Introduction to Photonics
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So good morning everyone, we will continue from where we left off in the last lecture, we said the learning outcome for this week is actually identifying the fundamental principles of photon optics and quantify photon properties, but before we understand proton properties, we need to understand electromagnetic properties of light. So that is what we started doing yesterday, we said okay, light actually travels as electromagnetic waves so it has to satisfy Maxwell's equations.

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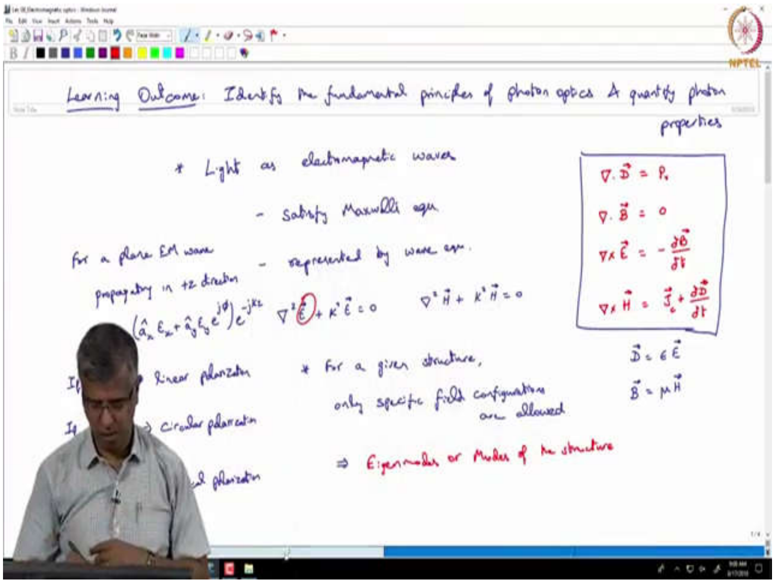


Learning Outcome: Identify the fundamental principles of photon optics & quantify photon properties

* Light as electromagnetic waves

- satisfy Maxwell's eqn

For a plane EM wave propagating in +z direction

- represented by wave eqn.

$(\hat{a}_x E_x + \hat{a}_y E_y e^{j\phi}) e^{-jkz}$

$\nabla^2 \vec{E} + k^2 \vec{E} = 0$ $\quad \nabla^2 \vec{H} + k^2 \vec{H} = 0$

If ⇒ linear polarization

If ⇒ circular polarization

polarization

* For a given structure, only specific field configurations are allowed

⇒ Eigenmodes or Modes of the structure

$\nabla \cdot \vec{D} = \rho_v$

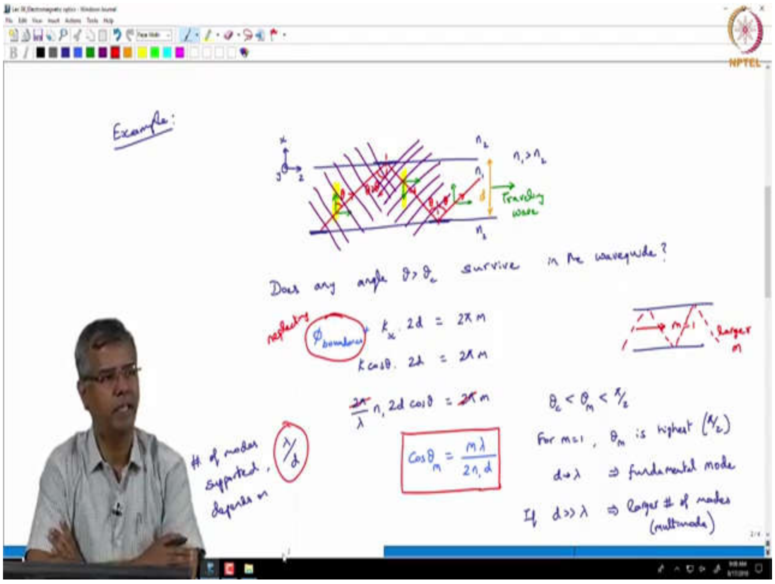
$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$

$\vec{D} = \epsilon \vec{E}$

$\vec{B} = \mu \vec{H}$



Example:

Does any angle $\theta > \theta_c$ survive in the waveguide?

reflecting $\phi_{boundary} + k_x \cdot 2d = 2\pi m$

$k \cos\theta \cdot 2d = 2\pi m$

$\frac{\cancel{2\pi}}{\lambda} n_1 2d \cos\theta = \cancel{2\pi} m$

$\boxed{\cos\theta_m = \frac{m\lambda}{2 n_1 d}}$

$\theta_c < \theta_m < \pi/2$

For m=1, θ_m is highest $(\pi/2)$

$d \sim \lambda$ ⇒ fundamental mode

If $d \gg \lambda$ ⇒ larger # of modes (multimode)

\# of modes supported, depends on λ/d

And then we saw how Maxwell’s equation’s would be satisfied for a particular structure, we took the example of an optical fiber, right, so we said if you consider an optical fiber, you can say that the light is essentially, you know from a ray picture, you can say that light is confined within a waveguide through total internal reflection, but then we saw that you know, you really have to consider them as waves and so you need, it basically accumulates phase as it is propagating and to satisfy Maxwell’s equations you need to satisfy the boundary conditions and as far as the boundary conditions are concerned, we were saying that you will have to have a consistent pattern of interference that is travelling all along the waveguide.

