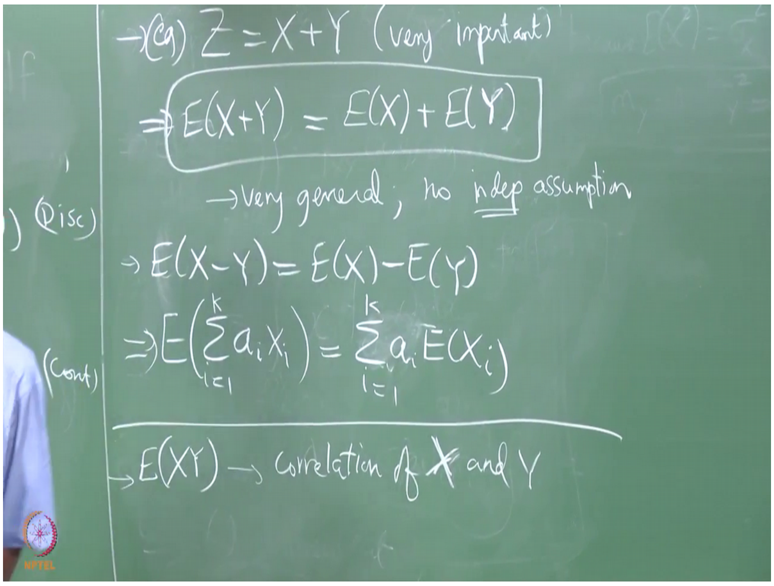
Probability Foundations for Electrical Engineers
Prof. Aravind R
Department of Electrical Engineering
Indian Institute of Technology, Madras

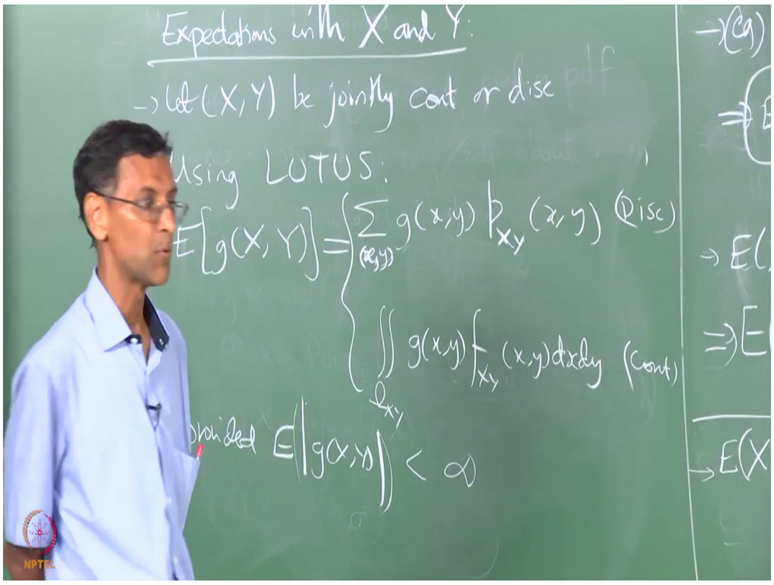
Lecture – 68
Correlation and Covariance

(Refer Slide Time: 00:18)



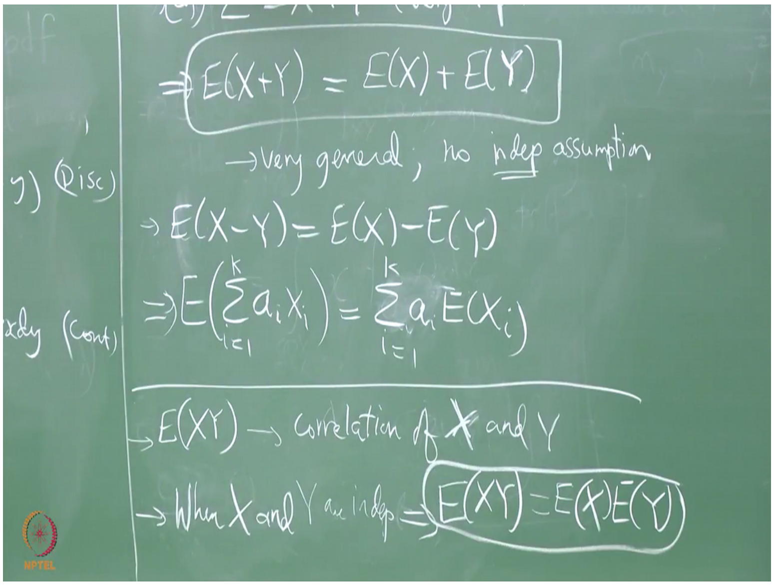
So, let me write this E of X, Y without calling it a name without giving a name to X Y this is called a correlation of X and Y I am sure you people have use the word correlation. In some other context, but the E of X Y is a precise mathematical terminology for is a meaning of correlation right.

