Introduction to Photonics
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Photon Interaction – 2
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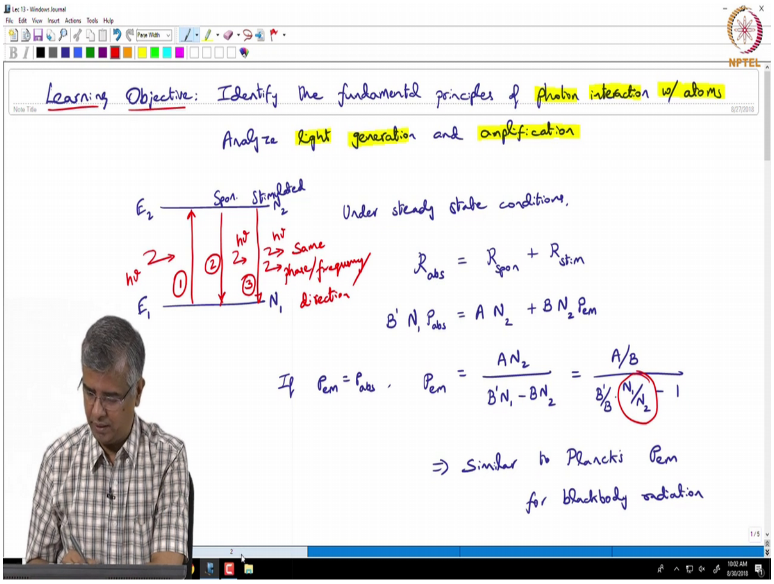

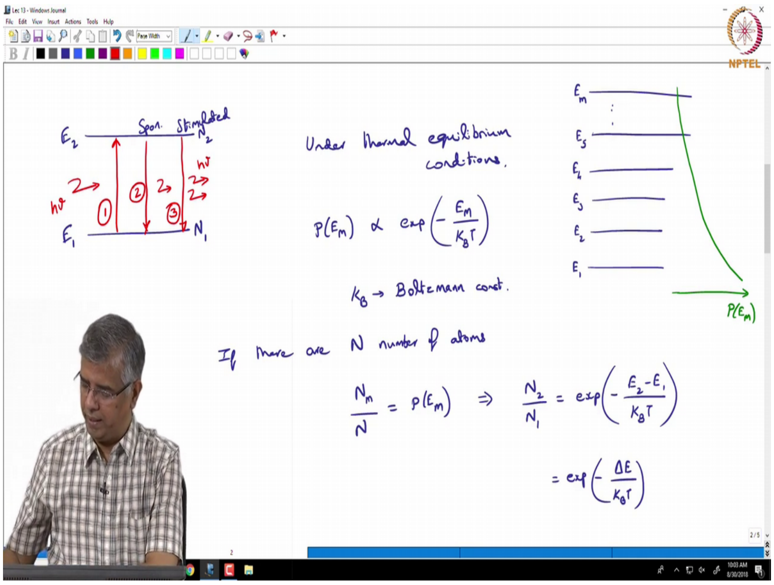

Okay, good morning and welcome to yet another exciting session on introduction to Photonics, so we have gone on to looking at the fundamental principle related to photon interaction with atoms that is the exciting part, your exciting material with photons right and through that process we can actually learn about light generation and amplifications, so that is the learning objective that we have and with that in mind you know we started looking at an elementary atomic system consisting of two energy levels wherein we were tracking three different processes you could have absorption and then with absorption process you are taking an atom to a excited state.

Essentially you are taking an electron from the valence orbital to one of the higher orbitals, at that higher orbital we said it cannot stay there forever, is going to have to come back to the original state and it can come back through by emitting a photon and if it is emitting a photon after spending some characteristic lifetime there which calls it spontaneous emission but we could also have an emission process which is triggered by an external photon that came across this excited atom and that processes is what we call as stimulated emission.

Where we said we have if you come in with H new you one photon with H new and if H new corresponds to the energy difference between the two energy levels that we are considering here then we go out with the a clone of the incoming photon which essentially has the same phrase, frequency as well as direction right, the same phase, frequency as well as direction you could get.

So you are essentially having a process that is a known as amplifications, so essentially stimulated emission is a key process for amplification, so then we started looking at this system under a steady state conditions, what are the different rates of this processes and then we came up with an expression for this emission probability density which is an important factor that determines how much of the light is undergoing stimulated emission versus a spontaneous emission and, then we got an expression like this and then we said okay this is similar to Einstein actually, realise that is similar to what plank had derived for blackbody radiation and through that we were going to we started getting into looking at you know what blackbody radiation meant, we started going back to looking at Boltzmann law and realising that the probability of finding an atom it in excited state EM is always going to be lesser where compared to probability of finding an atom at a lower energy level.

