Introduction to Photonics
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Tutorial Photon Optics

Ok good morning, welcome to Introduction to Photonics, so far we been actually looking at properties of photos in terms of the photon energy we are looking at photon momentum, what does it mean to, what is the priority of finding a photon at a particular location and related to that we were also looking at photon uncertainty. So what I would like to today is you know just see if you can gather all the things that we have learnt and see if we can solve problems ok related to photon optics.

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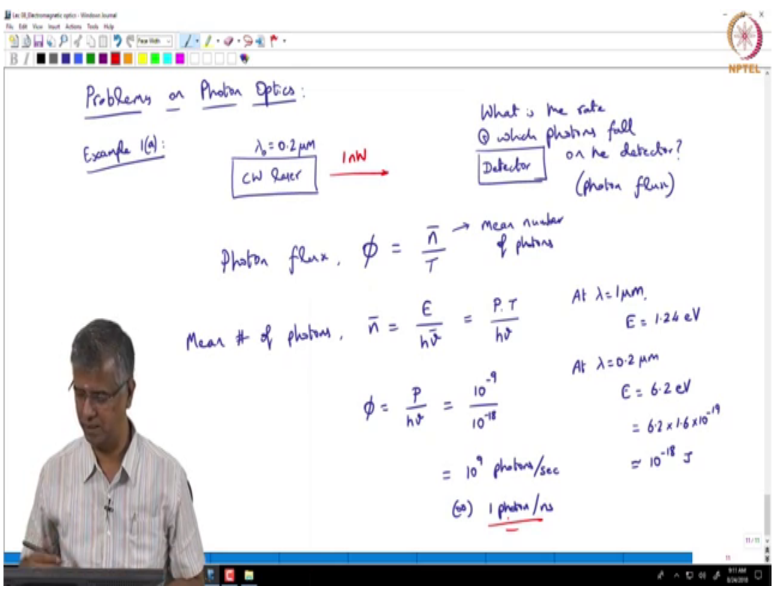

So lets take the example of example let us say 1 a right so you are told that you have a C W laser right you have a C W laser, so what does C W mean? C W is corresponding to continuous wave ok as suppose to the other type of laser which would be like a pulse laser ok so if its emitting pulse laser then pulses you will say it is a pulse laser otherwise t is a continuous wave laser. Ofcourse we will find that it is actually sort of a mis-normal because we talk about continuous wave as in continuously propagating as wave but what we are actually going to look at is the photon properties of this laser photon emitting properties of this laser.

So you are told that this continuous wave laser has wave length of 0.2 microns and it is emitting radiation which corresponds to 1 nano watt of power ok. Now what you are asked to figure out is if you put a detector over here then what is the rate at which photons fall on the detector. So what is the rate at which photons fall on the detector is what you are asked to figure out. So what we see is in conventional wave optics you deal with quantities in terms of power, energy, energy density, power density which is intensity and so on and what we will see in the corresponding photon picture is will talk about number of photons, we will talk about photon flux which is actually what is asked here.

What is the rate at which photons are arriving at a particular point so that we call a photon flux, what is the photon flux density? Ok per unit area what is the photon flux that we get at a particular point? So those are the quantities that we will start looking at ok. So for example here what you are asked is the photon flux so photon flux lets say you denote by is capital Phi is going to be given by the mean number of photons falling on the detector per unit time ok. So n bar here corresponds to mean number of photons right, so what is n bar? If you know the energy applied ok can we find what is the mean number of photons?

So mean number of photons n bar how is it related to the energy? If your energy of your radiation is E divided by h mu which is we energy corresponding to a single photon right so now we can if you are given power or energy you can actually figure out that in terms of number of photons ok and of course I say h mu I may also use h mu bar where would I use that? If I have a if I am given a broadband source right like what we were looking at yesterday maybe your source has got multiple frequencies in it and then you have a center frequency right that center frequency you make take as mu bar ok.