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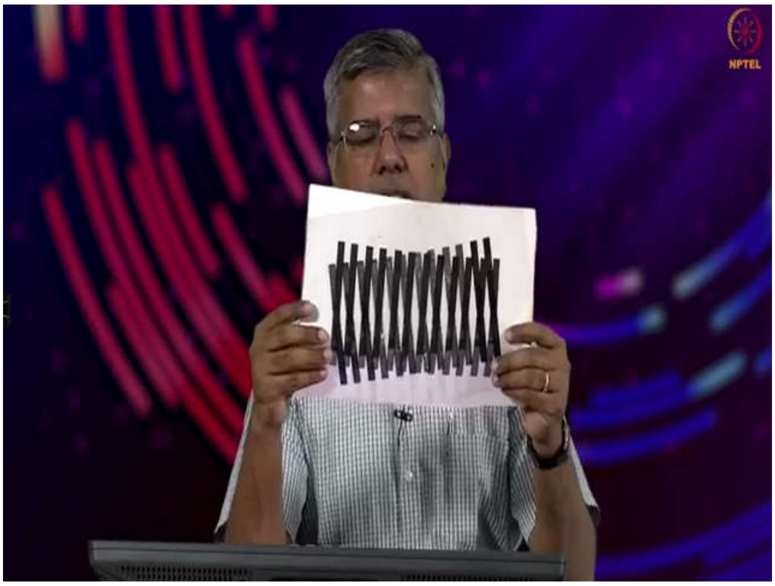

So yesterday I wanted to actually show you this is something that I experienced as a student and it stuck to my mind, but let us say, this is one plane wave and we are trying to explain how this interference happens and how all these different modes develops, so let us say, this is one plane wave travelling from this direction, from here to here and now, we have reflection from another plane wave right, and the reflection is such that, you know they have a very small angle between them right, so I will move one with respect to the other, you see, let us say that the bright regions are where constructive interference happening and the dark regions are where the destructive interference is happening.

You can see that, there is one pattern that is consistently developing across this, so this you may think of as your fundamental mode, and as you have steeper bounces, you are having one wave come across the other wave at an angle and then you can start seeing multiple, you know constructive and destructive interference fringes that are developing and the key thought is that, what we are talking about. When we are talking about, you know satisfying the boundary conditions is that this pattern, you if it has to satisfy Maxwells equation, it has to propagate all along the length of the fiber. Okay, so as you go higher and steeper and steeper angles, you have higher and higher order modes that are forming, but if you go to a wave that almost, along the axis of the waveguide, then you have only one, you know basically mode propagating and that corresponds to a bright fringe at the centre and it goes towards the darker fringes at the edges of the waveguide.

So this is sort of a very graphical way of seeing this interference and of course, you might have understood this, you know through this picture itself right. So we were talking about different modes and of course this is a very simplistic picture just to show how different modes essentially take different angles, if you just go back to the ray picture, which is probably easier for some of you to understand, it takes different angles and so we just came up with what should be that angle for a given mode.

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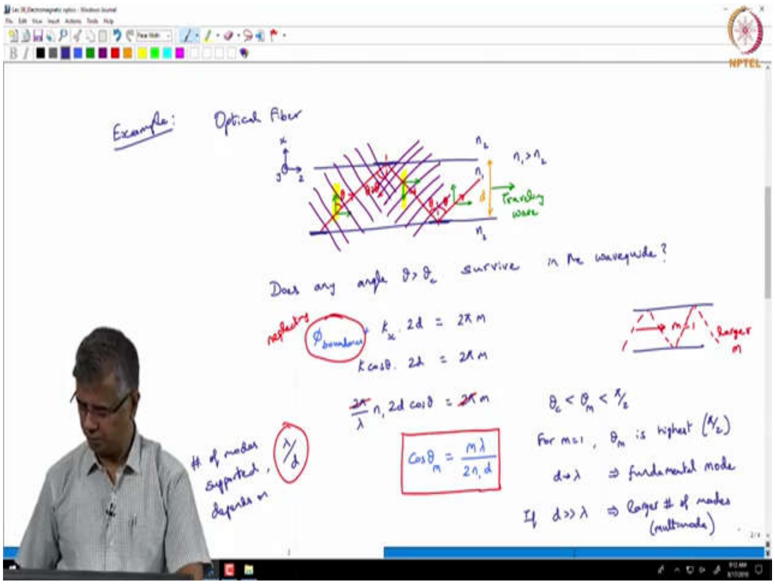

Example: Optical Fiber

Does any angle $\theta > \theta_c$ survive in the waveguide?