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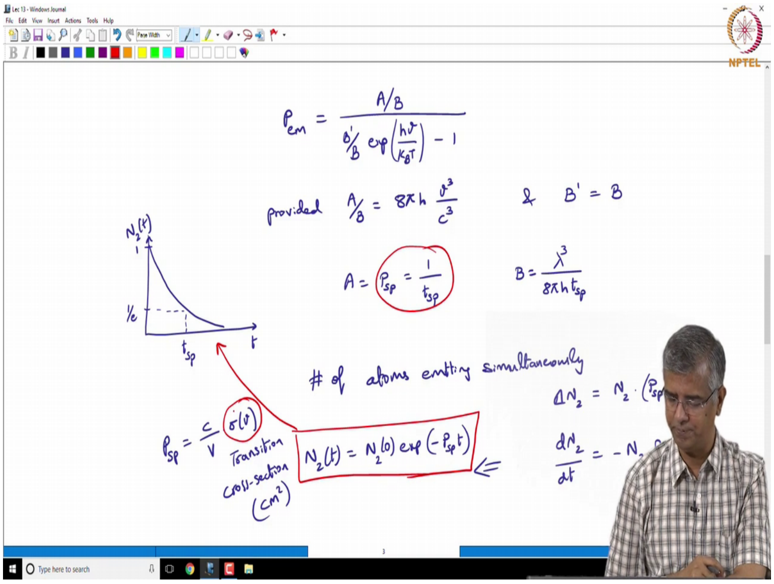

So we were essentially saying that the probability of finding an atom at an excited state is going to be lesser compared to finding an atom at a lower energy level and then we went on to look at spontaneous emission in little more detail and there was basically saying that a spontaneous emission happens such that the higher energy level is depleted continuously and so the depletion is actually an exponential, so why it is an exponential? You know maybe we can expand that on a little bit so if you consider the number of atoms that are emitting simultaneously at any particular time T.

Let us say that fraction of atoms emitting is delta N2 okay that certainly is going to depend on the number of atoms already at that higher energy level, so it depends on having a certain number of atoms so the higher energy level and then it is going to be emitted with a certain probability right, so the probability density that we are defined is PSP and then that is happening over let us say a time period of delta T right.

So this is actually over at discrete time interval but if you actually look at the continuous emission you basically say that DN2 over DT you can write it as minus N2 PSP, why I am putting minus there? Because it is decreasing right, so it is obviously when you are emitting this photons in a spontaneous manner you are actually depleting that population in that excited state, so this would essentially provide a solution which is given by N2 of T equals to N not or whatever was the population at time equals to 0 maybe we can write it as N2 of 0 right.

So a time equals to 0 exponential of minus PSP multiplied by T right, so that the solution you get for spontaneous emission and that is what we are essentially representing over here, where PSP is what we have identified over here, PSP is equal to 1 over that spontaneous lifetime okay or rather that spontaneous lifetime corresponds to the time where that into population goes to 1 over E value of whatever it was a time equal to 0 right.

So this of course you know we are also see, will start looking at these other quantity wherein we say PSP for a given volume is going to be given by C over V multiplied by sigma of mu okay, where sigma of mu this is what we are going to use as we go on we will not look specifically at the probability density every time we will actually look at other refer to this sigma value which is responding to what is called transition cross-section okay, so we will instead of representing in terms of probability will represent it in terms of what is called a transition cross-section okay.

So the transition cross-section is typically expressed in terms of metre square or centimetre square right and clearly in this case volume is an metre cube and you know in probability density is actually less, so just to normalise or just to make sure that the units are agreeing V are probability density actually corresponds to the rate at which this spontaneous emission happens, so make sure that things are you know the dimensionally correct, you know we have that C which is actually the corresponding to the speed of light that is in metres per second.

Okay, so we will be that when we are quantifying lot of this probabilities we will start expressing that in terms of this sigma which is called the transition cross-section, if it is transition cross-section corresponding to a emission we will call the sigma E, if it is transition cross-section corresponding to absorption we called that sigma A okay but what we realise in this case is you know that both host cross-sections are equal for the case that we have taken here which is considering just two energy levels but if you have multiple atoms, multiple energy levels those transition cross-section values can be different as well, we will look at that in more detail as we move on.

