Introduction to Photonics
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Three Level Systems

Good morning, so welcome back to introduction to photonics. So we have been talking about analysing light generation and amplification, so we go to the point of looking at an example case of where we need an optical amplifier we took the example case with an optical fiber communication system and we defined this specific gain medium that is erbium which is doped in silica as a possible gain medium to achieve amplification at 1550 nanometers, why are we picking 1550 nanometers? So the losses for the fiber will be very low, so that is why we are looking at that wavelength and then we are going around searching for an amplifier that can possibly or compensate for the losses in the optical fiber during propagation.

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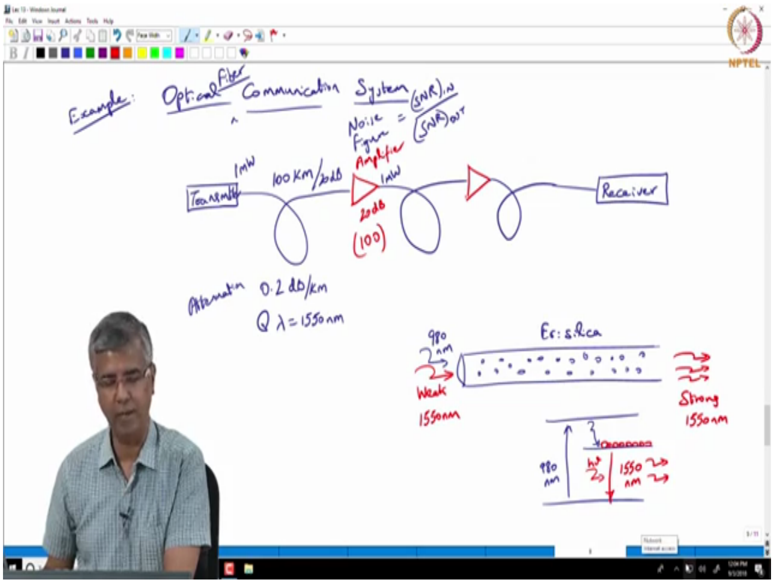

And so we took the example of an erbium silica fiber amplifier odium doped fiber amplifier in short it is called E D F A and then we jumped on to understanding this amplification process itself.

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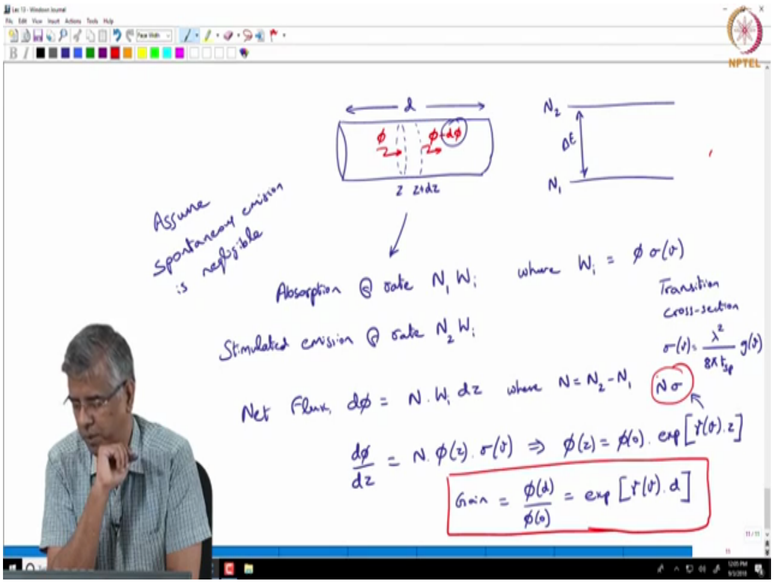

