Introduction to Photonics
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Lecture No 02
Diffraction & Interference

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The learning outcomes of this course are to:

1) Identify the fundamental principles of photonics and light matter interactions.
2) Develop an ability to formulate problems related to photonics, photonic structures, processes and analyze them.
3) Identify processes that help to manipulate the fundamental properties of light.

So let us go back and have a quick recap of where we were yesterday.

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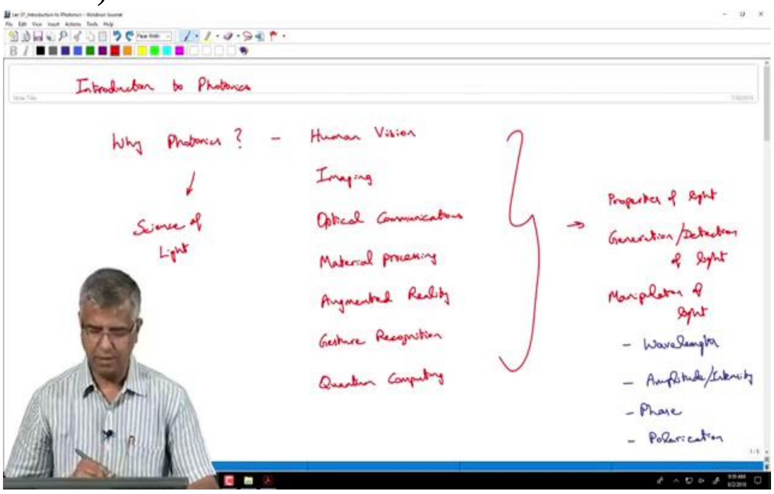

We started with a general statement, as to why we should be interested in photonics. We listed down a number of applications where photonics play a central role and then we went on to look at how we try to analyze the photonic processes.

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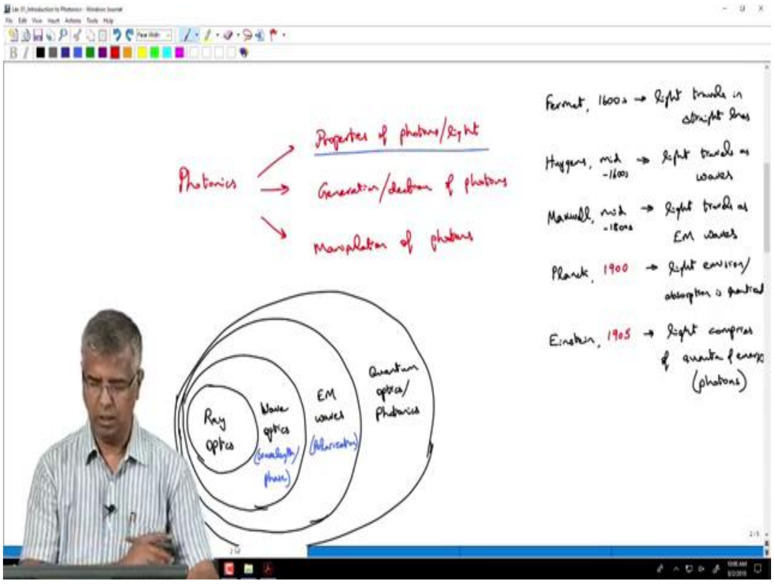

The photonics processes can be analyzed progressively using the following methods:

1. Ray optics – which uses Fermat's principle that light travels in straight lines

   Ray optics cannot explain concepts like wavelength and phase of light, so we go in for

2. Wave optics

   Since Wave optics cannot explain polarization, we go in for

3. Electromagnetic optics – which started with Maxwell - making his declaration around the mid 1800s- that light travels as electromagnetic waves.

   And as this did not explain quantization, we finally have

4. Quantum Optics/Photonics - Planck actually came up with this paper in 1900, that essentially was saying that light emission and absorption are quantized. And that was followed by Einstein's observation, that light itself comprises of quanta of energy which later on was termed as photons.

So, we will start with examples in ray optics and then proceed to examples with wave optics and beyond that, we will get on to electromagnetic optics and, eventually to quantum optics,.

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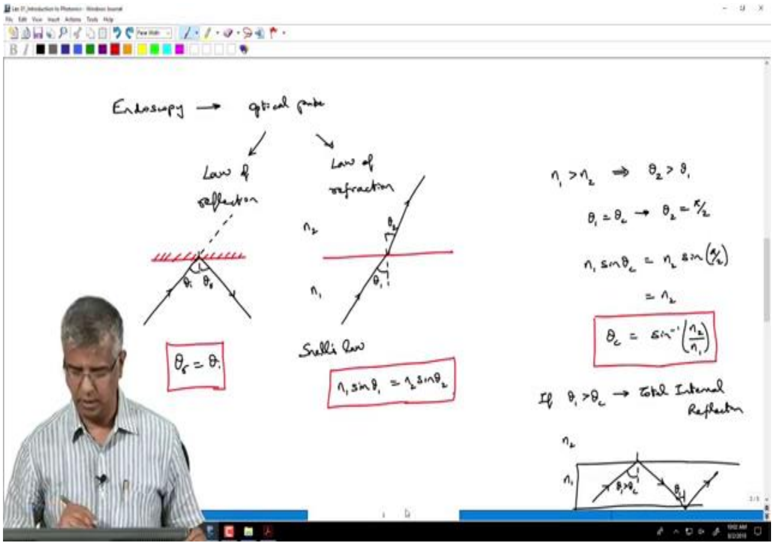

And as far as ray optics examples are concerned we take the example of endoscopy where we need an optical probe for doing endoscopy. It is fairly simple to design this optical probe.

We just need to know the law of reflection and law of refraction.

Endoscope:

In this structure, you have a material with refractive index n_1 which is surrounded by another material with refractive index n_2. We could have total internal reflection at the interface if the angle at which the light ray hits the interface is greater than the critical angle. Now, if you have a consistent structure where you have this interface between n_1 and n_2 and those two interfaces are parallel to each other, then we will have consistent guiding of light within this structure.

What we are typically interested as far as the endoscope (a light guiding structure) is concerned is -what is the maximum cone of angles that we can pick up using this structure?

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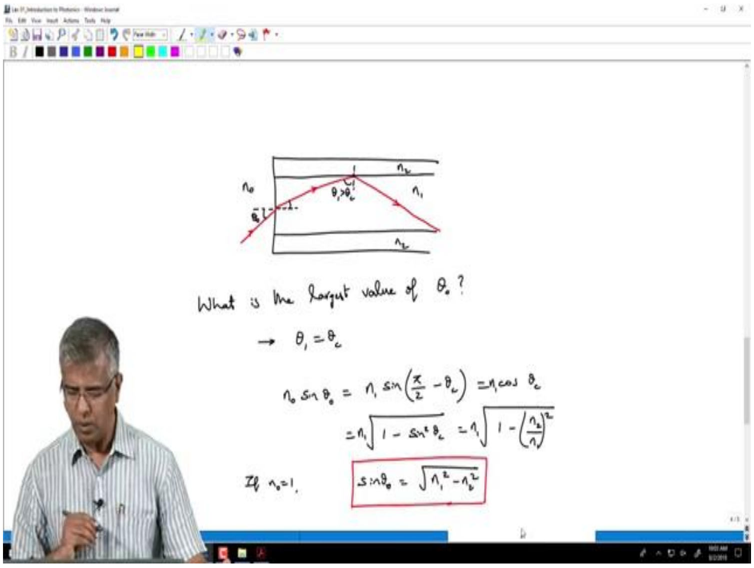

For total internal reflection to occur, the incident angle should be greater than the critical angle at the interface between n_1 and n_2.

And then if you trace it back, as far as launch of light into this structure is concerned, what we will have to essentially look at is what is happening at this interface between n_0 and n_1.

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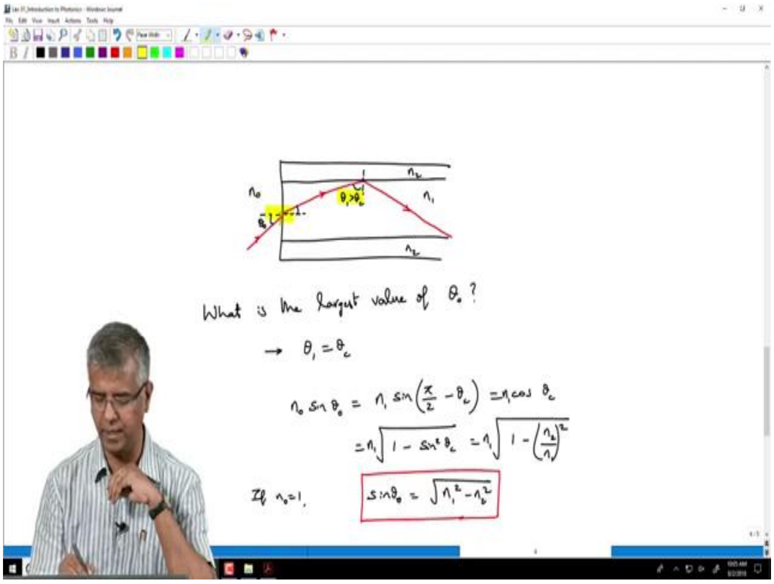

Now, typically the refractive indexn$_0$forthe outside medium is air and then you are going into glass material, with refractive index n_1.

You apply Snell's law over here andtry to find out the maximum angle which can be supported at this interface such that the light is guided through the structure. So the limiting condition for such a guiding is $\theta_1=\theta_c$

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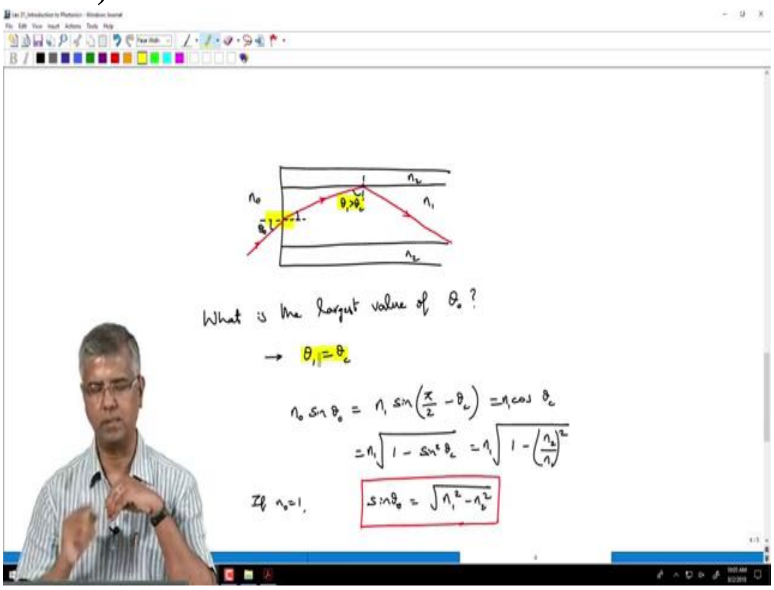

We need to have an incident angle greater than θ_c for light to be guided, and if you look at Snell's law at this particular interface, for very small angles of θ_0, for example the limiting case is where θ_0 is going straight down this dotted line over here, the light ray is going straight down, there is no problem. Light will go straight through this wave guide.

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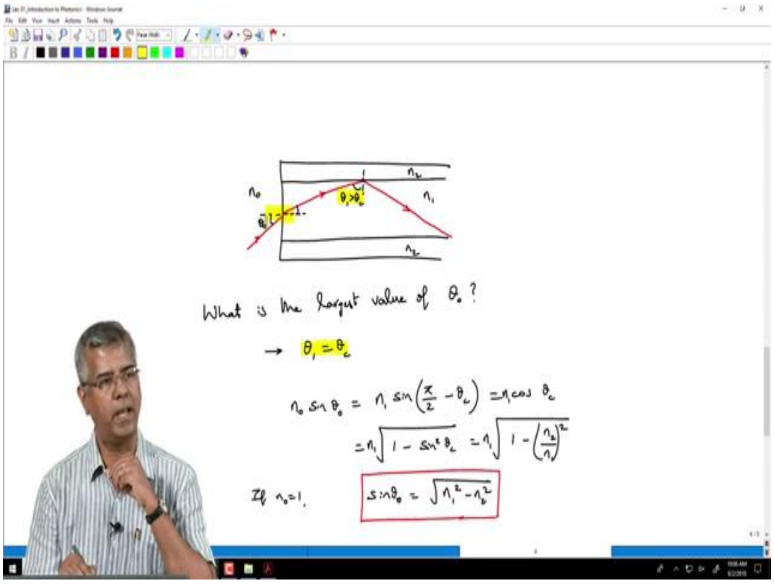

And as you increase θ_0, you get to a point where your angle at this interface is going to become smaller and smaller.

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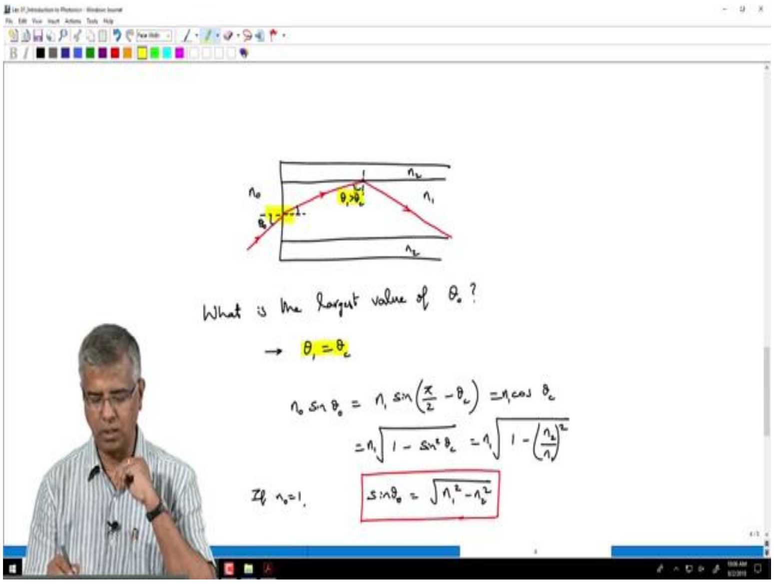

And if the angle θ_1 is becoming less than θ_c, then there is no guiding of light. So the limiting condition would be $\theta_1 = \theta_c$.

So, if I am able to find the corresponding angle θ_0, then I would say that anything within that cone of angles defined by θ_0, is going to be guided in this structure. Anything outside of θ_0, is going to be such that $\theta_1 < \theta_c$, then it is not guided by this structure.

So this cone of angles that is allowing light to be guided in the structure is called the numerical aperture of this endoscope.

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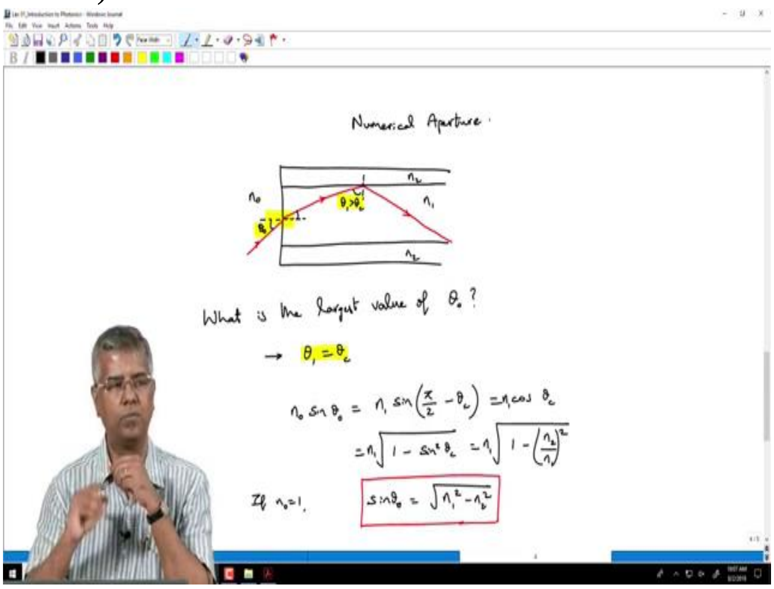

So what we are actually trying to get to is - what is the numerical aperture of this light guide? To do that, we apply Snell's law at the input interface, so

$n_0 \sin \theta_0 = n_1 \sin \theta_1$ where θ_1 is the refracted angle.

you would realize that if the angle made between n_1 and n_2 is θ_c this θ_1 has to be $\pi/2 - \theta_c$

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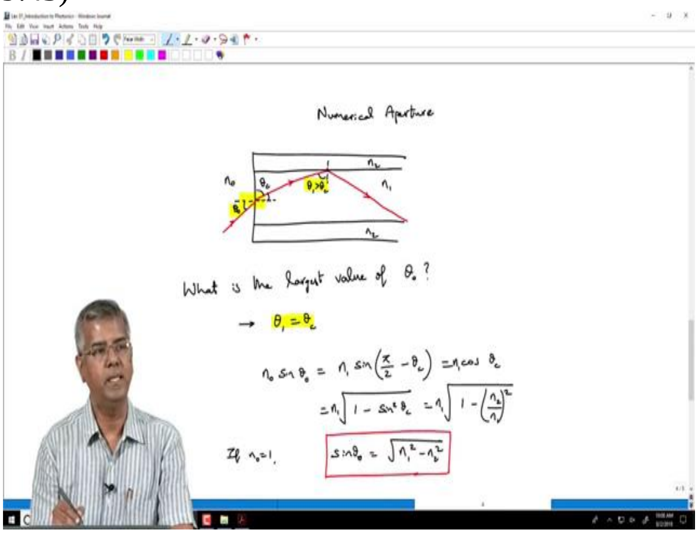

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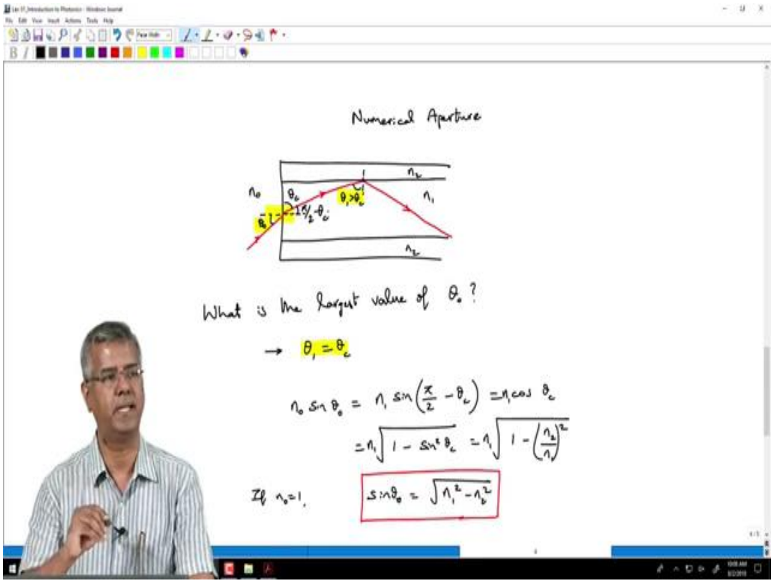

So the RHS has to be $n_1 \sin(\pi/2-\theta_c)$.

Since $\sin(\pi/2-\theta_c)=\cos\theta_c$, and $\cos\theta_c$ can be written in terms of $\sin\theta_c$ as

$\cos\theta_c = \sqrt{1 - \sin^2\theta_c}$ we get

$n_0 \sin\theta_0 = n_1\sqrt{1 - \sin^2\theta_c}$

We saw in the last lecture that $\sin\theta_c = n_2/ n_1$. So by substitution, we get

$n_0 \sin\theta_0 = n_1\sqrt{1 - (n_2/ n_1)^2}$

and if we take n_1 in common, and consider n_0 corresponds to air, i.e $n_0 = 1$ then we get this simple expression for $\sin\theta_0$,

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

What this tells you is that if you want a very large numerical aperture, what should you have? What do you want from an endoscope? Normally you want a very large field of view, so you can see things on either side over a fairly long angular spread, right?

So how is it enabled as far as the structure is concerned? n_1 has to be very much larger than n_2

$$n_1 >> n_2.$$

So if we have a large index contrast between the two media, you can essentially support a large numerical aperture. So how is this realized?

You basically have a cylindrical wire; with refractive index n_1, let us say it is made of glass. And you coat it with a polymer, with a much lower refractive index.

So glass, you know you say refractive index of 1.5, and that is actually a very loose definition, because that is what you hear in high school textbooks - that glass has refractive index of 1.5, water has refractive index of 1.33 and so on.

But you have to take that with a pinch of salt, because in reality, that refractive index is actually dependent upon wavelength. Remember this thing about how you form a rainbow? How do you, get a rainbow naturally?

Sunlight consisting of different colors, is going through this raindrop, which can be modeled as a prism made of water. Water essentially has a different refractive index, a slightly different refractive index for each of those colors. So when you apply Snell's law, each of those colors separate out, in terms of the angle of refraction, and that is essentially what you see as dispersion which causes the rainbow.

So, in general, that is the key thought that you should have in mind - that the refractive index of material, or in more basic terms - the permittivity of the material, the dielectric response of the material, is frequency dependent or wavelength dependent or, in layman's language - color dependent.

The refractive index of a material is dependent on Color

So, it depends on the color as to what is the refractive index. But nevertheless, if you say that you have a glass central structure surrounded by a polymer structure which is of much lower refractive index, then you can make an endoscope with a very large numerical aperture, with a very large field of view. So, we have now seen how to use Ray optics principles in designing the optical probe of the endoscope.

With ray optics we just saw how to design an endoscope. We can explain dispersion in a prism, You can actually do optical system design,

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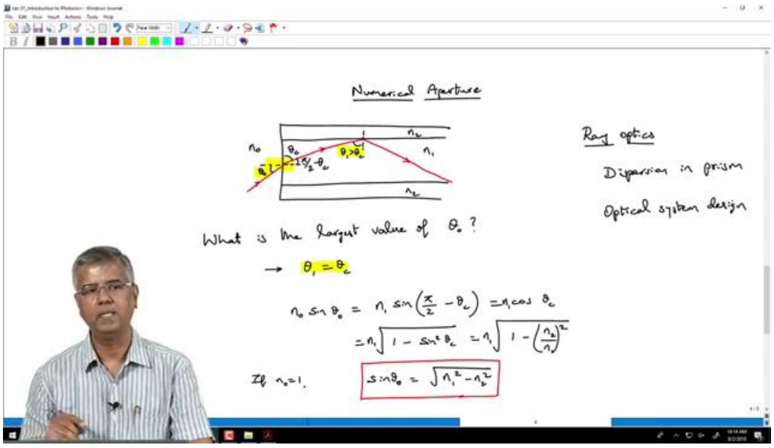

Which consists of multiple lenses, multiple mirrors and so on. You can do all that optical system design and go all the way up to designing a telescope using Ray Optics.