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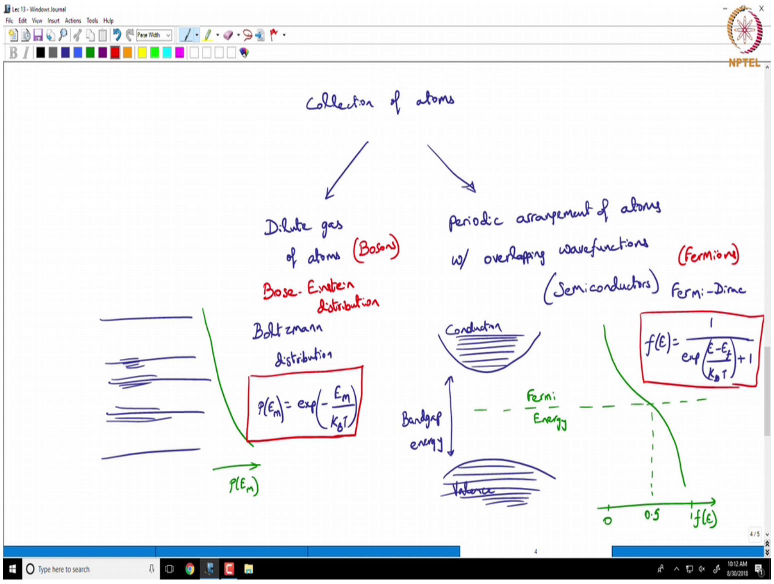

So beyond that we went on to looking at, so we were defining what type of atomic systems that we are going to deal with, we said okay there are atomic systems which follow the fermi dirac distribution and they are atomic systems where under thermal equilibrium conditions it actually corresponds to the Boltzmann distribution but we will introduce another distribution here which is also, which is called the Bose Einstein distribution right.

So you know we will see that we could describe the emission statistics from these dilutive gas of atoms in terms of Bose Einstein distribution, so from that prospective the emission from these are known as bosons whereas the emission from these are typically called as Fermions, so somebody add this question about photons as bosons the other day that the particles that satisfy fermi dirac distribution are called fermions and the particles that satisfy Bose Einstein distribution are called Bosons.

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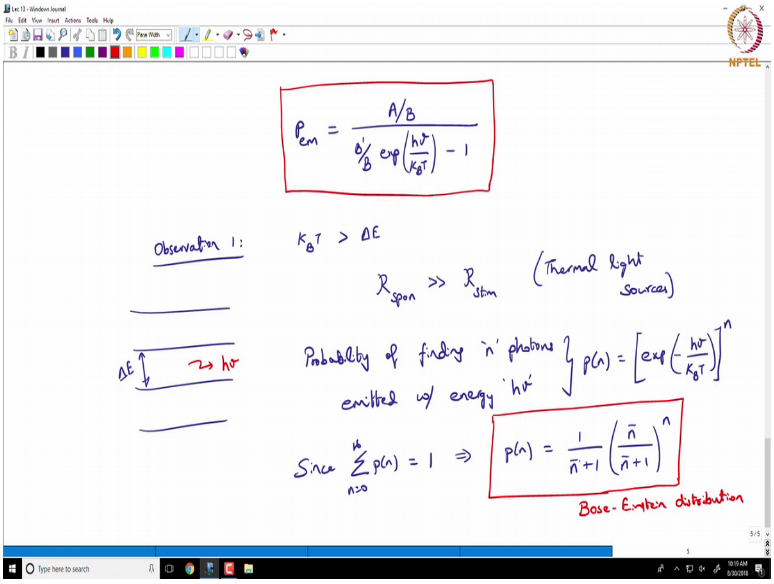

Okay, so we were looking at these two classes of systems from which light could be emitted and then we came up with this expression for the thermal equilibrium condition and we were starting to understand what this expression means right and the first case we were taking was when the thermal energy is greater than the energy difference between two energy levels, so if you say you are system as all this energy levels, let say with energy difference of delta E.

Okay, if you are thermal energy is such that is greater than delta E right, so what that means is you going to have a lot of electrons occupying the higher energy levels right, so nevertheless what we find is because you have all this electrons that are, so if the electrons are having that higher energy level than the probability of emission is a really high, so that is what mathematically you will see also right, so this will correspond to probability of emission very high but nevertheless what will find is at the spontaneous emission is more probable come back to simulated emission and this is the characteristic we will find with thermal light sources.

Okay, so or what you are talking about at the end of last session was at this is the condition for incandescent light sources, so you just heat up that material and two fairly high temperature, so that you have a lot of electrons going to higher energy levels and then you have a probability that this will emit photons but nevertheless, because of the fact that they are following the Boltzmann distribution, the Boltzmann distribution what is it say, what is it say about N2 verses N1.

So N2 is always going to be less than N1 right because of the probability of you know occupying a higher energy level is lesser compared to the probability that you will find that electron at a lower energy level, so N2 is a less than N1 as far as thermal equilibrium conditions are concerned, so because of that we will see that the spontaneous emission rate is more probable back to the simulated emission rate.

And if you were to go into this in a little more detail what we will find is the probability of finding N photons emitted with energy H new right, so we are basically talking about emission with energy H new corresponding to the energy difference between the two energy levels that we can call as P of N right but as far as spontaneous emission is concerned each of those emission E1s is actually an independent E1.

