inversion and you have lot of signal photons what happens? In this region towards the later part of the fiber you have very little pump light that means very few atoms are actually excited to that higher energy level so most of them are sitting in the ground state and then you



have 1550 nanometre photons coming through so what is going to happen, what is more likely to happen?

Stimulated absorption because there are not too many photons I mean too many atoms in that excited state everything is in the ground state so the probability of actually having that absorption event so you have here you know 1550 nanometre radiation come along and that energy difference corresponds to 1550 nanometre. So you have absorption of that light happening.

So you have what is called reabsorption of that light happened beyond a certain distance and then it might actually beyond that distance you know if you have longer fiber you would lose signal photons. So this process is called reabsorption and reabsorption is a major issue in three-level lasers but what can you say about this in a four-level laser? What can I say about reabsorption in a four-level laser system or amplifier system?

So we are considering stimulated absorption as this process, right going from the lower energy level to the higher energy level, but in this case what can we say about atoms in that lower energy level? Very few atoms in the lower energy level because we have a fast nonradiative transition relaxation that is happening, so everything is going to the ground state.

So now if I come in with in this case 1064 nanometre radiation very little absorption is going to happen. So you do not have this issue of reabsorption in a four-level system you do not worry about that, okay but in a three-level system you have to worry about it because if you are not exciting your (( ))(23:45) atoms they are all sitting in the ground state and they can actually reabsorb the signal, okay. So that is something to keep in mind.

So there is a question of optimization of your length there is a question of optimization of the length in sense that if you have such a long length that you have very few pump photons towards the later part of the fiber it is actually not helping you so you have to do some modelling to see how your absorption is happening and for a given pump power, so you typically consider the maximum pump power that is available to you and for that pump power you see how much you will be going to be your inversion at the father end of the fiber and then you say okay that is you know I will cut off the fiber at that particular length so you can you do not have so much reabsorption.

So the question is you know can we actually reenergize the erbium doped fiber at that particular point and that will be a very good idea, so you would actually you could think about reenergizing the fiber at that particular point. In fact what could be done is instead of just pumping from one end why do not you pump from both ends? And that would be a very good idea as well, so you can actually keep your excitation level relatively uniform across this fiber so that is called bidirectional pumping.

So now we are starting to look at some advance issues in advance configurations I am glad you guys are thinking like this but that is what you need to do to have uniform excitation and that is a good way of making sure reabsorption is not happening. But the other point about counter directional pumping let us examine that if I am launching my pump from this direction then I have very high inversion at the later part of the fiber and I have very low inversion at the front part of the fiber and that works out very well, why?

Because when you have you know the weak signal launched from the left side there are very few signal photons coming in so they need very few excited atoms to achieve you know the gain they need very little inversion to achieve gain and as they propagate towards the right side your signal photons are getting multiplied and they need more excited atoms more inversion to sustain their amplification.

So you would say from the perspective of utilizing your inversion efficiently you want to do counter directional pumping so the pump configuration that you would be doing in your laboratory this week just pay attention to that that is actually counter directional pumping. So the question is in counter directional pumping reabsorption does not happen the chance of that happening is very low of course you can argue that if my pump excitation is very low on the left side of the fiber then you have more atoms on the ground state which is this thing, so the few photons that are coming in they are getting absorbed.

So what you have to ensure is that you maintain a certain level of inversion, so you do not once again it does not mean that you can by counter propagation direction you can make the length arbitrarily high or long fiber, you still have to cut off the fiber so that the already weak photons do not get lost you know in that absorption. So those are all like I said fairly advanced issues that you can get into and you can look in lot more detail but may be as far this course is concerned we will not go into so much of the specifics.

But I will try to equip you to evaluate these conditions in a little more precise manner so that is what we will try to do now. Okay, so the question is what is this dependence how does that signal you know gain as a function of length, how do you evaluate this?

Now to evaluate this you actually have to consider one particular point which we have so far sort of neglected you know so when we were looking at these rate equations we made one assumption which is actually not you know practically speaking it is not correct, what is that assumption? So we used the same when we were looking at simulated absorption, simulated emission we use the same rates  $W_i$  okay and  $W_i$  we defined as you know  $\Phi_s$  multiplied by  $\sigma_e$  as far as the emission is concerned so this is  $\Phi_s \sigma_e$  and this one is actually  $\Phi_s \sigma_a$  and we are actually considering the signal photons so I will (subscript) superscript with s.

So far we have assumed that  $\sigma_e$  equals to  $\sigma_a$  that allowed us to get further get some more insights into this amplifier system in a simplistic manner but time has come now to you know get rid of that assumption let us actually look at how  $\sigma_e$  and  $\sigma_a$  would be. And for that we have previously considered this erbium absorption and emission cross sections and what do you see here? What can you say about  $\sigma_e$  and  $\sigma_a$ ?

They are not typically equal, in fact there is only one particular point where they overlap and you can say okay at that particular point around 1540 or something like that 1535 may be at that particular point they are equal but in general they are not equal and what this also tells us is the absorption process is more probable where at shorter wavelengths and emission process is more probable at longer wavelengths.

So you are more likely to get gain on longer wavelengths that is something that we were absorbing later on but as far as the how the signal is getting amplified now we have to consider this in a little more detail. So we will look at this, now what we want is at this point is  $P_s$  of L, we want to know what is the gain that you have in the system which is given by  $P_s$  of L over  $P_s$  of 0, okay.