A telescope is nothing but a series of lens elements that are put together. You can go all the way up to designing the Hubble telescope.

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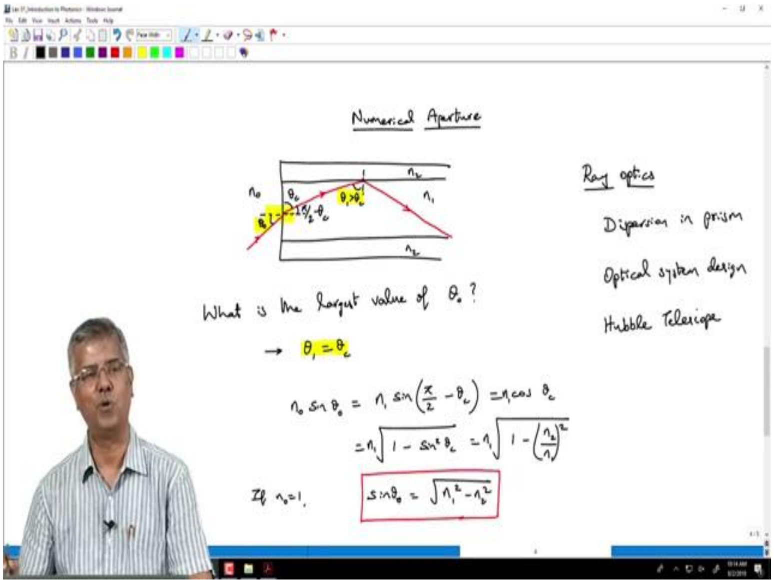

You know what a Hubble telescope is?

This is the telescope that people put in space, which is capturing images of the galaxy, deep out in space. So something as sophisticated as that could actually have the basic design of the telescope which can be achieved by just ray optics principles. That is the power of considering light as something that just travels in straight lines and you are able to deal with how it propagates through multiple interfaces.

So let us get a feel for designing a lens system. How do you use ray optics to see how light propagates through the systems?

So let us say you have an optical system, one of the key things that you define is an optical axis, basically the central line that joins all the optics within that system.

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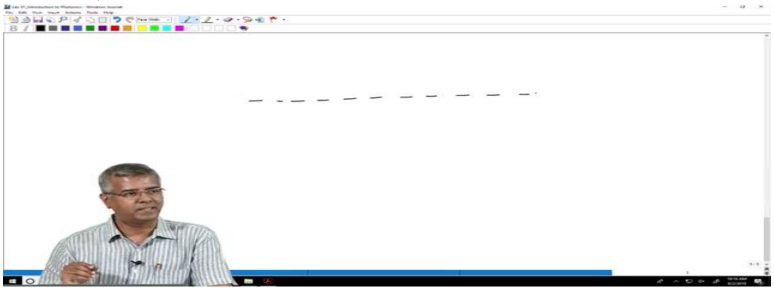

So let us say, it consists of a lens here, another lens

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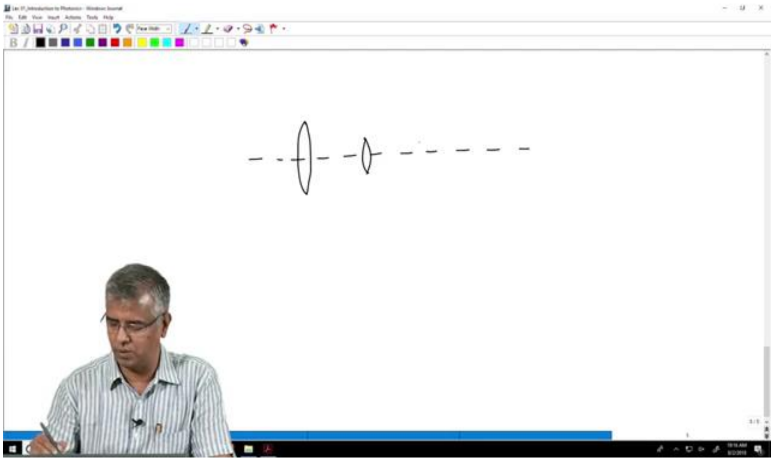

over here, let us say those two are what are called biconvex lenses

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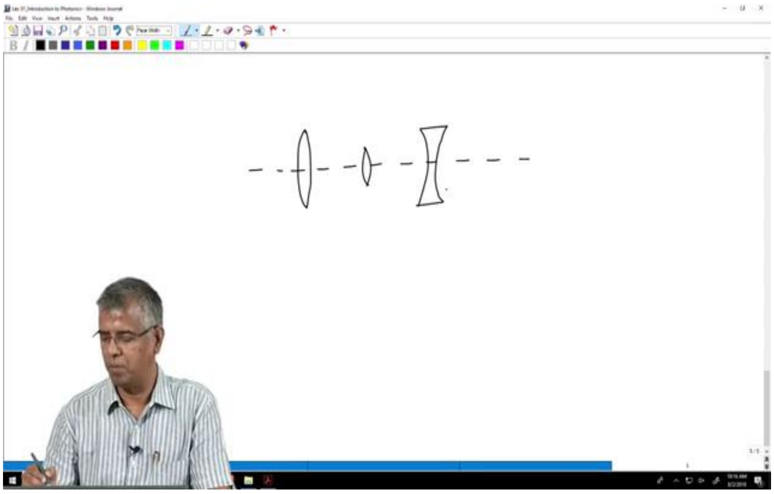

and this is actually a biconcave lens and may be another lens

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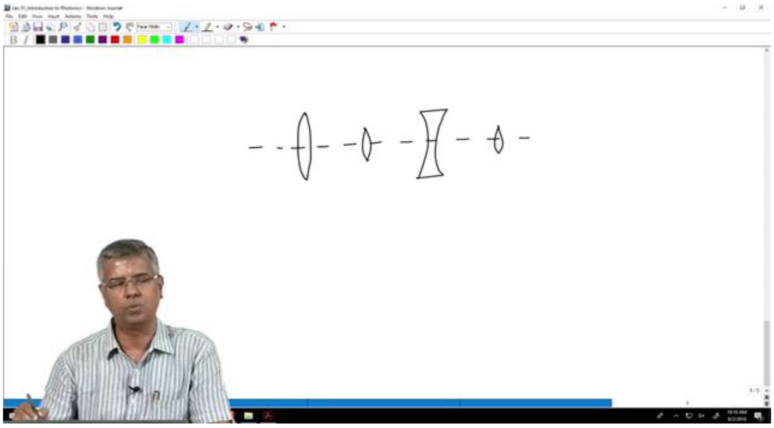

over here, right.

So if you want to analyze a system like this, what you want to know effectively is, if I consider a plane over here and a plane

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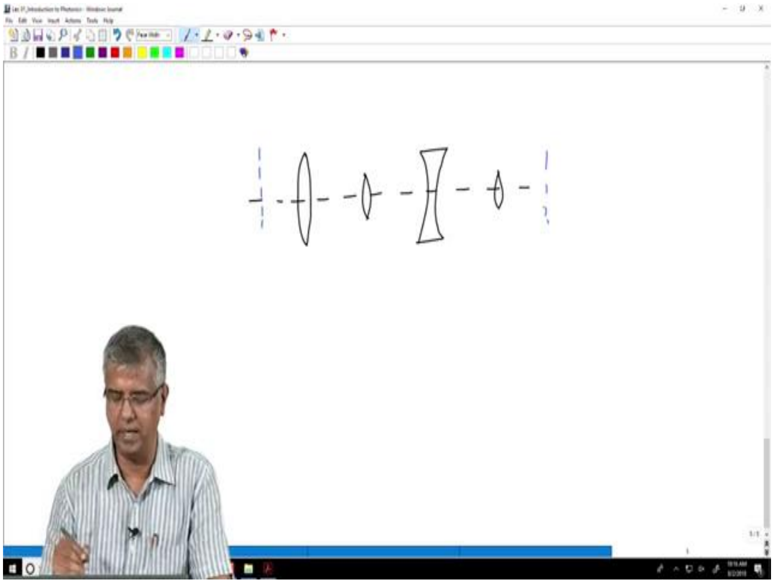

over here, Ok I want to look at a ray that is incident on this plane.

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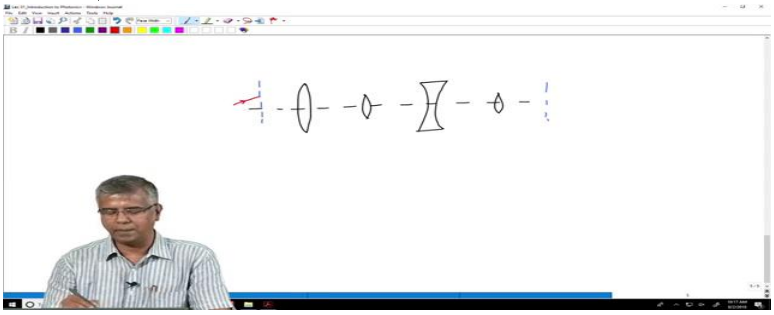

What happens to the ray as it propagates through the system and specifically I am interested in how the

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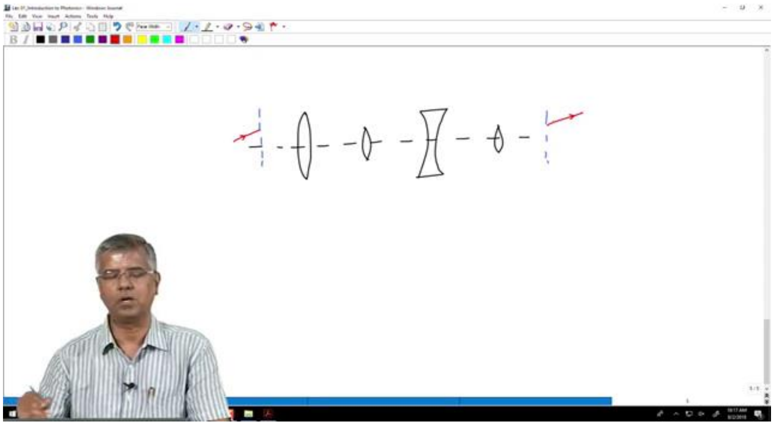

ray comes out of that optical system, right? That is a typical problem that we look at. So what you could do is define the distance from the optical

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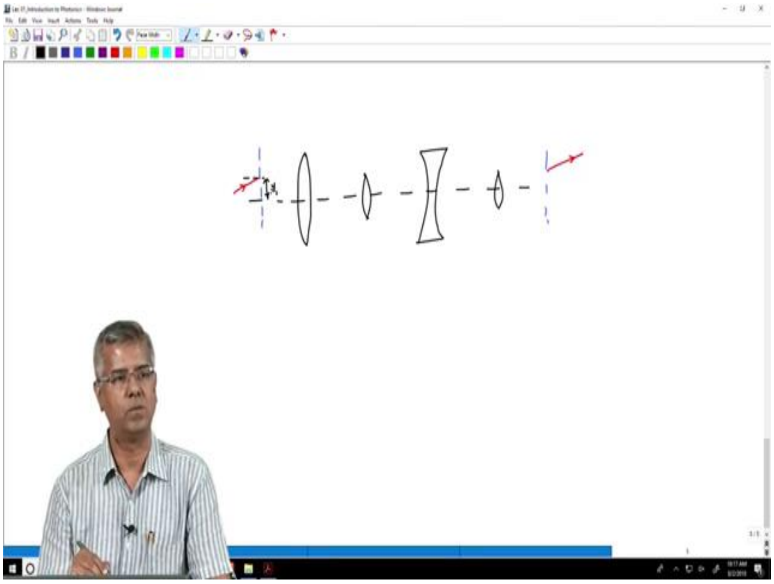

axis y_1 and let us say it

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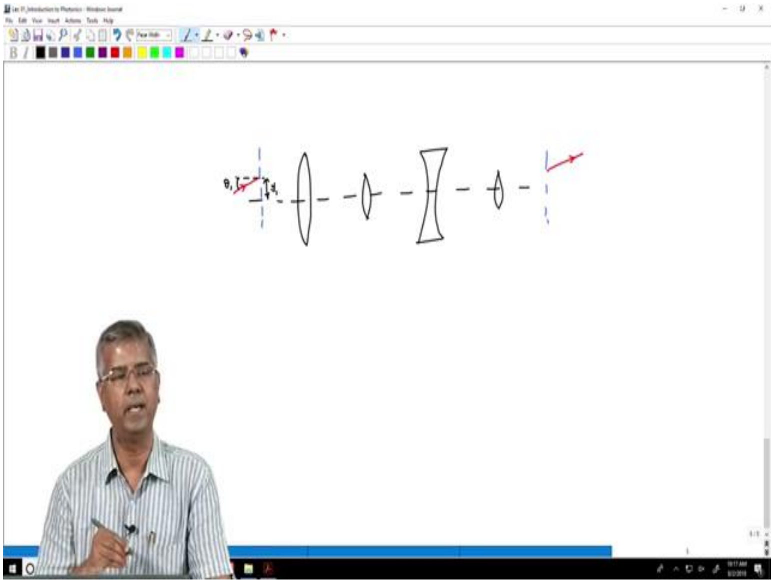

is making an angle $\Theta_1$with respect with the optical axis.

And over here

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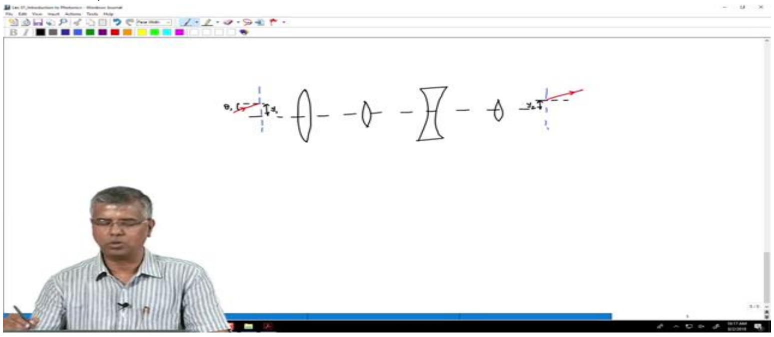

you go to y_2 and this is exiting

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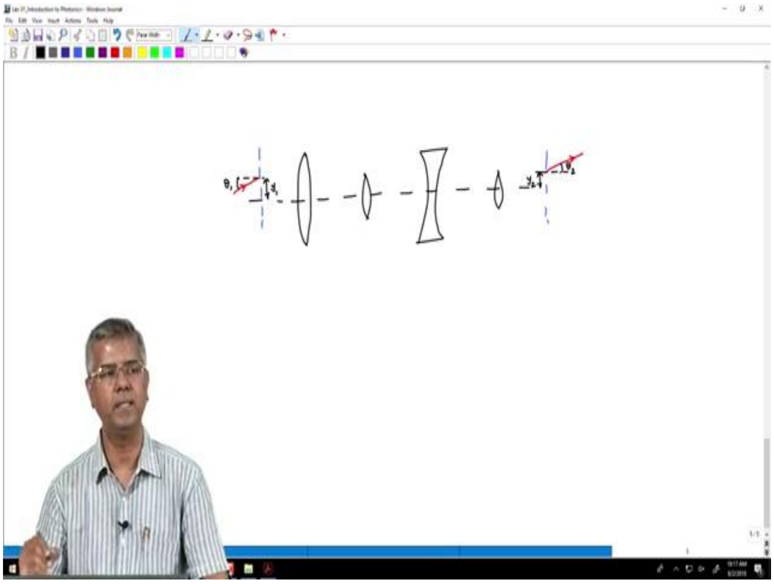

the system with an angle θ_2. So the idea is, if you are talking about a linear homogenous system, where it is homogenous within the lens, or within the propagation between the lenses it is all homogenous medium. If you consider a medium like this, you can actually write the output y_2 in terms of the input parameters.

Basically you say it has some dependence on where the ray is entering the system, so y_1 and it has got some dependence onwhat angle the ray is entering the system, θ_1.

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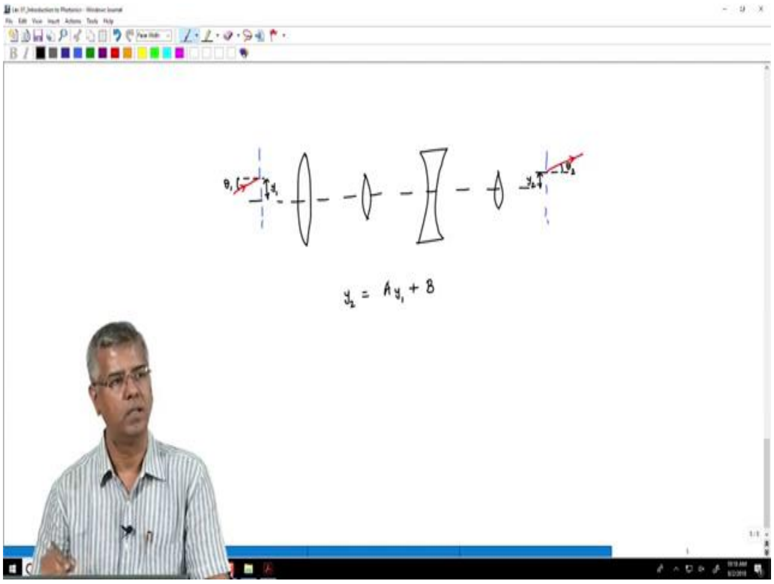

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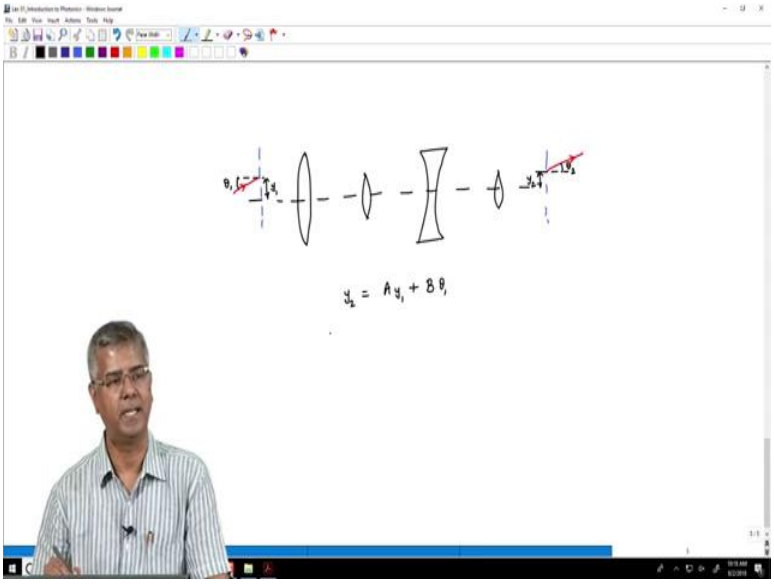

So $y_2 = Ay_1 + B\theta_1$

Similarly if you want to find θ_2, that again has a linear dependence

So you have

$\theta_2 = C y_1 + D\theta_1$

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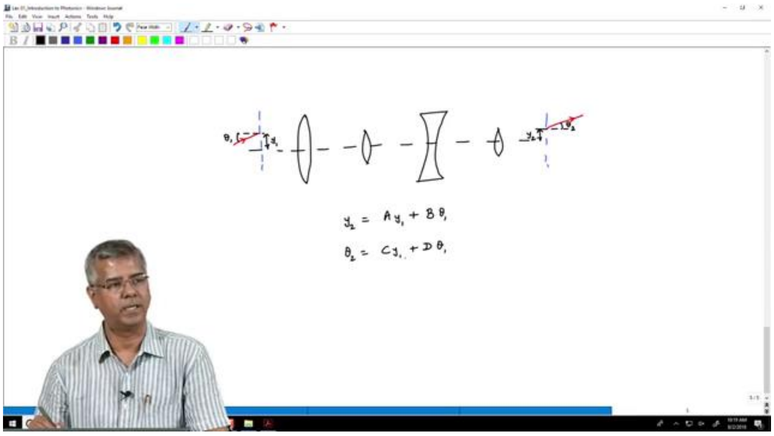

Ok to the point that you can write this in matrix form$\begin{pmatrix} y2 & \theta 2 \end{pmatrix}$ is what you

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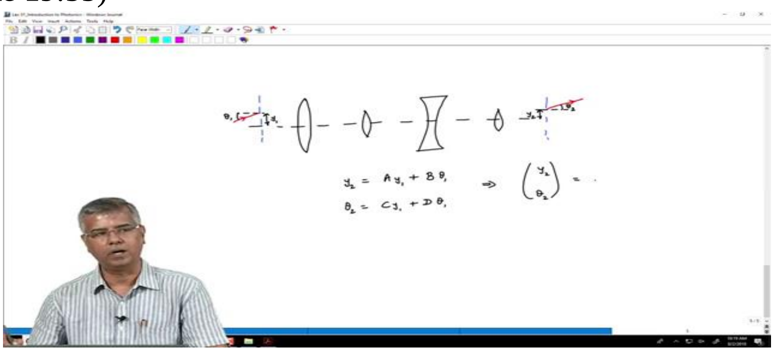

want to find out, $\begin{pmatrix} y1 & \theta 1 \end{pmatrix}$ is the input and then you have this matrix$\begin{pmatrix} A & B & C & D \end{pmatrix}$which defines the optical system

So, in matrix format

$(y2 \quad \theta2 \ ) = (A\ B\ C\ D\ )(\ y1 \quad \theta1 \ )$

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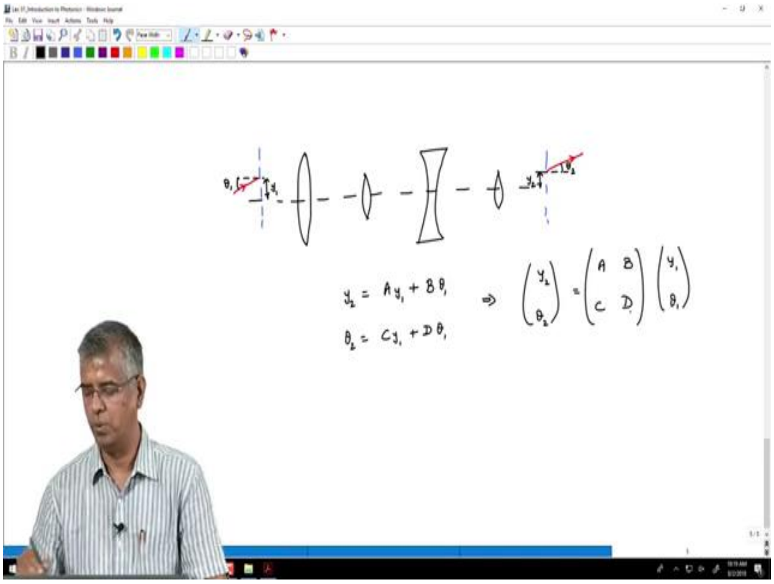

So if you can model the propagation of a light ray within each section of this optical system, through each surface of this optical system then, essentially let us say this is, what you call as the ray matrix

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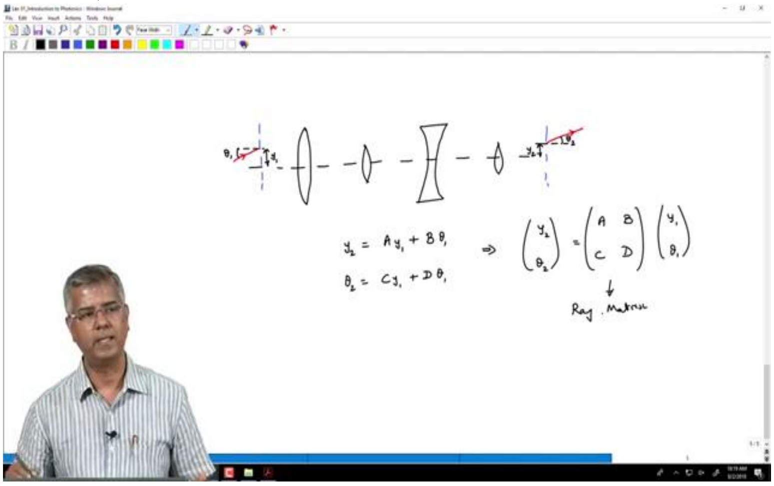

and the ray matrix is actually going through multiple sections, multiple surfaces and each one of those has its own$(A\ B\ C\ D\ )$Matrix

So, let us say there are n such occurrences, M_n, M_{n-1}, and so on, up to M_1, this would be the effective matrix that defines this entire thing.