Okay one is not dependent on the other, so you can the probability of emitting one photon would correspond to this value and so probability of emitting N photons since it is statistically independent will correspond to that expression, exponential of minus H new over KBT that would be to the power of N, however since when you sum all this probabilities, probability of emitting 1 photon, 2 photon, 3 photon and all of that.

If you sum for an equal to 0 to infinity that is got to be equal to 1 right, since that sum of all this probabilities will have to be equal to 1, you have essentially an expression or under this condition you have an expression for P of N which can be derived as 1 over N power +1, N power over N power +1 to the power of N, this actually was what originally S C Bose had derived okay and then he got that validated by Einstein, so Einstein also supported that theory and then you know.

So they come up with this, this is actually what we call as Bose Einstein distribution okay and in such a case, so we are talking about thermal light sources right it is actually following this Bose Einstein distribution.

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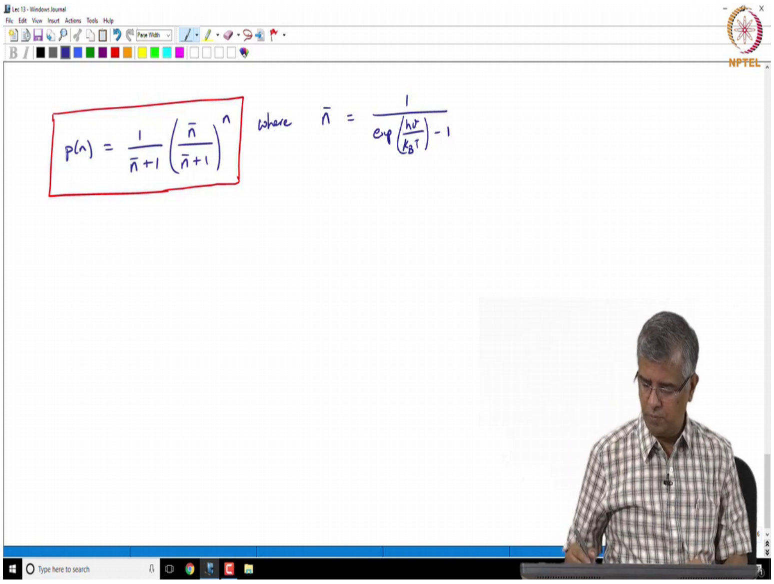

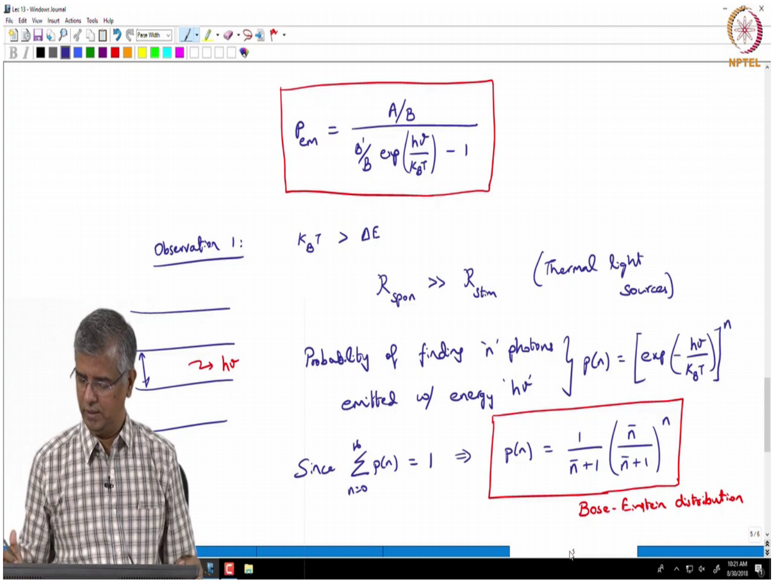

And where we can quantify N power, so N power is now going to be given by 1 over exponential of H new over KBT -1 right, so in this where N power is given by this expression, somebody has a question here yes, so what we are saying is you are having a system with a limited number of atoms, it consisting of a limited number of atoms and we are all excited with some thermal energy right, so they are all going to the higher energy level right and from that higher energy level that you know they will emit photons is actually a certainty.

So the what we are saying is they will emit a photon but then we are saying, you know what is the probability that you will find at any particular instant that N photons are emitted, so they may be emitted at different timescales, so they will be emitted as a certainty that is what the submission is saying and but they will be emitted at different time scales and they will be emitting at, you know different number of photons also, so the probability of emission, the total emission has got to be equal to 1 but the individual probabilities you can define right, yes.