Assume spontaneous emission is negligible

Absorption @ rate $N_1 W_i$ where $W_i = \phi\, \sigma(\nu)$

Transition cross-section

$$\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

Stimulated emission @ rate $N_2 W_i$

Net Flux, $d\phi = N \cdot W_i\, dz$ where $N = N_2 - N_1$

$N\sigma$

$$\frac{d\phi}{dz} = N \cdot \phi(z) \cdot \sigma(\nu) \Rightarrow \phi(z) = \phi(0) \cdot \exp[\gamma(\nu) \cdot z]$$

$$\boxed{\text{Gain} = \frac{\phi(d)}{\phi(0)} = \exp[\gamma(\nu) \cdot d]}$$

So we looked at an elemental gain medium just involving transition between two levels and for that sort of a medium we defined we found the gain the expression of the gain is given by exponential of gamma multiplied by the length of the amplifier where gamma itself is expressed as N into sigma where N corresponds to the inversion that you can achieve, so that is essentially the number of excited ions versus the number of ions at the ground state that is N 2 minus N 1 and multiplied by sigma where sigma we define as the transition cross-section and we assumed that the transition cross section is actually equal for both the you know emission as well as the absorption, right.

So that is how we came up with this common term, we will of course go back and question this because.

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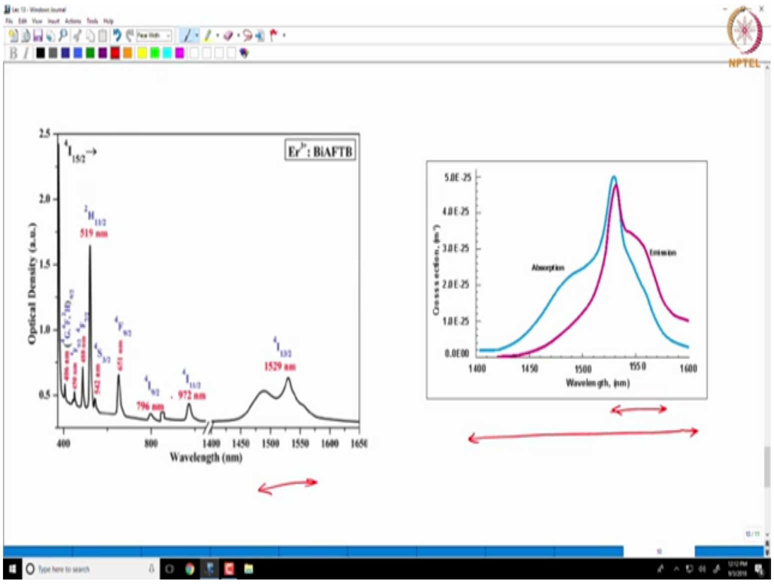

If you look at the absorption cross section absorption emission cross section of erbium you find that when you look at the cross section as a function of wavelength you find that the absorption cross section which is given as blue is different from the emission cross section which is given in purple, ok. So we may have to you know deal with these terms separately ok, so you have to instead of mentioning as a common term as sigma we may have to qualify that as sigma A and Sigma E Sigma A corresponding to absorption cross section and sigma E corresponding to emission cross section.

Just looking at this what do you think the relative values of sigma E and Sigma A should be if you want to make amplification would you get amplification across this entire band that is mentioned, so we are talking about a band that in this case represented in 1400 to 1600 nanometers, do you think it is possible to get amplification across the entire band? So where do you think you are likely to get amplification?

Student is answering:

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So you obviously need inversion for getting amplification but that inversion is also you know that it is not just the inversion by itself that inversion is multiplied by the transition cross section and you want more of an emission than absorption if you want to get gain, right. So you can essentially say this region where the emission is greater than the absorption is likely the region where you can get gain, ok.

If you have a absorption cross section or the probability of absorption more than the probability of emission then it is going to be very difficult to get gain you understand that, so we will come back and qualify that but as of now we assume that sigma A and sigma E are equal which is probably happening as far as this graph is concerned for a particular wavelength at maybe around 1530 or so, ok.

So that is one thing to keep in mind we will we will like I said we will go into a little more detail later in today's lecture but the other thing I wanted to give you a perspective off is the graph on the left side which is plotting what it calls us optical density as a function of wavelength, optical density you can look upon that as absorption as a function of wavelength for a erbium crystal erbium doped crystal, the crystal name is that BiAFTB does not matter what it is but this is a characteristic absorption spectrum for erbium and what do you see? You see multiple peaks in that absorption spectrum.

So what does that signify when you see multiple peaks like this in the absorption spectrum? It has multiple energy levels so right, so it can an atom can absorb a photon which has multiple wavelengths right and each of those wavelengths that it can absorb corresponds to a particular transition particular value of delta E energy difference between the ground state and the excited state.