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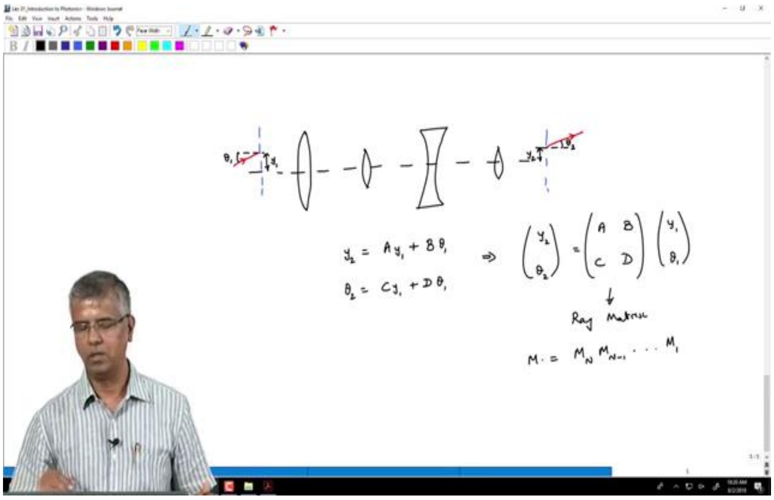

Just take one quick example and see how this works. I will take a very simple example in the interest of time. So let us say

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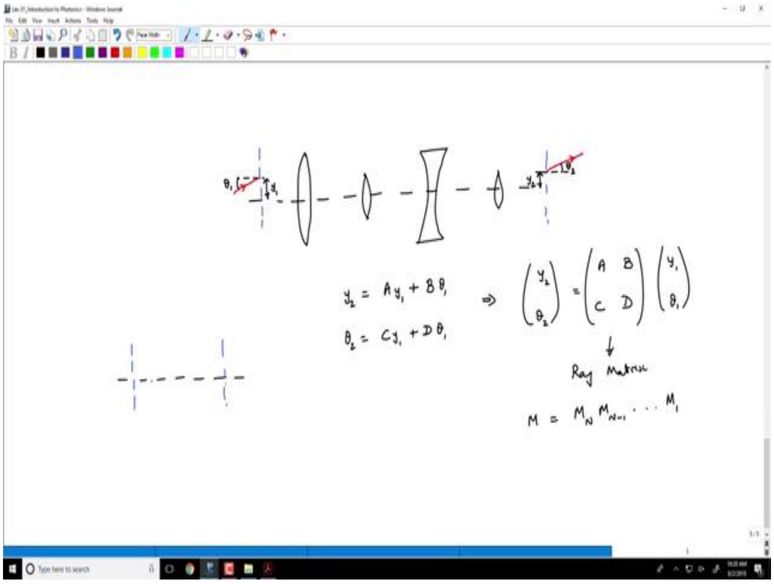

I just have a ray that is going straight through the system, Ok. I want to define how the propagation happens through free space -Without any elements coming to the picture. So how would the $(A\ B\ C\ D\ )$Matrix look for, something like this?

So I can essentially write, so this is y_1, this is y_2,

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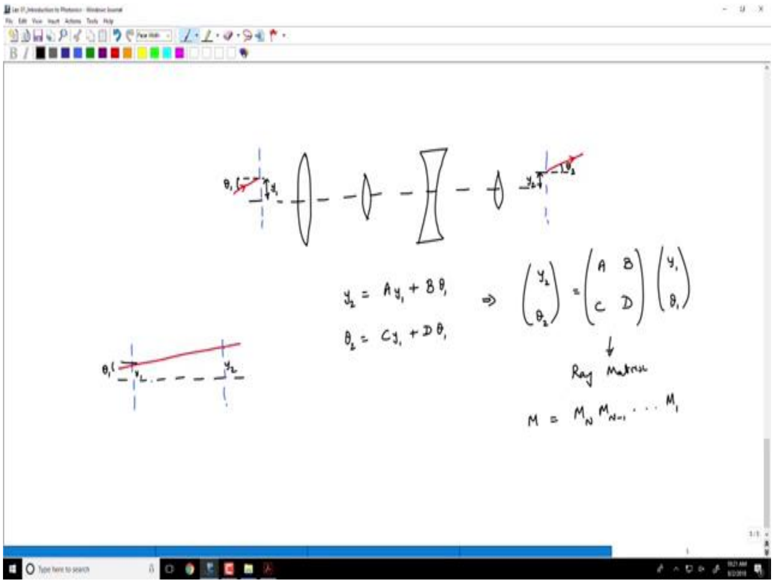

this is Θ_1, this is

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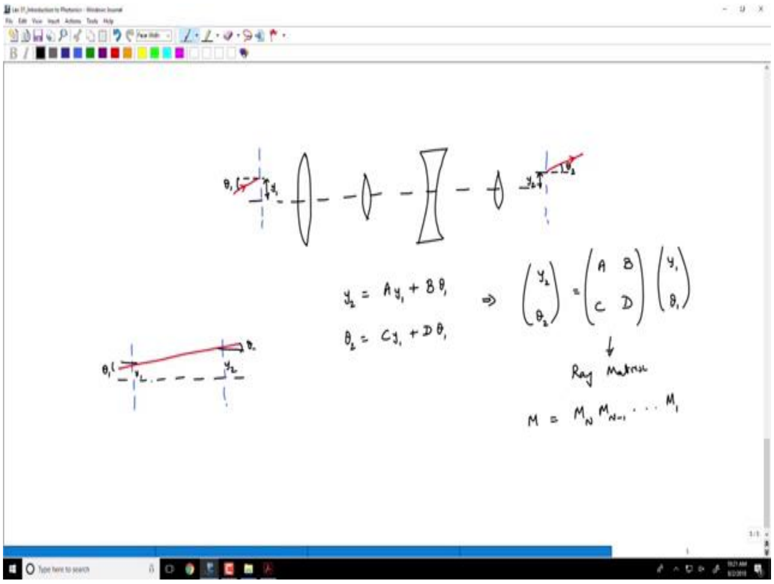

Θ_2, Ok.

So we know that $\Theta_2 = \Theta_1$,(as it is free space)

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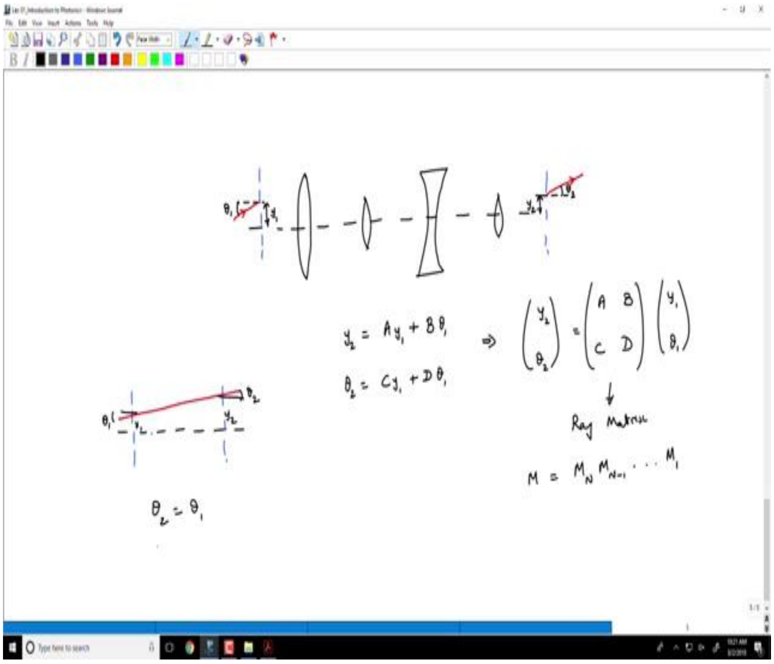

and what is y_2? So whatever this distance of propagation

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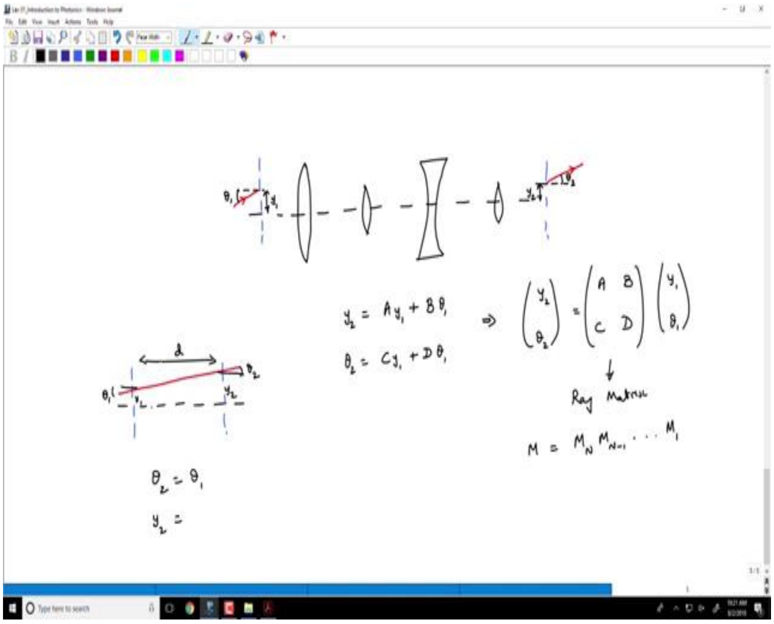

is, let us say that corresponds to d, so you essentially say this is going to be given by

$y_1 + d\,\Theta_1$ and $\Theta_2 = \Theta_1$

So we have

$$y_2 = y_1 + d\,\Theta_1$$
$$\Theta_2 = 0 + \Theta_1$$

So the$(A\ B\ C\ D)$matrix corresponding to this is A corresponds to value of 1, B corresponds to value of d, that is the distance of propagation in this medium. C is 0, Ok that is a coefficient of y 1 and that is 0, and D is 1. So the matrix for

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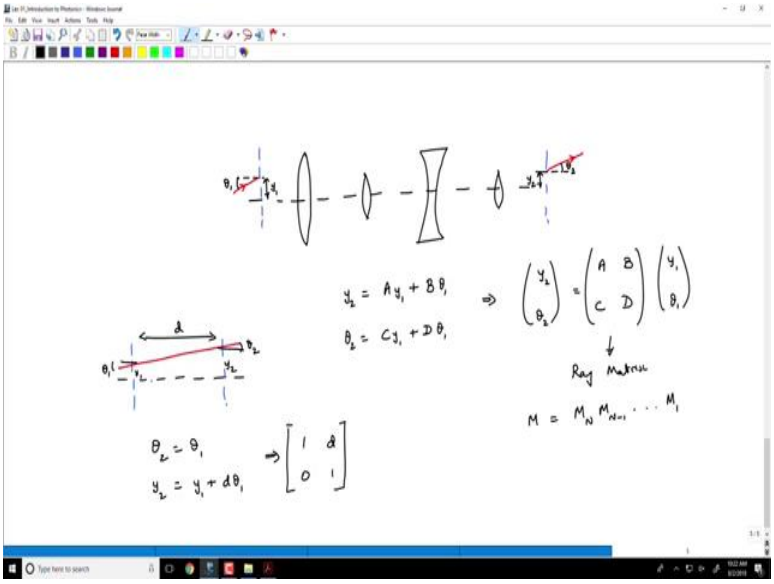

a simple propagation through free space corresponds to this.

i.e$(A\ B\ C\ D) = (1\ d\ 0\ 1)$

So you can do this for a lens as well, Ok and especially, there is one approximation which becomes very handy in these sort of situations. This approximation is called the paraxial approximation.

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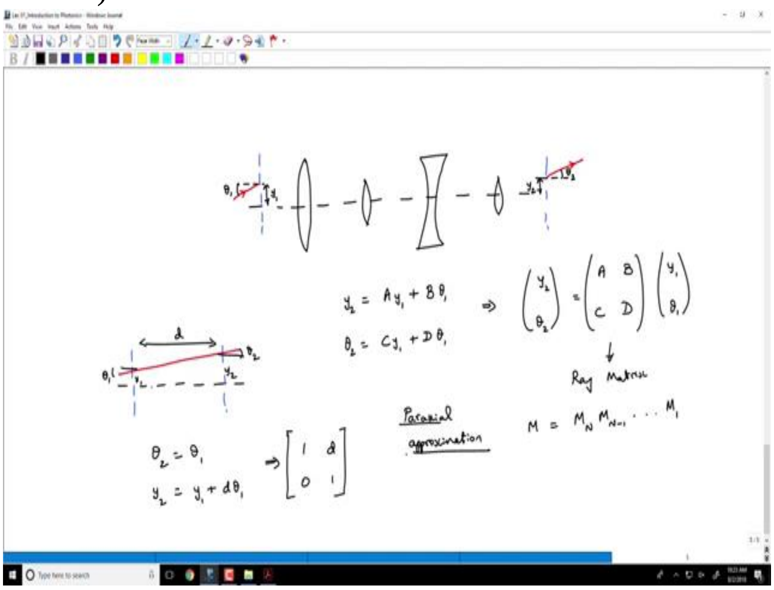

Par axial, right so it is something to do with axis, so what does paraxial mean? It essentially means that we are considering rays to be having a very small angle with respect to the optical axis, Ok

So in the paraxial approximation, if you write sin Θ, when Θ is small , the value of sin $\Theta = \Theta$, so sinΘ can be approximated as Θ.

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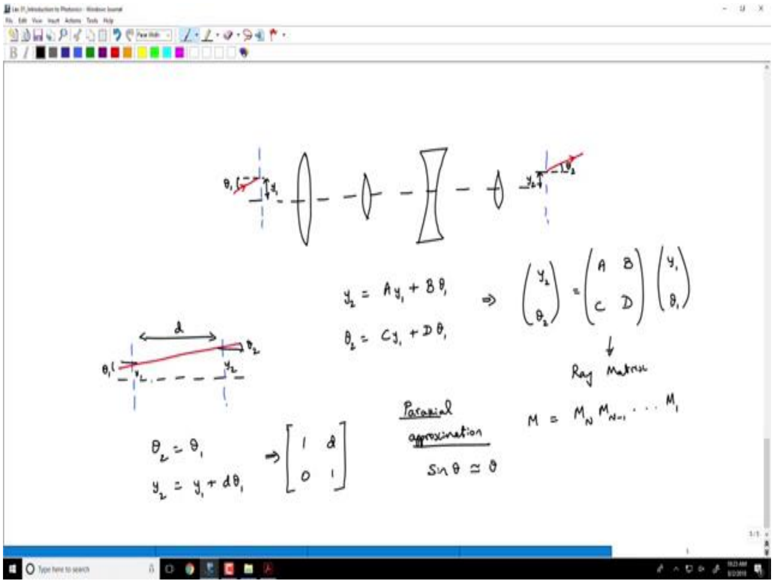

So how does that help? Because as you are propagating through this, you know optical system you are encountering surfaces and at each surface you are applying Snell's law. Snell's law says

$$n_1 \sin\Theta_1 = n_2 \sin\Theta_2.$$

But you do not want to put all these sin, cos things within this matrix. So Snell's law will become $n_1 \Theta_1 = n_2 \Theta_2$in the paraxial approximation.

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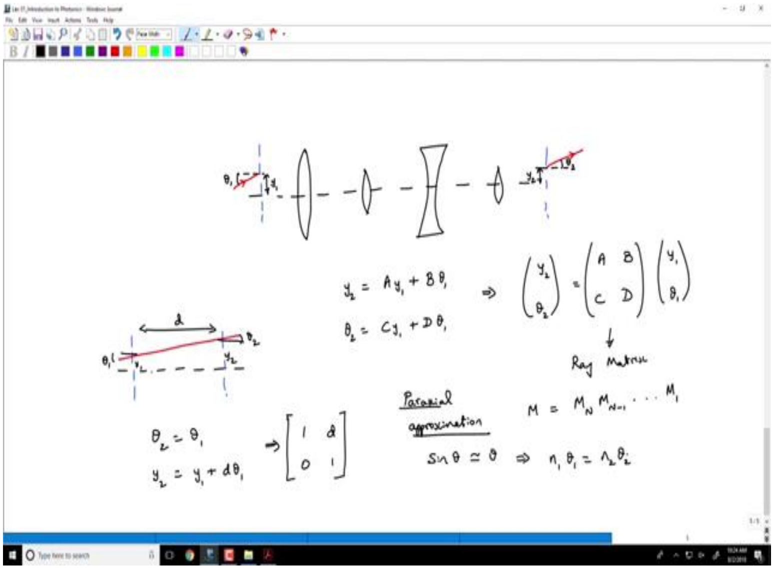

Then it is easy , form a corresponding matrix and then

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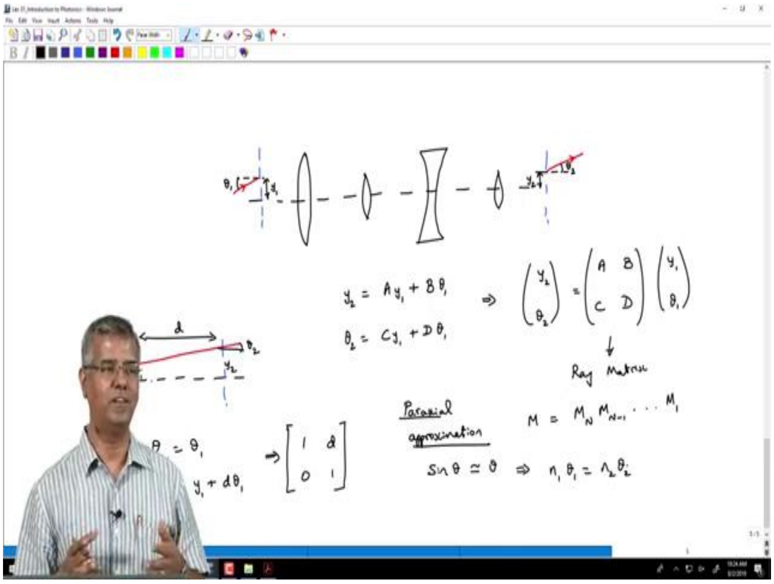

go through this thing. So what is the disadvantage of this? Obviously you cannot account for rays that are making a very large angle with respect to the optical axis.

So if you have a very large numerical aperture - like what we were trying to do with the endoscope - that cannot be modeled here.

But if you are talking about modeling a telescope, which is seeing something that is happening, thousands and millions of kilometers away, light coming from there is going to be fairly aligned to the optical axis of your telescope.

So paraxial approximation works very well if you are looking at really distant objects.

Can you apply this for the microscope?

Probably not - you are trying to magnify a small object and you have a very large spread of angles within that and so it is not easy to apply to a microscope,

So that just gives you a general thought of how far you can take ray optics, Ok. Any questions before we move on?

Professor: Why is the...?

(Professor – student conversation starts)

Student: 0:26:30.2

Professor: So all of these, once we put it this form, all of these are linear transformations, right, so the entire system becomes a linear system. And we are considering homogenous material so, so yeah we are taking essentially a linear response from the system. That is the basic assumption, Ok.

(Professor – student conversation ends)

So let us move on and

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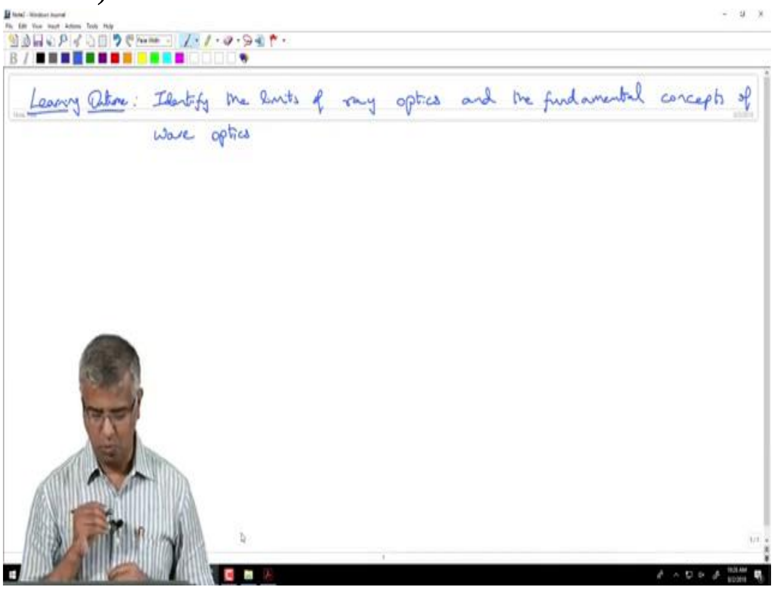

move back to what we wanted to carry on with, you know for the rest of today's lecture, Ok. And that actually takes us back to this tiny tit bit that I gave at the end of the last lecture, what did we do? What did I ask you to do? Right, there is one word to explain that.