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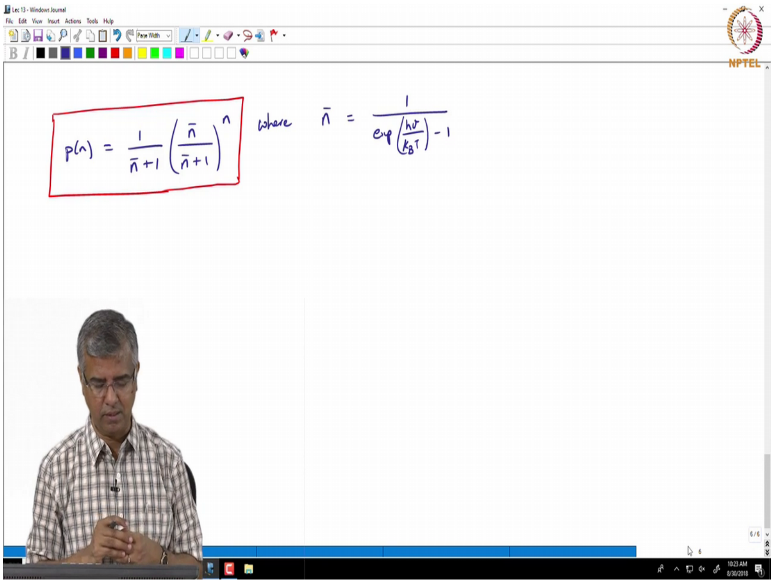

So if we, actually plot this P of N, so you rather, you know we have had this discussion before where we were looking at the probability of emitting and photons in a, for a generally light source right and we were actually talking about, you know lastly you are talking about that probability if you have N power equal to 10 it does not mean that you will exactly emit only 10 photons, there is a finite photon probability that you might actually be emitting only you know 5 photons or is it finite probability that you are emitting zero photons within a particular interval right.

So we were looking at that picture for a generally light source that could be basically a laser type of light source but now we are looking at thermal light sources okay and in thermal light sources we are saying that a, you know we have this sort of distribution.

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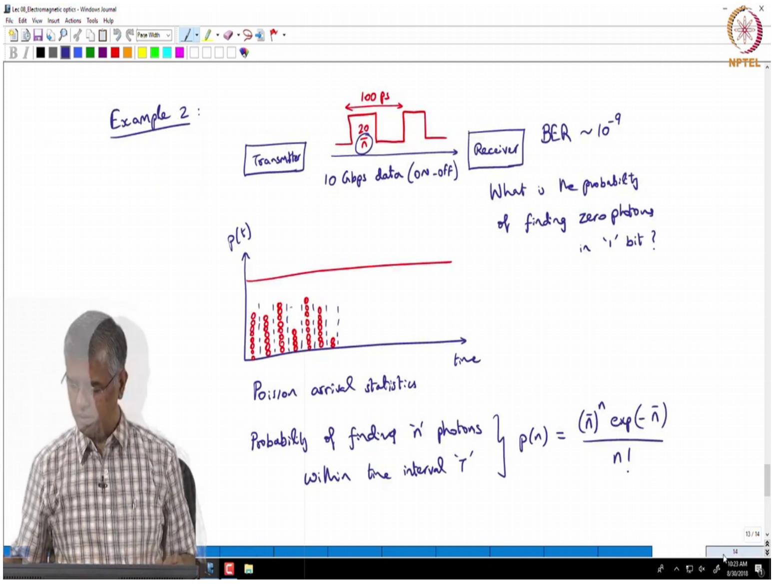