$\phi_{boundary}$ + $k_x \cdot 2d = 2\pi m$

reflecting

$k\cos\theta \cdot 2d = 2\pi m$

$\frac{\cancel{2\pi}}{\lambda} n_1 2d\cos\theta = \cancel{2\pi} m$

$$\boxed{\cos\theta_m = \frac{m\lambda}{2n_1 d}}$$

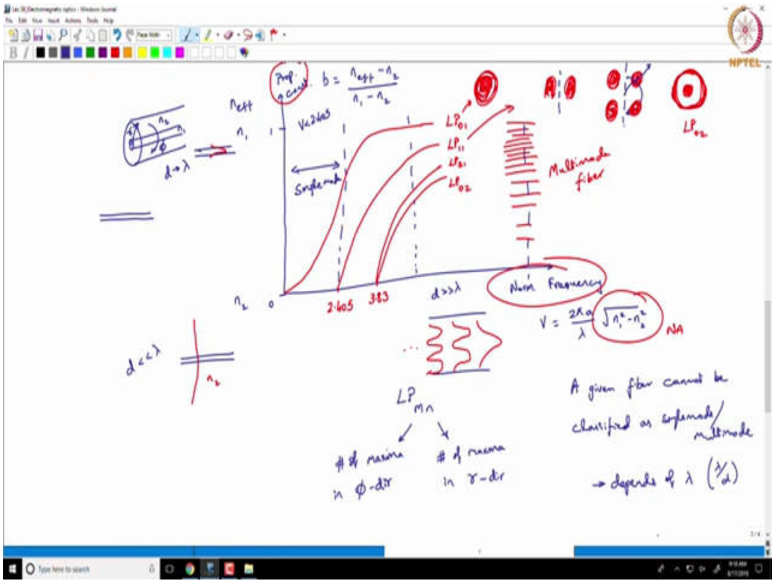
# of modes supported, depends on λ/d

$\theta_c < \theta_m < \pi/2$

For m=1, θ_m is highest ($\pi/2$)

$d \to \lambda \Rightarrow$ fundamental mode

If $d >> \lambda \Rightarrow$ larger # of modes (multimode)



Prop. const. $b = \frac{n_{eff} - n_2}{n_1 - n_2}$

$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$

LP_{mn}

# of maxima in ϕ-dir

# of maxima in r-dir

A given fiber cannot be classified as singlemode/multimode

→ depends of λ (λ/d)

Right and but if we had it to look at the true picture, you know analyse this cylindrical dielectric waveguide that an optical fiber represents, if we had to do this more rigorously by solving the wave equation for cylindrical coordinates and looking at all the solutions, what you are essentially trying to get is, this propagation constant at the function of this normalised frequency. Okay, so we want to see at any given wavelengths which are the modes that are propagating right, and we cut out a lot of details in getting to this point but nevertheless just to understand, you know what are the properties of these modes in this waveguide, so if you look at this picture we saying that the fundamental mode would correspond to a Gaussian pattern, so which will essentially have a maximum intensity at the centre and it would trail off as it goes towards the edges, as it goes away from the centre and then we said okay beyond V number of 2.405, you have the next mode and so on right, so and a higher-order modes and so on right.

So, yes very good point, the question is about polarisation, how does this work out here. So I did not mention actually what LP means here, in this case LP means linearly polarised modes, so it so happens that the final modes that come out, you know this LP mode, in reality it is not a single mode, there are two different configurations consisting what is, in technically it is called the HE_{11} modes, so that is HE_{11} odd and HE_{11} even mode, together constitute this LP mode and when we look at the field configurations of this total mode, LP mode the electric field is actually polarised in a linear manner okay, so that is why these are called linearly polarised modes, but again to understand the actual structure, we need to solve through the entire, you know from the wave equations which are not doing as far as this course is concern. Okay.

So one thing that I would also mention is you know, if we are considering a very thin waveguide, thin as in d the diameter of the waveguide, which we have actually defined as 2a, if $d << \lambda$ right, then you would have a case where probably something like this is happening, we have a fairly large evanescent field, and maybe I should just write this here, so this is probably, what you will happen when $d << \lambda$, what is it corresponds to in terms of V number? V number will be a very very low value.

So we are somewhere over here right, so effectively you have a very little energy propagating the core, there is a large evanescent region and when d approaches λ, that is the situation we have, where you know that it is fairly nicely confined and then there is this situation where $d >> \lambda$ and in this case you not only have the fundamental mode, you have the higher-order mode and then the next higher order mode and so on, so you have all these modes, you know being confined within this waveguide. Okay.

So that is what is happening for $d >> \lambda$. So just to illustrate that, but the key point that I wanted to make, maybe I said that towards the end of last class, but I did not emphasise enough is a given fiber cannot be classified as single mode or multi-mode. Okay, it actually depends on λ and specifically, you know this λ/d factor, so what is the dimension of the waveguide with respect to the wavelength. Okay.