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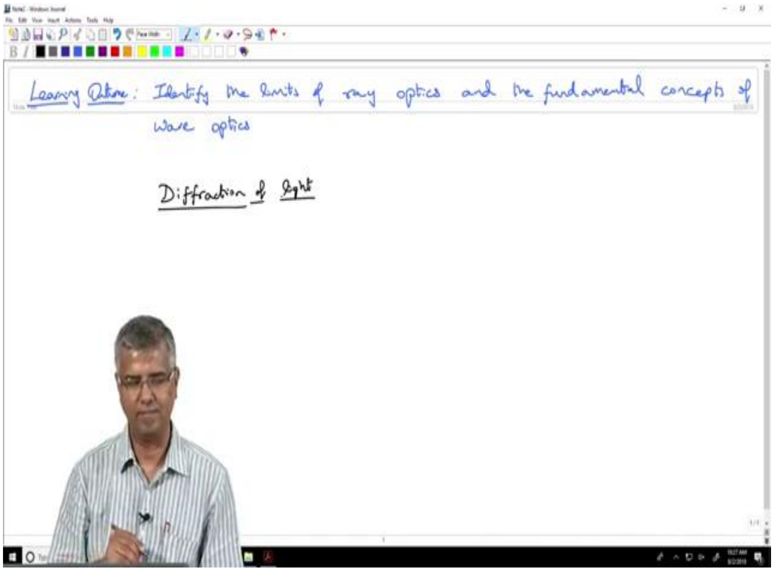

Right, so diffraction of light is what is happening.

So what is diffraction? Now whatever we have been saying so far or whatever we have been seeing so far is that when you have a large opening, you can use ray optics

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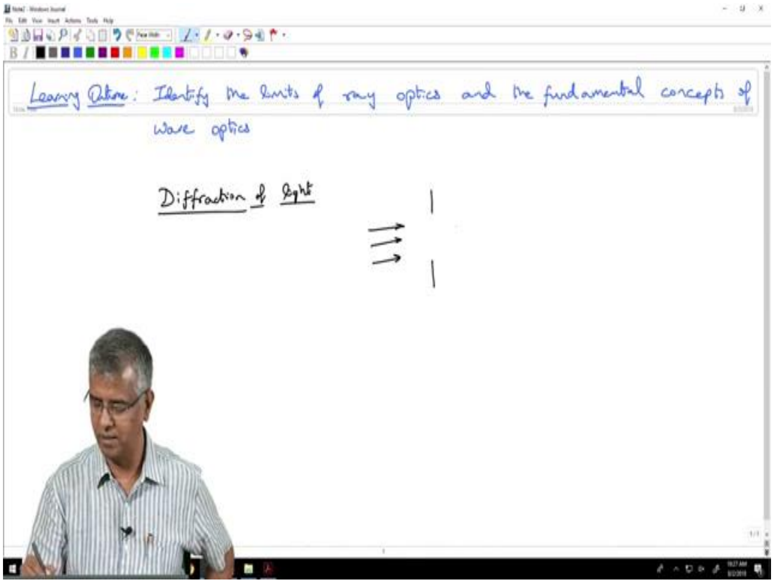

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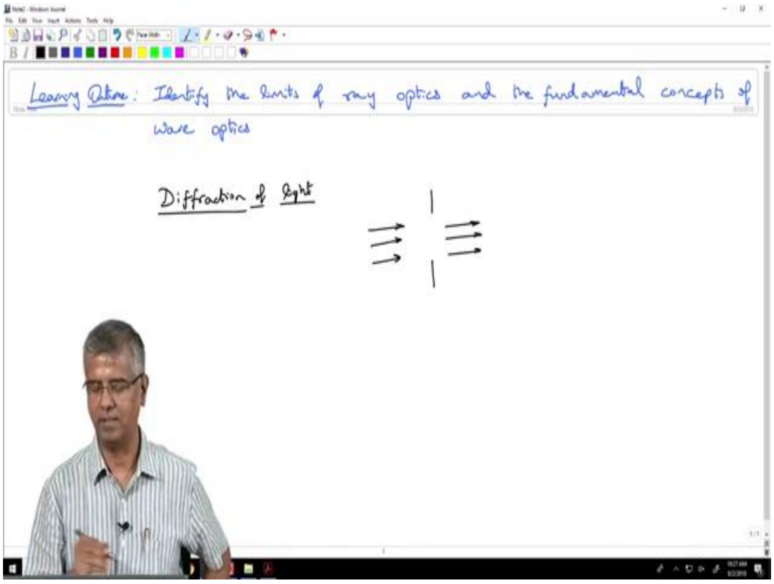

to see what is happening on the other side, right. And when I say large, what exactly do I mean? Large is how large?

(Professor – student conversation starts)

Student: comparative to wavelength 0:28:14.4

Professor: Very good, so it is comparative to the wavelength of light.

(Professor – student conversation ends)

So if you have an opening with,

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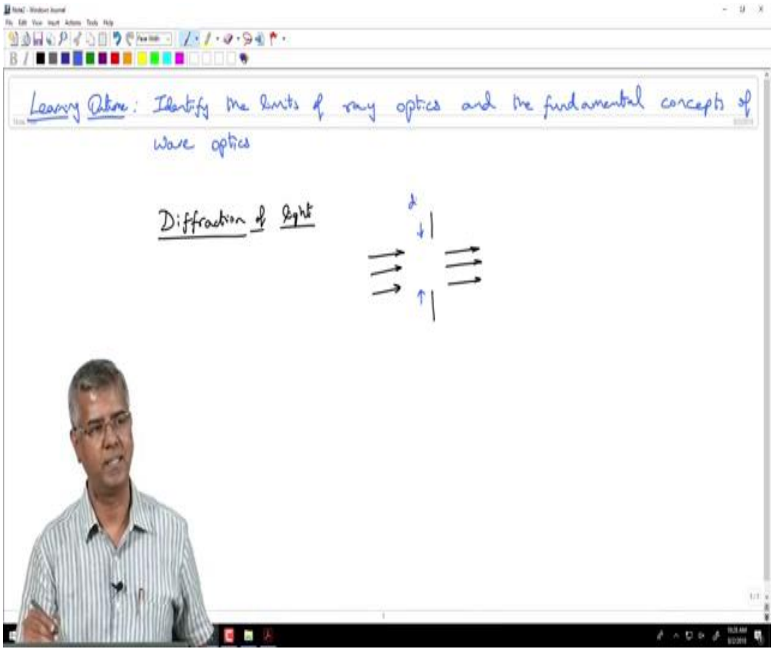

let us say d as the dimension that is far, far greater

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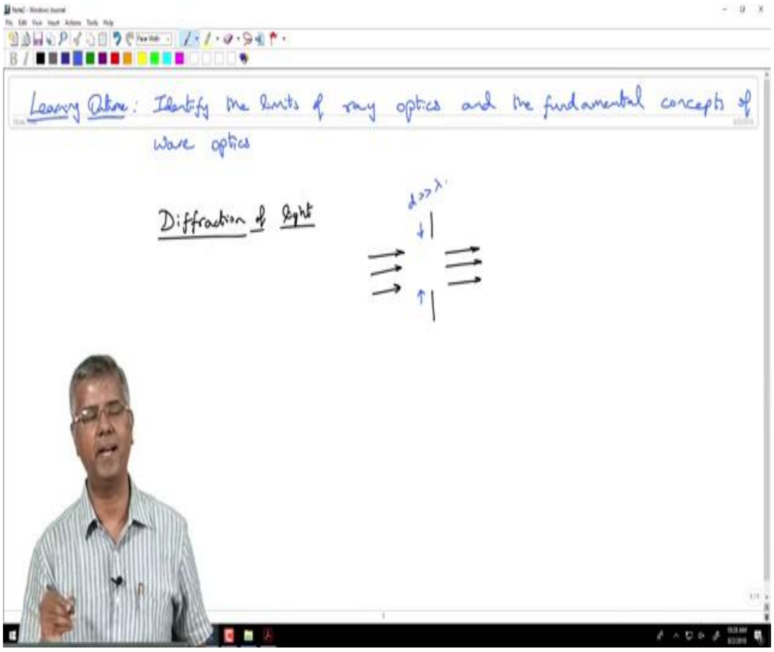

than the wavelength of light, right, so you can explain everything by ray optics. But what happens if you have a very tiny aperture,

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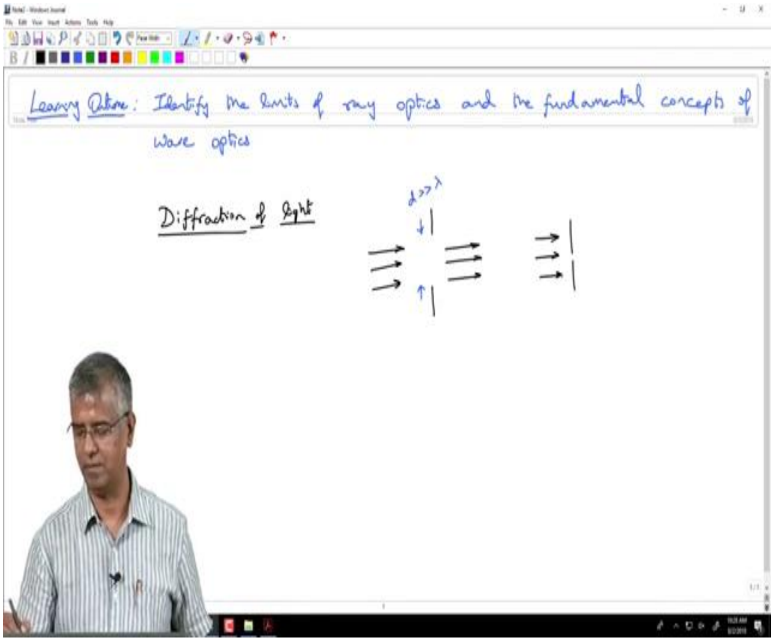

very small aperture so in this case d approaches the wavelengthλ, Ok

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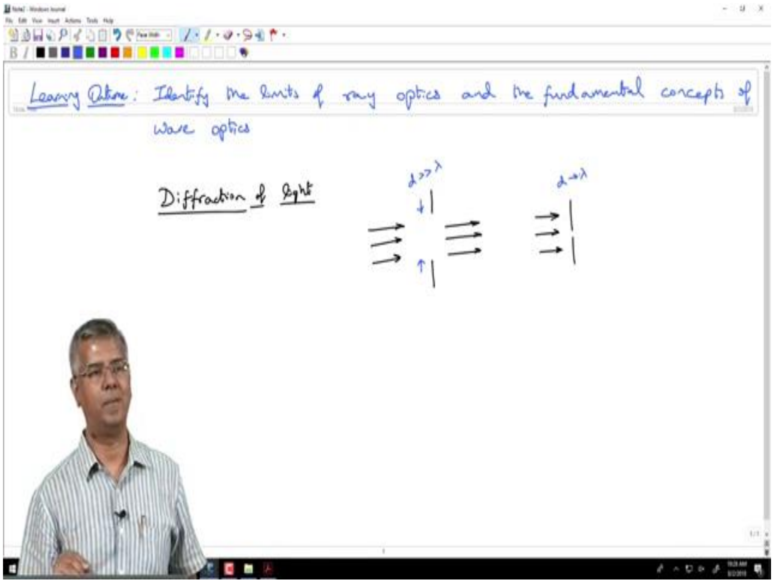

.

As d approaches λ there is very little light that is going through but we are not worried about how much is the intensity of light. We are worried about characterizing the property of light beyond that point. And this is what Huygens did, you know much earlier, several centuries ago.

He actually said that light

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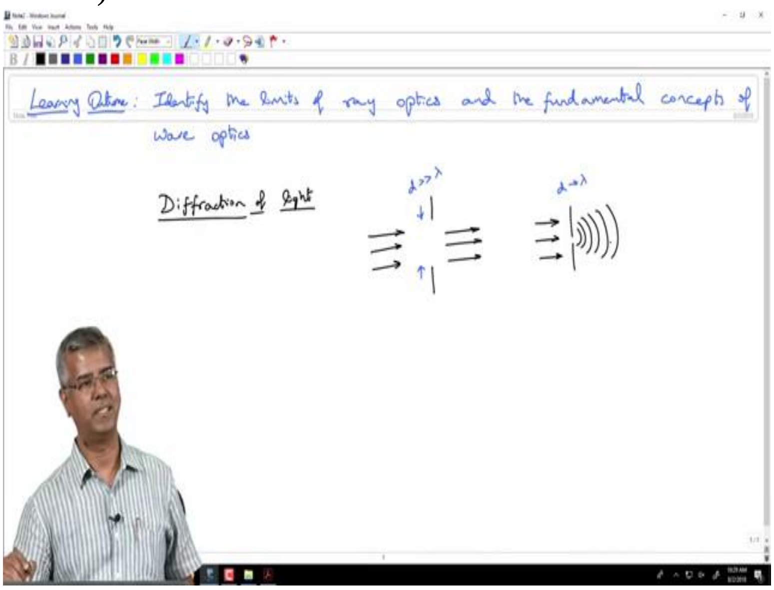

propagates through the structure as waves. So his hypothesis at that particular point was that light propagates as waves very much like a pebble dropped in water, right, so you see waves that are

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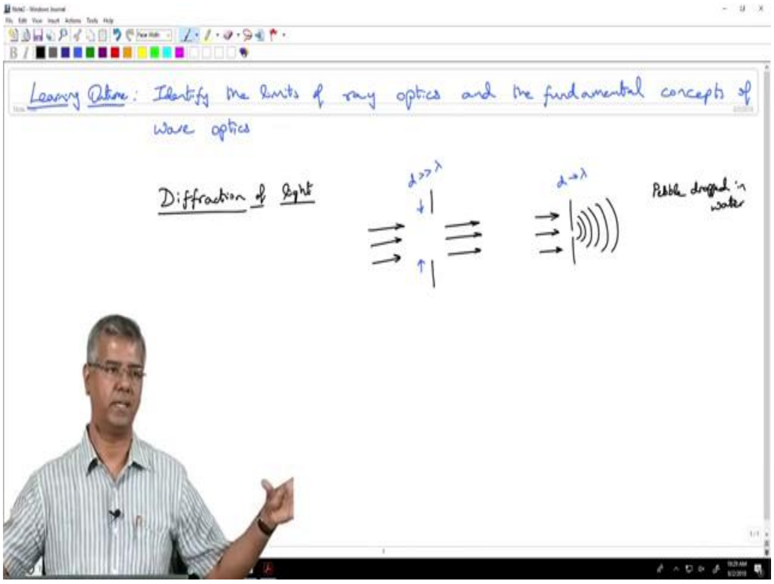

going out from the point where the pebble has gone through the water surface.

And another example could be sound from a loudspeaker. So you have this loudspeaker blaring out and you can hear that sound over a very wide region because, you know sound is propagating as waves from that source and it is, it is actually spreading around.

So ray optics breaks down, when you consider features that are comparable to the wavelength of light. That is a key thought that you want to carry on.

Ray optics is limited when light is actually having to deal with structures where the feature sizes are comparable to the wavelength of light.

So you have to jump over to wave optics and you need to understand how light propagates as waves. And any problem where you are dealing with propagation of waves, where does it start? Where would you start any formal problem which is dealing with waves, propagation of waves?

The wave equation, right.

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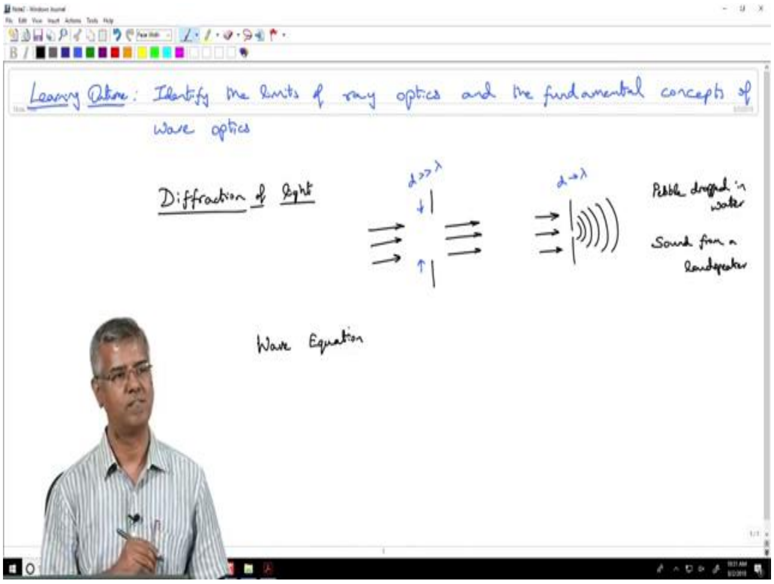

So what does the wave equation tell you?

Basically $\nabla^2 U$, let us say U corresponds to the description of the light wave.

$\nabla^2 U - 1/c^2 (\delta^2 U/\delta t^2) = 0$

That is your wave equation.

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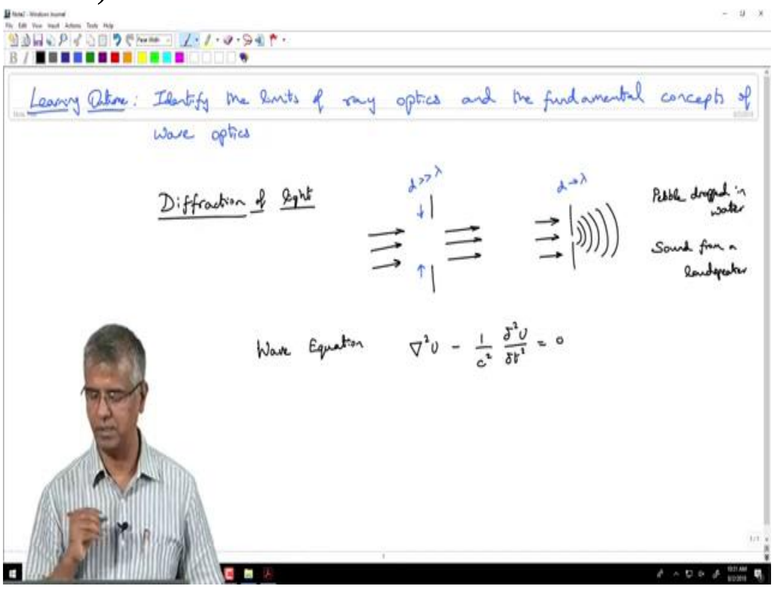

Now of course you are familiar with this in a slightly different manner.

Lot of you would have seen this in electromagnetics. In electromagnetics you would have seen that U is replaced by E or H,

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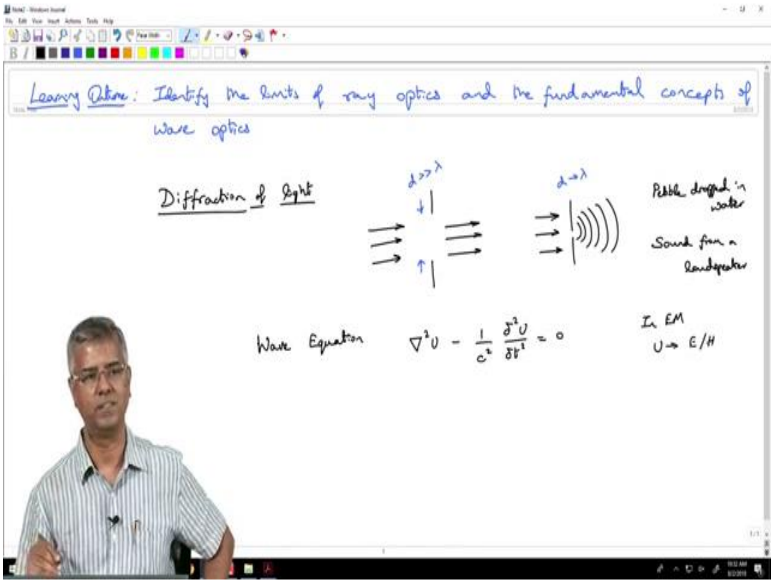

electric or magnetic field. So if instead of U, if you substitute E or H you get the wave equation which, from Maxwell's equations, you know couple of steps you get to the wave equation. So that is the same format that we have.

Ok there is something, one approximation, that you can take at this point in terms of finding a solution for the wave equation. And that approximation is associated with the general observation that when you look at waves, these waves are normally periodic in nature, right.

So, when you drop a pebble in water, you see these waves are essentially periodic in nature. So you can actually go on to describing them as time periodic signals

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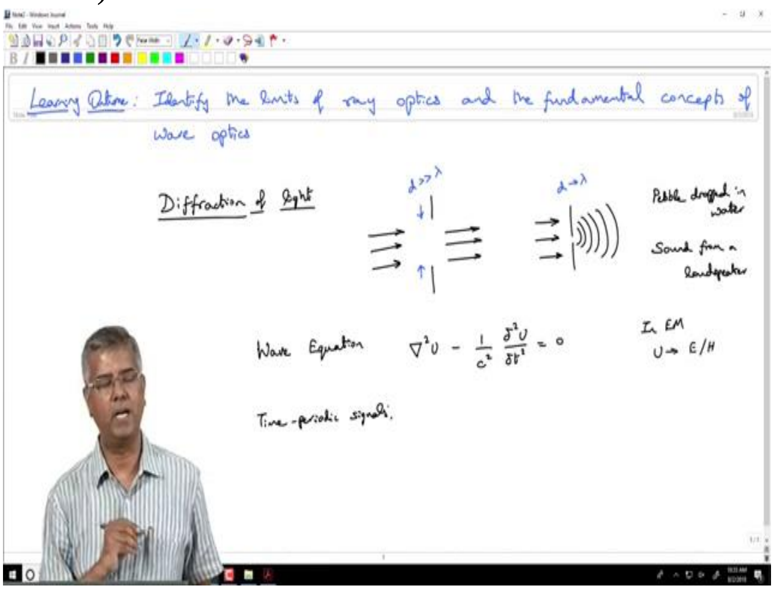

and what is the advantage of looking at them as time periodic signals?

If I am looking at the solution for U in terms of, let us say Cartesian coordinates x, y, z and t, the time dependence, then this can be written as U of x, y, z and then the time dependence comes out as $e^{j\omega t}$.