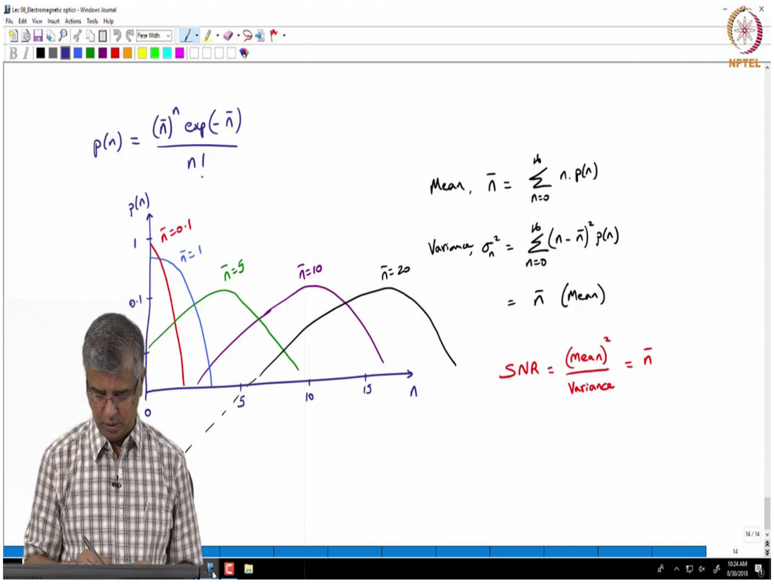

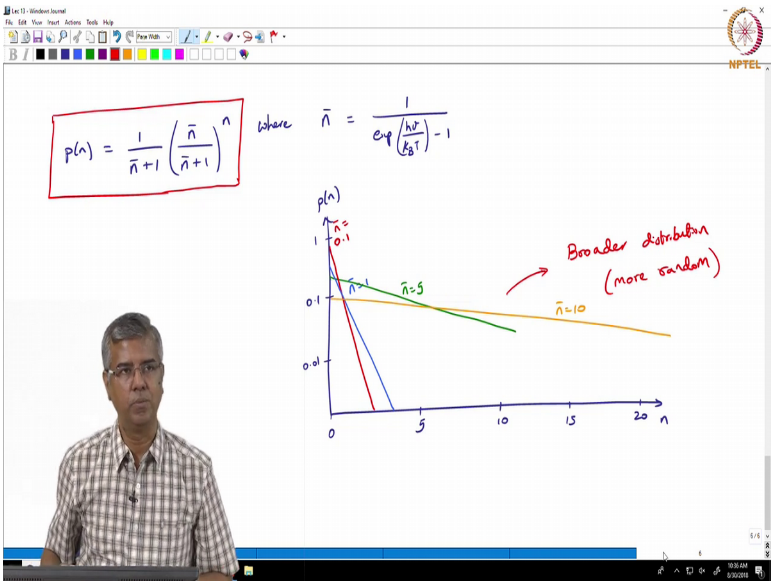

So we were saying that generally with light sources we follow arrival statistics follow, a poison distribution and that is why we were at P of N as a function of N but now we are talking about different type of light source which is just thermally excited okay and for thermally excited light source is if we look at P of N as a function of N okay, P of N let say this corresponds to 1 and 0.1 and 0.01 and lower than that and N corresponds to 0 say 5, 10, 15, 20 and all that okay.

In this case what will find is that for N power equal to 0.1, so let me just see what colors are used, so used to write blue, green, so for 0.1 so this is basically N power value, N power value of 0.1 if you plot this expression is going to look like this and then for N power equal to 1 right, it will be like this and then for N power equal to 5 right and so on, so if you go to N power equal to 10 may be something like this and so on.

So what do we see here? As far as you know your normal light sources are concerned, when I say normal light sources I am talking about what you have in a laser for example, the output of a laser, you have these statistics which say that okay this is a finite probability outside of that N power value, so if you say 10 is my N power here, the mean number of photons emitted is 10, so you will have a finite probability of finding 10 photons in any given interval that but that is the highest probability that you have.

Okay and then it is actually going down drastically beyond that right, we were actually net picking and we are saying that probability of finding zero photons in a given interval is actually a fairly small number but still a finite number, we were saying that right but now look at this picture, what can we say about this picture, the distribution is much flatter okay, which source would be more random, which is more random, here this source is much more random than the other source okay because statistically if you consider for example N power equal to 10, there it actually goes to a maximum and falls of in either side of N equal to 10 right.

But here you just quite flat which means that there is actually, in this case what it saying is that even though we have N power equal to 10, there is actually a probability that you will find only 0 photons emitted within a particular timeslot okay that probability is actually even higher than finding 10 photons emitted, so it is like completely random sort of emission okay, so we were previously discussing this randomness in terms of a quantity, what were we discussing? How did we characterize this randomness? In terms of a particular phase we were using.

Coherence right, we were talking about that statistical, the randomness of light emission where characterizing in terms of coherence, so what we can say is thermal light sources are relatively incoherent compared to these other light sources like in the case of a laser for example okay.

So thermal light sources as in the case of sunlight or in the case of incandescent light which are basically blackbody radiation is fairly incoherent, fairly you know random light sources is what we were saying, so the key point is it is broader distribution or in another case more random emission is what we are talking about, so what we are saying is that emission first of all you can say at this point this is, this many photons are getting emitted.

So that actually is an uncertainty but the specific question is why is it that emitting zero photons is more probable than emitting higher number of photons and that is basically because when you look at the probability that we have here you know, we are saying number of photons since they are statistically independent, the number of photons that you emit is going to be, if it is N number of photons, it is to the power of N, so it is actually statistically less probable that you will have that many photons emitted at the same time okay.

The key thing is that we are looking at you know, yes the number of photons that are emitted simultaneously or within a very small timescale, so what we are saying is, so the question is are we implying that most of the time there is nothing emitted, that is not what we are saying, I mean there is a emission happening clearly but the probability that within a particular timeslot you find zero photon emitted is more, so when we are looking at light emission from an incandescent lamp there is a continuous, I mean what we see sunlight is not like sunlight goes on and off right.

