Probability Foundations for Electrical Engineers
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Lecture - 64
Properties of Expectation

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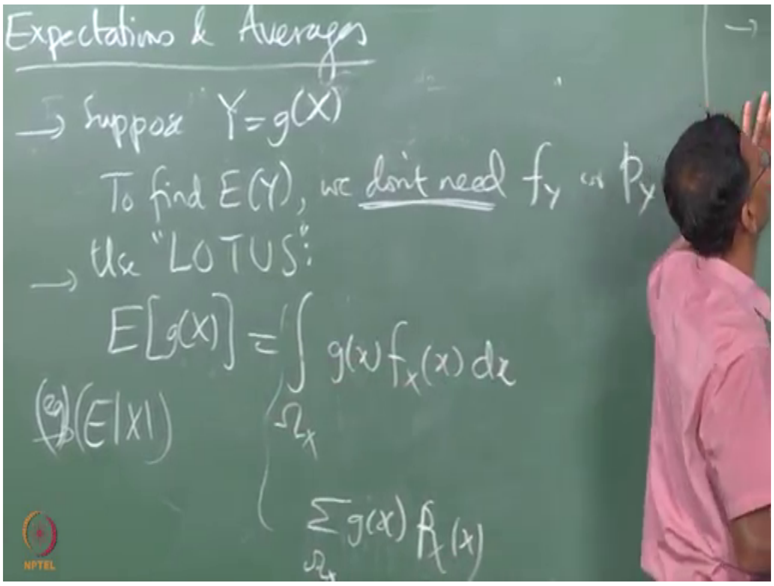

That is what supposing I say Y is g of x and this is a advantage of doing transformations before expectations because this is no longer new to you; if I hit you on the head, if I put this let us say immediately after defining x you do not know what the hell is g of x right.

Now, you know you do not worry about g of x it is just another random variable and you have done enough examples and problems you understand what that right how do you handle this.

Now, to find the mean E of y do I have to find to go through the PDF route that is find f y answer is no. You do not need you guys yes you were always do it right there is a long route you can touch your nose either by doing it like this or like this right, but the thing to do is so unless you need the PDF you know of the derived quantity y you do not have to compute it certainly you do not have to compute the PDF the output PDF of this transformation to just to simplifying this mean value.

Instead what is what we have we have what is called lotus. Does anyone has anyone has seen the expansion that this term this is a very nice acronym and this I have seen is used in so many places that I am sure is good it is there is a there in Google search also I have not done it the expansion of “LOTUS” in this case happens to be law of the unconscious statistician. Main thing is unconscious statistician you unconsciously you do it, what do you think how do you think I should go about finding g of E of g of x.

Student: (Refer Time: 02:10).

I just substitute what in place of x what do I do.

Student: g of x.

I put g of x unconscious I do not even I do it without even a second thought is not it. I do not even need y I can just say E E g of x I do not even have to capture the output of g of x and some y right I just write E of g of x as what integral or sum over omega x in case of in case this is continuous. So, I am assuming of course, that so I am assuming that this g is well reasonably well behaved which allows y to be purely a continuous and this integral to be written.

And of course, it is it is a summation for this is in fact, I think makes more is really easy to understand the discrete case because if x is discrete then y of g of x is also discrete and the PMF we if you go back and easily interpret this using PMF’s. Let me put p here you can easily right for if you say it is a one to one transformation right.

You want y into py of y, but what is y it is one of the values of g of x and py is p of p of g of x px of sorry py of g of x is equal to px of x and there is a one to one transformation from x to g of x by a right conservation of probability or transfer of probability and even if 2 or 3e things mapped to the same g of x this summation will take care of that this g of g of x would make will be common right.

If there are two values of x mapping to the same g of x right then this g of x will be common and this summation will clearly be exactly equal to sigma y p y, but in any case remember lotus as you do this even without giving it a second thought so as naturals that you agree or not even without my saying it you people said the answer right at least some of these people said more a little more vocal then people folks out there. But say let me also ask you agreed it is almost a.