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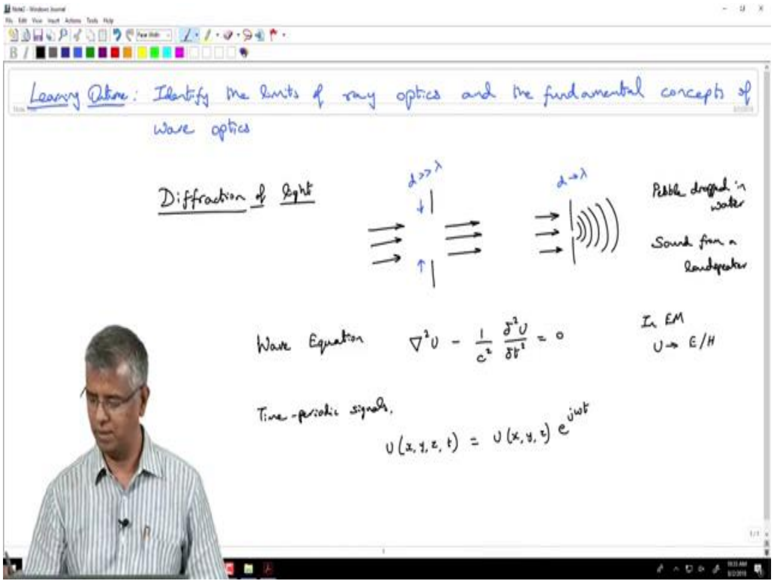

Now, $e^{j\omega t}$ = cos(ωt) + sin(ωt)

So it is basically representing a sinusoid.

So we are saying it is basically a time periodic signal which corresponds to a sinusoid and we represent it this way. What is the advantage of representing this way?

If I differentiate this with respect to t, then that will give me **jω,** and **U** will remain the same.

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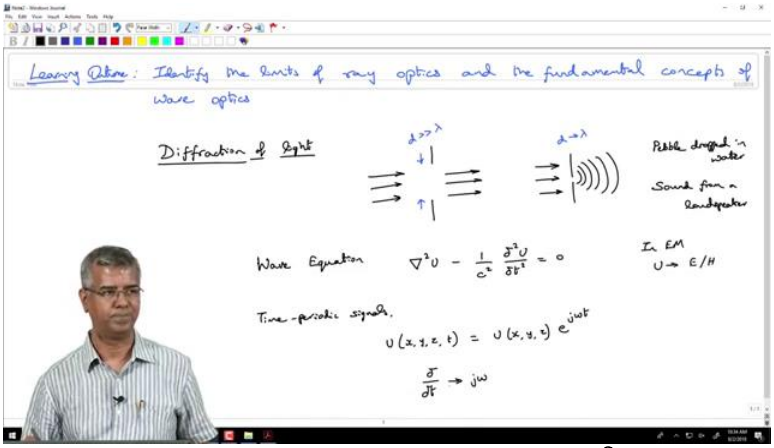

And if I want to do the second derivative that is nothing but $-\omega^2$

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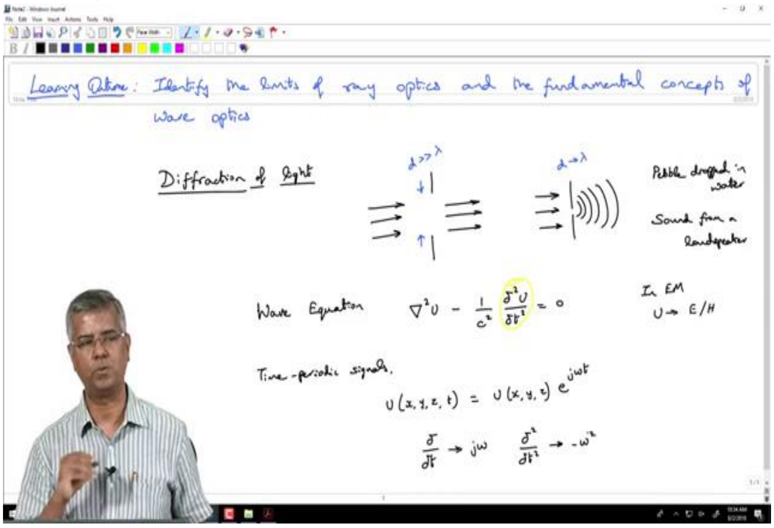

So instead of this second differential $\delta^2/\delta t^2$ here, I can substitute with $-\omega^2$ which gives me

$$\nabla^2 U + k^2 U = 0 \text{ where } k = \omega/c$$

So this is my wave equation for a time periodic case

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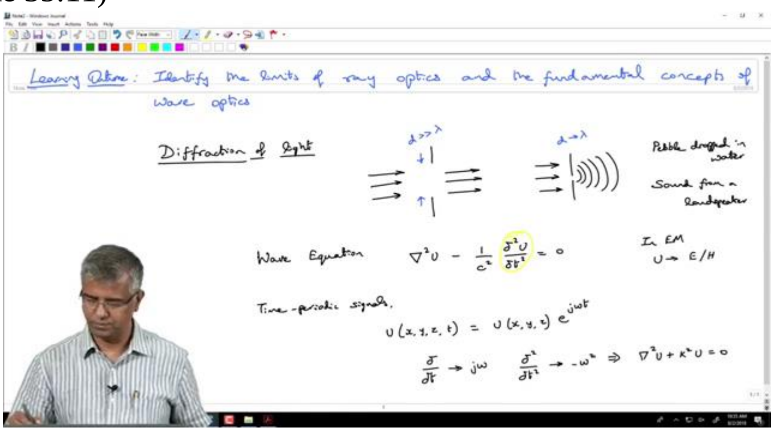

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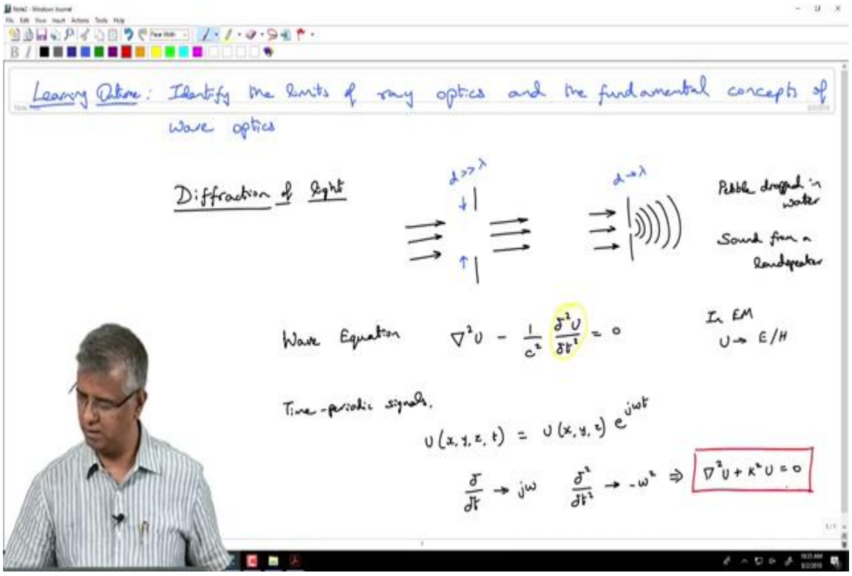

ω is nothing but the angular frequency, so you can write $\omega = 2\pi f/c$

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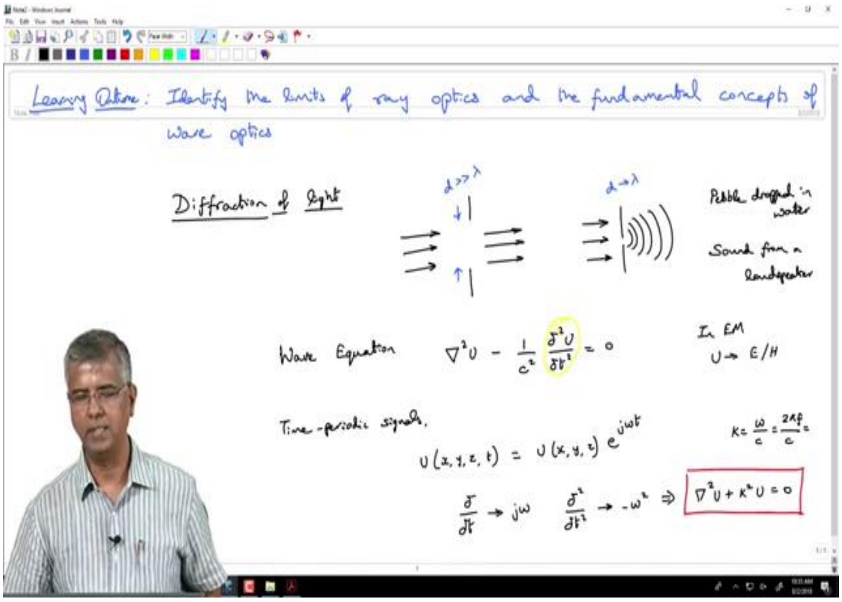

andwhat is **c /f ?**

c is the velocity of that electromagnetic wave, sorry in this case this light wave and

f is the frequency,

so **c / f** would correspond to **λ**. So you can write $\mathbf{k = 2\pi/\lambda}$

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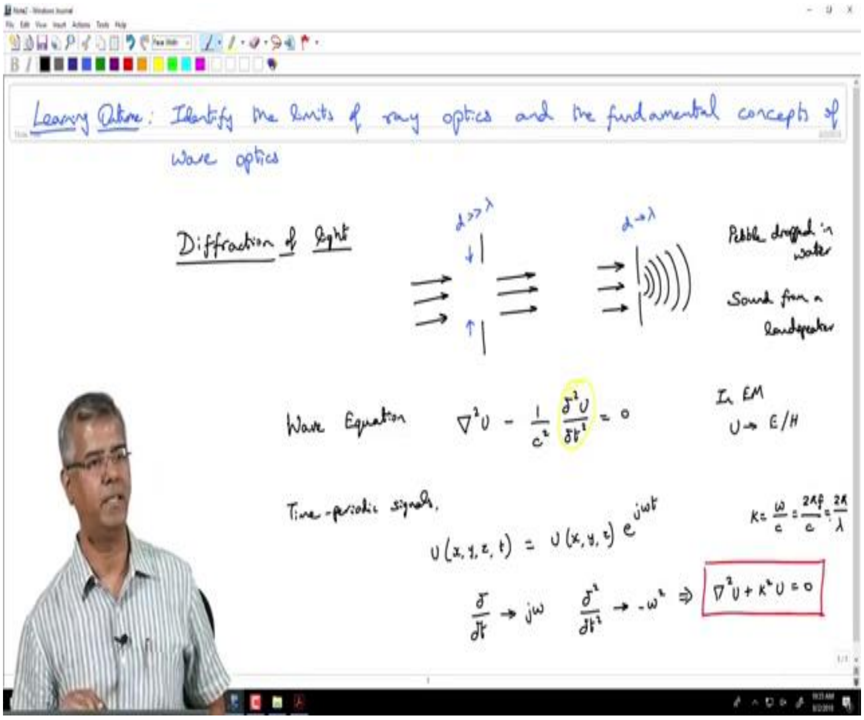

So what does that mean?

Essentially if I look at the solution of this, let us say, this corresponds to a wave that is propagating in the positive z direction,

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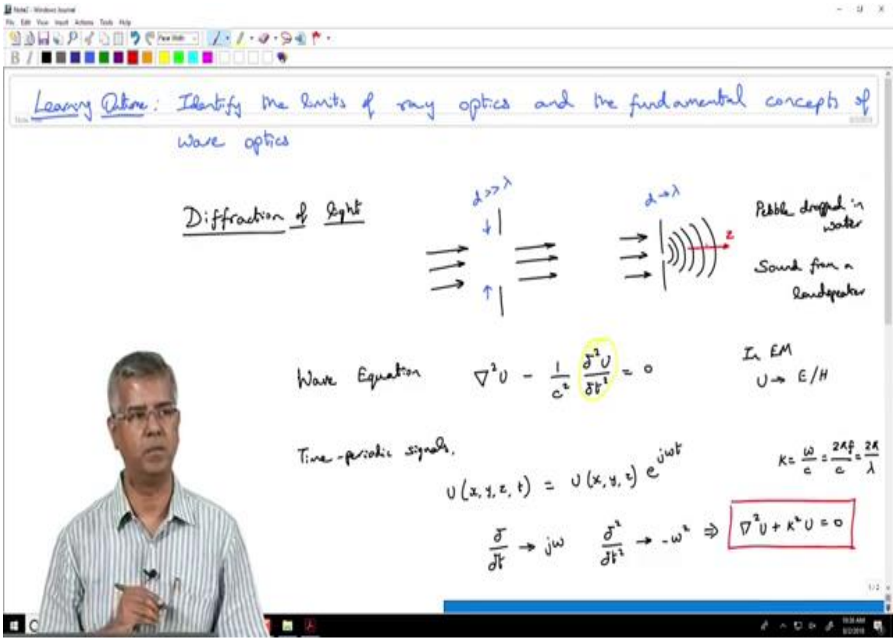

then the solution of this can be written as U(r) where r can be some radial parameter, this is given by $U(r) = A(r)\, e^{-jkz} e^{j\omega t}$

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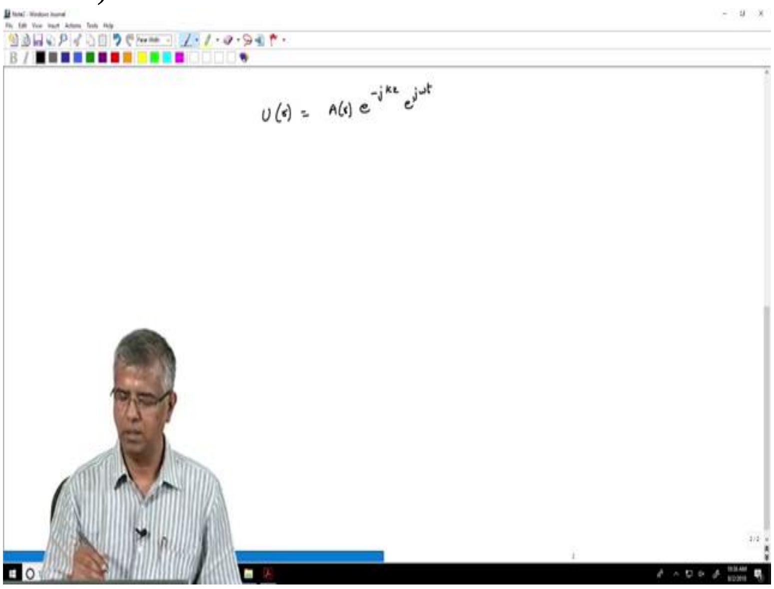

Ok

So how do I get minus j k z? If you

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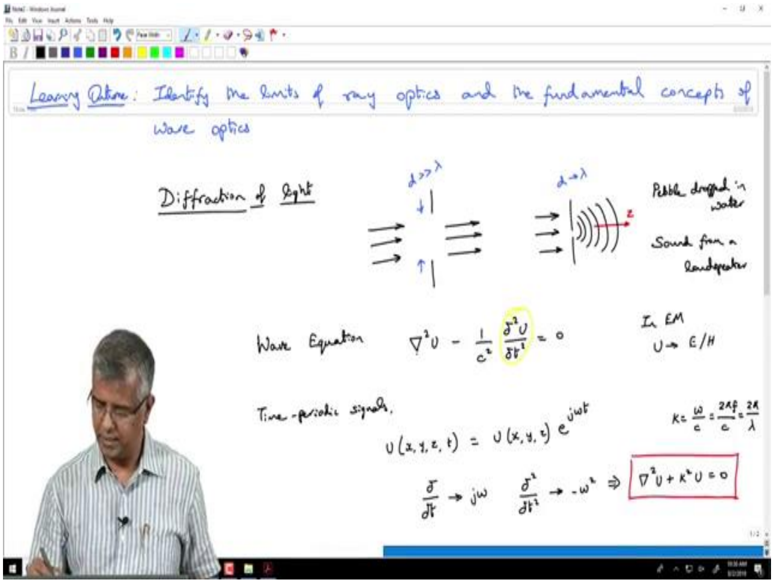

go back here this ∇^2 is actually a Laplacian, Ok. So in Cartesian coordinates this corresponds to $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ right?

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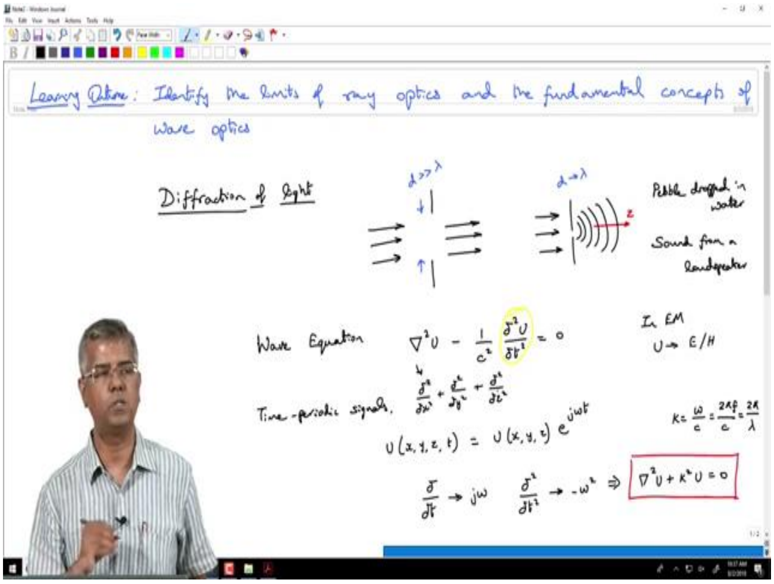

Now if I say it is propagating along z and I say it is actually propagating with very little loss, Then we can say that in the z direction, there is only an accumulation of phase in that term.

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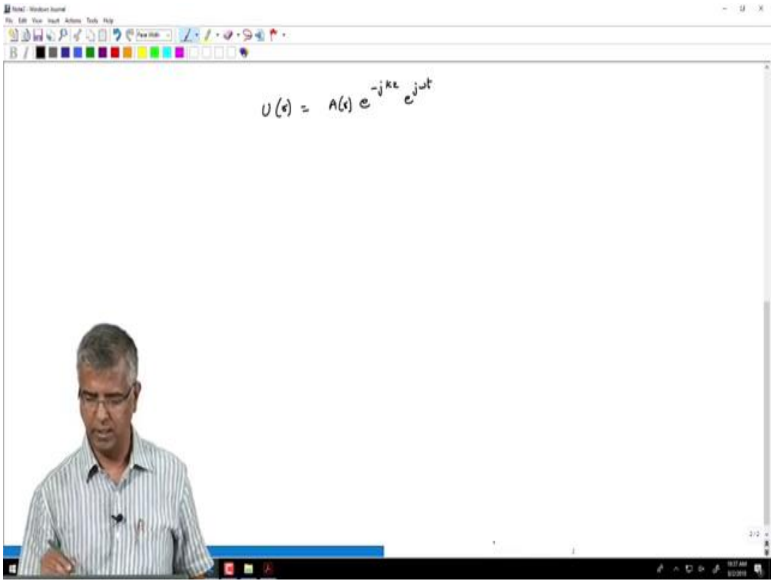

And that is what we get over here.

So if I actually look at this in the z direction what I would find is, this is basically you are looking at U (r)

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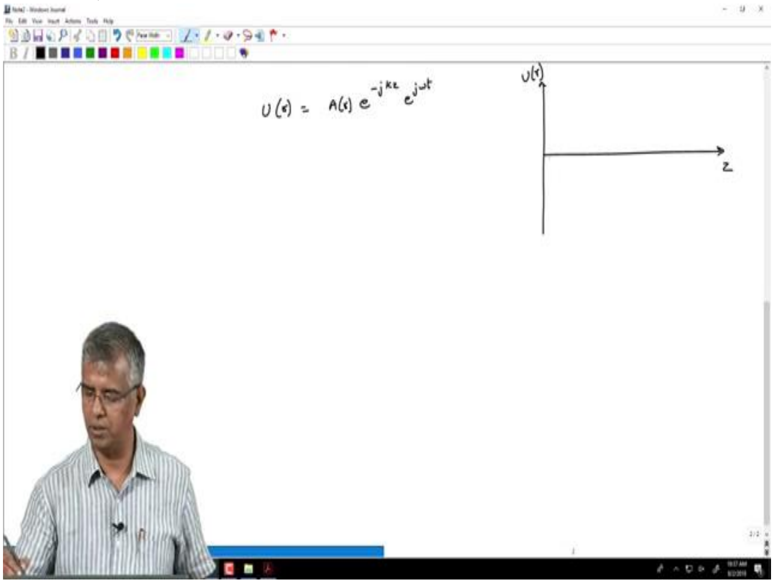

what you would find is it is

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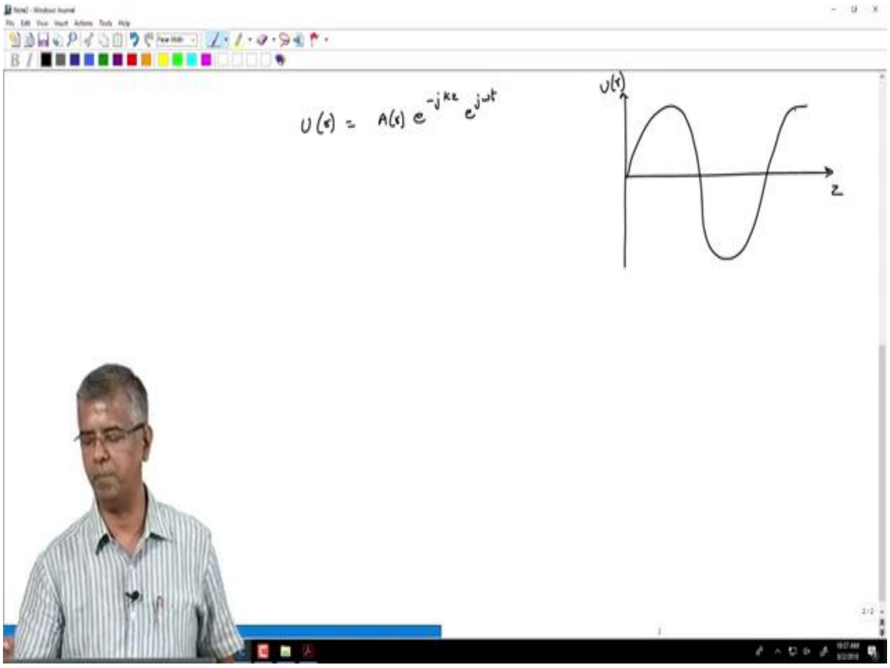

actually varying in some sinusoidal fashion and this, from where it goes, from 0 and goes to 0 again, what does that correspond to, in space? - Wavelength