It is in a perceivable manner because we are time averaging all of our thing is a lot of emission that is happening, however you can take sunlight and you can maybe you can go to nanosecond time interval that there is a good chance that you will find, you know zero photons compared to chance of finding one photon, two photon and all of that, you will have to consider that this is the entire distribution we are talking about and it is flat, another words if I were to draw this for N power equal to 20 to look even flatter, what is that value?

It is telling me that there is equal probability of finding zero photons as that is finding hundred photons right, that is what, it is quite random is what we are saying okay, so it does not mean that no photons emitted at any particular time, I mean you have photons emitted but the probability of finding zero photons is slightly higher and then when you consider large number of photons that is actually a flatter probability.

So we are saying that the emission is just random that the key point right, what we are saying is we are choosing a time interval but assuming that all this propagation happens in sort of a that statistics of propagation are not distorted, we can map whatever observation is to the time of emission, so from that perspective I am talking about right, so but we are essentially looking at I mean looking at it from a observation you point typically okay.

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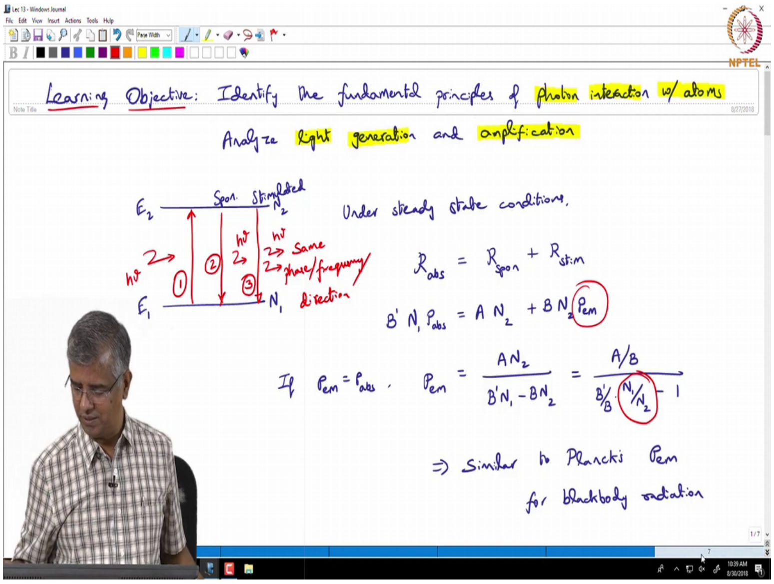

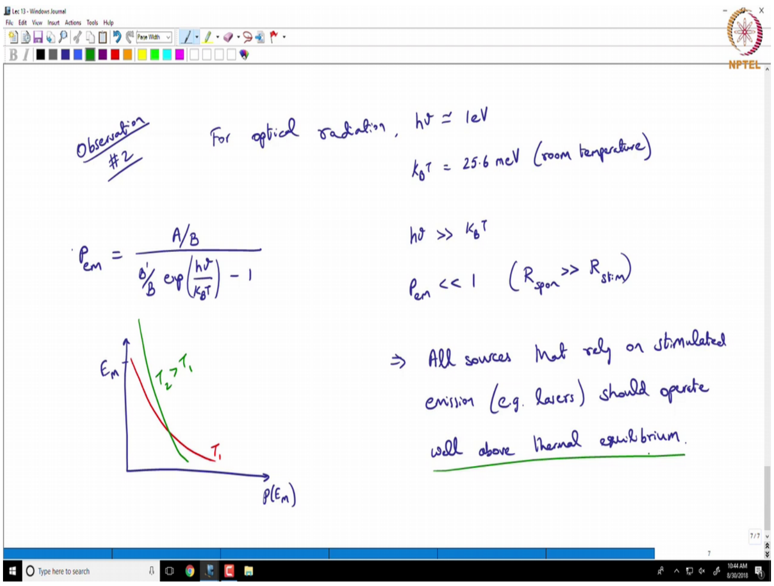

So that is one observation, observation number 2 is the case where, if you consider optical radiation right for optical radiation H new, what can we say about H new? What sort of energy do we tend to see? For example what is the energy for photon with a wavelength of one micron? 1.24 electron volt right, so it is in the order of an electron volt, so you can say this is in the order of electron volt but what is the thermal energy? KBT is 25.6 mille electron volt at room temperature right.

So what we see is H new is far far greater than KBT, if H new is far far greater than KBT what can we say about this emission probability density, so those is going to be fairly large value, so you know then emission probability density is going to be far far less than one right, so and as a corollary you can say that since emission probability density you know shows up here right, so which is corresponding to the stimulated emission term, you can say that the spontaneous emission is going to be more probable than stimulated emission right.