So to evaluate that gain we need to now understand how this amplification is happening, so what is the gain at any particular point along this fiber? So if I am, so what I have to track is the signal power or I could actually track the signal flux because this can be written as  $\Phi_s$  of L divided by  $\Phi_s$  of 0 that is the signal flux that is coming into the fiber and then we have a signal flux that is going out of the fiber, what we can do now is as far as the signal flux is

concerned I can write an expression for  $d\Phi_s$  over  $dZ$  because that is what is describing how my signal flux is you know varying along the length of the fiber and that I can write as  $N_2$  I will not use  $W_i$  anymore because I want to bring out the fact that  $\sigma_e$  and  $\sigma_a$  are different.

So I would have to write this as  $\sigma_e$  at the signal wavelength multiplied by  $\Phi_s$ ,  $\Phi_s$  is photons per second the small  $\Phi_s$  we are using as photon flux density this is the larger  $\Phi_S$  the capital  $\Phi_S$  which corresponds to photon flux. So this minus the simulated absorption term which I would write as  $N_1 \sigma_a \Phi_s$  at that particular point, okay now you have to remember this which are constants and which are variables in this expression? Which are the constants?  $\sigma_e$  and  $\sigma_a$ , they are constants for a given amplifier system, whereas everything else is variable.

So I actually if I were to write it out properly I would say  $N_2$  of  $Z$  and then  $\Phi_s$  of  $Z$  and  $N_1$  of  $Z$   $\Phi_s$  of  $Z$ , you understand that and what determines  $N_2$  of  $Z$  and  $N_1$  of  $Z$ ? You certainly it depends on the total amount of the doping that you have done but what else the pump excitation that you have at that particular time that particular location. So that is what makes this a little more interesting.

So since I can write  $P_s$  as  $\Phi_s$  multiplied by  $h\nu$ , energy over time is power and flux is 1 over time it is 1 over second so  $h\nu$  is I would say  $h\nu_s$  because that corresponds to the signal photon, so the signal photon energy multiplied by the flux gives me power so I can just write this as  $dP_s$  over  $dZ$  equal to  $P_s$  multiplied by  $N_2 \sigma_e$  minus  $N_1 \sigma_a$ . So instead of writing in terms of photon flux I am writing in terms of photon power so I mean the signal power  $P_s$ , okay so instead of  $\Phi_s$  I am just substituting  $P_s$ .

So if we look at this and I can write the overall gain now based on this I will just go over to the next page for that so my overall gain which is given by  $P_s$  of  $L$  over  $P_s$  of  $0$  is now going to be exponential of this factor you have  $N_2 \sigma_e$  minus  $N_1 \sigma_a$  but this is not a constant now so when I want to look at the overall gain what do I have to do? I have to integrate under that curve, right. So I would have to integrate from  $0$  to  $L$   $dZ$ , okay this entire thing now is sitting within an exponential, do you understand how I wrote that?

Just go back to this so the solution for this is going to be an exponential and since we have just a positive sign here we are just saying exponential of this but what we are interested in that is the solution of  $P_s$  of  $Z$ , what we want is  $P_s$  of  $L$ , so  $P_s$  of  $L$  if you want to figure out

then that I would have to basically look at that how it is accumulated over that entire length so I have to integrate over that entire length to get this so if I want to have a gain coefficient a gain coefficient at any particular point this would be my gain coefficient, this entire thing now is my gain coefficient which we previously said corresponds to  $\gamma$  of  $Z$ .

But if I want to find the gain the overall total gain I will have to integrate the gain coefficient over that entire length and that is what we are doing, previously when we were discussing this sort of case what we did was we assumed uniform excitation if there is uniform excitation  $N_2$  of  $Z$  is actually a constant you know you can just say it is  $N_2$  and  $N_1$  of  $Z$  is constant  $N_1$  and so that integral will yield you what? Integral over  $dZ$  will yield you just  $L$  so we just multiplied this by  $L$ .

So I can make a special case here say for uniform excitation case I would just say  $G$  is exponential of  $N_2 \sigma_e s$  minus  $N_1 \sigma_a s$  multiplied by  $L$ , okay do you understand this? So for uniform excitation it will be like that but for if you want to consider the fact that you are launching may be from only one end of the fiber and then you have a exponential decaying pump then you may have to deal with this sort of situation, okay.

So this also tells us something very very important to realize if  $\sigma_e$  the emission cross section is greater than  $\sigma_a$  mathematically you can say that  $N_2$  need not maybe I will just emphasize that need not be greater than  $N_1$ , does that make sense? To get gain what you need is that the gain coefficient be positive that entire value needs to be positive so it does not mean that you necessarily have to have  $N_2$  greater than  $N_1$ , you have an inversion but in these regions in these regions over here you have  $\sigma_a$  is almost 0,  $\sigma_e$  is a finite value, okay.

In those regions  $N_2$  need not be very high compared to  $N_1$ , it need not be greater than  $N_1$ ,  $N_2$  can be a fraction of  $N_1$  and you can still get gain because what matter is that this factor overall is positive, okay. So conversely I can also say that  $\sigma_e$  if it is less than  $\sigma_a$  s if the emission cross section is less than  $\sigma_a$  s that does not mean that you cannot get gain normally you say oh absorption is higher than emission cross section so you cannot get gain no, what it just means is that sufficiently high  $N_2$  over  $N_1$  can provide gain.

So if you just have you just if  $\sigma_e$  is less than  $\sigma_a$  you just need to overcome that with a higher fraction of these (45:50) atoms in excited state compared to what is in the ground state so this is the question, you need population inversion for lasering to happen in

the case of  $\sigma_e$  equal to  $\sigma_a$ . So far we had been saying that, but inversion the definition of inversion is slightly modifying, we are saying that the inversion is not just number densities you have to factor in that (46:24) cross section also, okay. So that is way (46:31) a lot of time I will continue this in the next session.