So in this case you are given power right given optical power how do you convert energy to power? What is the relationship? Power times the power integrated over time is energy right so P times T divided by h mu and so now we need to find out h mu for the given laser the laser is oscillating with 0.2 micron wavelength right, so we know that lambda equal to 1 micron what is the corresponding E? Would it be C? 1.24 electron volt right, so you are asked to figure out the energy of a photon at lambda equal to 0.2 micron, what would this energy be?

So it is five times lower wavelength so the energy should be five times higher right so 5 times 1.24 that would correspond to 6.2 electron volt right. So lambda at 0.2 micron is 6.2 electron volt and if you convert that into joules what do you multiply that with? 6.2 multiply by 1.6 into 10 power minus 19 charge of an electron and that would give roughly about 6.0 to (())(08:40) roughly about 10 so that is about 10 power minus 18 joules right. So if I am what I am actually out to calculate is the photon flux so I go back to photon flux and I substitute instead of n bar I substitute this expression so then what you get is essentially the T in the numerator and denominator will cancel each other.

so what you get is this is going to be equal to the power divided by h mu ok since I am having only one wavelength given I can just take it as a h mu so this is so the power that is given is 10 power minus 9 right 1 nano watt and h mu you figured as 10 power minus 18 ok so that corresponds to 10 power 9 photons per second right. So the photon flux is 10 power 9 photons per second which is corresponding to the power of 1 nano watt at a wavelength of 0.2 microns. So this is actually you can also say that this means that I have one photon arriving per nanosecond right. One photon arriving per nanosecond on the average so this would be an interesting experiment I was hoping I could give you a demo of this you have done this before.

So you take a laser and you know you take maybe a mile watt power level laser like what you guys have been working with for your demos right it is in the order of few mile watt so you put an attenuator in front of it you attenuate that to this lower levels 1 nano watt power levels and your detector let us say it is a high speed detector what I mean by that is it is capable of picking up light radiation at nanosecond time scale ok you take that out of a detector and you convert that detector will give you a corresponding photo current which you convert to voltage and you feed that voltage to a speaker ok. Now what do you expect?

You expect to hear a top a pop, a pop every you know nanosecond right so you will basically ofcourse you not you know hearing at nanosecond time scale you don't have that so what you will actually hear drrrr you know this continuous thing so you can actually listen to photons falling on the detector so that is actually a very beautiful experiment we have done that before I will see if the TS can arrange for that it is not too difficult but it just means I need to find the high speed detector and connect it to a thing so you can listen to photons falling on the detector when you go to real low level of photons with high speed detector ok.

That is when you believe ok all this is true there are actually photons that are coming and falling on the detector anyway so as far as what we were ask in this problem you know we just converting traditional quantities like power and energy into you know photons in number of photons and photon density and so on.

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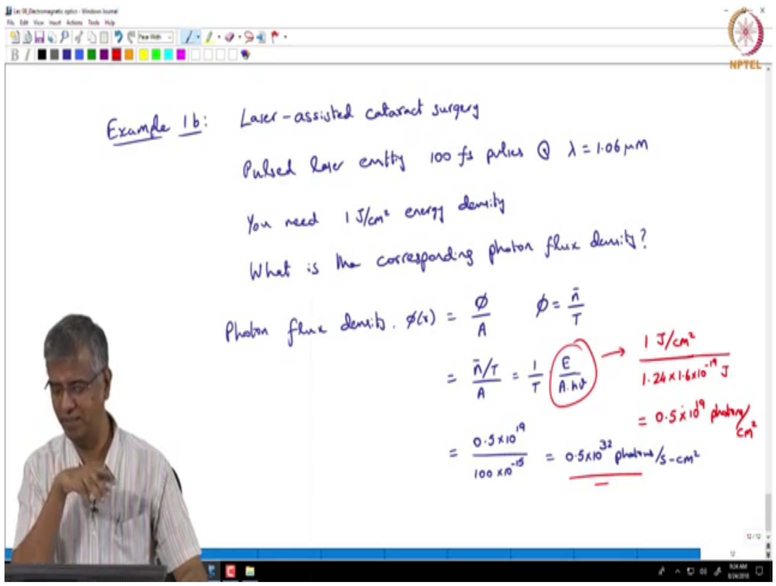