Student: Simple extension.

A simple extension.

Student: (Refer Time: 04:23).

Anyway take a minute to appreciate it. Now why does this mod of x coming I wrote E of mod of x what g of x it I use for that mod of x.

Student: (Refer Time: 04:39).

Just mod x itself, but if I use y y y equals mod x I would get exactly the same thing here.

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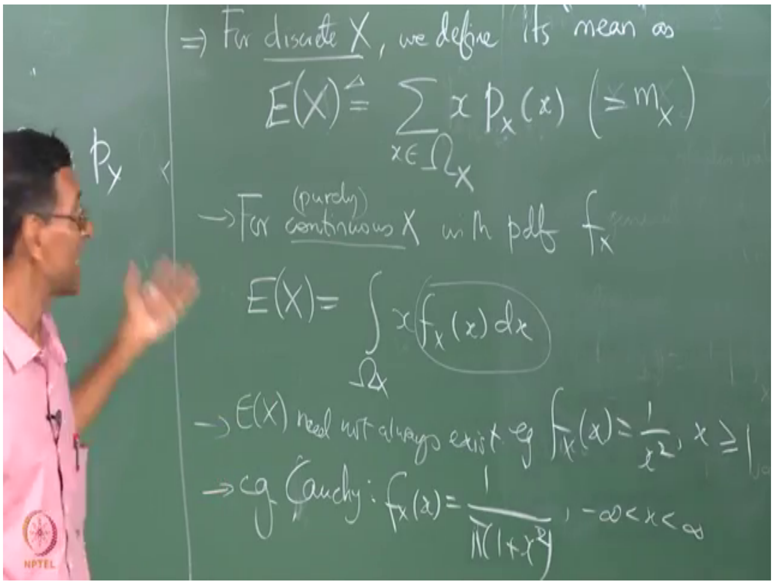

So, if I want to put a mod sign here that is E of mod x all I have to look considered as a function mod x that will exactly give me mod x sigma over of mod x px or integral mod x fx and that is has to be nothing, but this E of mod x.

So, which this explains E of mod x again this these integrals are not guaranteed to exist very clear if E of x itself does not exist there is no guarantee that E of g of x some off some arbitrary gs again going to exist I can have an explanation increasing g or something so; obviously, you can cook up any g and these integrals and summations will be finite only if omega x is a finite interval or a finite collection of points alright.

So, then you do not have to worry , but as long as you say right either plus infinity right it goes up to plus infinity or the entire real line then these integrals cannot always cannot be guaranteed to exist. So, again what is a test what is the implicit assumption we are saying if you are saying that this is finite; that means, that you have ensured that by putting a mod around g of x there is a there sum or the integral converges to some finite value right only if that is finite then we say that E of g of x is given by this expression.

Otherwise you say E of g of x does not exist at all ok. What is the use of this g of x come on you tell me beyond E of mod x; what is the next important you are taken you understood the mean value why did I have to introduce all these E of g of x now apart from mod x. What is the statistic with your come your next stage of mod x that you care about they.

Student: Variance.

Variance, but before the vary to all study the variance I need one more basic quantity which is.

Student: (Refer Time: 07:00).

E of.

Student: x square.

E of x square so, please wake up and right I want you to tell me that you want right the next thing after E of x is E of x square.

Student: yes sir.

Right.