So all these Peaks are saying that I have multiple energy levels so far we have been just considering two energy levels but in reality you have to consider all these different energy levels when you are trying to find the gain of this amplifier, so you can essentially achieve an inversion potentially by pumping at any of these wavelengths where there is a peak because at all these wavelengths you can absorb energy and that absorption will take the erbium atom from a ground state to a higher energy level and from one of the higher energy levels you can get emission and you can probably get stimulated emission also ok.

So this is typically the kind of picture that we are going to be looking at so we will as we move on in today is lecture and the next we will start considering three level systems four level systems and so on and try to find out what are the characteristics of those type of atomic systems, any questions about this before we move on? So the emission it is not given but you can so the emission in the 1550 band which corresponds to this region is provided and but you can imagine that wherever there is absorption there is a possibility of emission also ok.

So you can probably make a laser around 972 nanometers or 800 nanometers or this high peak over there at 590 nanometers that corresponds to green, so we have made green lasers out of this erbium systems as well right, so there is a possibility of emission as well although that is not indicated in this particular picture ok.

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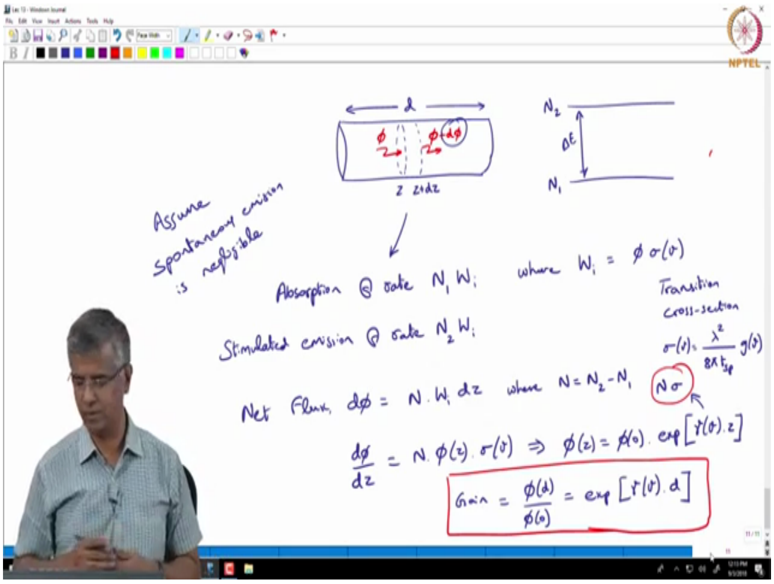

Assume spontaneous emission is negligible

Absorption @ rate $N_1 W_i$ where $W_i = \phi\,\sigma(\nu)$

Transition cross-section

$\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$

Stimulated emission @ rate $N_2 W_i$

Net Flux, $d\phi = N . W_i\, dz$ where $N = N_2 - N_1$ $N\sigma$

$\frac{d\phi}{dz} = N . \phi(z) . \sigma(\nu) \Rightarrow \phi(z) = \phi(0) . \exp[\gamma(\nu) . z]$

$$\boxed{\text{Gain} = \frac{\phi(d)}{\phi(0)} = \exp[\gamma(\nu) . d]}$$

But when we come back to this system right and we sort of left off without completing this because we started with a requirement of 20 dB gain from our amplifier ok and so now we can you know throw in specific numbers and try to see what should be the length of the amplifier that you need to achieve 20 dB gain, ok. So let us go on to substitute some numbers.