So let us move on to another example, example 1 b lets say and this is actually related to something called laser assisted cataract surgery so you know we have actually started a company that makes fiber lasers you don't know what are fiber lasers maybe I will come back and talk about that later on. But we make fiber lasers and we make pulsed fiber lasers and this fiber lasers are being looked (())(14:05) work with one of the research institute which is doing I research as to whether this laser can be used to ablate tissue in the eye ok so that you can basically make a very precise incision in one of the tissue layers in the eye and you can prove that you can do cataract surgery ok so that is what laser assisted cataract surgery is all about and in this case we use a pulse laser ok so the pulse laser you know it is emitting pulse laser emitting 100 (())(15:02) pulses.

What is the (())(15:06)? 10 power minus 15 seconds right (())(15:12) which is 10 power minus 12 so 100 (())(15:15) pulses. At wavelength of 1.06 microns why that specific wavelength will come back and look at that later when we are discussing lasers but let us say this is emitting this pulses at 1.06 microns and you are told that you need 1 joule per centimeter square energy density to create a to basically punch a hole through the tissue layer ok so this is a process that is called photo disruption that you are doing where you are actually cutting tissue with this pulse lasers and why did you so you can always use a high power laser and cut tissue why do you want to use very short pulse laser? Any idea?

If you are continuously depositing energy on the tissue it will be absorbed by the tissue and then there is a heat that is generated, that heat will you know spread around the tissue and it will end up charring the neighboring tissue also there will be a lot of collateral damage that is happening. But if you are using 100 (())(16:57) pulses what are you doing? You are packing a whole lot of photons within very-very small duration so it goes and hits and it does the work of you know getting absorbed and then it is rupturing the tissue but since it is all happening within that 100 (())(17:21) scale it doesn't the heat actually builds up over millisecond times scales.

So if you are doing much-much shorter than that there is very little collateral damage that is happening so you can get very precise cuts if you are using such short pulses ok but you need you told that you need 1 joule per centimeter square energy density so the question is, what is the corresponding photon flux density? Ok so that is what you are asked to figure out and photon flux density what do you think that corresponds to? At a particular location lets say it is small phi of r would be what? Would be photon flux per unit area right, so that is what photon flux density is and we know that from our discussion in the previous problem we know that phi corresponds to n bar over T ok so you can substitute that over here and what you are given is actually energy density right.

So you can basically say this is going to be equal to n bar over T over A right but n bar over A so I will keep 1 over T outside right and n bar over A can be expressed n bar is in other words is basically E over h mu so you can write this as E over A times h mu right and what I have given you are given energy density which is E over A right and so if you substitute that and then you can actually and you are all you also given this time so based on that you can figure out. So lets just first of all figure out this quantity so this is given by E over A which is 1 joule per centimeter square divided by h mu, h mu for a lambda 1.06 microns, what do you think it be?

It is around something around 1.24 electron volt right so this is basically 1.24 electron volt and then you convert that to joules right, what do you do? Multiply by 1.6 into 10 power minus 19 so that will be in joules right. So if you do the math you will probably get something in the order of 5, 0.5 into 10 power 19 and this will be in terms of photons per centimeter square right so that will be the quantity here and to get the photon flux density you just substitute that into this other expression which is basically you say 0.5 into 10 power 19 over time the interaction time here is 100 (())(22:13) seconds right so 100 into 10 power minus 15 so if you do the math that will come out to be about 0.5 into 10 power 32 photons per second centimeter square so that is the photon flux density that you need.

Now that is actually sounds like really large number but you know what probably make sense is that it is not the photon flux density that you are interested in you are looking at the photon density so you are actually you are already packed so many photons. How many photons are you packing? You know something the order of 10 power I out it as 9 it should be 19 here right, so that is 10 power 19 photons per centimeter square but you have not doing a whole over a centimeter square area that will essentially take out your eye completely.