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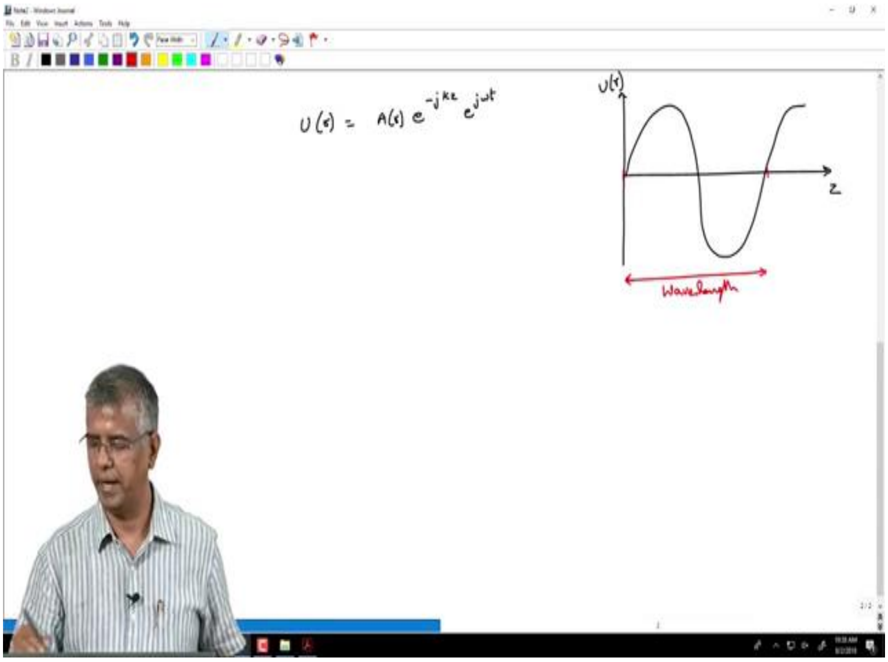

and over a wavelength how much phase does it accumulate? It accumulates a phase of

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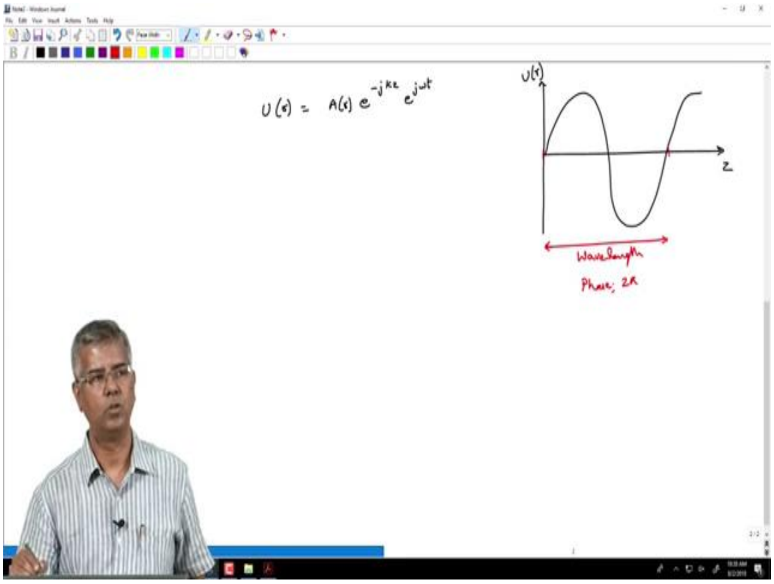

2 π radians.

So effectively what we are saying is this term is

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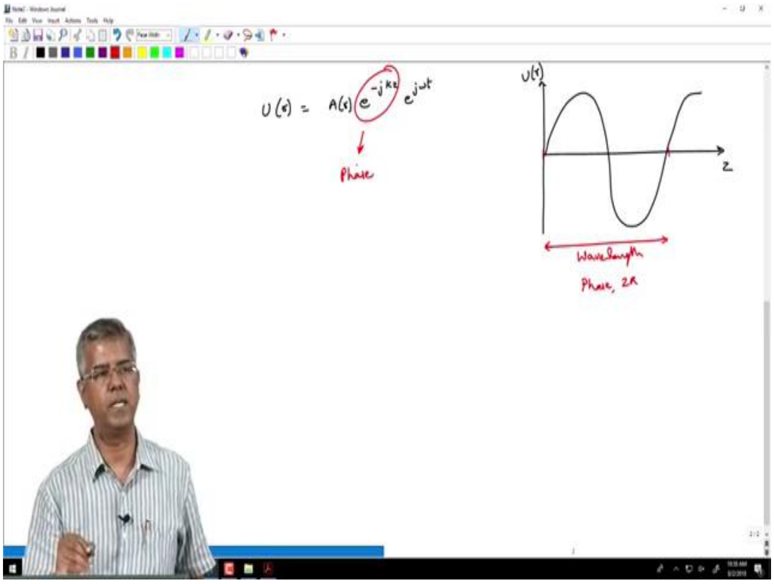

representing the phase that the wave accumulates as it is propagating, Ok.

And this $e^{j\omega t}$ term is actually a fairly boring term. I mean it is representing the time dependence but it is actually periodic waves. So you know that is not changing when it is going through material, right -linear material.

So we can choose to just consider the wave as a phasor. So U can be considered as a phasor and if it has an amplitude A, then, U can be represented as A $e^{j\phi}$

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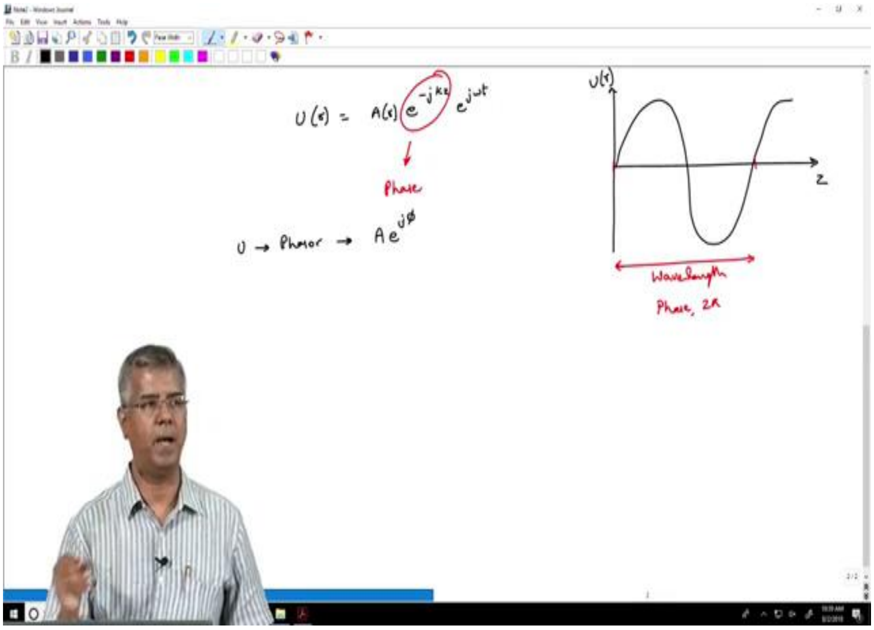

Ok because we are interested in tracking the phase which actually is changing during the propagation of that light.

So I can just represent this in a very simple form, the phasor form and U in general can be complex quantity. So you can represent the, plot the real part of U

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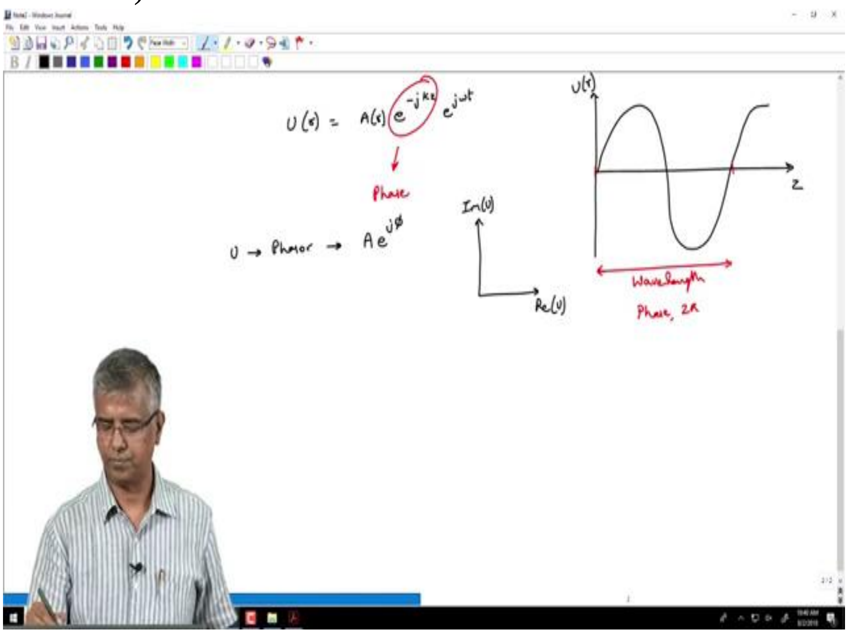

against the imaginary part of U. So when I am plotting like this, what type of plot is this? It is the plot in the complex plane. What do we call this?

You have heard of a polar plot?

(Professor – student conversation starts)

Student: Polar plot

Professor: Right, so you essentially have a polar plot where this is a phasor with

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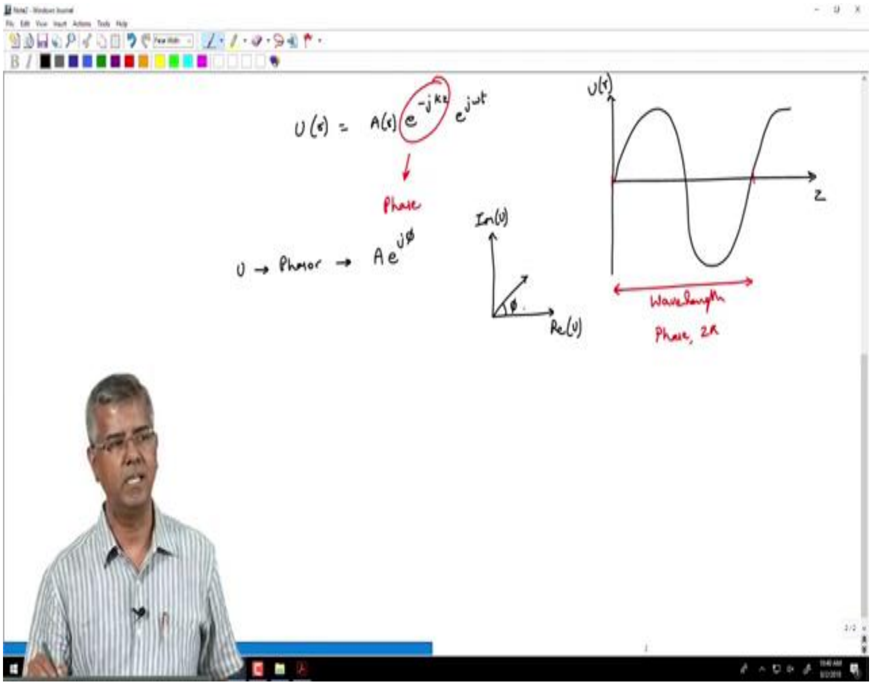

an angle ϕ and,

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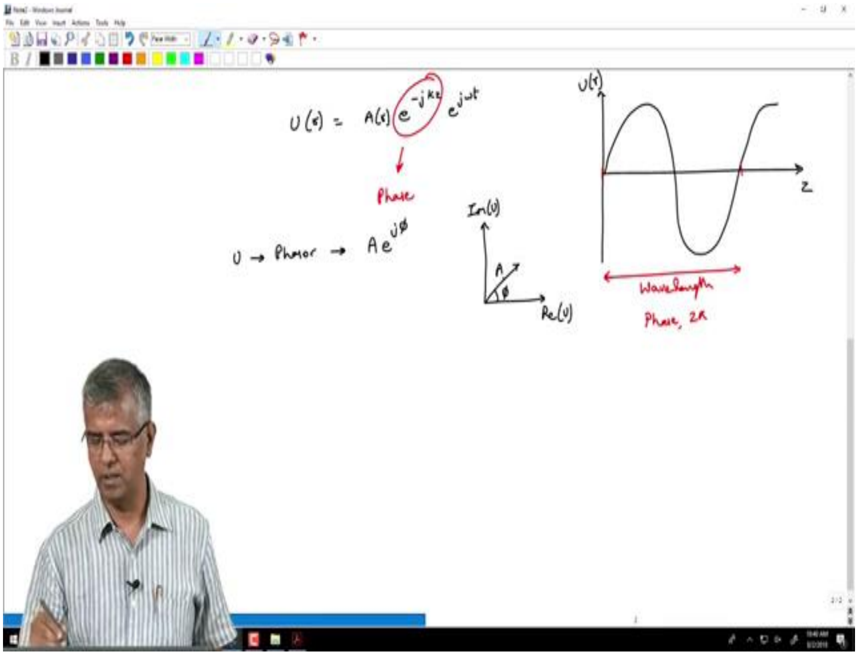

you know it has got a magnitude A.

(Professor – student conversation ends)

So you can represent this in terms of a phasor. And now if you want to define propagation we know it is actually going through a sinusoid, so you can say that as it propagates it accumulates phase, right, and it is basically going round and round in this phasor.

So it basically goes around and it is repetitive, you know just indicating that it is actually a time periodic function, right. So now with this, we can go ahead and explain what happens when 2 waves come together, which is what, it is amazing this experiment was done in 1801, as early as 1801 so this person Young, Thomas Young did this experiment, right.

Thomas Young

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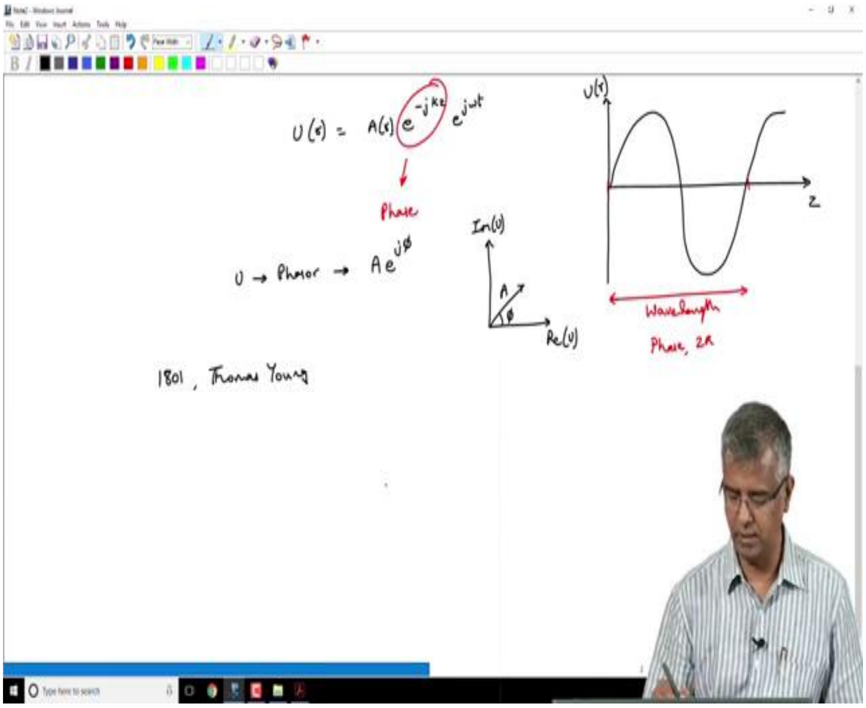

did this experiment where he basically

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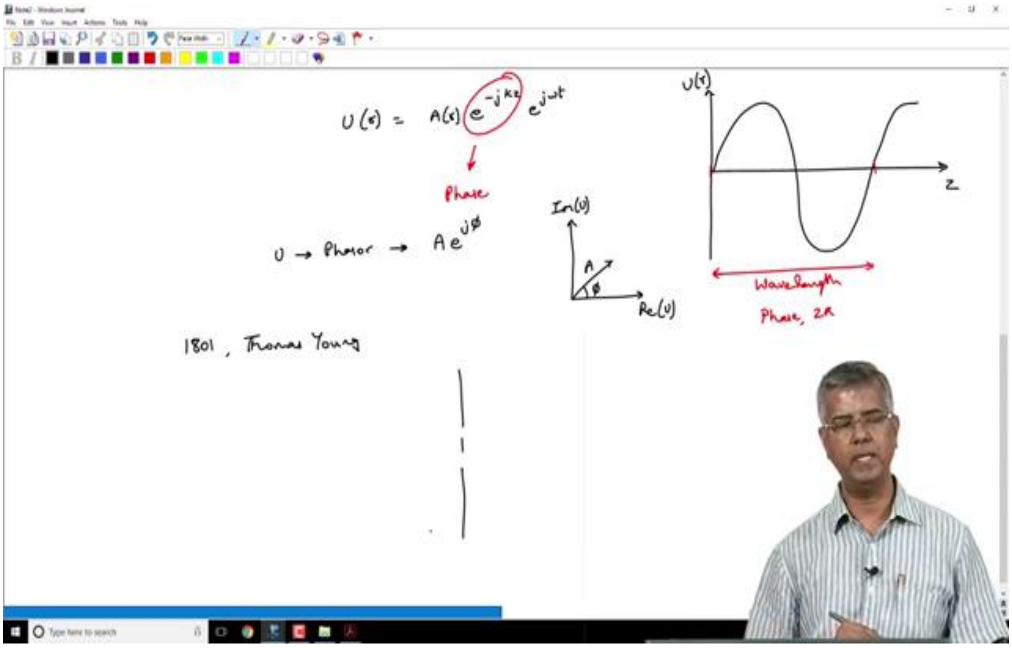

defined two slits, Ok which are separated by, let us say, center to center

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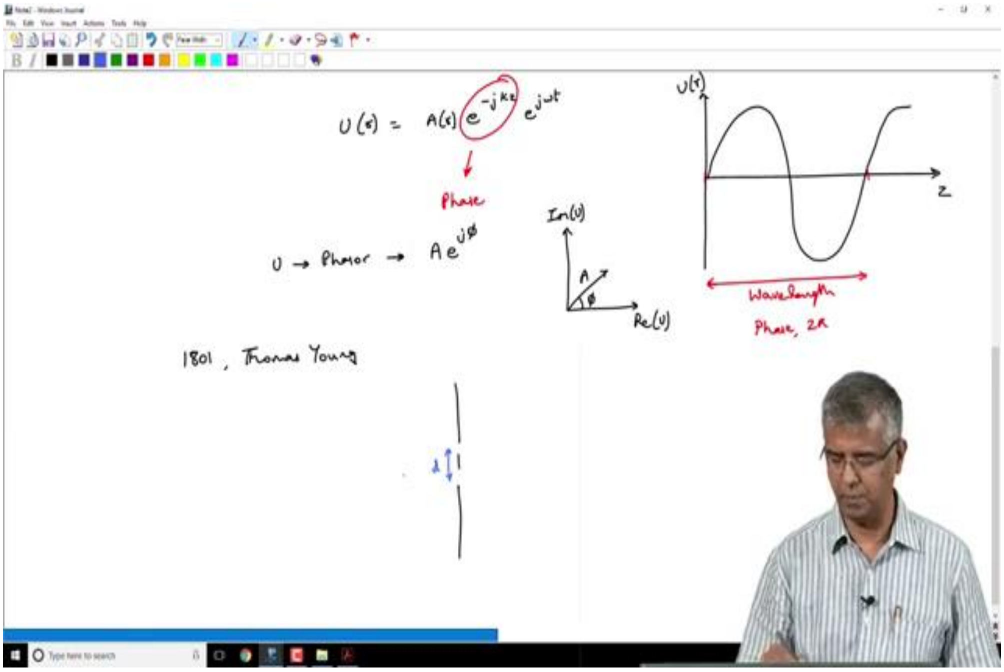

separation corresponds to d and then he was observing the propagation of light through this.

So you have a wave this is incident

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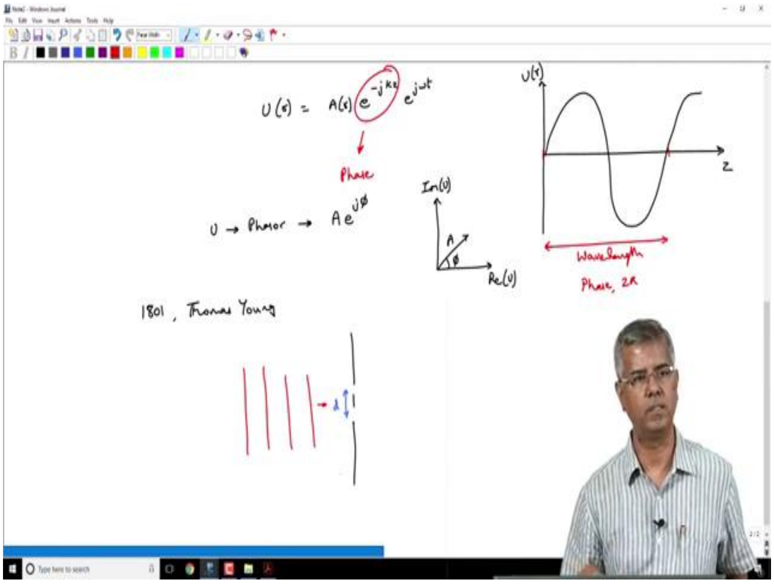

on this aperture. So what happens, and each of those slits were quite small, small in the sense it is approaching wavelength. So effectively what you expect is

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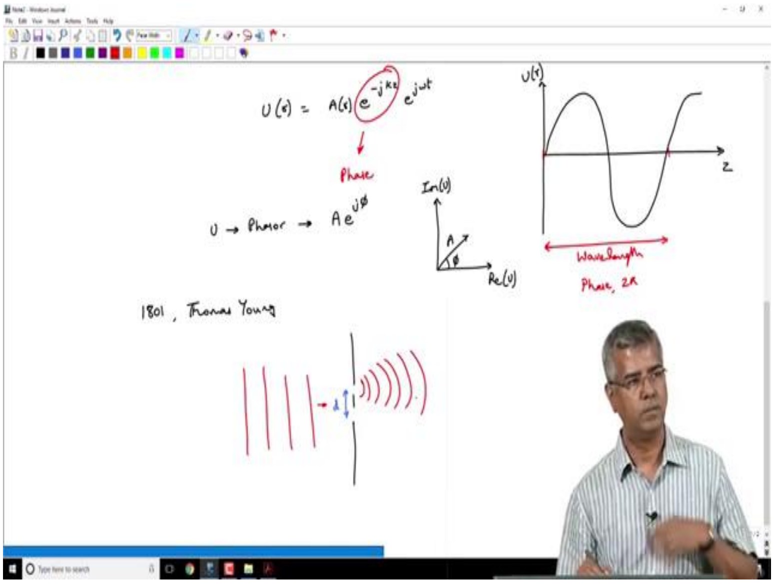

this goes on like this and then similarly you have another wave,

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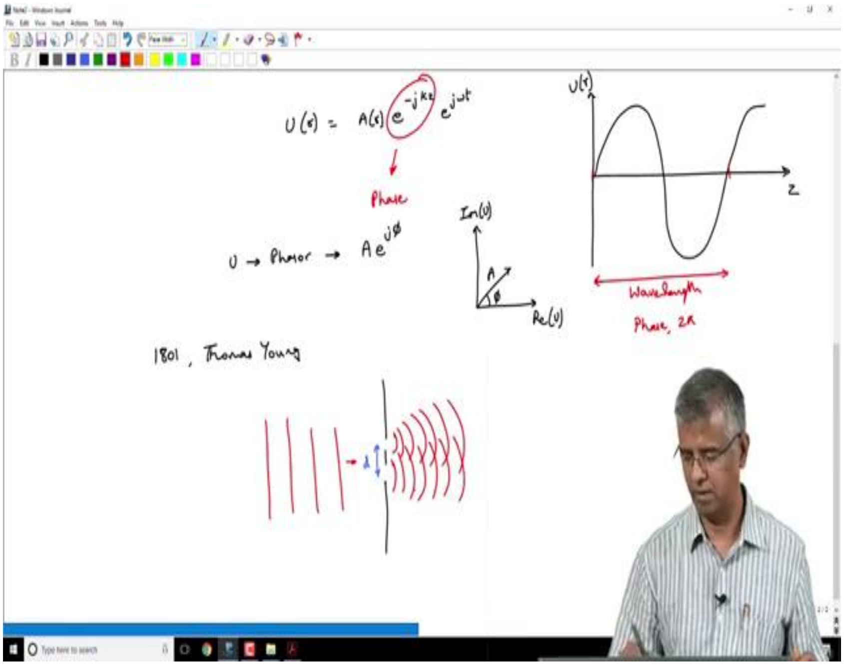

it is similar to, in a bucket of water where you drop 2 pebbles.