If low, if that emission probability is very low, so this is basically saying R spon is far far greater than R stim okay, so this is saying that for optical radiation you cannot stimulated emission or you have very low probability of getting stimulated emission unless what? All this expression has assumed one specific condition, what is that condition? So we say it is steady-state but then we said there is something, some other condition that we are considering thermal equilibrium conditions right.

So all this, so in this sort of a case optical radiation is not possible, stimulated optical radiation is not possible under thermal equilibrium conditions, so that is what is are saying right, so this implies that all sources that rely on stimulated emission, for example as in the case of lasers right should operate well above thermal equilibrium right, all sources that rely on stimulated emission should operate well about thermal equilibrium and maybe you can look at this in little more detail, under thermal equilibrium if you are looking at, finding you know an atom at a excited state right.

It is going to be something like, let say like this at a temperature T1 right, if you go to a temperature T2 which is greater than T1 right, the probability is like this, nevertheless if you are looking at an probability of finding an atom at higher energy level even a T2 or even a as you increase the temperature you will find that, that probability is always less are compared to finding something at a lower energy level right.

So under thermal equilibrium conditions you cannot N2 right and then N1 right, so that is what we are saying here, we cannot have N2 greater than N1 and so if we need to make N2 greater than N1, you need to be well about this thermal equilibrium condition okay, you need break free of this thermal equilibrium condition, so essentially what do I? What am I talking about? We need to bring in some other energy into the equation, we cannot rely on thermal energy because you just heating the material higher and higher, you will get more emission that is what we see, you take an incandescent lamp and you give more and more current to it, you crank up the current it becomes brighter and brighter right, But you know it still in thermal equilibrium condition which means that as far as optical radiation is concern it is all only spontaneous emission that we are getting okay.

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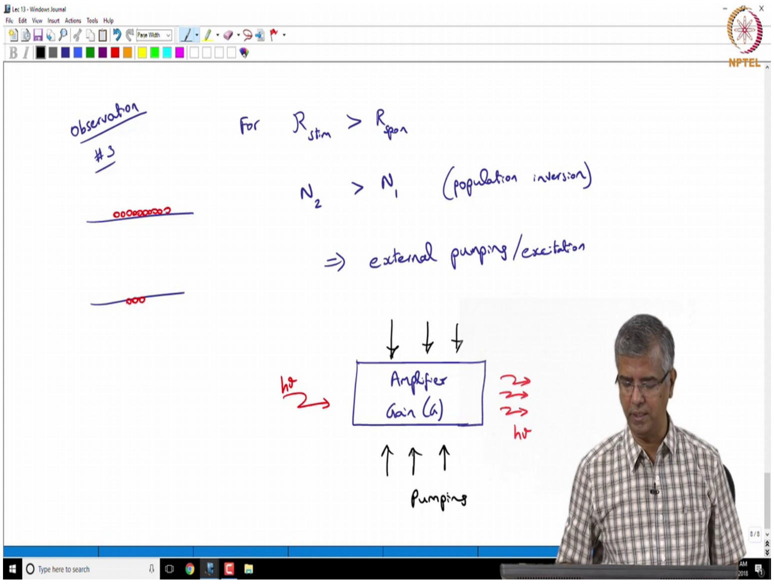

So that is our observation number 3 right, so if you want to get the stimulated emission rate to be greater than the spontaneous emission rate, you essentially need to have N2 greater than N1 right, so you need to have essentially more number of atoms at an excited state compared to the lower energy level and of course beyond that you also need these photons coming into stimulated the transition, so that is another thing that we need but basic condition you require is that, you know N2 should be greater than equal, greater than N1 right and this is a condition that we call as population inversion okay and this is accomplished by external humping or sternal, excitation that and thermal excitation.

So what are we saying? We are saying that to get gain, optical gain and another words multiplication of photons in a medium you need stimulated emission right, and to get stimulated emission unit external excitation, so if I am building an amplifier with some gain G, so what does that mean? I am coming in with some photons H new, I am going out with lot of photons at that same frequency, the same energy right and to achieve this what I needed now is some external pumping and this external pumping can be in terms of optical radiation right.

So you can come in with optical radiation to excite to a higher energy level right and from their you could have stimulated emission or you could have electrical means of pumping right, as in the case of semiconductors what we have is, by electrical injection of carriers you can have a lot of electrons in the conduction band and holes in the valence band and then you could have recombination of this electrons you know with the holes which result in optical radiation okay.

So you could in that case it is actually electrical energy that you are putting in but here the other case you can think of it as optical energy okay, so you need some external energy source to get over this thermal equilibrium condition and at this stimulated emission possible okay and that actually takes us into this exciting topic of optical amplification which will look into more detail in the following lectures okay, so let me stop at this point and will continue in the next lecture.