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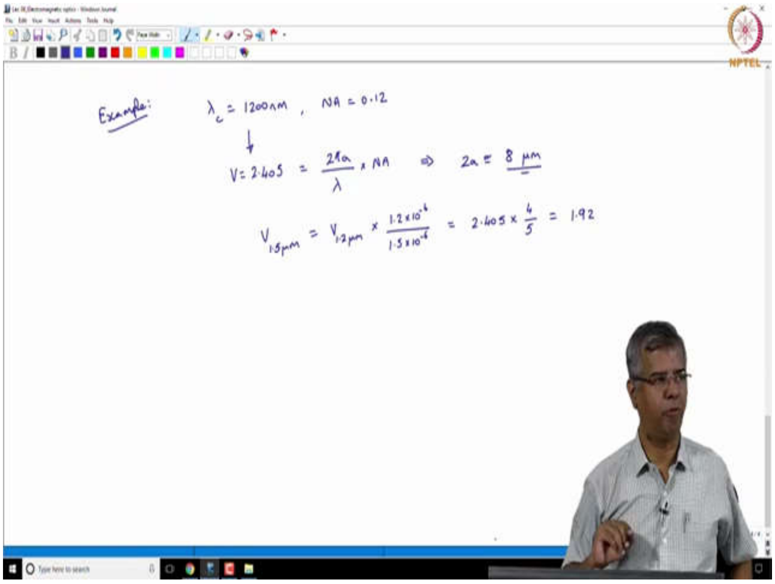


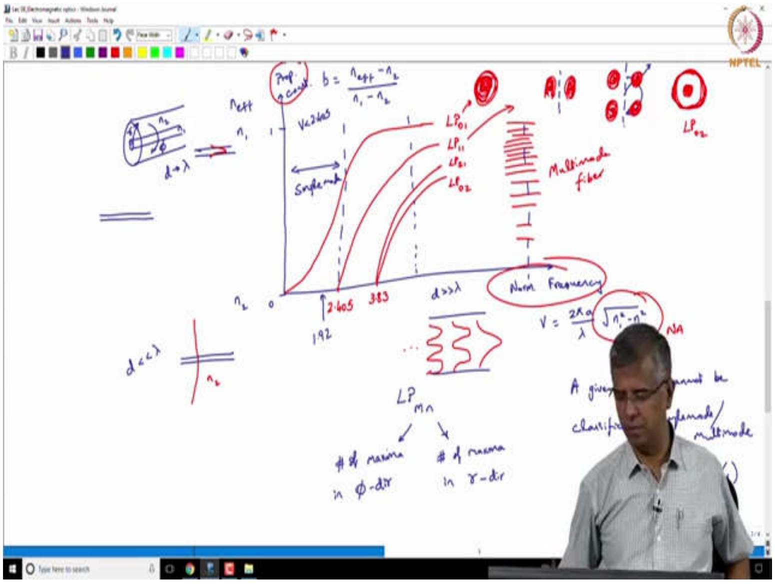
Example:

$\lambda_c = 1200\,nm$, $NA = 0.12$

$\downarrow$

$V = 2.405 = \frac{2\pi a}{\lambda} \times NA \Rightarrow 2a \approx \underline{8\ \mu m}$

$V_{1.5\mu m} = V_{1.2\mu m} \times \frac{1.2 \times 10^{-6}}{1.5 \times 10^{-6}} = 2.405 \times \frac{4}{5} = 1.92$



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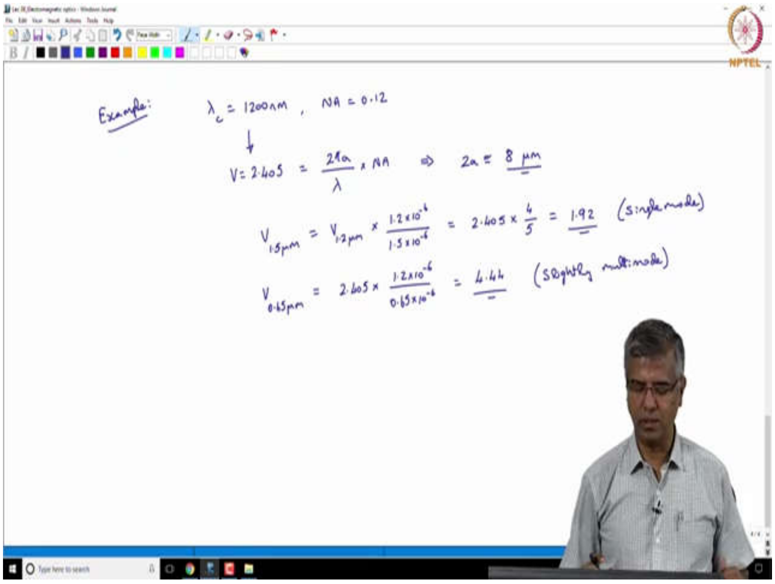
LP_{mn}