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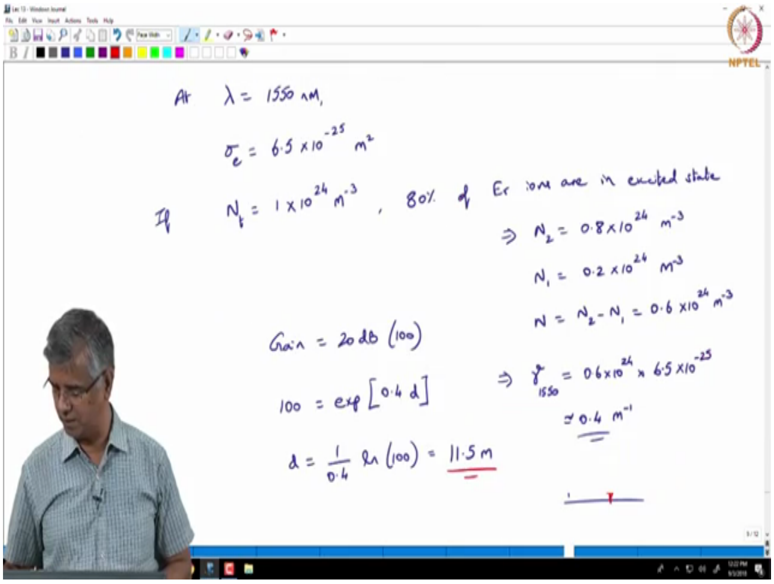


At $\lambda = 1550$ nm,

$\sigma_e = 6.5 \times 10^{-25}$ m^2

If $N_t = 1 \times 10^{24}$ m^{-3}, 80% of Er ions are in excited state

$\Rightarrow N_2 = 0.8 \times 10^{24}$ m^{-3}

$N_1 = 0.2 \times 10^{24}$ m^{-3}

$N = N_2 - N_1 = 0.6 \times 10^{24}$ m^{-3}

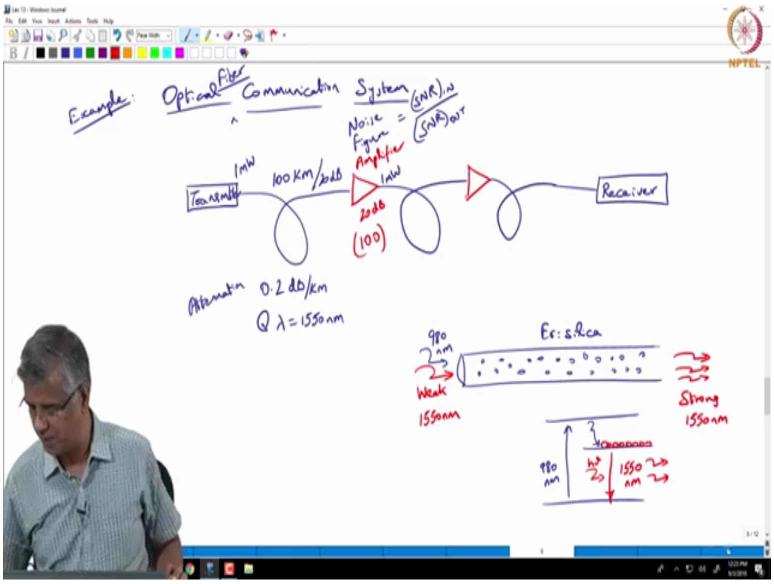
$\Rightarrow \gamma_{1550} = 0.6 \times 10^{24} \times 6.5 \times 10^{-25}$

$\simeq 0.4$ m^{-1}

Gain = 20 dB (100)

$100 = \exp[0.4\, d]$

$d = \frac{1}{0.4} \ln(100) = \underline{11.5 \text{ m}}$



So let us say we have at lambda equals to 1550 nanometer let us say you have an emission cross section responding to 6 point 5 multiplied by 10 power minus 25 what are the units of emission cross section.

Student is answering:

Centimeter square.

Centimeter square or meter square, in this case it is actually expressed in meter square ok and if we are told that the fiber that we have has uniform doping level ok and that is that doping density is given by 1 into 10 power 24 per unit volume ok 1 into 10 power 24, so just think about these numbers for a second I mean the absorption cross section is a fairly small value right.

So you say ok that the probability of that happening is very small but then you consider the number density of erbium atoms it is at such level you know it is you are talking about 10 power 24, so maybe the probability of this absorption or emission event happening is not so bad right, because the numbers that we are and a number density of erbium ions that we are looking at is fairly high, ok.

So we are also told that 80 percent of the erbium ions are in excited state, right. So you are somehow managed to you know pump it externally and maintain 80 percent of your ions in the excited state ok, so what is the N 2 now total is 1 into 10 power 24 80 percent of that is excited point, so this is going to be point 8 10 to the power 24 per meter cube right and what is the N 1 point 2 if these are the only two states that we are considering N 1 is point 2.