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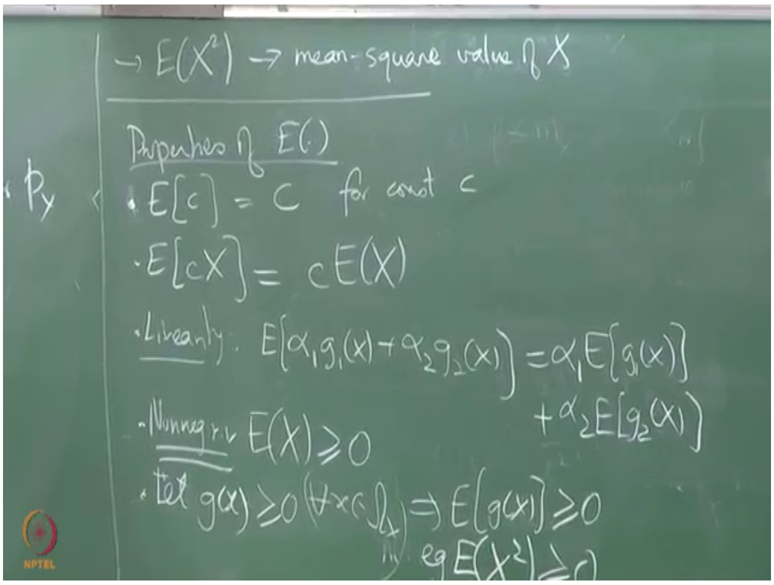

So, E of x square is if E of x is a mean value E of x square is called the mean squared value.

Student: (Refer Time: 07:20).

Mean of the square I am not sure if I has said d here or not, but means let me not I do not think mean square value let me just say this is not in general the variance right, but it is the for the variance we have to do more work we will come to this variance maybe tomorrows class not today.

But E of x square is just a special case of E of g of x and in any case for a real random variable mod x squared is same as mod x whole squared. So, you do not have to worry about either the integral would side way diverge or converge that itself tells you whether have of x squared exists or not right you do not have to do any special test for this alright.

So, that is so what I will do in today’s class is to write to wind up by looking at some properties of the expectation operator then we will study the variance tomorrow. Let me list out some simple properties of the expectation which I wish we can do at this point this is going to be an incomplete list right we will keep expanding on this as we go along. So, just with whatever you have study in half an hour today.

What properties can we write let me not list them. The expected value of a constant is equal to the constant first of all is very important this is a case of what g of x equal to c this always exists if c is finite you do not care about even right the E of x itself may not exist, but E of c always exists because sum of a probability distribution or integral of probability distribution is 1 that is without that you do not have a probability distribution. So, therefore, E of c is always c ok. So, then what is E of c X ?

Student: c of c (Refer Time: 09:55).

C into.

Student: E of x.

E of x right and this requires c E x to exist right, but let me not keep on us again someway we have broken record each time right we if we write E of x we are assuming that there is no problem with that number. So, this should not be any should not right pose any problem to you all you have to do is stick in in place just remember this whenever right whatever comes here you put here.

And; obviously, if cx you have c will come out and you will be left with the standard integral for E of x then they have important linearity property this is also linearity property, but let me write the more more general linearity E of let us say if I take if I have two functions alpha g 1 x plus alpha 2 or c 1 plus c 2 whatever what right what is this.

Student: Alpha 1 E g 1 of x plus alpha 2 E g of x.

Right so it is all very easy right then E of x for non negative random variables which means what x does not take negative values E of x if it exists has to be at least more than 0, it cannot be negative. These now up to here it is all equality now we are going to look at some bounds for non negative random variable obviously, there is no negative term in the sum or integral and the resulting thing has to be positive.

When this if you say this non negative and E is somehow E of x turns out to be 0 then; that means, x must be 0 with final probability 1 x is degenerate it is not random at all, but that is a very trivial you know it is a exceptional case. So, let me not then if g of x is positive this is right ok. So, let the let g of x be positive for x in omega capital omega x this is of course, like x squared which is positive for all non negative for all x.

But, but it does not have to you not have to consider x squared in general you can look at just any g of x which is positive for omega x alone then E of E of g of x must be also be non negative this implies that E of so which E g let me write at the bottom here E of x squared remember this E of x squared can never be negative no matter what x what distribution you have for x ok.

So, I think we will stop here for today and so, this is by no means incomplete list we will expand it as we go along.