Both the pebbles hit the water at the same time and you have these waves that are coming across and then they, they add with each other at some point, right. And so then they may cancel each other at some other point.

So let us say this is our observation plane. So what do we see over here, so let us actually define an optical axis that goes through this

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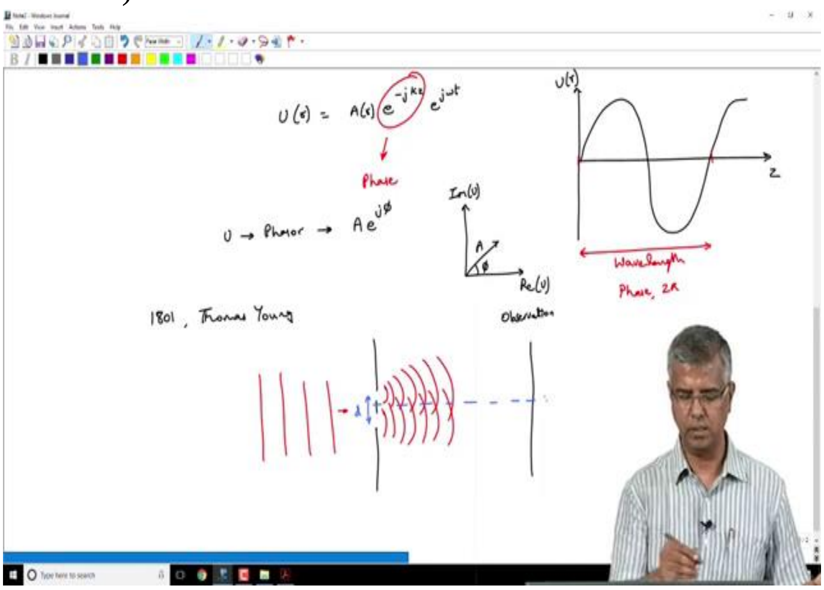

and what you see on this side is a fringe pattern like this

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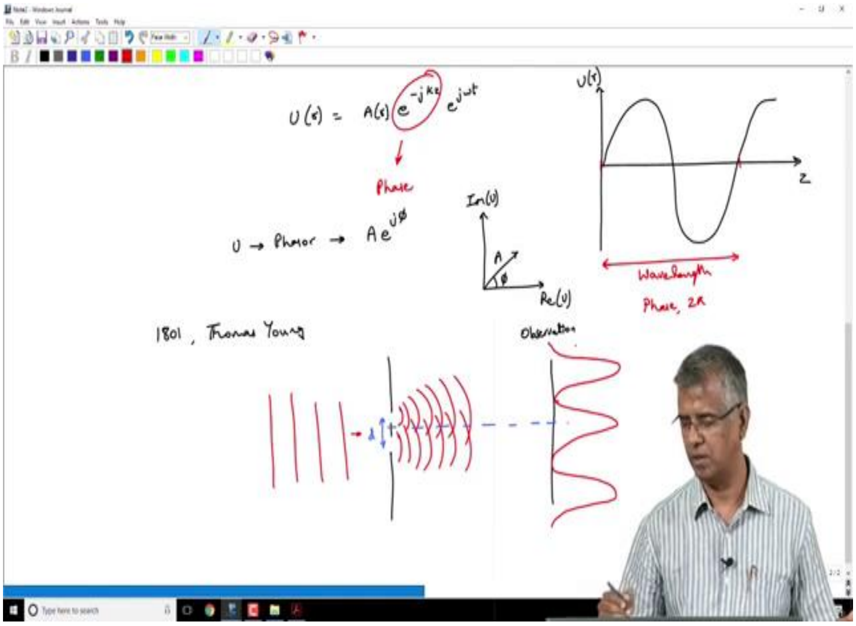

which goes to a maximum and minimum alternatively, Ok. So how do you explain this fringe pattern?

Essentially if you look at the total intensity over here, let us say that corresponds to I

I corresponds to,

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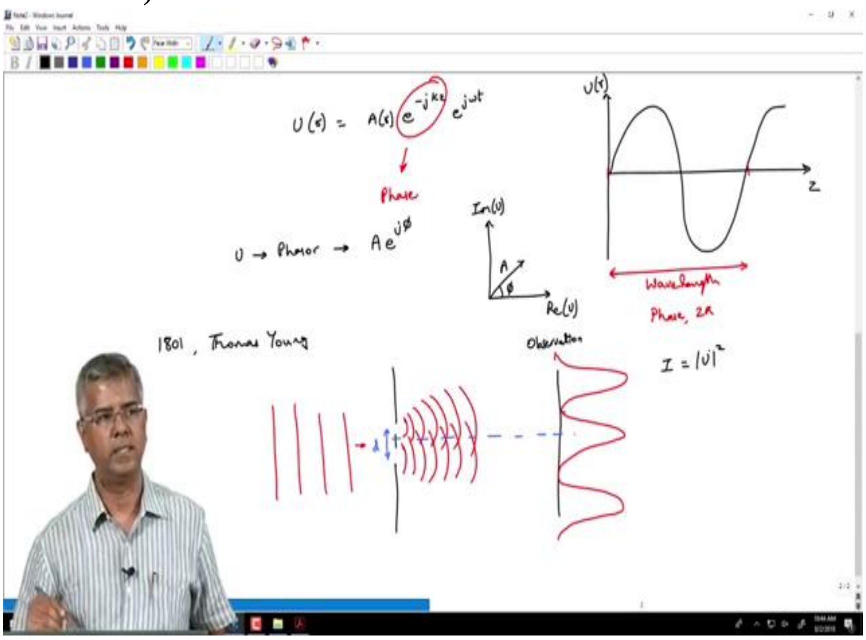

Let us say the field is represented by U so the magnitude of the field and square of that corresponds to I but this is now represented by two different waves, one wave which is represented by U_1 is $A_1 e^{j\phi_1}$

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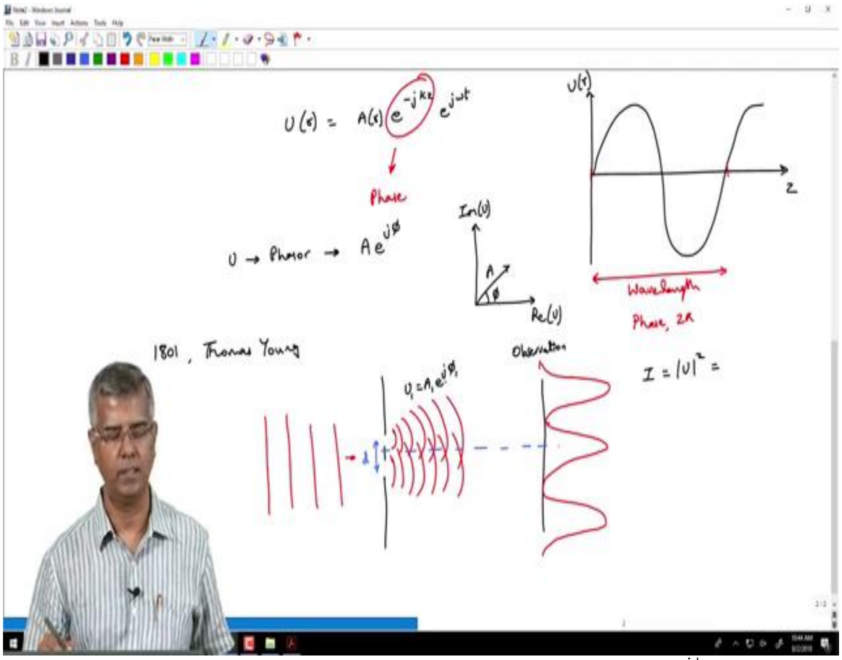

and another wave which is represented by U_2 corresponding to $A_2\, e^{j\phi_2}$

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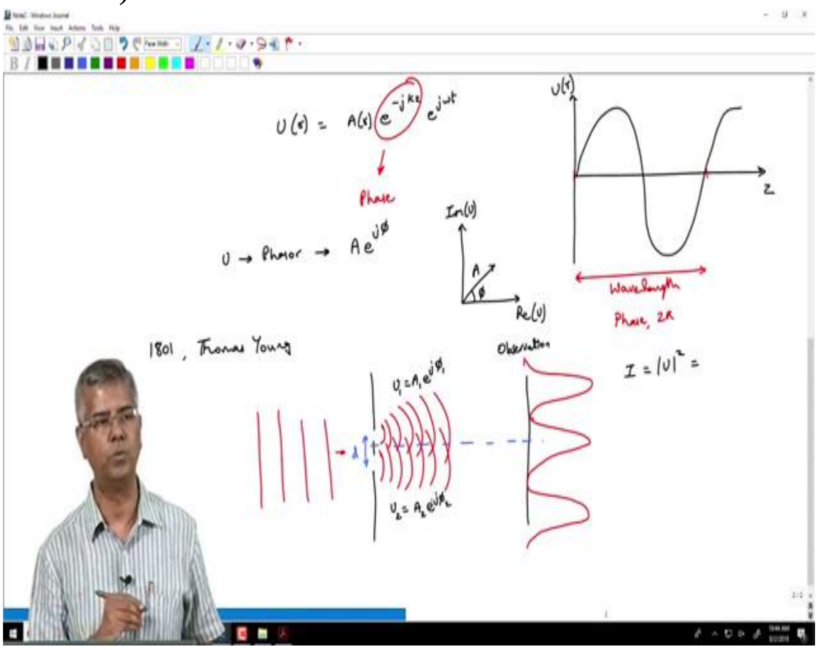

Ok

So U is consisting of contributions from both those waves so U_1 plus U_2,

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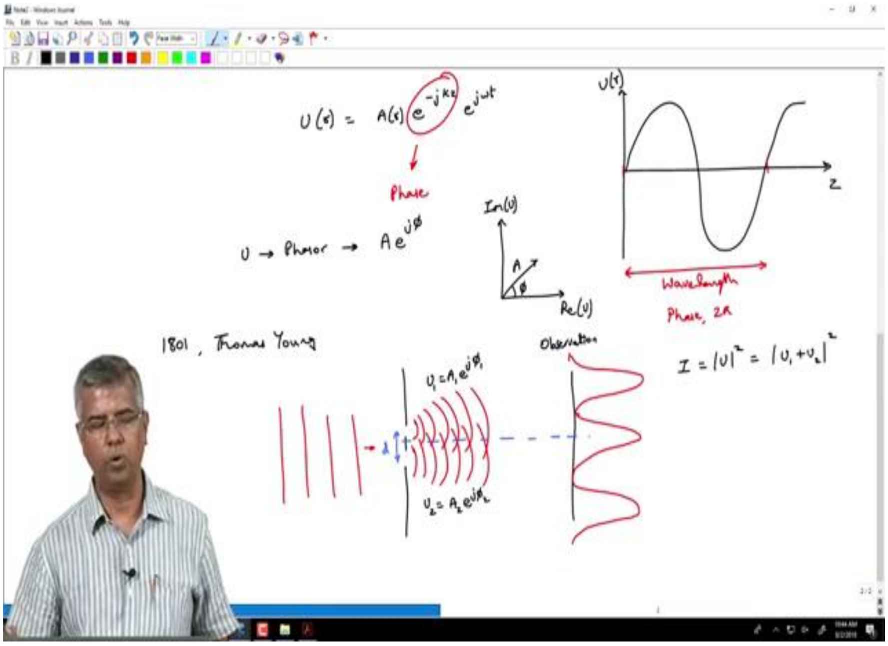

and both are complex quantities, right so when you do this square what you get is U_1^2

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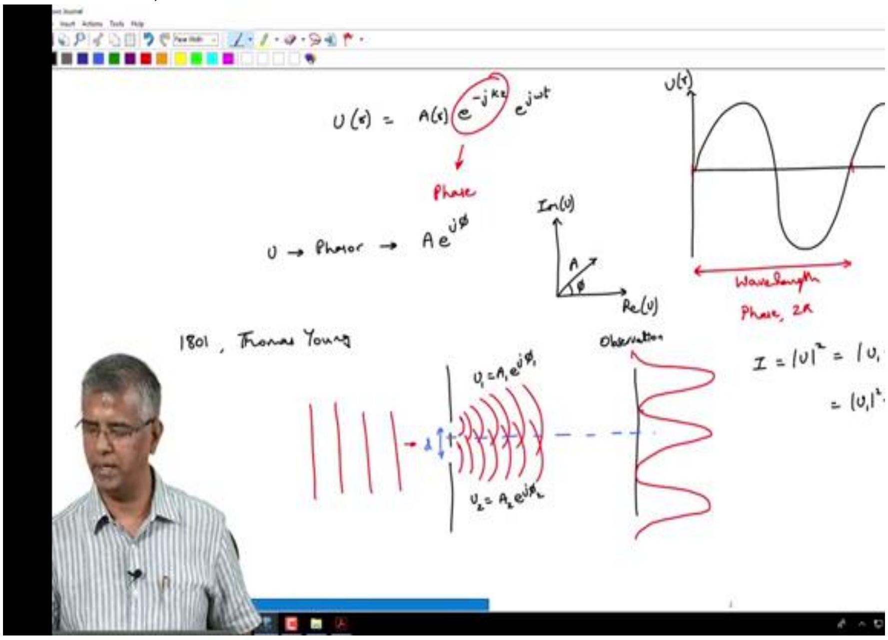

plus U_2^2 and then the beat terms between them.

Since it is a complex quantity what you will get is $U_1 U_2^* + U_1^* U_2$

i.e we get $U_1^2 + U_2^2 + (U_1 U_2^* + U_1^* U_2)$

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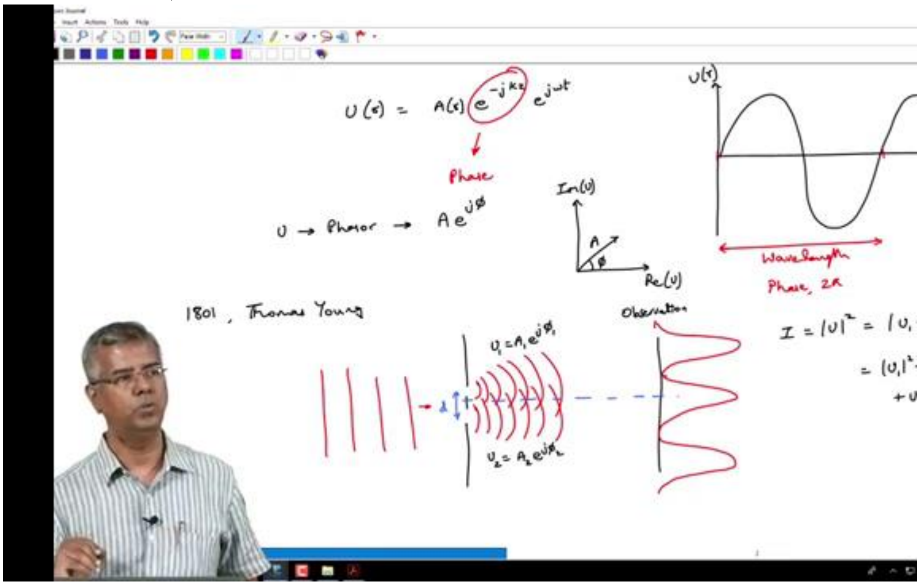

right, so you substitute the respective expressions, the phasors for U_1 and U_2, what you get is this one, U_1^2 is going to correspond to I_1, which gives the intensity of wave 1, U_2^2 corresponding to I_2, plus $2\sqrt{I_1 I_2}\ e^{j(\phi1 - \phi2)}$

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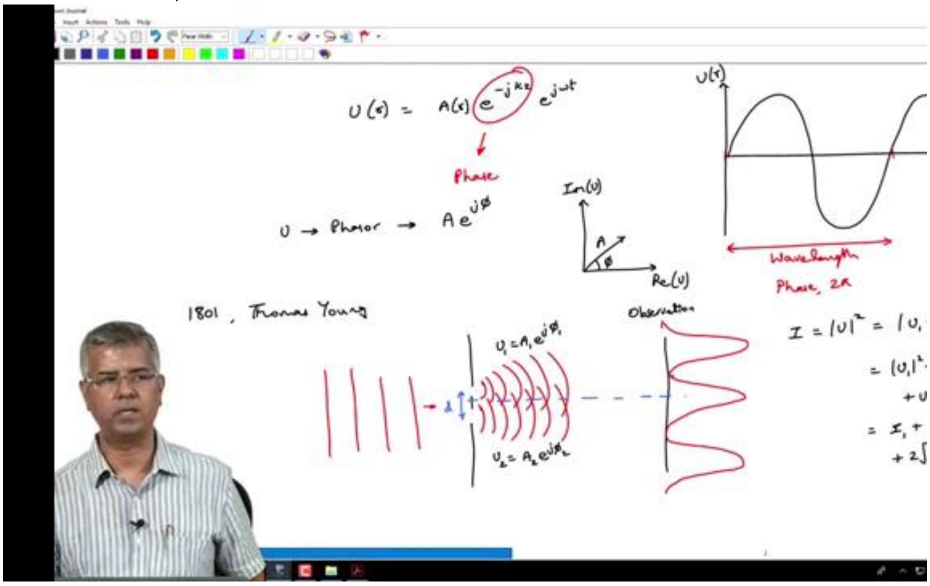

because we are looking at the conjugate of U_2, right

And similarly the other term is going to be $2\sqrt{I_1 I_2}\ e^{-j(\phi1 - \phi2)}$

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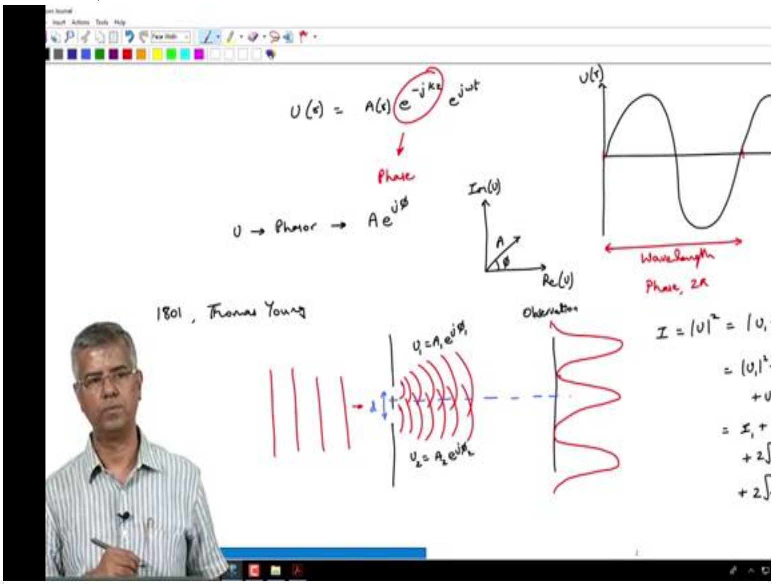

So those are going to be the two beat terms and you can simplify that and I am going to slide over here to do this. So I can write my total intensity I now. So it has got a common beat term root of $2\sqrt{I_1 I_2}$ and then you are adding these two terms, $e^{j(\phi 1-\phi 2)}$ and its conjugate, right.

So when you add those two what do you get?