So what is the inversion? Yes point 6 to 10 power 24 per meter cube, so you have that many numbers of sites which are available for stimulated emission ok, so that those that inversion that population inversion is what can give rise to gain, right. So if this is the case this implies that gamma at specific wavelength of 1550 because the transition cross section is given for this wavelength at 1550 gamma is given by N into sigma right where n is the inversion and sigma is their transition transection.

So in this case it is N is point 6 into 10 power 24 sigma is 6 point 5, 10 power minus 25 point 4 right point 4 and just say it is approximately point 4 or the units?

Student is answering:

Per meter.

Per meter, right. So the gain that you are available the gain coefficient I should say that is available is point 4 what does it have to be multiplied with to get the actual gain? The length of the amplifier right, so what we are told is that the gain required is 20 dB which is a factor of 100, right. So we can say that 100 equals exponential of point 4 multiplied by the distance d for the amplifier or in other words d is going to be given by 1 over point 4 log of 100, right and if you is substitute log of 100 is about 4 point 56 is something like that, so if you substitute those numbers what you will find is about 11 point 5 meters, so for this case we need a erbium doped silicon fiber of length 11 point 5 meters ok to achieve a gain of 20 dB, you understand this.

Mind you one of the assumptions that we have taken as far as this problem is concerned is that well of course that the emission and transition cross sections are equal right that is one assumption we will come back and question that assumption but the other assumption is that 80%percent of the erbium ions are in the excited state all across the length of the fiber right only if that is the case then you can say this otherwise you will have to see what is the inversion at each location and you know you have to account the inversion at each location or what is the corresponding gain coefficient at each location and based on that you would have to do that calculation.

So what I am talking about is if you go back and look at this picture over here we said both our pump as well as the signal is actually launched from one end of the fiber ok, so the pump is actually getting absorbed as it propagates down the fiber ok and what can you say about how the pump will get absorbed what would be the pump power as a function of distance? How would that look like? It will be decreasing and it will be exponentially decreasing, right.

So if you were to be rigorous about this I would say that you know if I launch a certain pump power it is going to be exponentially decreasing acid propagates down the fiber and correspondingly the inversion that you get will also be exponentially decreasing, so the inversion is not uniform in a real case ok but for simplicity in our problem we have assumed that it is uniform, ok.

I just wanted to give you that perspective our problem says okay 80 percent of ions are in the excited state all along the length of the gain medium, so you somehow managed to pump from the side of your fiber and get them all excited get the erbium so excited which is practically a very challenging thing to do but for the sake of this question we have assumed that, ok.

But this is just an illustration to say if I have if I know that so many ions are excited then I can actually find out what is my what is the gain of that system, so when we look at this sort of a system we see that is there is only two levels and of course they have a certain rate at which absorption takes place certain rate at which simulation takes place and we are tracking only the number density whereas we are assuming that W i which is the transition the stimulated absorption of stimulated emission transition rate is the same ok.

But this although it helps us in this understanding this is not a practical case right because if you have two only two energy levels you have to pump at that wavelength at a particular wavelength and you need to extract the signal at the same wavelength ok and if the transmission cross sections are the same then you almost have assuming that all your ions are excited the best case you have equal population for N 2 and N 1 because both the transition equally probable it is like tossing a coin, right.

So you will have equal probability for N 2 and N 1 and which case you know it is not possible to get your gain amplification in a two-level system ok. So you cannot normally have just two levels for your amplifier or your laser ok, so what you normally have is what we were trying to project in the case of erbium right you would have another energy level that is also participating in this process.

So you essentially pump to a higher energy level lose some energy due to non radiative relaxation and then you have a inversion built to provide amplification ok. So let us look into such a model let us evolve from just a two-level system to a three level system and let us see how things work out in a three level system ok.