# of maxima in ϕ-dir

# of maxima in r-dir

A given [illegible] cannot be classified [illegible] singlemode/ multimode



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$V_{0.65\mu m} = 2.405 \times \frac{1.2 \times 10^{-6}}{0.65 \times 10^{-6}} = \underline{4.44}$ (slightly multimode)

So let's just take an example to illustrate that. Let us consider an example where you are given an optical fiber, whose cut-off wavelength is given as 1200 nm. This happens to be, you know the design of a standard single mode fiber that you use for telecommunications. You know all your optical communications is happening on a single mode fiber, which will have a cut-off wavelength around 1200 nm and let us say the numerical aperture is also given as 0.12. So can you find the diameter of your waveguide, if you are given this information?

How do you proceed with that? At the cut-off wavelength, what is the V number? Yes, V equals to 2.405, which is actually given by $\frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda}(NA)$, okay, so if you substitute $\lambda = 1.2$ μm is the wavelength, then if you substitute all the values given, you can find out that 2a would roughly correspond to about 8 μm, okay.

So let us say we have this waveguide, in fact this is one of the waveguides that, you know we will be using in the demo and the lab experiment this week, so this is a standard telecom fiber. Let us see first of all, what would be the, if you have a telecommunication wavelength, telecommunication happens at 1.5 μ and beyond normally, so if you want to look at what is the V number at 1.5 μ? You can simply write it as

$$V_{1.5} = V_{1.2}\frac{1.2\times10^{-6}}{1.5\times10^{-6}} = 2.405\times\frac{4}{5} = 1.92$$

So you have a V number of 1.92 at 1.5 μ wavelength, so where are we now, 1.92 is somewhere over here right, so it clearly behaves as a single mode fiber, you know as you go to longer wavelengths.

Similarly, if you want to look at the V number at 600 nm, what would that be? If you use 600 nm right, V number would have increased, specifically what is factor by which it increases, 600 nm is a factor of 2 lower, so V number would have doubled right, so you could have said okay V number would correspond to 4.8 and that would constitute a multi-mode waveguide, but in our experiments we use source which is, you know 650 nm, so if you look at the V number at 650 nm, that is once again $2.405\times\frac{1.2\times10^{-6}}{0.65\times10^{-6}}$ and that would work out to be a value somewhere around 4.4.

So in this case at 1.5 it clearly works like the same fiber right, it works as single mode fiber, whereas in this case it works as a slightly multimode fiber and I should recall a funny episode related to this. So know you we have this kit here, this experimental kit, where the vendor essentially, setting up similar experiment and he sells some fiber at single mode fiber, uses red wavelength, so okay, so he sells some as single mode fiber and he sells something as multimode fiber and then he sells something a special fiber he calls that, which is slightly multimode and you know over period of time we ended up damaging that fiber, so we were going for replacement and he was actually charging us a lot of money for that replacement and then we looked at this saying okay, this slightly multimode fiber is what we normally uses telecom fiber and we have lots and lots of telecom fiber in our laboratory, so why do not we just used that over here and that is what you are doing, you know if you are doing that experiment today, you are using one of those normal telecom fiber, so there is no magic about designing particular fiber for particular application and all that, it may be something that was designed for some other application, if you are wavelength is appropriate, you may still be able to use it.

So do not buy into things which say, oh this is single, generally single mode fiber, multimode fiber, if they are saying single mode fiber, watch where the cut-off wavelength is and where you are with respect to the cut-off wavelength and then you can decide whether it single mode for you or not.

Student: Sir, cut-off wavelength it will design in minimum value or maximum value.

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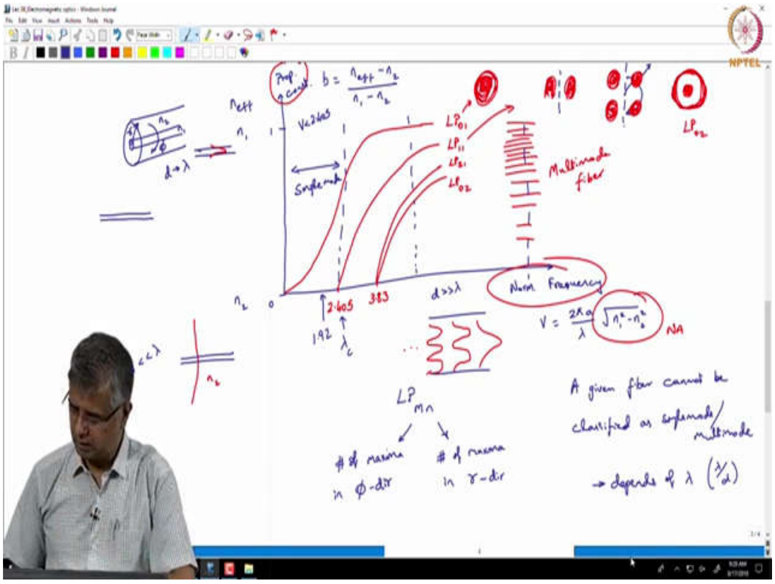

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Prof. Balaji: So the question is what is the cut-off wavelength imply? The cut-off wavelength implies that, this particular point right, so this is where λ_c is, so the λ corresponds, which gives you for a given fiber, λ gives you 2.405, so if you go higher V number, you have the possibility of the next mode also showing up, which is what we are calling as slightly multimode fiber, but if you have lower V number, which is corresponding to longer wavelength, longer wavelength than the cut-off wavelength, then you can be sure you are working in single mode regime.