(Refer Slide Time: 00:56)



So, this; obviously, this is a special case of this where I am going to stick in here the product x, y or here. Now, fine yes; I have x, y, but can it simplify in general this right; it you cannot simplify that sum or that integral the double sum of the double integral you can simplify only in the case that this joint is a product of the marginals E X and Y are independent.

(Refer Slide Time: 01:47)



So, when X and Y are independent; I am going to use right at the bottom here, I am hoping that; whatever right at the bottom here is fully capped you know recordable without any. So, when x and y are independent what can you say about E X Y.

Student: (Refer Time: 01:52).

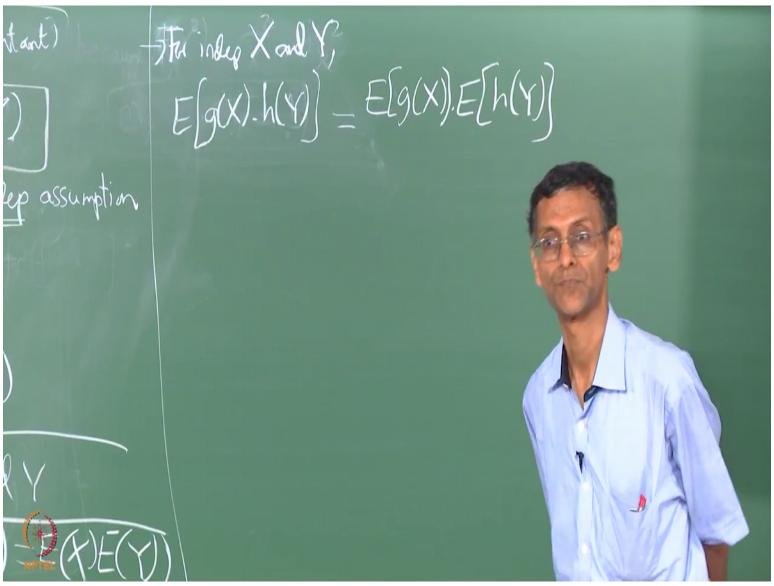
This implies what this becomes p X X into p Y Y, f X X f Y Y, and then you get and then this will always be a rectangular grid this will always be a rectangular region. So, you can always integrate over Y alone and X alone. So, what happens to E X Y.?

Student: E of X E of Y.

Right this implies that E this; the correlation is a product of the means. So, this is an implication which goes only in this direction for now right; do not as I will show you not too far maybe latest by tomorrow this does not imply independence is only this implication for now goes only this direction. Let us not take right, I am looking at the most general thing right not taking specific examples right.

So, we are now just looking at general independent X and Y only or not independent right in they can when they are not independent general this is not true, but the independence case can be stretched even more we only pointed out that if X and Y are independent g of x and h of y are also independent.

(Refer Slide Time: 03:30)