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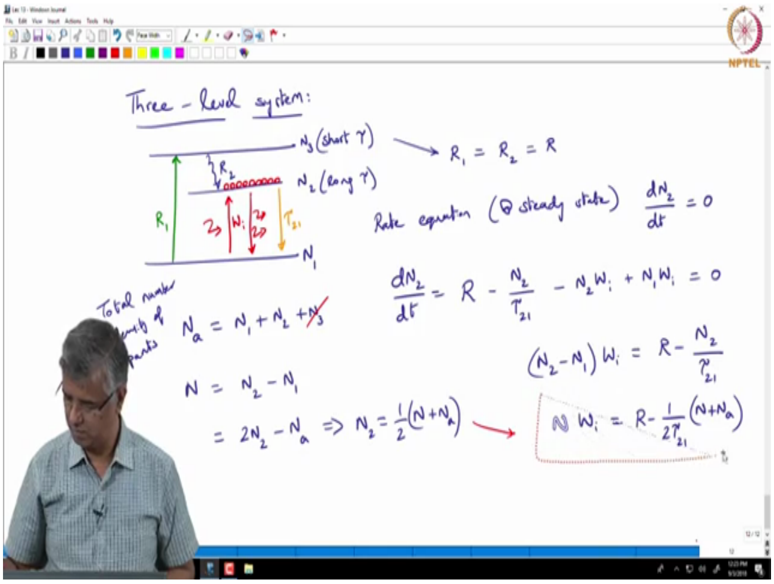


Three - level system:

$\rightarrow R_1 = R_2 = R$

Rate equation (@ steady state) $\frac{dN_2}{dt} = 0$

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}} - N_2 W_i + N_1 W_i = 0$$

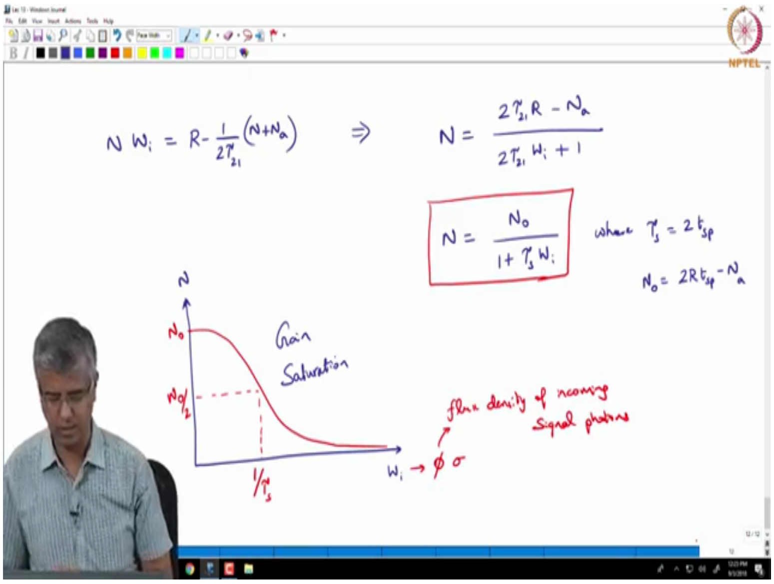
$$(N_2 - N_1) W_i = R - \frac{N_2}{\tau_{21}}$$

Total number density of dopants

$$N_a = N_1 + N_2 + \cancel{N_3}$$

$$N = N_2 - N_1$$

$$= 2N_2 - N_a \Rightarrow N_2 = \frac{1}{2}(N + N_a) \rightarrow \boxed{N W_i = R - \frac{1}{2\tau_{21}}(N + N_a)}$$



$$N W_i = R - \frac{1}{2\tau_{21}}(N+N_a) \Rightarrow N = \frac{2\tau_{21}R - N_a}{2\tau_{21}W_i + 1}$$

$$\boxed{N = \frac{N_0}{1+\tau_s W_i}}$$

where $\tau_s = 2t_{sp}$

$N_0 = 2Rt_{sp} - N_a$

So let us just consider three level atomic system ok in which we you have a ground state, you have an excited state, so you have your pumping happening maybe I can use a different colour for this may be pumping is happening let us say at a rate R 1 right, so you have pumping happening at rate R 1, it goes to this energy level and from there let us say it rapidly decays to another energy level ok.

So and if you are told that suppose this is the N 1 this is N 2 and this is N 3 if you have a condition that n 3 corresponds to a short lifetime very short lifetime this is the case of erbium also right, so erbium you have this N 2 level which has a long lifetime N 2 for erbium in silica could be in the order of 10 milliseconds right whereas N 3 the energy level corresponding to the 10 3 that is actually the order of a fraction of a millisecond ok, so it is got 100 micro second.