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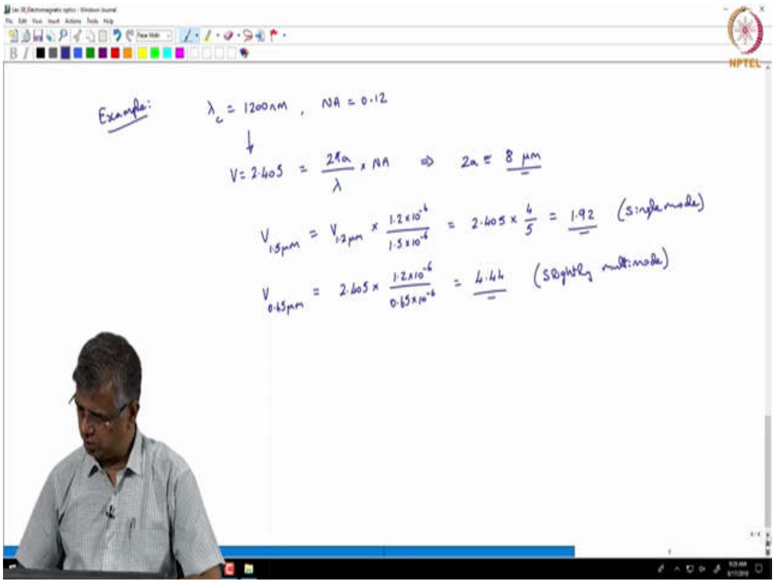

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So that is one part, the other part is, you know, how do we couple light into an optical fiber? Okay, so let us just quickly look at that before we jump onto looking at photons.

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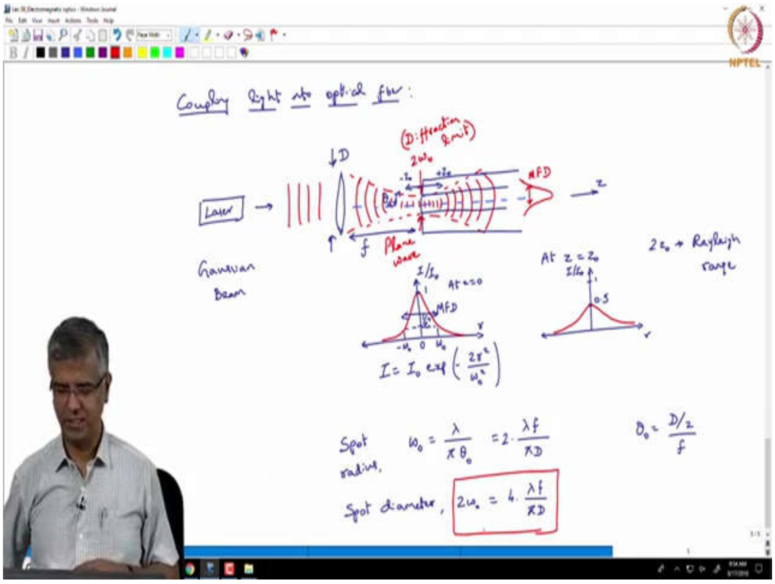

Coupling light into optical fiber:

$I = I_0 \exp\left(-\frac{2r^2}{w_0^2}\right)$

Spot radius, $w_0 = \frac{\lambda}{\pi\theta_0} = 2\cdot\frac{\lambda f}{\pi D}$ $\quad \theta_0 = \frac{D/2}{f}$

Spot diameter, $\boxed{2w_0 = 4\cdot\frac{\lambda f}{\pi D}}$

So, coupling light into optical fiber, what we normally do is, let us say we are basically coupling laser light into an optical fiber. You have a laser, let us say it is emitting close to plane waves, so what comes out of that is, these plane waves and, you know it comes out with a particular radius, the laser that we have has got a radius of few millimetre, but then it has to couple into an optical fiber, let us say we are coupling into a single mode optical fiber, okay what is a size of single mode optical fiber, for the wavelength that we are considering? It is in order of few microns right, so a few millimetres need to be fitted into a waveguide which is few microns, you cannot just stick the fiber directly in the path of the beam because it may couple something but that would be a very, very small fraction of the light that is generated by the laser

So what do you have to do, if you want to fit this into the waveguide, yes, you said lens. So you basically put a lens over here, sort of which brings the light into a focus and what does the lens do? This is a plane wavefront, okay it goes through a material that is thick at the centre and thin at the edges, so what does that do? The phase fronts will be curved right, so you would have basically the phase fronts that are curved like this, why? Because the central part of the wave is going through a thicker material, so it is going to be phase delayed with respect to what is happening at the outer edges, right and that makes the beam come to a focus and then it should diverge out.