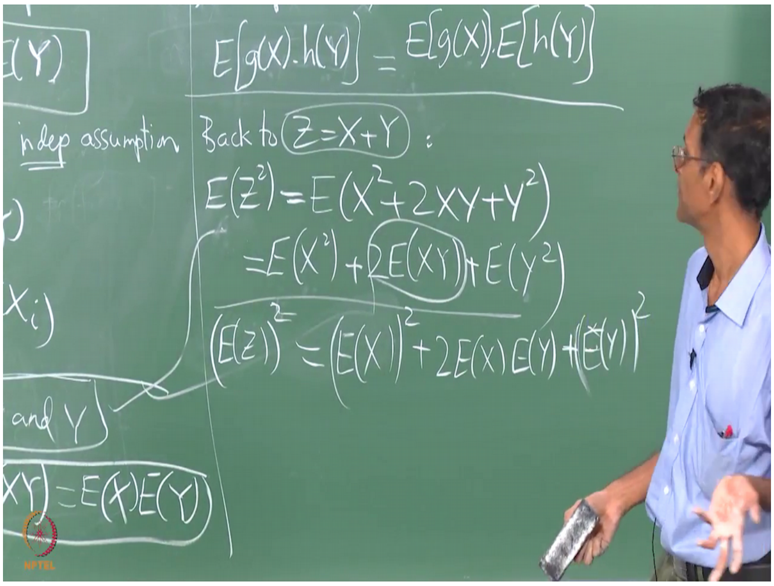


So, you can say one more thing right; what can you say? For independent X Y this E of g X multiplied by h Y remember this g is not the same as this g, because this is a function of two variables here some, but I am just I do not want to keep on inventing subscripts and so, on. So, what is this going to be? G X and h Y are independent as we already pointed out for when X and Y depends.

So, this is going to be again it depends only on the marginal statistics of X and Y. So, this will be E of g multiplied by; again this load as principal comes in right is just used left and right and all of this. And notice the way just one E for everything; it is absolutely unambiguous because you look at this and say this is E of g of X it uses only the PDF of x finished this is here there is a X here Y here, but. So, it has to in general use the joint PDF or joint PMF right is this there clear everybody ok.

So, this is a correlational; why is this correlation so, important in the in the variance calculation of the sum you go back to. So, I will draw a line here.

(Refer Slide Time: 05:12)



So, I say back to; now I will put Z equal to X plus Y alright. Now I want to call it a name Z here same thing. So, what is E of Z squared?

Remember, we are only dealing with numbers random variables are after all numbers right. So, algebra always applies; that squared is X plus Y whole squared which is x squared plus.

Student: Surprise surprise.

Surprise surprise is X squared plus two XY plus Y squared right seven standard identity brought back to use in the form of random variables any problem? Ok. Now linearity again; what version of linearity should I use i. In fact, I did not you know. So, now, we have a what you might think of as different functions of X and Y in general; not exactly this kind this involves many many different random variables actually I wanted to write something like this I put a i of gi of x comma y, but ended up somebody said linearity and I got sidetracked and many I put many random variables, but that is there is nothing wrong with this right.

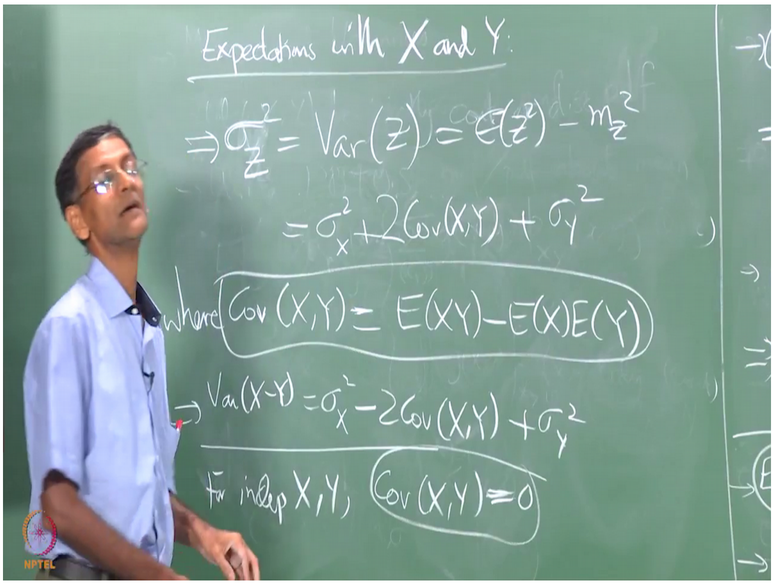
But I mainly I wanted what I want to put here as ai gi of X comma y again the same kind of thing here. So, this is E now you see how it simplifies right in. So, this first term depends only on the marginal PDF of X the third term depends only on the marginal PDF of Y, but this term here is exactly this E of XY I am not assuming independence, I am assuming just this. Please do not throw any independence there is no independence right as of now.