So if you have this condition what you can say is if I say this is happening at R 2 you can say that whatever rate this would imply that whatever rate I am pumping I am rapidly giving up that energy through non radiated relaxation, so that R 2 which corresponds to the rate at which that relaxation happens is almost as rapid ok, if you have a very short lifetime for that excited state right.

And so then you can represent that entire thing with a common rate R ok and once you relax to that energy level E 2 where the number density can be represented as N 2 especially if you have a very long life time for this energy level you can say that basically all the erbium atoms or all the atoms in this particular system are sitting in this higher energy level and from there you know there are different events that are possible one is that it can actually do a spontaneous emission right.

So maybe just so I did the squiggly line for non radiative relaxation but spontaneous emission is actually a radiative relaxation, so you can you can either have spontaneous emission and let us say that is happening with the time tau 21 with a rate defined by 1 over tau 21 the you can have spontaneous emission or you could have stimulated emission right, you could have stimulated emission you could also have stimulated absorption, ok.

Let us consider this rate to be W i ok that is a common rate at which both the absorption and emission can happen between those two energy levels, ok. Now to find out how the what we are interested in this how is the inversion changing as a function of certainly pump power but also as a function of the incoming signal photons right this is the signal photons that you want to amplify in this medium, ok.

So you are trying to multiply these signal photons but we are interested in the inversion or through that the gain that is available from the system for different levels of signal photon density ok, so that is what we are trying to achieve. So to do that you have to write what is called the rate equation, so you have to track all these rates ok, so let us go ahead and try to write the rate equation for a system like this and let us say we are looking at the rate equation at steady state ok, so we are not looking at a transient phenomena right.

We have turned on the pump we have allowed all the transients to settle down and then once you settle down in the steady state we are interested in knowing what is the gain that we have for such a system but what is steady state mean? Steady state means that essentially if you are looking at the number density right N 2 how it changes with respect to time what would that be in steady state that should be equal to zero right because there is no change happening that the pump level is held constant and you are extracting a certain gain from this system so in steady state that dN 2 over dt should be equal to zero.

So let us write an expression for dN 2 over dt, so the rate at which you are pumping atoms to a higher energy level right where you are populating that energy level E 2 what is the rate R right, that is the rate at which you are populating that excited in state and from that excited state you may lose some population to spontaneous emission right, so that is going to be given by N 2 over tau 21 right, since you are losing that number density you have a minus sign corresponding to that right.

You may lose some population due to stimulated emission and that rate is given by N 2 multiplied by W i but you also have a way of gaining population that is too stimulated I am absorption what will be that rate N 1 times W i, you are understand how we are writing this rate equation this is very important for the discussions that we are going to have you know as we move on.

So you just you just tracking the rate at which different events are happening. So the question is are we considering the inversion as constant yeah this is we are not actually doing this as a function of length yet ok, so we are just looking at one particular location and at that location you know the atomic system that we have is having a three level three energy levels with these properties and we are just trying to find out what is the inversion at that particular location, so the length dependence does not come into the picture yet.

So of course you can rewrite this and say N 2 minus N 1 multiplied by W i is equal to R minus N 2 over tau 2 1 right, where N 2 minus N 1 once again you can write it as N which is what we want to find out ok, so that is good but on the right hand side you still have N 2 can we write N 2 in terms of N and the total number of you know atoms so the total number density let us say is N a, ok.

So if we say n a is equal to N 1 plus N 2 plus N 3 but what can we say about N 3 for this specific thing that we are considered since it short lived there in steady state N 3 is almost zero right. So I can knock off N 3 from this and then say N a equal to N 1 plus N 2, so from that perspective if you if that is the case then if you look at the inversion, inversion is given by N 2 minus N 1 and N 1 can be written as N a minus N 2, so this will be this can be rewritten as 2 times N 2 minus N a right and this would imply that if I can write N 2 in terms of the inversion and N a, so N 2 is nothing but one half of of N plus N a right, so there is no magic in this, this is just straightforward expression.

So the question is what is the N a? Na is the total number density of the dopant, dopants that we have these dopants are the ones that are giving us gain, right. The dopants are have these energy levels appear mentioned here ok. So now I can substitute this into this other expression over here, so from here I can say that N into W i is going to be equal to R minus 1 over 2 times tau 21 multiplied by N plus N s right and so in this case we know tau 21 for a given atomic system let us say we know what it is already been characterized what is that spontaneous lifetime, Right.