So you are actually doing holds some microns holds right. So when you do it in terms of microns square right its maybe 100 microns square so the number of photons is in the order of 10 power 12 photons but still huge number you are talking about trillion so million, billion, trillion, trillions photons right you are packing within 1 pulse and bullet ok and you may actually require multiple pulses to do the precise cutting so you are like it is like a machine gun so just hitting with bullets and this bullets are extremely powerful I mean you are talking about packing that many photons within a very short time scale.

So this tissues have no chance it will rupture in no time but the key point here is because it is all this interaction is happening in within a very short time scale the thermal in a buildup is very-very low so you can get very precise cuts. Now it could be your eye it could be this wooden table here it could be you know a metallic part where you want to make a very precise incision you want to actually do some patterns in a metal some micro machining that you want to do in a metal all of that you can do if you have (())(25:19) pulses ok. How we generate this (())(25:23) pulses we will have to wait I mean there is still several weeks away we will come back and look at that in more detail but if you have a such a source you know this what you are dealing with I am just trying to give you a picture in terms of photons. Any questions before we move on?

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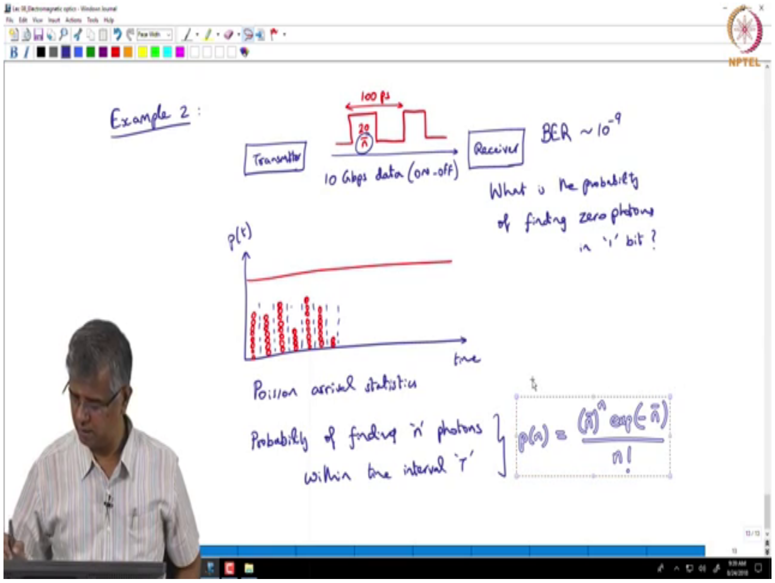

Ok lets now look at another interesting example, example 2, where we are looking at optical communication right, so in optical communication you have basically transmitter which is typically a laser based transmitter and you are communicating information in bits right, let us say you are transmitting data at 10 Giga bits per second ok and you are transmitting in on off format. What does on off format mean? One would correspond to the laser being on you have bunch of pulses and bunch of photons and off is the laser is off right so you should not have any photons in the off condition right.

So you are transmitting pulses like this and if you have transmitting a 10 Giga bits per second your pulse period here the time period between two successive pulses is 1 over 10 Gig, 10 Gig is 10 power 9 so 1 over 10 Gig be 100 Pico seconds. So you know within that 1 bit which is in the order of 50 Pico seconds you either turn on or turn off the photons ok. What you are told is in this case you have 20 photons that is the mean photon number n bar right is 20 photons and obviously the off duration that would be zero photons ok.

So that is what you have transmitted and what you are asked to do is in your receiver right what is the probability of finding zero photons in a 1 bit? Why we are interested in finding that? Obviously if you detect only zero photons in a 1 bit then that corresponds to an error so you want to find out what is the the typical quantity by which you, you know you (())(29:23) the performance of a communication system digital communication system is this quantity called the

BER, BER is Bit Error Rate so you want to look at the error rate that you are having in the communication and typical bit error rates in communication in optical communication you try to look at n power minus 9 ok.