So $e^{j\theta} = \cos\theta + j\sin\theta$ right. So you are going to get, sorry I made a mistake here, so there is no 2in the conjugate terms over here, that is just $\sqrt{I_1 I_2}$,right

So $U_1^2 + U_2^2 + (U_1 U_2^* + U_1^* U_2) = I_1 + I_2 + \sqrt{I_1 I_2}\ e^{j(\phi 1-\phi 2)} + \sqrt{I_1 I_2}\ e^{-j(\phi 1-\phi 2)}$

$= I_1 + I_2 + \sqrt{I_1 I_2}\ (e^{j(\phi 1-\phi 2)} + e^{-j(\phi 1-\phi 2)})$

$= I_1 + I_2 {}^{+} \sqrt{I_1 I_2}\ * 2\cos(\phi 1-\phi 2)$

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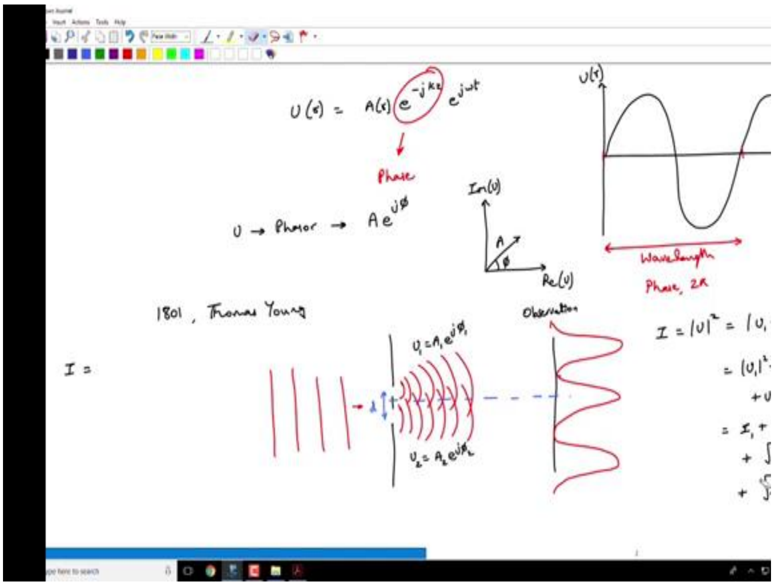

So when you add these two, you get a 2 cosθ term because the sin terms are canceling each other.

So you have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos(\phi 1 - \phi 2)$$

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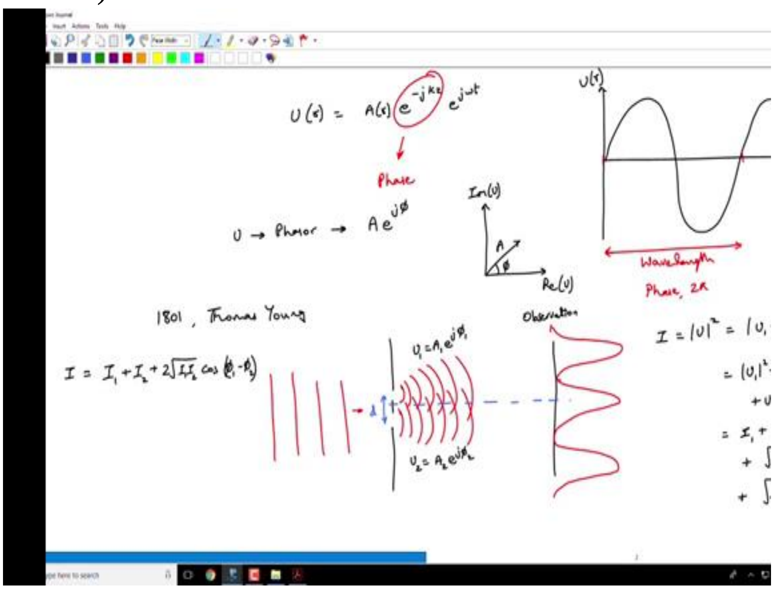

Ok so if $I_1 = I_2$ let us say is equal to I_0 then what do you get?

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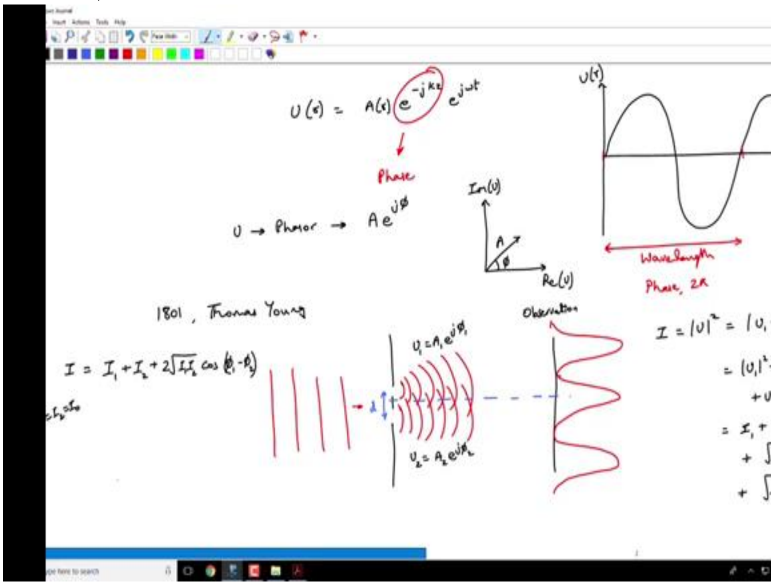

So, when $I_1 = I_2 = I_0$

$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\ \cos(\phi 1 - \phi 2)$

So,

$I = 2 * I_0 (1 + \cos \Delta \phi)$,

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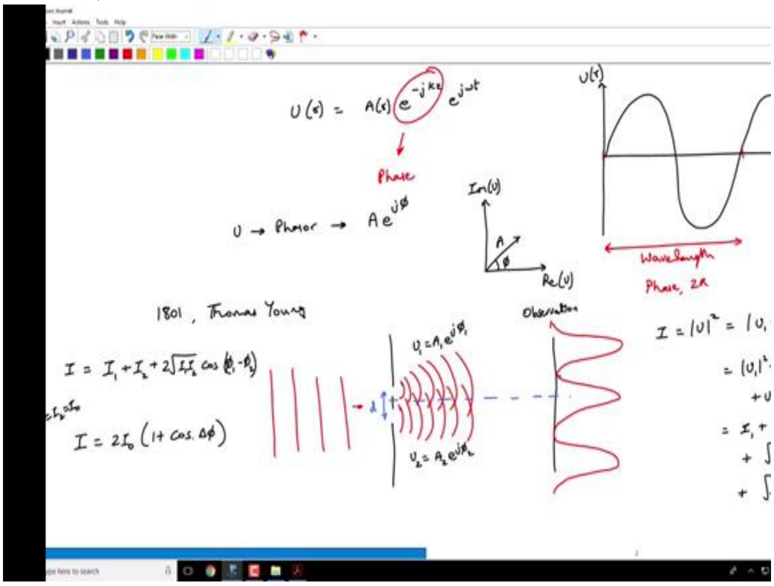

right. So that is in effect

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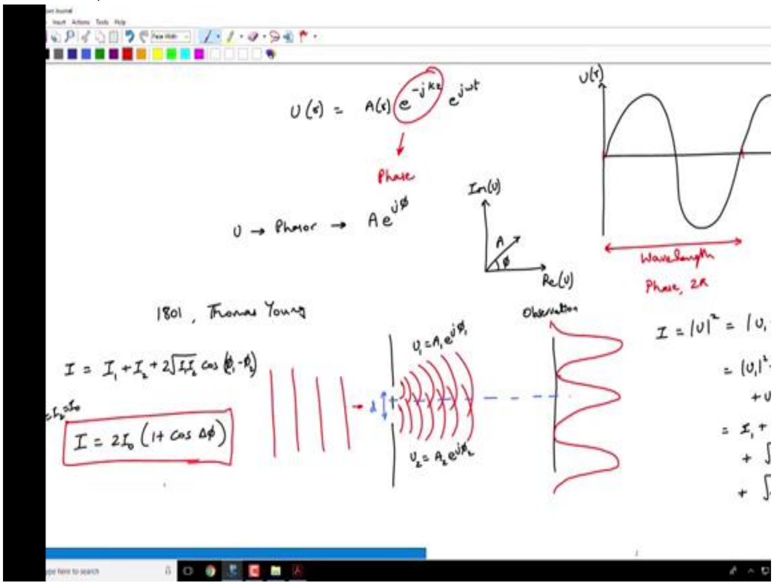

the, response that you would see as a function of $\Delta \phi$, Ok.

I will just leave you with this thought, essentially if you plot, you know this I as a function of $\Delta \phi$.

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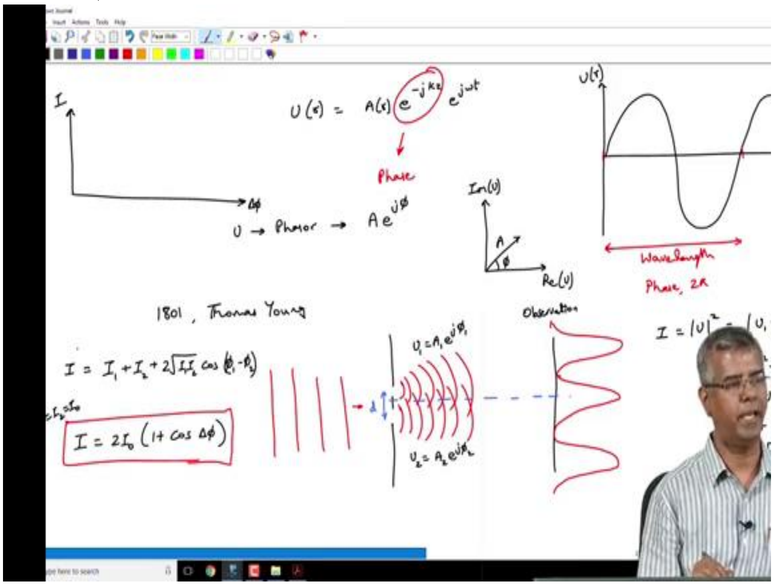

then what you get is, it basically goes through, you know sinusoidal function when $\Delta\ \phi$is 0, then this corresponds to the maximum, that corresponds to $2I_0$, and then it is going to become a zero at some point, go to the maximum, go to the zero at some point and so on.

Where does it go to zero?

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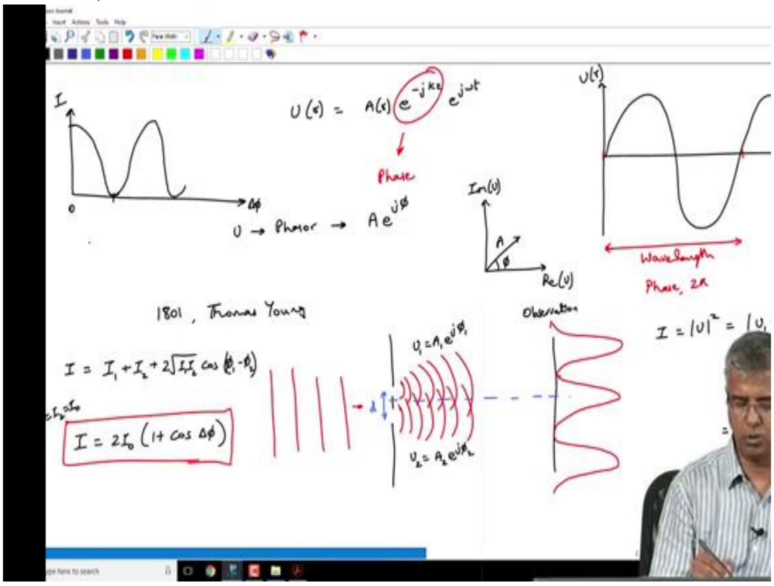

I becomes 0 when $\Delta\ \phi = \pi$, right

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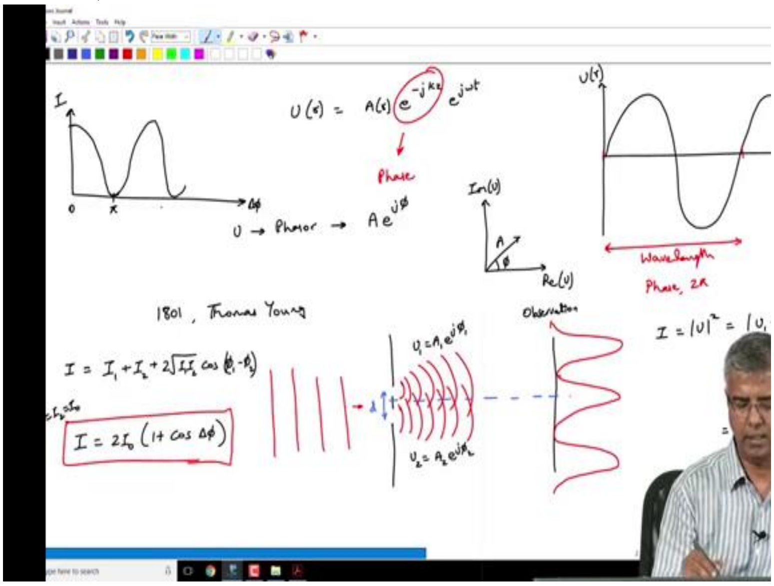

and similarly you know when $\Delta\ \phi= \ 2\pi$ it will go to the maximum

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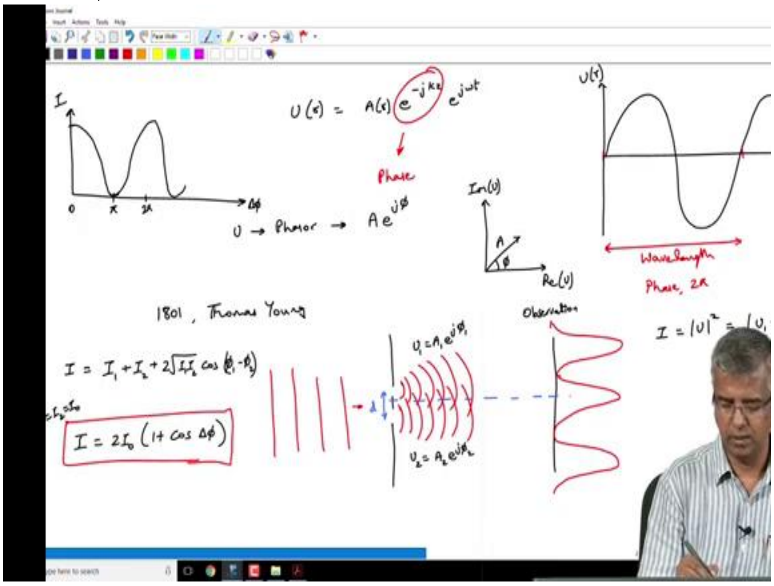

and 3π itwill go to minimum

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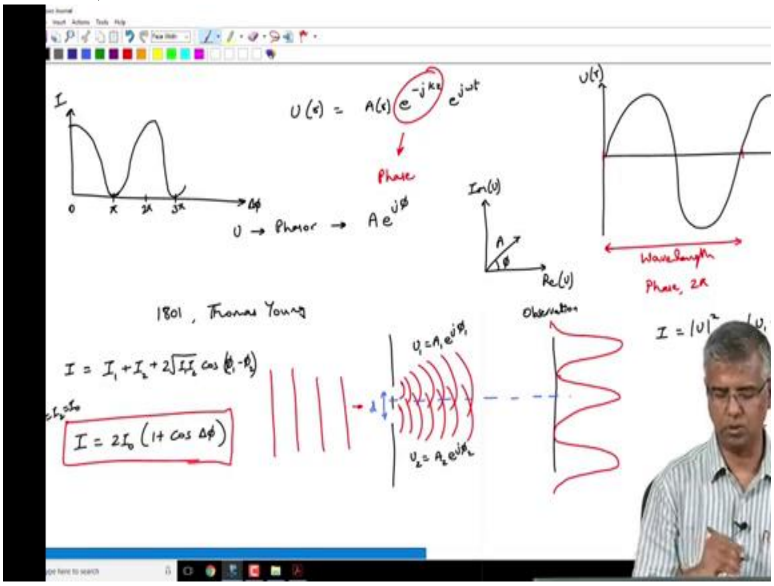

and so on. So what does that tell you?

These are representative of constructive interference

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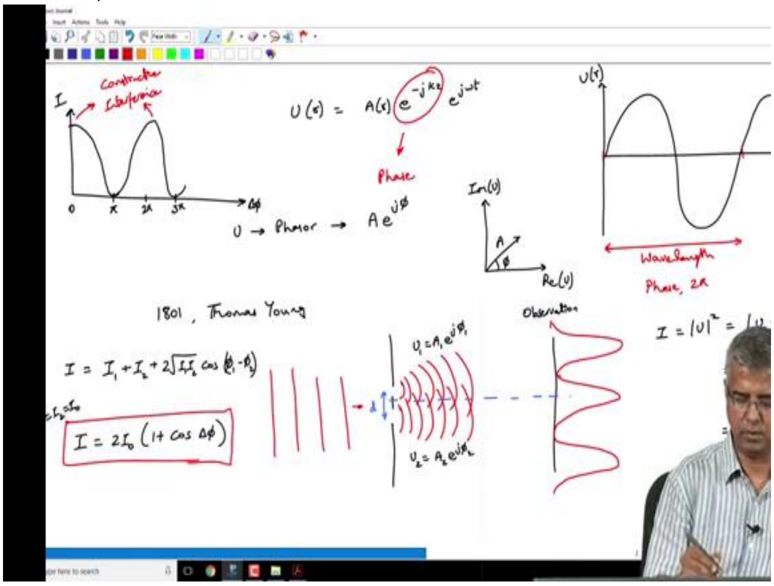

and these points are representative of destructive interference.

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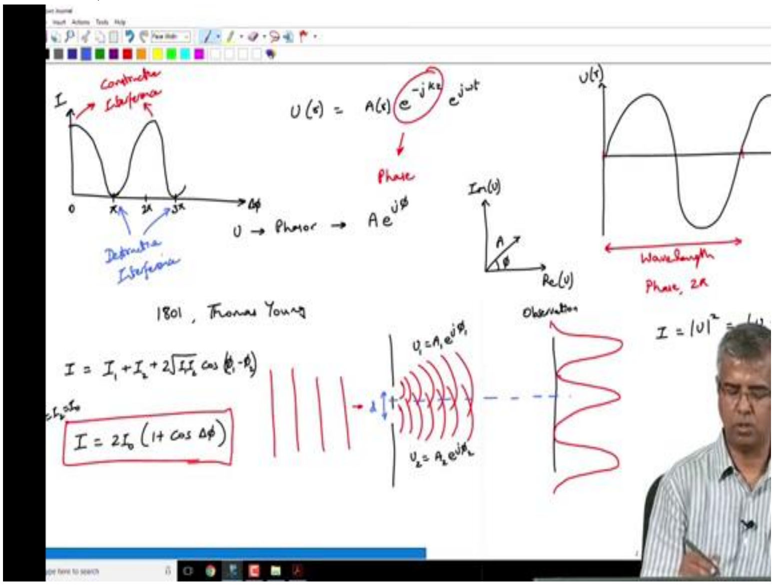

So constructive interference happens, well let us first finish destructive interference. Destructive interference happens when Δ ϕ equals to odd integral multiples of π,

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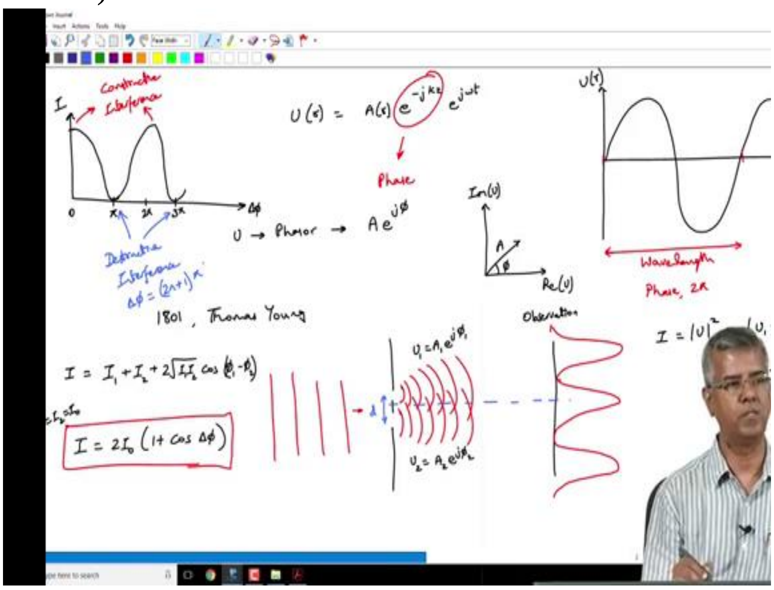

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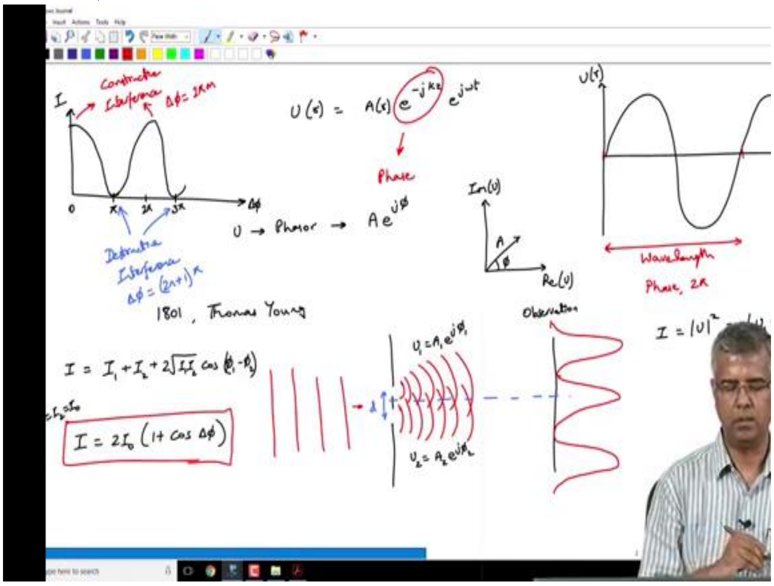

andconstructive interference happens when Δ ϕ equals to even integral multiples of π.

Key thought:

Destructive interference happens when Δ ϕ equals to odd integral multiples of π,

Constructive interference happens when Δ ϕ equals to even integral multiples of π.

So that is of course, you know that intuitively but if you go through the wave picture, you know you can show this. But the idea here and the idea that we are going to propagate forward is that, to check this constructive and destructive interference criteria, you dont have to model that entire wave, Ok.

You just model the propagation phase that it accumulates. And you just compare the phase between the light beams that are coming together, and based on that you can actually see constructive and destructive interference.

And just working backwards, it all started with saying that light has this, when light approaches features whose sizes are comparable to the wavelength of that light, then, it actually exhibits wave phenomena which means that it undergoes this diffraction.

It actually bends around these apertures and that can give you, his sort of things where, once you consider them as secondary wavelets those wavelets can come together and interfere with each other and it can give you constructive and destructive interference.

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For those of you that are taking this online course we will have a demo of this experiment which looks at diffraction of light and using the property of diffraction of light how to measure certain feature sizes.

So that is what we are going to see and moving forward, going towards next week we are going to look at this in little more detail and we are going to look at something else that is very important, which I have not touched much here, that is - the property of the light source.

So you start defining that, for all this to happen the way it is projected - you need to have a coherent light source. So then what is the meaning of coherence? How do you quantify coherence? You know those are the things that we are going to see in the upcoming week. So let us stop with this point, Thank you.