Now the question is, if you are doing this in ray optics, you will basically put a ray that is coming down this way, ray coming up this way and it will show as if that rays are going to focus to a tiny tiny spot. I mean it is going to come to a point and if it is mini small point, but can you focus light to that small a spot, what you think? Have you tried focusing light? Yes, so some of you have done this yesterday and what you find is, it does not allow you to go to a particular spot. There is actually a certain, minimum diameter, let us call this $2w_0$, this is a minimum diameter that you can get to and why we get to a minimum diameter, you know explaining that, you have to go back to looking at this right. What is this signify, we are basically looking at the diffraction of light.

You can say that the minimum waist diameter that you are going to get is going to be limited by the diffraction of light. There is going to be something call the diffraction limit that you are going to get into, now how do we quantify that diffraction limit and then based on that, how do we figure out, how to couple light into an optical fiber? Let us look at this and before we do that, there is something else we want to do is continue with this phase fronts. This is actually going to, if it is converging like that, it is going to diverge in a similar manner on the other side and what you think is happening at the centre. Clearly the curvature is changing sign right, so you can say that at the centre it is going to be close to a plane wave or it is going to be a plane wave at the exact centre of the wavefronts okay.

So that is interesting because when it comes to a single mode fiber and we saw how the single mode pattern would look like, what can you say about the wave vector for the single mode fiber, how many wave vectors does a single mode fiber represent the light that is propagating. Well it is a single mode, so it has to be one wave vector and that is going straight down the centre of the waveguide, so what is that constitute in terms of the wavefronts? It is a plane wavefront right, so what you have in a single mode waveguide is actually a plane wavefront that is moving all the way across. So that is an interesting thought because when you look at any of these optics, when you are trying to focus light to a spot, it is get to a plane wavefront only at that spot and then beyond that it starts showing this curved behaviour, if you want to capture that plane wavefront and transmit over a long distance right, this optical fiber is a very nice way of doing that. It basically, you capture it, you put that optical fiber right at the focal point, in which case you can be sure that you have a plane wavefront and that is an essential condition.

Because if you come in with any other curved wavefront only the plane wave component, that is oriented along the centre, only that component will couple down the fiber, all the other component will just radiate off. Because that is not, the waveguide says no-no you are not allowed here right, so only that component is thing and so only when you put it at the focus, you have a plane wavefront and that way front is caught and that is consistent with the mode allowed by the waveguide that is consistent with the Maxwell's equations for that particular waveguide and then that is actually propagated all the way through, okay.

So before we get to some of the specifics, let us just examine what does this laser light represent? Now we said all waves, you solve, what is called the Helmholtz equation and get a solution for the Helmholtz equation. One of those solutions corresponds to a Gaussian beam, so whenever we talk about these plane waves, we are talking only about the wavefront, what is a wavefront once again? The collection of points which have the same phase okay, if it is represents a plane, then the call as a plane wavefront right.

So there we are only representing the phase of the light, so far we have not talked about the amplitude distribution in the transverse direction right, so if I am talking about a Gaussian beam, what does that represent? What you think the amplitude or the intensity distribution is like? That is a Gaussian shape, so you know that the centre is going to have maximum intensity and it is going to fall off. And it is interesting that the light modes that you get, light beams that you get from a good laser which is operating at, what is call a fundamental mode, this Gaussian mode is actually, we will see later on, is actually a fundamental mode for this waveguide itself, for the laser itself. That mode is consistent with the mode, the fundamental mode for this waveguide okay. The waveguide actually, if you, I was mentioning yesterday, if you do the solving of the Maxwell's equations rigorously, you will find that, the field shape corresponds to Bessel function in the core region and a modified Bessel function in the cladding region, so together it will look like a Gaussian beam, so it been nice to have a Gaussian beam source to couple into an optical fiber because not only as the phase fronts have to be consistent but the shape of your field pattern or the intensity pattern that also has to be consistent with the waveguide.

So you cannot take, let us say laser with a very poor wavefront, let us say you are taking LED light right, it is got very poor spatial coherence, a wave front is like all wobbly and all that. You cannot take that and expect to couple a lot of light into a single mode fiber, okay because that light source essentially has a non-planar wavefront, okay it has got multiple angles and only one of those angles is consistent with, what is supported in the single mode fiber.