So you want to achieve a BER of 10 power minus 9 ok, 1 in a billion bit can be in error but not more than that, did you have a question? No, ok, right so we are trying to achieve that and to achieve that you need to understand what is the probability of finding zero bits zero photons in a 1 bit. So it is given that you are transmitting a mean number so the key point is it is this n bar is actually a mean number of photons of 20 what is the probability that you don't have any photons in that 1 bit at the receiver? You think from sending 20 photons and something (())(30:41) there so you say that it is impossible that you will have you know zero photons corresponding to that.

But we will look into that little more detail now. The general misconception about light when we measure light is that we typically do what you do in the lab is actually you measure using a power meter right and the power meter essentially shows that it gives the power as a function of time so over a period of time you say the intensity is constant right, so that is what you are seeing but what is the time scale with which the power meter is providing its value doing the measurement it is typically you know its got some integration time and all that millisecond is typically typical time scale that you have right.

In the millisecond time scale everything looks constant you get the perfect reading you know 3.14 and so put them the reading 3.14 mili watt but does that mean that you have a uniform stream of photons heating that power meter, actually not because if you look at if you actually do this you know if you look at smaller sections in time specially if you go down to kind of the time scale that we were projecting in the previous case you will find that ok in this time slots so many photons came, in this time slots so many photons came, in this time slots maybe a few more photons came and maybe you would have a time slot where only a few photons came and then another time slot where there is much many more number of times slot that came and so on right.

You get the picture so what I am trying to illustrate here is that even though your power may show up as you know uniform in millisecond time scales if you go down to nanoseconds Pico seconds time scales what you may actually find this that it is not uniform really ok. Why is this the case?

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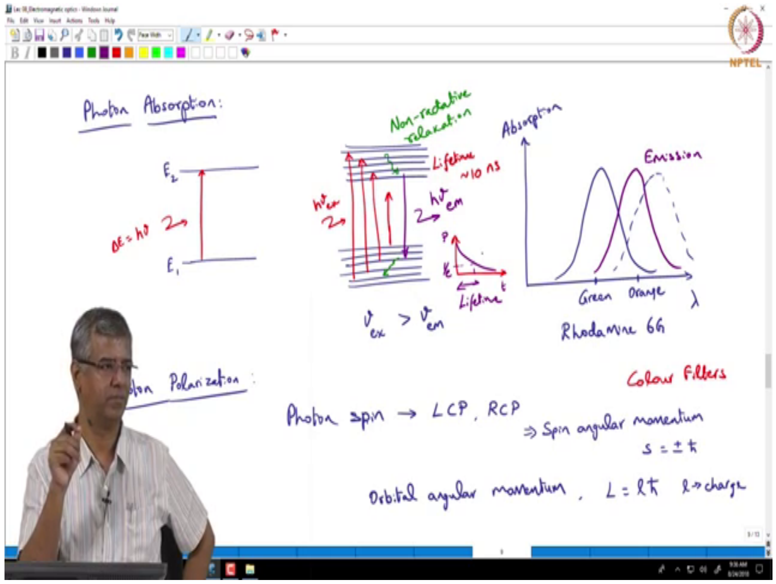

I will just give you a clue this point we don’t have a lot of what do you call lot of data points to say why but I will just give you a clue here. We are looking at this picture of photon absorption and emission and when we talked about a particular thing over here there is a lifetime over here right. So there is a lifetime that it spends in the upper energy level before it comes down to the lower energy level.

Now that lifetime for (())(34:22) is about 10 nanoseconds ok in fact we were trying to do a demo where we can measure this lifetime but unfortunately that didn’t work out because we cannot generate pulses that are much faster than 10 nanoseconds with the equipment that we have with us. But nevertheless the lifetime is approximately 10 nanoseconds right for (())(34:53). So what is this lifetime mean? Does it mean that every electron that goes to the upper energy level it spends exactly the 10 nanoseconds then after that it emits not really. It actually it is a probabilistic event I was mentioning that this absorption is a probabilistic event, emission also is a probabilistic event.