So we know tau 21 for a given atomic system, we also know what is a doping density, we know N a and let us say we know what our pumping rate is right, that just corresponds to how much what is the rate at which we are sending in pump photons, so if the all those are known then the only unknowns are N and W i so i can actually find out what is the expression of the inversion as a function of W i, ok.

What is W i now? That is the stimulated absorption of the stimulated emission rate right, how is W i expressed? We looked at in the last lecture what is W i mean? That corresponds what is it depend on that corresponds to the flux density of photons that are coming in and what else the cross section right, the absorption or emission cross section ok, so W i corresponds to that.

So essentially what I can do is I can now track inversion as a function of the flux density of photons that are coming in and mind you I am NOT talking about pump photons because pump photons are separate I am you know tracking that through R, what are the photons that I am talking about here? These correspond to the signal photons ok, the photons that need is to be amplified in the system, ok.

So I can just rewrite this maybe we can just carry this over right, so from this I can say this implies that N, I can just rearrange some terms and say this will correspond to 2 times tau 21 R minus N a divided by 2 times tau 21 W i plus 1 right. So what I am doing is all the factors corresponding to n I am taking to the left side and the other terms I am retaining on the right side and then just simplifying from there, so I get an expression like this.

So this I can write as N equal to N naught divided by 1 plus some term Tao s multiplied by W i because I want to plot N as a function of W i, so of course where I can say where tau s would correspond to two times tau 21 ok, so that is just a timescale that corresponds to 2 times tau 21 and tau 21 is actually the spontaneous lifetime from that particular energy level, energy level 2 and N naught is nothing but 2 times R into tau 21 I can write it as t sp in fact this one also I can write as t sp, t sp I am substituting instead of tau 21 but it is just saying it is a spontaneous lifetime, right.

So N naught is given by 2 times R times t sp minus N a ok but if I plot this it actually gives me some very important result that needs to be kept in mind, right. So N I am plotting as a function of W i how is that going to look, so it is of course when W i is 0 that is going to have the value of N naught right and then as you increase it is going to go down you know the value of N is going to go down and it is going to take the specific value of N naught over 2 at what point and when you get this, when W i equals 1 over tau s, right.

So at a particular value of W i it gets to N naught over 2 and just to emphasize what is W i indicate W i is given by phi times sigma right, so if sigma is the transition cross section and phi corresponds to the flux density of incoming signal photons, right. A sigma is constant right for a given atomic system that sigma is a constant but phi is a variable right, so what is this graph tell us? This graph tells us that initially I don not have any signal photons coming in but I am pumping, right and so I am pumping and I am maintaining a certain inversion.

What are that inversion? What does it correspond to N naught? Right when W i is zero and I have a certain inversion that is what we are talking about 80 percent of the inversion I mean 80 percent of all the ions are sitting in the excited state that was just an example normally it is not that higher number ok but nevertheless you have a certain inversion but now I start sending in some signal photons that needs to be amplified.

What does that do according to this graph that actually reduces the inversion that is available to you, why? Because they have been depopulated right so if you are getting amplification right and you if you are actually sending in more and more photons if you are sending let us say 100 photons and let us say they are all spread in time the first few photons they say the first 10 photons comes and it is easy and not, ok.

It sees this large inversion most of the ions are in the excited state, so it gets lot of gain but in that process what does it do it depopulates that excited state it goes to the ground state, so for the remaining photons that are coming in you do not have as much an inversion available to you right unless of course you pump harder and replenish this thing but that actually takes some finite time, so immediately after an amplification happens that inversion gets reduced in other words this is what we call us this leads to what is called as gain saturation because reduced inversion means reduced gain.

So as you increase the incoming flux density of signal photons you are reducing the gain that is available to you ok. So let us stop at this point and this is actually a very important concept to understand because as we go on to not just looking at amplifiers but as we go on to looking at lasers also we will keep revisiting this concept ok and this is what we have done for a three level system.

Now we will go on to consider a four level system and see how things could be different in that case ok, so that is what coming up in the next lecture.