Now, ok. So, what is; So, how do, but variance it is not just this right variance this is minus what

Student: (Refer Time: 07:59).

Which is what? What is E of Z the whole square? From here; it is E of X the whole square plus 2 E X times E Y right plus E of Y the whole square correct. So, now, if I take the difference; what do I get? I think I can I can go back here.

(Refer Slide Time: 08:53)



Sigma variance of Z; by the way the sigma Z squared is a bit inconvenient to write down. Another notation we are going to use. So, variances to write var and put the name of the put the random variable within brackets which is not a i it I some some ways cleaner, because avoid subscript and the square also superscript and subscript they both avoided in this quasi you know English mathematical notation right, but it is fine.

So, this is basically this is E of Z the whole squared minus the mean squared; which is that it is what this is sigma X squared plus sigma Y squared you write at the end; what is the term in the middle? The term in the middle is called the covariance of the twice covariance of X comma Y. So, if I write this as two c o v X comma Y what is that covariance of X where this covariance of X comma Y is what? Tell me.

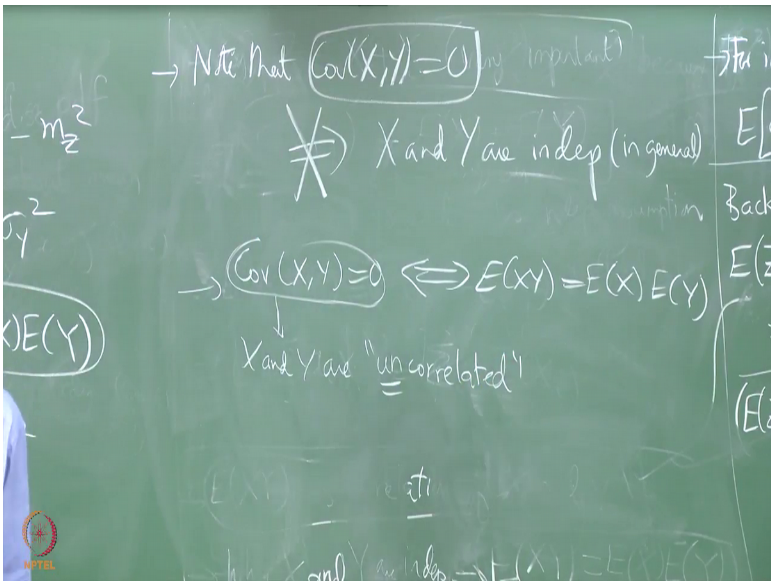
Student: (Refer Time: 10:06).

It is the expectation of XY minus or m X m Y right. So, this is the formula for the variance of the sum, if I took the difference instead of the sum what would be the variance of X Y X minus. So, what would be the resulting variance fine if I wanted to right now look at the power of this notation I can just simply write variance of X minus Y; I do not have to give it a name u or v or anything you just really write like this; this will be turns out again these two will come always to the positive sign this term alone will become negative.

Please go back you do the derivation you will see only this term becomes a minus. So, this becomes sigma X squared; minus two covariance XY plus sigma Y square. So, using this covariance idea you can write out not only the sum the variance of the sum, but also the variance of the difference. Supposing X and Y are independent; now we will bring any independence, then if X and Y are independent then covariance is always 0. If this covariance is 0, then the variance of the sum is equal to the sum of the two variances the variance of the difference is also equal to the sum of the two variances.

In fact, the last thing I want to say for today is whether or not X and Y are independent right if their covariance is 0, that they write like the X and Y the way the covariance can be 0 independent or regardless of the independence of X and Y note that I just last thing, because I started exactly fifteen minutes ago I need my full fifteen minutes right. So, lasting.

(Refer Slide Time: 12:39)



So, this in general does not imply that X and Y are independent covariance X Y Z can be zero without X and Y being independent in general. All it says is covariance X of 0 is the only thing it says is that if this coherence is 0, it only means this E of X Y equal to E X into E Y this is the only thing that it means and there is a special term for this X and Y are called what? Uncorrelated ok.

So, we will stop here