So you can only tell in terms of some mean probability right over some mean time over which this emission happens in fact this lifetime is defined as if you look at if you send a pulse of light for the excitation that you will have an emission corresponding to that which will be high initially because a large number of electrons are at high energy level and then some of those electrons are starting to emit so your florescence if you look at the florescence as a function of time you will see that it is actually coming down in an exponential manner because there is a finite reservoir that is built in with every photon that is emitted that reservoir is getting emptied.

So when you look at the rate at which the power is emitted as a function of time it will come down exponentially and the 1 over E point is what you figure out is the florescence lifetime ok. So what that says that the photons are emitted continuously well it is ofcourse quantized wave packets that are emitted but they are emitted at different times so you cannot control this is a quantum event you can control it to say that emits (())(37:13) at that this particular Pico second right. So it actually you can track the envelope of that but you cannot tell this photon is going to come at this particular time. So we just building on that in saying that when you think about light emission we think about light it is falling on me you are detecting that in an integrated manner.

You not seeing photon by photon if you go down to a Pico second times scale you probably see that this photons are coming at some random manner random it is emitted and some random times scale right so they are not coming uniformly ok that is the point. So your arrival statistics at the receiver ok that actually follows what is called a poison distribution. So since this photons are independently emitted ok the arrival status you have what is called poison arrival statistics ok and it is actually not too difficult to you know derive this, this formula for Poison Arrival Statistics it is one of the example problems it is given in your textbook.

But you can show that probability of finding n photons within time interval in a any given time interval right you denote as P of n, in this case what you have done is you have sent a mean number of photons n bar ok so if you take the mean of all this small circles that I have drawn here that would probably correspond to mean of 6 ok across all this thing so lets say you have sent 6 photons what is the probability of finding one photon in one of this time slots? Right that is what we are tracking and that for Poison Statistics works out to be n bar to the power of n exponential of minus n bar divided by n factorial ok.

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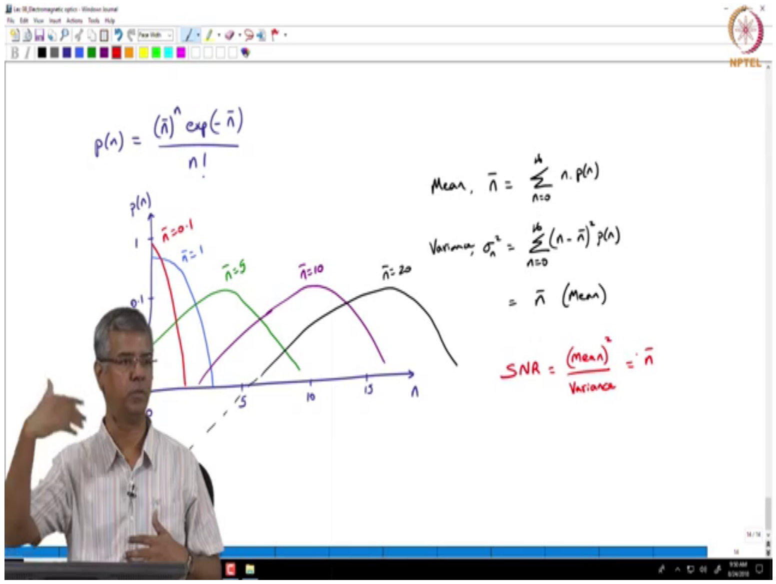

So it is this is basically the Poison Statistics, so let me actually select this and I will just ok so that is the probability that we have ok. Now if I try to plot this ok then we may actually get a clear picture of what this means what this Poison Arrival Statistics mean right so here I have n and I am plotting P of n 1 and let us say a plot is in log scale so 0.1, 0.01 and so on and n is 0, 5,10, 15 and so on ok. Now let us plot this function for n bar of 0.1 ok so average within an interval is 0.1 so you say how can I express it as a fraction? Well you cannot like pair the photon and say ok this much is going here that much is going there, no, it is not like that.