So you cannot do that, whereas you if you have a laser which is emitting its fundamental mode, which happens to be a Gaussian beam is a solution of, fundamental solution of the laser cavity, then you can couple a lot of light into this optical fiber. So we can define a few things over here. We can basically say okay, let us define this optical axis and if we look at this, at the focal point, if we look at, what is the intensity pattern that you get? We clearly said that has got to be Gaussian shape, so it goes to a maximum and then it comes to a minimum, so let us say this is $\frac{I}{I_0}$, where $I = I_0 \exp\left(-\frac{2r^2}{w_0^2}\right)$ okay, so it will be the intensity pattern, so clearly this would have a value of 1 at the centre. What would w_0 correspond to in this picture here? $\frac{1}{e^2}$ value, so where $\frac{I}{I_0}$ goes to $\frac{1}{e^2}$, the corresponding radius would be called as the mode field radius and the corresponding diameter would be called as the mode field diameter.

I said mode field diameter but okay let us just take it as the Gaussian diameter because when I am defining an optical fiber, I am defining an optical fiber such that the core region is like this, the cladding would be somewhere like this right, so I want to put the fiber core over here at the focus region and I am saying that the mode that supported by this fiber is Gaussian in shape and that diameter, the mode field diameter within this fiber - that has to be consistent with the diameter of my Gaussian beam, the focused Gaussian beam, so that I can have maximum, so this also is like a Gaussian shape right and this will also have a mode field diameter, so if this diameter and this diameter are matching, that is when I have maximum coupling into the optical fiber.

Do you understand that? The optical fiber will have certain mode shape for its fundamental mode and that mode shape is roughly a Gaussian shape and so that Gaussian is going to have a diameter, a characteristic diameter, which we call it, is a mode field diameter and that mode field diameter and the mode field diameter of my incoming beam or the spot size at the focus of the incoming beam has to be consistent for maximum coupling. So it is basically the overlap of these two fields that determines what is the coupling efficiency that I achieve. If I have a larger diameter than, clearly you know the focus diameter is larger than clearly, you know you can see that part of the light is not going to be coupled into the fiber. But there is a misconception in general about this. People say if the diameter is smaller then this, than the size of the fiber that the mode field diameter of the fiber, then I should have, you know high coupling as well right. No it does not work out that way because what matters is the overlap integral of this field and that field okay. As long as that overlap integral is not matched, you do not have good coupling efficiency, But intuitively you think yes I will just make sure that the spot size is lesser than the you know size of the mode field diameter of the fiber and then I should have maximum coupling. Okay, so it does not work out that way.

The other part that is typically important is the range over which, the focused beam maintains a near planar wavefront. I said near planar because it can be plane wavefront only at the centre but it is close to a planar wave front around the centre, so that is actually defined by a parameter which is called the Rayleigh range. So what is the Rayleigh range? So it is such that across this so is going from $+z_0$ to $-z_0$, okay, it maintains a near, you know planar wavefront and the Rayleigh range is defined such that at, so this is at $z = 0$, I am saying this direction is z right, at $z = z_0$, the overall intensity pattern is like this, it takes a value, this is 1 when we are defining $\frac{I}{I_0}$, this is 1 and this is $\frac{1}{2}$ okay, the overall intensity actually comes down by a factor of 2, that is, what is known as z_0 and this $2z_0$ is what we are calling as the Rayleigh range.

If we go back to, now defining w_0, so this is the spot radius. If you look at the spot radius, now that has a dependence on what, clearly that as a dependence on the angle at which I am trying to come to a focus because if I have a shorter focal length, lens you can imagine that is going to get focused toward tighter spot size okay, I supposed to lens that is, you know that is, that has a very long focal length, so you can basically write this as, this spot size, if we go through that analysis, you can write the spot size, spot radius is given by $\frac{\lambda}{\pi\theta_0}$, where θ_0 in this picture would correspond to, if we say the size of the beam is *D* and the focal length of this lens, if we call that *f*, we can say that, especially if $D >> f$, you can write this as basically $\tan\theta_0 = \frac{D/2}{f}$.

So you can substitute that over here and what you get is $w_0 = 2\frac{\lambda f}{\pi D}$ and if we are talking about the spot diameter, which we are typically interested in or the diameter which we want to match to the mode field diameter of the fiber, that is going to be given by $2w_0 = \frac{4\lambda f}{\pi D}$ okay, so what does that tell you, and of course you can get a smaller spot diameter, if you go to shorter wavelength, which is very very important concept to when it comes to what something to do with semiconductors. If you are making semiconductor chips right, you want to define features in your semiconductor chip and you go through a process called lithography.

So if you want to get better and better resolution in your lithography, you want to go shorter and shorter wavelength, so you see in the semiconductor industry, you know using blue light than ultraviolet radiation, now they are talking about x-rays, using x-rays for your lithography so that you can get finer and finer features right and that is related to this, you can get a much smallest spot size as you go to a much smaller lambda but you also want to, you get a smallest spot size with smaller focal length and a larger beam to start with okay. *D* corresponds to the diameter of the beam; we can catch up in the next lecture okay. Thank you.