You are basically saying that the probability of finding a photon over 10 consecutive times scales this time intervals you know is one so it could appear in any of those 10 time slots ok so that is what m bar or 0.1 but interestingly if you plot this you would find that you have a very good probability that you will find no photons in a given time slot right there is a still a very good probability that you find I mean you will find one photon as a finite probability that could find one photon in the time slot or you could even have two photons in a time slot. There is a finite probability however small the probability is there is a probability.

Why are we tracking this? Because problem requires us to find zero photons when you have a one bit so there you have n bar of 20 ok and you are asked to figure out what is the probability that you find zero, zero photons in the time slot? So if we work further I may chose a different color and if you if I say n bar of 1 that is going to look like this right I am just giving you a general perspective don't pay too much attention to the specific numbers here and if I go to n bar of 5 it is going to look like that and similarly if I go to n bar of 10 and if I still go on to what we are looking for n bar of 20 would have probably look like this.

So ofcourse we have a scale here the wise scale where we have only up to 0.01 but if you just extrapolate over here you will find that it goes to a fairly low value. So what we are actually trying to find out is this point over here. So you have bar of 20 right that has been sent by the transmit that is the average that you have sent but you could have a probability that zero bits are there in that time slot ok you have a finite probability. If you work out this expression over here what you find that this probability corresponds to 2 into 10 power minus 9 ok.

So 2 times out of 10 power 9 bits right so that is a billion bits ok so if you have collected 10 power 9 bits then you may find that there are two bits that this sort of occurrence is happening ok that seems like a very small number but it still significant in communications because of what? Because you are transmitting a 10 Giga bits per second so within a second you may actually see two of this events ok. So that is the thing but ofcourse (())(46:44) it is relevant because of the high speed at which we are transmitting information but the key point is you cannot say that I am transmitting 10 photons in this slot and expect that you know all this slots are having only 10 photons, it could be 10, it could be 9, it could be 8, it could be 12, it could be 20, it could be 0.

Each of this have a finite probability ok so when we are talking about photons streams this is what this is one of the important concepts that we need understand that (you) cannot control cannot time the photons so precisely ok. Just one quick thing before I end this if I just go on to looking at for a Poison Distribution if you look at the mean number of photons that would correspond to you have all this possibilities that it could be zero or (all the way) to infinity but the probability of those (())(48:00) so m into P of n is the mean and more importantly if you look at the variants so this is the mean and this is the variance, variance for a Poison Distribution works out as infinity n minus n bar the whole square p of n and if you compute that interestingly what you find is that corresponds to n bar itself.

So the mean and the variance of a Poison Distribution are the same ok so that is an important you know aspect of poison distribution and the consequence of that is when you are looking at the signal to noise ratio most of the communication applications most of the practical applications of light we are interested in the signal to noise ratio so the signal to noise ratio calculated in terms of photons it corresponds to the square of the mean over the variance which is the noise right. So in this case that since you have a mean square divided by the variance which is the mean so your signal to noise ratio goes as mean (in the photon).

So if you want to improve your signal to noise ratio you want to have a larger mean number of photons right that is what it means as for say communication application concern ofcourse that means that you want to flood your you know your time slots with photons but it is not as simple as that because in practical systems we find that beyond a certain number of photons if you put into the medium, the medium starts behaving in a non-linear fashion ok that epsilon that we are talking about the permittivity which is the response of the medium for a given excitation given electromagnetic wave right that response it is linear up to a certain number of photons beyond that it is starting to behave non-linearly.

So you cannot extend this argument you know to a very large number of photons because you know you will start hitting this non-linear issues and that is one of the major research areas in optical communication today, how do you in a proven non-linearity from happening in the optical communication system and still get very good signal to noise ratio ok. Ok so lot of time now so lets stop at this and what we are going into now is looking more closely at the absorption and emission of photons how does that happen in different systems, what are the properties of those photons and so on. So will going to build towards amplifiers towards lasers, how to build lasers that is what’s coming up in the next few weeks ok, so thank you.